Lista 8 - Função Logarítmica

Projeto de Extensão: NIVELAUERJ

Cálculo Zero

Questão 1

Resolva as equações:

(a) $2^{3x-2} = 3^{2x+1}$

$$S = \left\{\log_{\frac{8}{9}} 12\right\}$$

(b) $7^{2x-1} = 3^{3x+4}$

$$S = \left\{\log_{\frac{49}{37}} 567\right\}$$

(c) $5^x + 5^{x+1} = 3^x + 3^{x+1} + 3^{x+2}$

$$S = \left\{\log_{\frac{5}{3}} \frac{13}{6}\right\}$$

(d) $2^{x+1} - 2^x = 3^{x+2} - 3^x$

$$S = \left\{\log_{\frac{2}{3}} 8\right\}$$

(e) $2^{3x+2} \cdot 3^{2x-1} = 8$

$$S = \left\{\log_{72} 6\right\}$$

(f) $4^x - 5 \cdot 2^x + 6 = 0$

$$S = \left\{1, \log_2 3\right\}$$

(g) $4^x - 6 \cdot 2^x + 5 = 0$

$$S = \left\{0, \log_2 5\right\}$$

(h) $3^{2x+1} - 3^{x+1} + 2 = 0$

$$S = \emptyset$$

(i) $3^{x+1} + \dfrac{18}{3^x} = 29$

$$S = \left\{2, \log_3 \frac{2}{3}\right\}$$

(j) $4^x = 2 \cdot 14^x + 3 \cdot 49^x$

$$S = \left\{\log_{\frac{2}{7}} 3\right\}$$

Questão 2

Resolva as equações:

(a) $\log_4(3x+2) = \log_4(2x+5)$

$$S = \{3\}$$

(b) $\log_3(5x-6) = \log_3(3x-5)$

$$S = \emptyset$$

(c) $\log_2(5x^2 - 14x + 1) = \log_2(4x^2 - 4x - 20)$

$$S = \{3, 7\}$$

(d) $\log_{\frac{1}{3}}(3x^2 - 4x - 17) = \log_{\frac{1}{3}}(2x^2 - 5x + 3)$

$$S = \{4, -5\}$$

(e) $\log_{\frac{1}{2}}(5x^2 - 3x - 11) = \log_{\frac{1}{2}}(3x^2 - 2x - 8)$

$$S = \emptyset$$

(f) $\log_{\sqrt{2}}(3x^2 + 7x + 3) = 0$

$$S = \left\{-2, -\frac{1}{3}\right\}$$

(g) $\log_4(2x^2 + 5x + 4) = 2$

$$S = \left\{-4, \frac{3}{2}\right\}$$

(h) $\log_{\frac{1}{3}}(2x^2 - 9x + 4) = -2$

$$S = \left\{5, -\frac{1}{2}\right\}$$

(i) $\log_3(x - 1)^2 = 2$

$$S = \{4, -2\}$$

Questão 3 ....

Resolva as equações:

(a) $\dfrac{1}{5 - \log x} + \dfrac{2}{1 + \log x} = 1$

$$S = \{100, 1000\}$$

(b) $\dfrac{3 + \log_2 x}{\log_2 x} + \dfrac{2 - \log_2 x}{3 - \log_2 x} = \dfrac{5}{2}$

$$S = \{4, 512\}$$

(c) $\dfrac{\log_3 x}{1 + \log_3 x} + \dfrac{\log_3 x + 2}{\log_3 x + 3} = \dfrac{5}{4}$

$$S = \{3, 3^{\frac{-7}{3}}\}$$

(d) $\dfrac{1 - \log x}{2 + \log x} - \dfrac{1 + \log x}{2 - \log x} = 2$

$$S = \{10^4, 10^{-1}\}$$

Questão 4 ....

Resolva as equações:

(a) $\log_2(x - 3) + \log_2(x + 3) = 4$

$$S = \{5\}$$

(b) $\log_2(x + 1) + \log_2(x - 2) = 2$

$$S = \{3\}$$

(c) $\log_3(5x + 4) - \log_3 x - \log_3(x - 2) = 1$

$$S = \{4\}$$

(d) $\log_{\frac{1}{2}}(3x+2)^2 - \log_{\frac{1}{2}}(2x-3)^2 = -4$

$$S = \left\{\frac{14}{5}, \frac{10}{11}\right\}$$

(e) $\log_{36}(x+2)^2 + \log_{36}(x-3)^2 = 1$

$$S = \{-3, 0, 1, 4\}$$

Questão 5 ....

Construa os gráficos das funções:

