AI engineering apps

IEOR 4574E001

Fall 2025

Introduction to large language models

Large language models

- Large language models are AI systems designed to process and analyze vast amounts of natural language data and then use that information to generate responses to user prompts.
- These systems are Trained on massive datasets using machine learning algorithms to learn the patterns and structures of human language.
- Large language models are becoming increasingly important in a variety of applications such as natural language processing, machine translation, code and text generation, and more.

Large language models

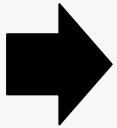
- Predicts the next token in a sequence
- Think of it as a completion engine
- Early examples: n-grams, Markov models, RNNs
- Core idea: probability distribution over words

Sentence

My favorite book is _ ?

Tokens

5444 8340 2392 382 _ 1423



Moby Dick	4.1%
War and Peace	3.9%
the one	1.4%
written	2.2%

Large language models

- LLMs scale up **parameters**, **data**, **compute.**
- Scale unlocks emergent capabilities (...probably).
- Foundation models: pretrained once, reused everywhere.

LLMs are Multitask

- From task-specific ML → general-purpose models
- Broad impact: coding, support, content, analytics

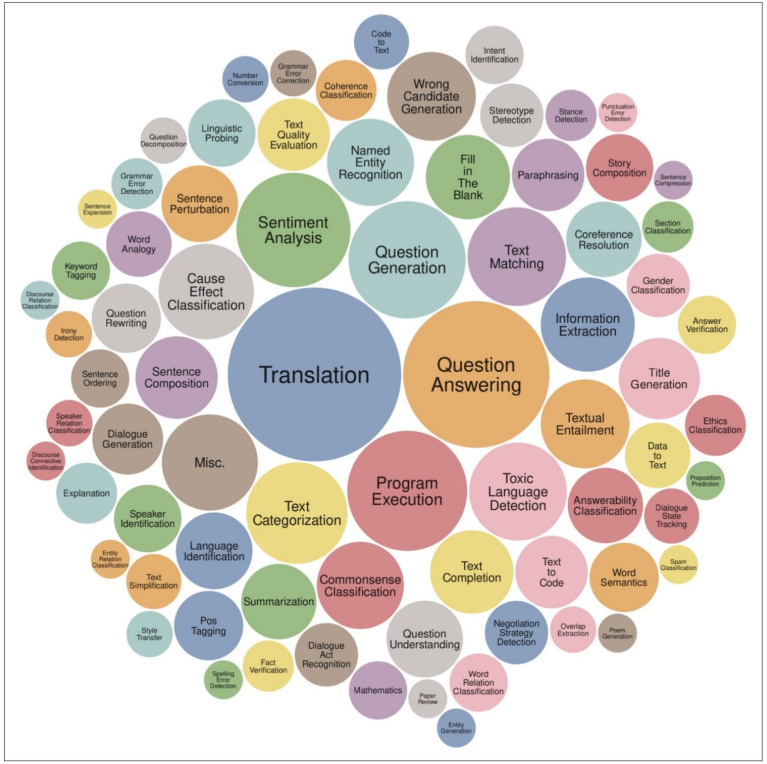


Figure 1-4. The range of tasks in the Super-NaturalInstructions benchmark (Wang et al., 2022).

From models to applications

- Reusable “foundation” you adapt, not retrain.
- API access lowers the barrier → faster product loops.
- Self-hosting and open models trade convenience for control.

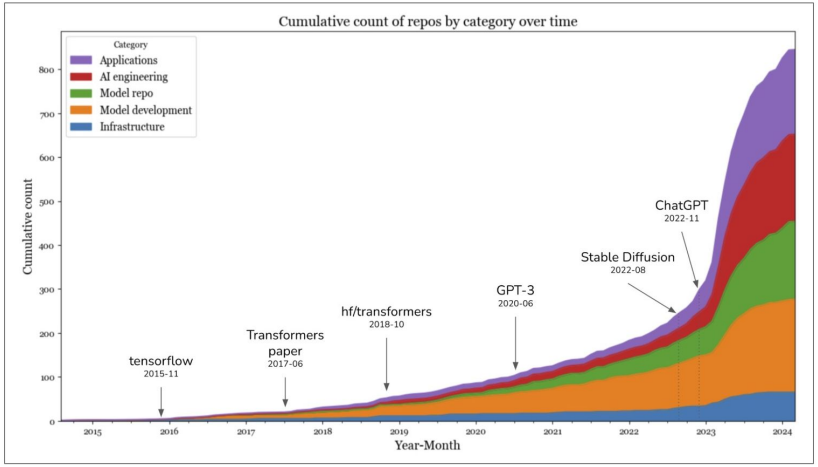


Figure 1-15. Cumulative count of repositories by category over time.

1950s–1990s

- Early NLP is mostly **symbolic**: hand-written rules and logic trees try to “translate” by following fixed procedures.
- Works in tightly defined niches; collapses when inputs drift outside the rulebook.

1990s

- Shift to **statistical NLP**: n-grams, HMMs, and probabilistic parsers start modeling language as distributions.
- Ambition outpaces hardware—compute and data hold progress back.

2000s

- **Machine-learning pipelines** (SVMs, CRFs) become standard.
- The web explodes; so does the pool of text to learn from.

2012 - 2016

- **Deep learning renaissance** (e.g., ImageNet/AlexNet) proves scale wins; sequence-to-sequence RNNs power early translation.
- Still bottlenecked by recurrence and limited context.

2017

- **Transformer** architecture lands: attention replaces recurrence, unlocking parallel training and richer context handling.

2018

- **BERT** (bidirectional encoder) sets a new bar for language understanding.
- Decoder-only pretraining (GPT-1) shows the promise of large, self-supervised generative models.

2019–2020

- **GPT-2** surprises with broad zero-/few-shot behavior; **GPT-3 (175B)** establishes few-shot prompting as a general interface to language tasks.

2022

- **ChatGPT** productizes LLMs: a chat UI + instruction tuning makes models useful to everyone, not just researchers.

2023

- **GPT-4** raises capability and reliability;
- Open-source families (e.g., **LLaMA**, **Alpaca**, **Vicuna**) accelerate and broaden access; multimodal systems gain traction.

