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Applied Statistics and Econometrics I

Practice problems - suggested solutions

For Q1 and Q2 see hand written notes.

3) Let $\bar{X}$ denote the mean of a random sample of $n = 100$ scores from a population with $\mu = 60$ and $\sigma^2 = 64$. We want to approximate $P(\bar{X} \leq 58)$. We know from the Central Limit Theorem that $(\bar{X} - \mu)/(\sigma/\sqrt{n})$ has a distribution that can be approximated by a standard normal distribution. Using the standard normal table we have

$$P(\bar{X} \leq 58) = P\left(\frac{\bar{X} - 60}{8/\sqrt{100}} \leq \frac{58 - 60}{.8}\right) \approx P(Z \leq -2.5) = .0062$$

Because this probability is so small, it is unlikely that the sample from the school of interest can be regarded as a random sample from a population with $\mu = 60$ and $\sigma^2 = 64$. The evidence suggests that the average score for this high school is lower than the overall average of $\mu = 60$.

4) See Greene solutions manual p.163: http://pages.stern.nyu.edu/~wgreene/Text/Greene_6e_Solutions_Manual.pdf

5) Let the joint density of two random variables x_1 and x_2 be given by

$$f(x_1, x_2) = \begin{cases} \frac{1}{6} X_1, & \text{if} \quad 0 \leq x_1 \leq 2, \quad 0 \leq x_2 \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Find $\mathrm{Cov}[x_1, x_2]$

As proven before,

$$Cov[x_1, x_2] = E[x_1 x_2] - \mu_{x_1} \mu_{x_2}$$

Let's look at each term separately:

$$\begin{aligned} E[x_1 x_2] &= \int_0^3 \int_0^2 x_1 x_2 \frac{1}{6} x_1 dx_1 dx_2 \\ &= \frac{1}{6} \int_0^3 \int_0^2 x_1^2 x_2 dx_1 dx_2 \\ &= \frac{1}{6} \int_0^3 x_2 \left(\frac{x_1^3}{3} \Big|_0^2 \right) dx_2 \\ &= \frac{1}{6} \cdot \frac{8}{3} \int_0^3 x_2 dx_2 \\ &= \frac{8}{18} \frac{x_2^2}{2} \Big|_0^3 \\ &= \frac{8}{18} \cdot \frac{9}{2} \\ &= 2 \end{aligned}$$

$$
\begin{aligned}
\mu_{x_1} \equiv E[x_1] &= \frac{1}{6}\int_0^3 \left(\frac{x_1^3}{2}\Big|_0^2 \right) dx_2 \\
&= \frac{1}{6}\int_0^3 \frac{8}{3} dx_2 \\
&= \frac{8}{18} x_2 \Big|_0^3 \\
&= \frac{4}{3}
\end{aligned}
$$

$$
\begin{aligned}
\mu_{x_2} \equiv E[x_2] &= \int_0^3 \int_0^2 \frac{1}{6} x_1 x_2 dx_1 dx_2 \\
&= \frac{1}{6}\int_0^3 x_2 \frac{x_1^2}{2}\Big|_0^2 dx_2 \\
&= \frac{2}{6}\int_0^3 x_2 dx_2 \\
&= \frac{2}{6}\int_0^3 \frac{x_2^2}{2}\Big|_0^3 \\
&= \frac{2}{6} \cdot \frac{9}{2} \\
&= \frac{3}{2}
\end{aligned}
$$

Therefore,

$$
Cov[x_1, x_2] = E[x_1 x_2] - \mu_{x_1}\mu_{x_2} = 2 - \frac{4}{3} \cdot \frac{3}{2} = 0
$$

As expected

6) See table :

			Y-Y_bar	X-X_bar	x*y		b0+b1*X	Y-Y_hat	Y_hat - Y_bar			
	Y	X	y	x	xy	x^2	Y_hat	e	y_hat	y_hat^2	y^2	e^2
	4	1	-3	-3	9	9	2.929	1.071	-4.071	16.577	9	1.148
	5	4	-2	0	0	0	7.000	-2.000	0.000	0.000	4	4.000
	7	5	0	1	0	1	8.357	-1.357	1.357	1.842	0	1.842
	12	6	5	2	10	4	9.714	2.286	2.714	7.367	25	5.224
Sum	**28**	**16**	**0**	**0**	**19**	**14**	**28**	**0**	**0**	**25.7857143**	**38**	**12.214**
Mean (Y_bar X_bar)	**7**	**4**								**ESS**	**TSS**	**RSS**
b1 = xy/x^2 = 19/14	**1.357**											
b0 = Y_bar - B1*X_bar	**1.571**											
R2 = ESS/TSS	**0.679**											
R2 = 1 - RSS/TSS	**0.679**											