(a) $f(x) = \log_2 x$

(b) $f(x) = -\log_2 x$

(c) $f(x) = \log_{\frac{1}{2}} x$

(d) $f(x) = -\log_{\frac{1}{2}} x$

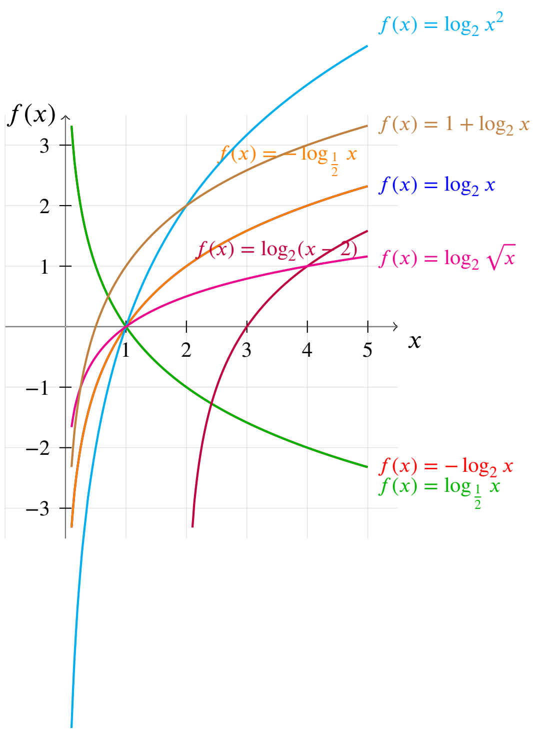
(e) $f(x) = \log_2(x-2)$

(f) $f(x) = \log_2 x^2$

(g) $f(x) = \log_2 \sqrt{x}$

(h) $f(x) = 1 + \log x$

Solução:
Os gráficos das questões (a) até (h) são:



Questão 6 ....

Determine o domínio das funções:

(a) $f(x) = \log_3(x^2 - 4)$

$$D = \{x \in \mathbb{R} | x < -2 \textbf{ ou } x > 2\}$$

(b) $f(x) = \log_2(1 - 2x)$

$$D = \left\{x \in \mathbb{R} | x < \frac{1}{2}\right\}$$

(c) $f(x) = \log_3(4x-3)^2$

$$D = \left\{x \in \mathbb{R} | x \neq \frac{3}{4}\right\}$$

(d) $f(x) = \log_5 \dfrac{x+1}{1-x}$

$$D = \{x \in \mathbb{R} | -1 < x < 1\}$$

(e) $f(x) = \log(x^2 + x - 12)$

$$D = \{x \in \mathbb{R} | x < -4 \textbf{ ou } x > 3\}$$

(f) $f(x) = \log_{(x+1)}(2x^2 - 5x + 2)$

$$D = \left\{x \in \mathbb{R} | -1 < x < \frac{1}{2} \textbf{ ou } x > 2 \textbf{ e } x \neq 0\right\}$$

Questão 7 ....

Resolva as inequações:

(a) $\log_3(5x-2) < \log_3 4$

$$S = \left\{x \in \mathbb{R} | \frac{2}{5} < x < \frac{6}{5}\right\}$$

(b) $\log_{0,3}(4x-3) < \log_{0,3} 5$

$$S = \{x \in \mathbb{R} | x > 2\}$$

(c) $\log_{0,5}(3x-1) \geq \log_{0,5}(2x+3)$

$$S = \left\{x \in \mathbb{R} | \frac{1}{3} < x \leq 4\right\}$$

(d) $\log_2(2x^2 - 5x) \leq \log_2 3$

$$S = \left\{x \in \mathbb{R} | -\frac{1}{2} \leq x < 0 \textbf{ ou } \frac{5}{2} < x \leq 3\right\}$$

(e) $\log_{0,5}(x^2 - 1) > \log_{0,5}(3x+9)$

$$S = \{x \in \mathbb{R} | -2 < x < -1 \textbf{ ou } 1 < x < 5\}$$

(f) $\log_{0,1}(x^2 + 1) < \log_{0,1}(2x-5)$

$$S = \left\{x \in \mathbb{R} | x > \frac{5}{2}\right\}$$

(g) $\log(x^2 - x - 2) < \log(x-4)$

$$S = \emptyset$$

(h) $\log_5(x^2 - x) > \log_{0,2} \dfrac{1}{6}$

$$S = \{x \in \mathbb{R} | x < -2 \textbf{ ou } x > 3\}$$

(i) $\log_{0,5}\left(x^2 - x - \dfrac{3}{4}\right) > 2 - \log_2 5$

$$S = \left\{x \in \mathbb{R} | -1 < x < -\frac{1}{2} \textbf{ ou } \frac{3}{2} < x < 2\right\}$$

Questão 8 ....

Resolva a inequação $\log_a(2x-3) > 0$, para $0 < a < 1$.

$$S = \left\{x \in \mathbb{R} | \frac{3}{2} < x < 2\right\}$$

Questão 9 ....

Resolva as inequações:

(a) $\log_3(3x+4) - \log_3(2x-1) > 1$

$$S = \left\{ x \in \mathbb{R} | \frac{1}{2} < x < \frac{7}{3} \right\}$$

(b) $\log_2 x + \log_2(x+1) < \log_2(2x+6)$

$$S = \{ x \in \mathbb{R} | 0 < x < 3 \}$$

(c) $\log_2(3x+2) - \log_2(1-2x) > 2$

$$S = \left\{ x \in \mathbb{R} | \frac{2}{11} < x < \frac{1}{2} \right\}$$

(d) $\log(2x-1) - \log(x+2) < \log 3$

$$S = \left\{ x \in \mathbb{R} | x > \frac{1}{2} \right\}$$

(e) $\log_3(x^2+x-6) - \log_3(x+1) > \log_3 4$

$$S = \{ x \in \mathbb{R} | x > 5 \}$$

(f) $\log_{0,5}(x-1) + \log_{0,5}(3x-2) \geq -2$

$$S = \{ x \in \mathbb{R} | 1 < x \leq 2 \}$$