2024

- Scaling a single monolith further is costly—focus shifts to **efficiency** and **reasoning**: Mixture-of-Experts, test-time compute, speculative decoding, **chain-of-thought** prompting.

- **Agent** patterns (tool use, retrieval, action) mature; open source keeps closing the gap with proprietary stacks.

Key concepts

- **Tokenization** (subword units).
- **Context window** (how much the model can “see”).
- **Sampling & stochastic** outputs.
- **Pre-training & Post-training** (SFT, preference tuning).
- **Limitations**: hallucinations, cost/latency, evaluation.

Training strategy

Training phases

Phase	Purpose	Data	Outcome
Pretraining	Learn general linguistic and world knowledge	Large unlabeled text corpus	Base LLM
Instruction Tuning	Learn to follow instructions/tasks	Supervised human-labeled data	Instruct model
Preference Optimization	Align with human values/preferences	Human feedback, rankings	Chat/assistant model
Domain/Modality Adaptation	Specialize or extend capabilities	Domain or multimodal data	Specialized model

Self-supervision unlocks scalability

- No manual labels: the data is its own target (next/masked token).
- Each sentence yields thousands of training pairs → labels '**for free**'.

Input: "The cat sat on the **[MASK]**"

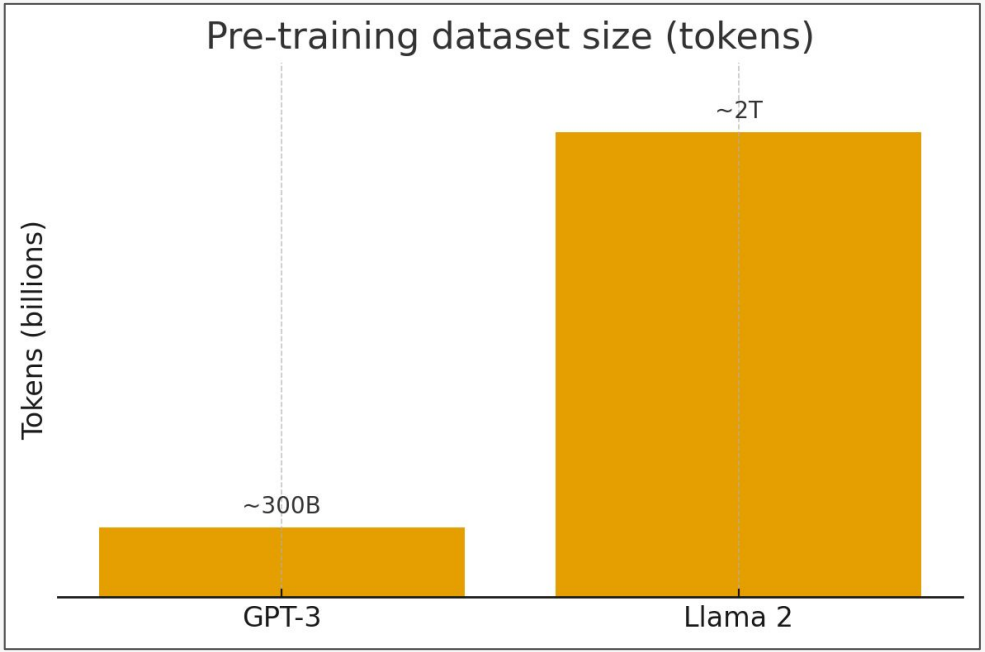
Label: "mat" (comes directly from the original sentence).

<BOS>	the
<BOS>, the,	cat
<BOS>, the, cat	sat
<BOS>, the, cat, sat	on
<BOS>, the, cat, sat, on	the
<BOS>, the, cat, sat, on, the	mat

Training set

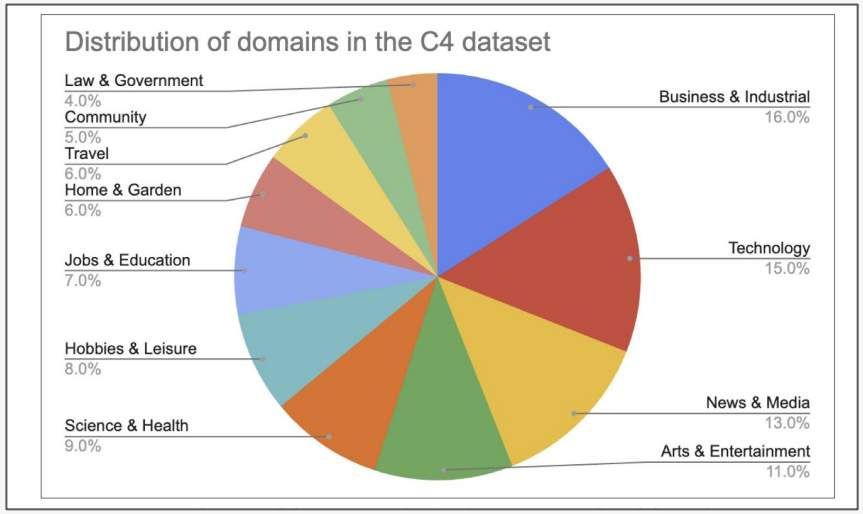
Self-supervision unlocks scalability

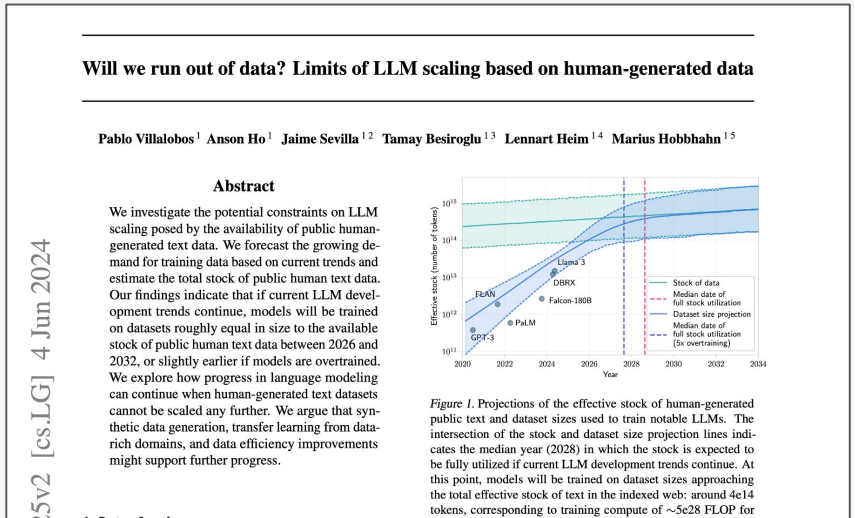
- Self-supervision allows for dataset sizes that are simply **infeasible to label by hand.**
- Remember the scaling laws paper by Kaplan et al. (2020).
- No manual labeling but filtering, deduplicating, domain weighting, synthetic data generation.



