Economics GA.1101.003
Applied Statistics and Econometrics 1
Fall 2021

Assignment 3

Your solutions to this problem set are due before 11:59pm on Tuesday, 23 November 2021 on Brightspace.

Problem 1 (3 points)

A multiple regression of y on a constant, x_1, and x_2 produces the following results:

$$\boldsymbol{X}'\boldsymbol{X} = \begin{bmatrix} 5 & 15 & 25 \\ 15 & 55 & 81 \\ 25 & 81 & 129 \end{bmatrix} \quad , \quad \boldsymbol{X}'\boldsymbol{y} = \begin{bmatrix} 20 \\ 76 \\ 109 \end{bmatrix}$$

$$RSS = 1.5, ESS = 26.5, n = 5, R^2 = 0.95$$

a. Calculate (i) the adjoint and determinant of $\boldsymbol{X}'\boldsymbol{X}$ and (ii) the regression coefficients $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$.

b. Compute a 95% confidence interval for β_1.

For each hypothesis test in c,d, and e below make sure you show: (i) the value of your test statistic, (ii) the relevant critical value(s), (iii) comparison of the test statistic vs. critical value(s), (iv) your conclusion. Unless specified, assume 95% significance levels i.e. α=0.05.

c. Test $H_0 : \beta_2 = 0$ versus $H_a : \beta_2 \neq 0$.

d. Test $H_0 : \beta_1 + \beta_2 = 0$ versus $H_a : \beta_1 + \beta_2 \neq 0$.

e. Test $H_0 : \beta_1 = \beta_2 = 0$ versus $H_a : \beta_1 \neq 0$ or $\beta_2 \neq 0$. Note that most software packages include the test in the default regression output as the *F-statistic*.

f. What inferences do you make about this regression based on the R-squared value and how do these inferences compare with those made based on the F-statistic in (e) above?

You can find statistical tables relevant to this problem here.

Problem 2 (4 points)

In the fourth quarter of 1966, UK's Labor government liberalized the National Insurance Act by replacing the flat-rate system of short-term unemployment benefits by a mixed system of flat-rate and (previous) earnings-related benefits, which increased the level of unemployment benefits. Dataset https://www747.github.io/NYU/HW3/problem_2_data.csv includes the UK unemployment rate U, vacancy rate V, and a dummy variable D = 0 before 1966Q4 and 1 after.

a. Import the dataset and estimate the model $U = \beta_0 + \beta_1 V + \beta_2 D + \beta_3 D \cdot V + \epsilon$

b. Holding the job vacancy rate constant, what is the average unemployment rate in the period beginning in the fourth quarter of 1966? Is it statistically different from the period before 1966 fourth quarter? How do you know?

c. Are the slopes in the pre- and post-1966 fourth quarter statistically different? How do you know?

d. Is it safe to conclude from this study that generous unemployment benefits lead to higher unemployment rates? Does this make economic sense?

Problem 3 (3 points)

Dataset https://www747.github.io/NYU/HW3/problem_3_data.csv includes quarterly data on the following variables:

$Y =$ quantity of donuts sold, dozens

$X_1 =$ the trend variable

$X_2 =$ average price of donuts, $\$/$ dozen

$X_3 =$ average price of cupcakes, $\$/$ dozen

$X_4 =$ average weekly family disposable income, $\$/$ week

a. Estimate the demand function $Y_t = \alpha_0 + \alpha_1 X_{1t} + \alpha_2 X_{2t} + \alpha_3 X_{3t} + \alpha_4 X_{4t} + \epsilon_t$ and interpret the slope coefficients of this linear model.

b. Estimate the semi-log demand function $lnY_t = \gamma_0 + \gamma_1 X_{1t} + \gamma_2 X_{2t} + \gamma_3 X_{3t} + \gamma_4 X_{4t} + \epsilon_t$.

c. Estimate the demand function $\ln Y_t = \beta_0 + \beta_1 lnX_{1t} + \beta_2 \ln X_{2t} + \beta_3 \ln X_{3t} + \beta_4 \ln X_{4t} + \epsilon_t$ and interpret the the slope coefficients of this log-linear model.

d. $\beta_2, \beta_3,$ and β_4 give, respectively, the own-price, cross-price, and income elasticities of demand. What are their expected signs assuming donuts are a normal good? Do the results concur with the a priori expectations?

e. How would you compute the own-price, cross-price, and income elasticities for the linear model?

f. On the basis of your analysis, which model, if either, would you choose and why?