Pre-training

- **Objective:** predict missing/next token (masked vs. autoregressive).
- **Scale:** no human labels → web-scale corpora.
- **Data recipe:** diverse, multi-domain (and increasingly multilingual).
- **Cost reality:** pre-training dominates total compute.
- **Output:** a powerful base model (not yet aligned).





Will we run out of data? Limits of LLM scaling based on human-generated data

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Abstract

We investigate the potential constraints on LLM scaling posed by the availability of public human-generated text data. We forecast the growing demand for training data based on current trends and estimate the total stock of public human text data. Our findings indicate that if current LLM development trends continue, models will be trained on datasets roughly equal in size to the available stock of public human text data between 2026 and 2032, or slightly earlier if models are overtrained. We explore how progress in language modeling can continue when human-generated text datasets cannot be scaled any further. We argue that synthetic data generation, transfer learning from data-rich domains, and data efficiency improvements might support further progress.

Figure 1. Projections of the effective stock of human-generated public text and dataset sizes used to train notable LLMs. The intersection of the stock and dataset size projection lines indicates the median year (2028) in which the stock is expected to be fully utilized if current LLM development trends continue. At this point, models will be trained on dataset sizes approaching the total effective stock of text in the indexed web: around 4e14 tokens, corresponding to training compute of ~5e28 FLOP for

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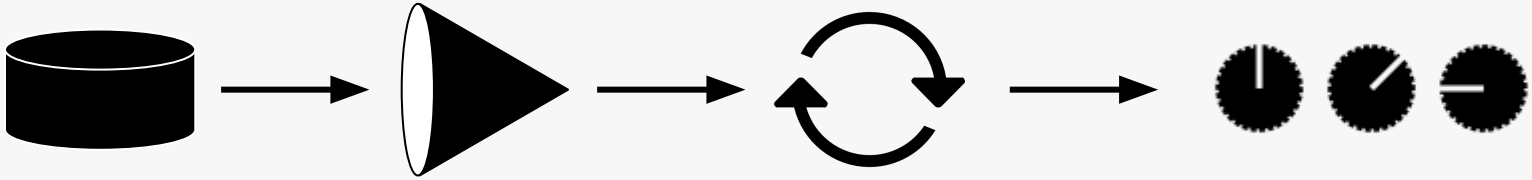
Post-training: make the base model useful



Dataset

Filtering, enrichment, cleaning, synthetic data

Pre-training

Post training

Supervised Fine Tuning (SFT) and preference tuning.

Runtime control: sampling, structured decoding, schemas.

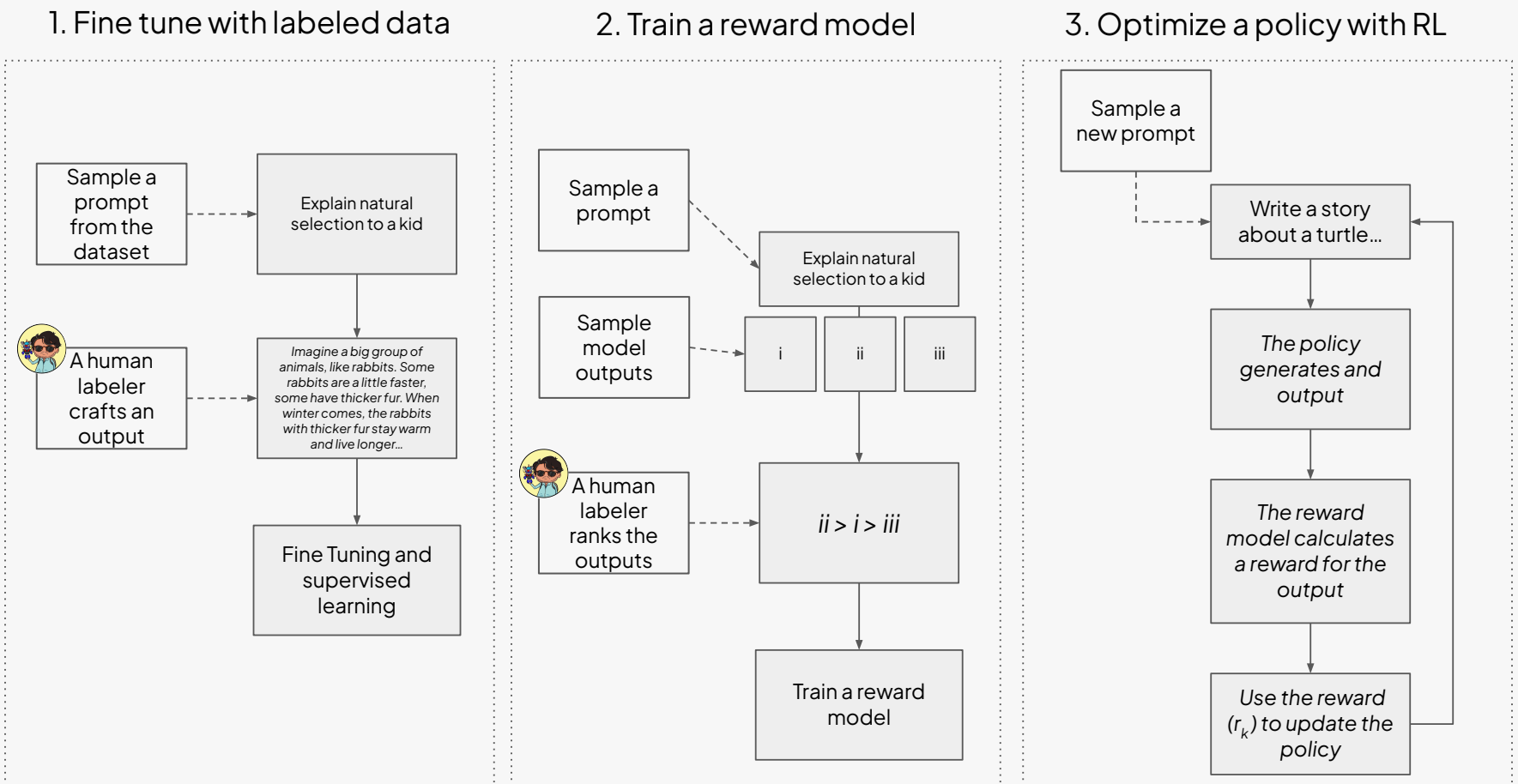
Supervised fine-tuning (SFT)

- Train on curated instruction → answer pairs.
- It teaches **formatting**, **task-following**, tone, start/stop, structure.
- **Data quality rules:** small and high-quality beats large and noisy.
- **Limit:** imitates examples well but is brittle on unseen tasks; can overfit to style.

Preference Optimization

- It trains a reward model with human preferences over multiple candidate answers.
- **Goal:** push behavior toward helpful, harmless, honest responses.
- **Two ways:**
    - **Reinforcement Learning from Human Feedback (RLHF):** learn a reward model from rankings, then optimize the policy with RL (e.g., PPO).
    - **Direct Preference Optimization (DPO):** skip the reward model and RL; directly increase the probability of preferred outputs.
- **Strengths:** steers tone, safety, refusal rules; balances helpfulness vs. caution.

Reinforcement Learning from Human Feedback



Summary

Stage	What Happens	Model Role	Learning Signal
SFT	*Learn to follow instructions*	*Generator (policy)*	*Explicit targets (supervised labels)*
Reward Model	*Learn to judge outputs*	*Evaluator*	*Human rankings*
RLHF	*Learn to optimize for quality*	*Generator (policy)*	*Reward signal from evaluator*

Probabilistic decoding: temperature, top-k, top-p

- After training, **generation depends on how we sample** from the probability distribution.
- **Deterministic** runs for **eval**; **stochastic** runs for **ideation**.
- Adjust **temperature**, top-k, or top-p to tune *diversity vs. reliability*.
- Use **frequency** and **presence penalties** to reduce *repetition* and *topic loops*.

Probabilistic decoding: methods

Parameter	What It Controls	How It Works (Mechanics)	Low Value Effect	High Value Effect	Typical Range
Temperature	Randomness of token selection	Scales logits before sampling	Deterministic, conservative, repetitive	More random, creative, variable	0.2 – 1.2
Top-k	Breadth of token choices	Only the top *k* most probable tokens are kept; probabilities renormalized	Focuses on most likely words → safer, less varied	Considers many tokens → freer, risk of nonsense	20 – 200
Top-p	Adaptive probability mass	Keeps smallest set of tokens whose cumulative probability ≥ *p*	Predictable, sometimes repetitive	Richer vocabulary, risk of drift	0.7 – 0.95
Frequency penalty	Repetition of phrases or words	Subtracts a penalty proportional to how often each token has appeared so far	Repeats common phrases	Suppresses repeated words/phrases	0 – 2
Presence penalty	Topic re-use	Subtracts a fixed penalty once a token has appeared at all	Sticks to prior topics	Encourages introducing new topics/words	0 – 2

Test-time compute: generate, verify, select

- n-best sampling: produce multiple candidates.
- Self-consistency and reranking with heuristics or verifiers.
- Trade-off: quality ↑ vs latency/cost ↑.

Structured outputs & tool-use scaffolding

- Constrained decoding for JSON/XML/regex/grammars
- Schemas and function/tool calling for APIs
- Benefits: reliability, parse-ability, guardrails

The probabilistic nature of AI

- Outputs are distributions, not facts.
- Expect variance; manage determinism for tests.
- Prefer verify-then-trust: logprobs, checks, or retrieval grounding.

Deep Dive into Language Models

Tokenization

- Tokenizers map **text to ids** tied to a model's vocabulary.
- **Strategies**: bytes, characters, words, subwords.
- **Subword tokenizers** balance vocabulary size with open-vocabulary coverage.
- Two sentences that look short to you may differ by a factor of two in tokens once encoded.
- Your tokenization strategy impact directly inference cost and latency.

lowercase and CAPITALIZATION

🤗

show_tokens False None elif == >= else: two tabs:" " Three tabs: " "

12.0*50=600

Subword tokenization strategies

Method	Training principle	Example output	Used in
BPE	Merge most frequent symbol pairs	`["un", "believ", "able", " stories"]`	GPT-2, LLaMA
WordPiece	Maximize corpus likelihood; mark continuations with ##	`["un", "##believ", "##able", "stories"]`	BERT
SentencePiece	Train on raw text (spaces = token "_"); optionally unigram LM	`["_un", "believable", "_stories"]`	T5, XLNet

Representing words as vectors: bag of words

Corpus

A collection of text documents

↓

Tokenize

Divide the text into smaller units called tokens, usually words or phrases

↓

Count word frequencies

Create a vocabulary of unique words and count the number of times each word appears in each document.

↓

Encode the data

Encoding the text data as numerical values by creating a vector for each document, with each element of the vector representing the frequency count of a particular word in the document

Corpus:

The dog is happy. The child makes the dog happy. The dog makes the child happy

↓

Tokenization:

D1: [The] [dog] [is] [happy]
D2: [The] [child] [makes] [the] [dog] [happy]
D3: [The] [dog] [makes] [the] [child] [happy]

↓

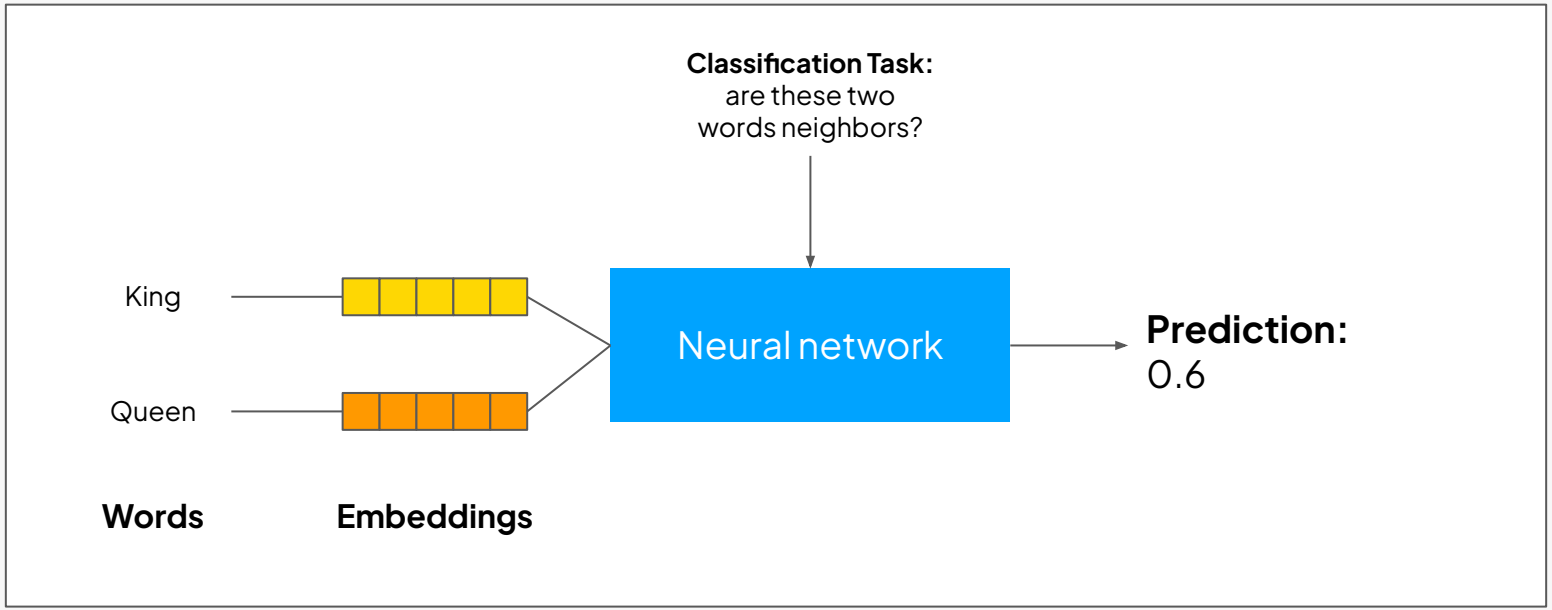
Documents	Counting word frequencies
D1	the: 1, dog: 1, is: 1, happy: 1
D2	the: 2, dog: 1, makes: 1, child: 1, happy: 1
D3	the: 2, child: 1, makes: 1, dog: 1, happy: 1

↓

Encode	child	dog	happy	is	makes	the	BoW Vector representations
D1	0	1	1	1	0	1	[0,1,1,1,0,1]
D2	1	1	1	0	1	2	[1,1,1,0,1,2]
D3	1	1	1	0	1	2	[1,1,1,0,1,2]

Dense vector embeddings

- **Meaning and context:** dense vectors capture distributional similarity ("you shall know a word by the company it keeps").



Dense vector embeddings

- Geometry encodes meaning: Use cosine similarity as a proxy for semantic relatedness.

$$\cos\theta = \frac{u \cdot v}{\|u\| \, \|v\|}$$

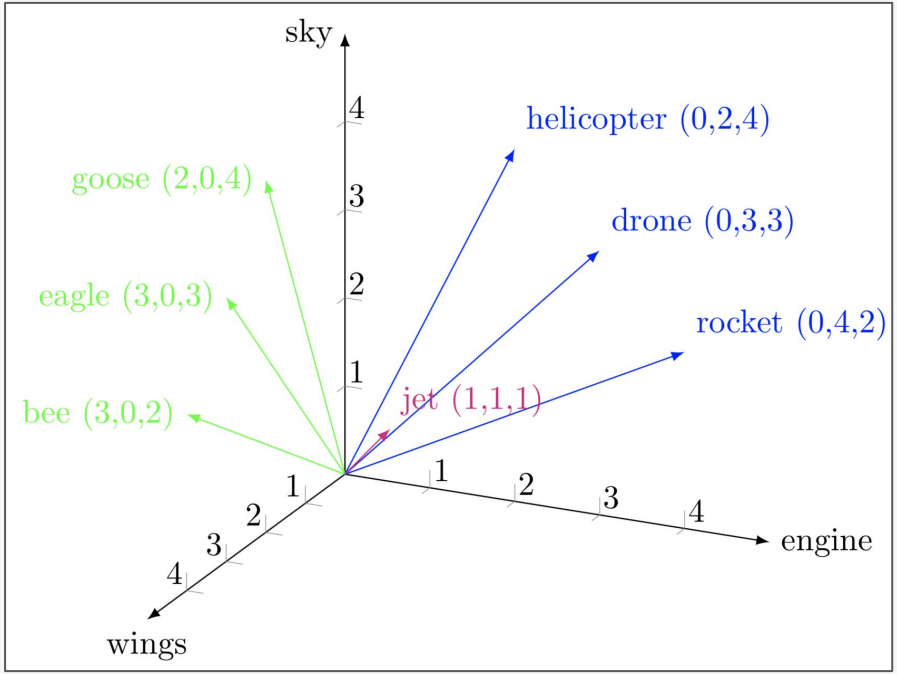


Image from Corpus Linguistics and Statistics with R
http://www.springer.com/gp/book/9783319645704

Types of embeddings

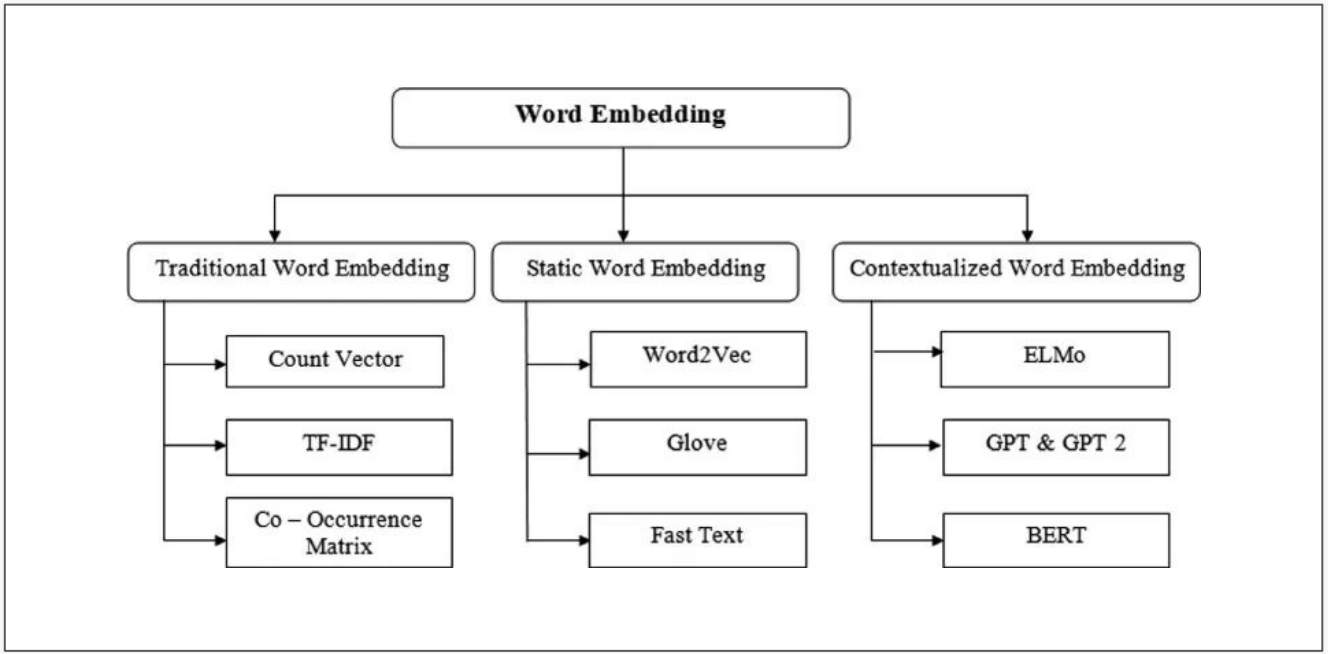
Positional embeddings: encode where each token appears in the sequence (order, not meaning).

Token embeddings: encode what each word is (same vector every time).

Contextual embeddings: encode what it means here (word meaning changes with context).

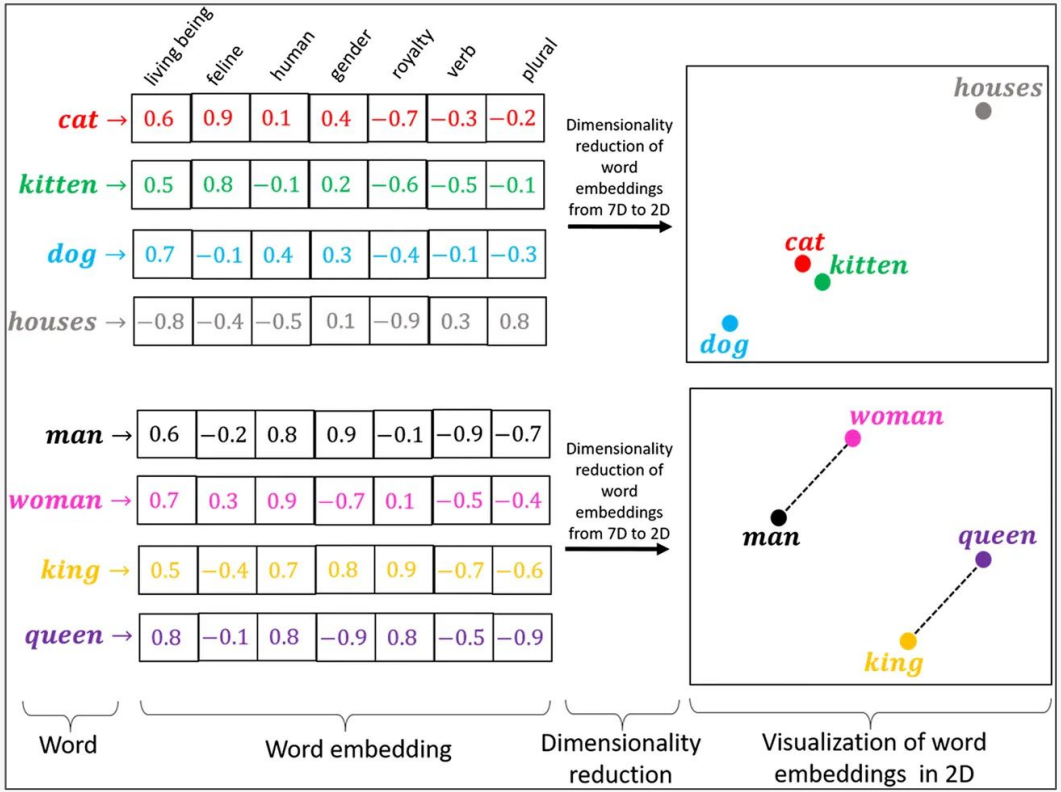
Sentence/Document embeddings: encode what the whole text is about (one vector summarizing overall meaning).

Types of word embeddings



Training static embeddings (Word2Vec)

- Training static embeddings (Word2Vec)
- Predict context from a center word (skip-gram) or vice versa (CBOW).
- Learn two embedding tables; score with dot products.
- Optimize logistic loss with negative sampling and backprop.
- Result: one vector per word type. Great for intuition, limited on polysemy.



Embeddings in practice (inside an LLM)

- You download: tokenizer, embeddings, transformer blocks, output head.
- Embeddings are necessary but not sufficient.
- Most capability lives in attention and MLP layers.
- LM head is often tied to the token embedding matrix.
- For search: use purpose-built sentence/document encoders; for generation: use the full LLM.

LLMs and Sequence Modeling

Architectures to predict sequences

n-grams → learn from counts. Models estimate probability from counts; simple and fast, but suffer from sparse statistics as n grows, and can't model long contexts or generalize beyond observed n-grams.

RNNs (2014) → learn from sequences. Recurrent nets replace fixed n with a hidden state, letting the model in principle use unbounded context. Practically, they struggle with vanishing/exploding gradients and long-range dependencies.

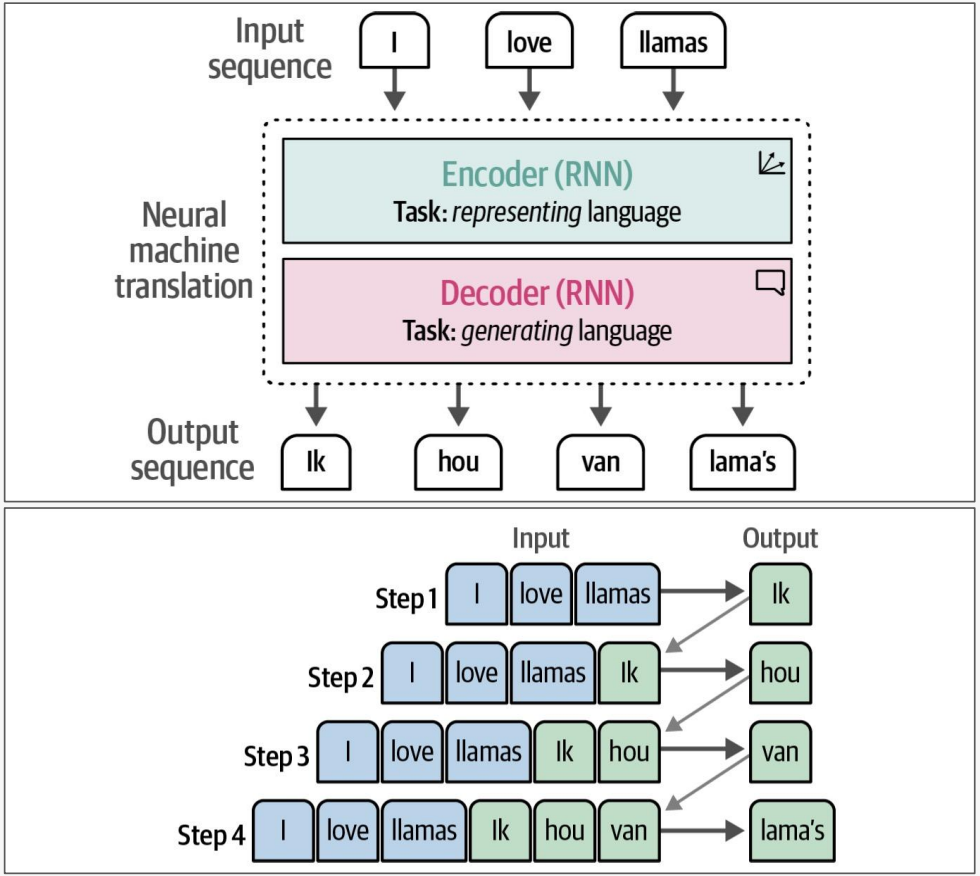
LSTMs → mitigate vanishing gradients. LSTMs add gates and persistent cell states, making RNN training more stable and improving long-range patterns. They power early state-of-the-art seq2seq systems, but are still sequential.

Attention (2015) → break the "thought-vector" bottleneck. Vanilla seq2seq compresses an input into one vector; attention lets the decoder soft-search over all encoder states each step.

Transformers (2017) → scale with self-attention. Self-attention replaces recurrence with direct token-to-token interactions; multi-head attention captures multiple relationships; positional encodings restore order; and the architecture trains in parallel over tokens.

Encoder–decoder

- **Encoder**: reads the input sequence step by step and compresses it into a hidden representation (context vector).
- **Decoder**: takes this representation and generates the output sequence, one token at a time.
- **Training**: decoder is guided with the correct previous word (teacher forcing).
- **Inference**: decoder relies on its own predictions (autoregressive loop).



Images from Jay Alammar, Maarten Grootendorst

Context-thought bottleneck

- **Seq2seq compresses the whole input into one fixed-size vector:** it works on short, simple inputs BUT drops details on long, complex sentences.
- **Symptoms**: wrong word order, missing entities, brittle agreement across clauses.
- **Motivation for attention**: let the decoder look back at all encoder states, not just one summary.

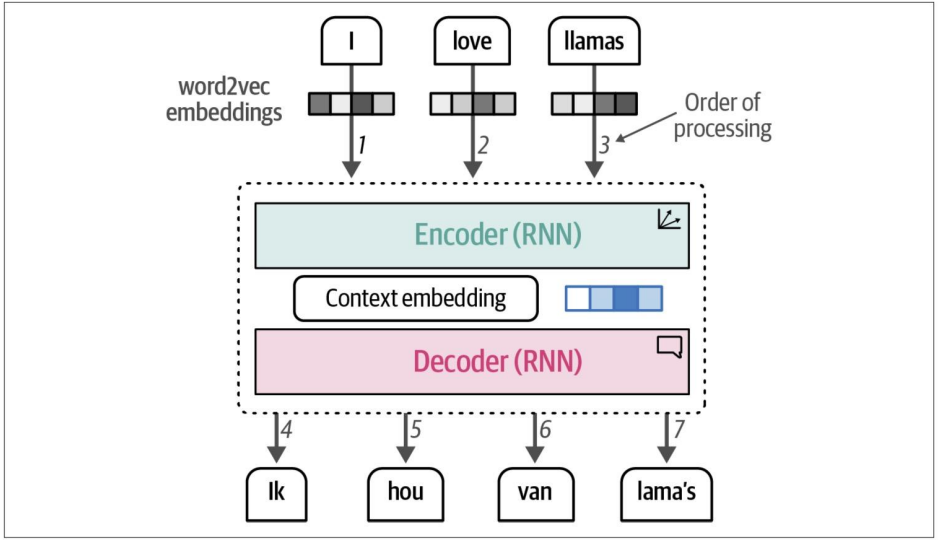
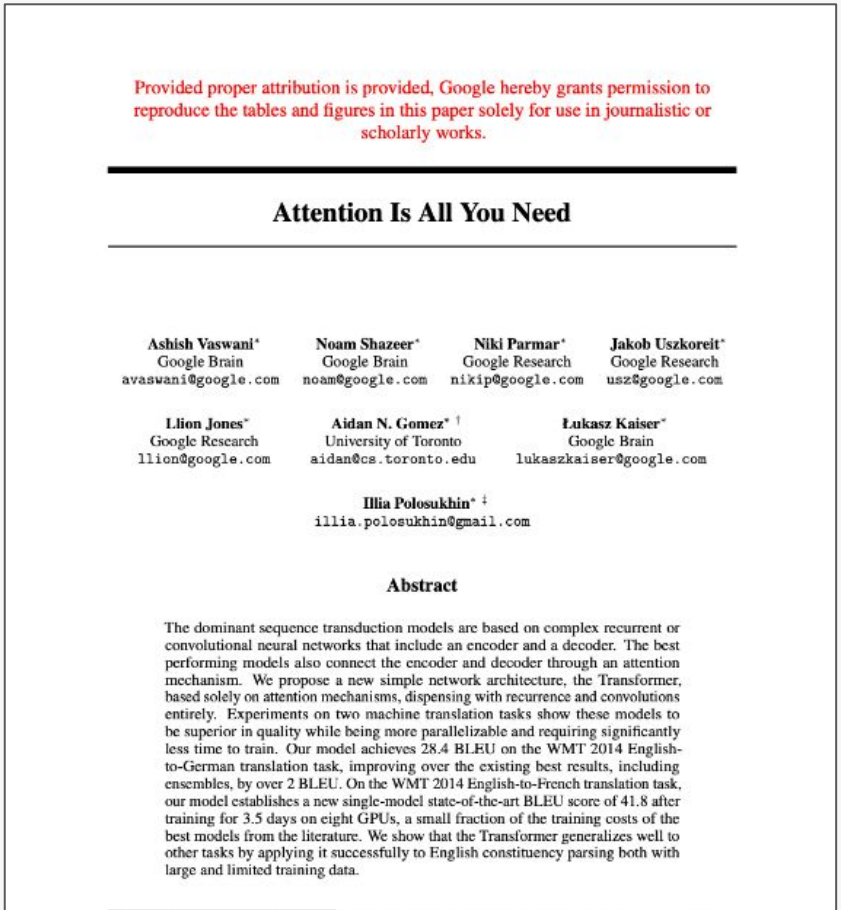


Figure 1-13. Using word2vec embeddings, a context embedding is generated that represents the entire sequence.

Images from Jay Alammar, Maarten Grootendorst

Attention is all you need

- In 2017, attention replaces recurrence in sequence models.
- **Key idea:** tokens attend to tokens directly (no more RNN loop).
- Attention unlocks parallel training, longer-range dependencies, clean scaling.
- This architecture enabled **BERT, GPT, T5** and the foundation model era.



Attention Is All You Need

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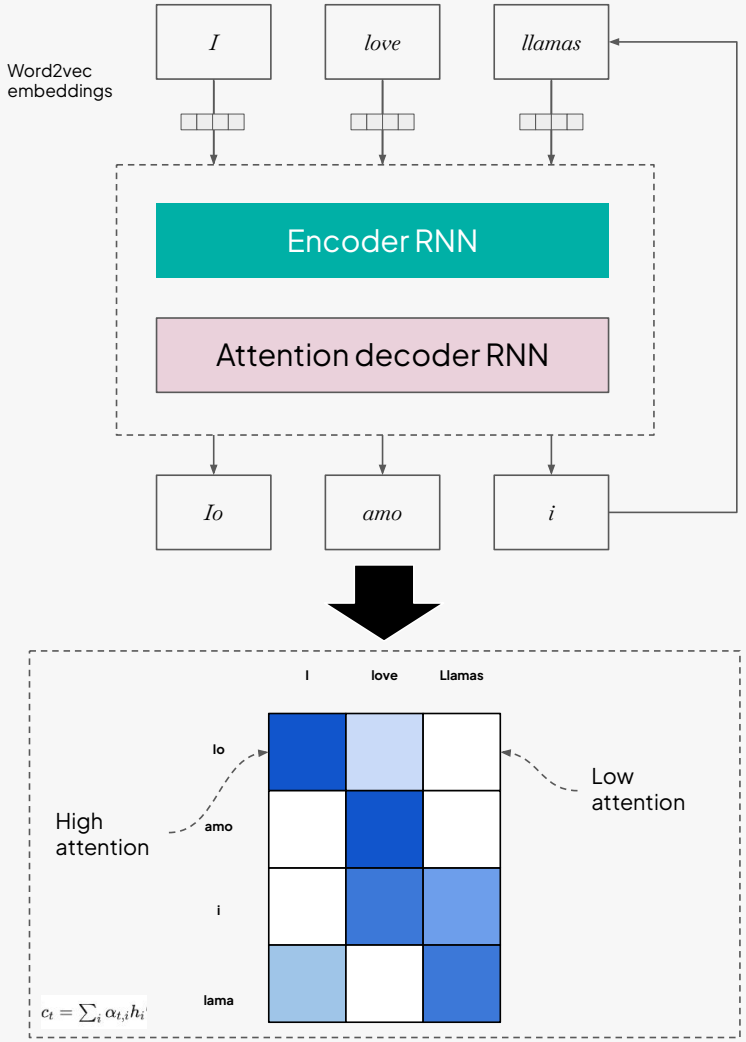
Abstract

The dominant sequence transduction models are based on complex recurrent or convolutional neural networks that include an encoder and a decoder. The best performing models also connect the encoder and decoder through an attention mechanism. We propose a new simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely. Experiments on two machine translation tasks show these models to be superior in quality while being more parallelizable and requiring significantly less time to train. Our model achieves 28.4 BLEU on the WMT 2014 English-to-German translation task, improving over the existing best results, including ensembles, by over 2 BLEU. On the WMT 2014 English-to-French translation task, our model establishes a new single-model state-of-the-art BLEU score of 41.8 after training for 3.5 days on eight GPUs, a small fraction of the training costs of the best models from the literature. We show that the Transformer generalizes well to other tasks by applying it successfully to English constituency parsing both with large and limited training data.

Attention in seq2seq

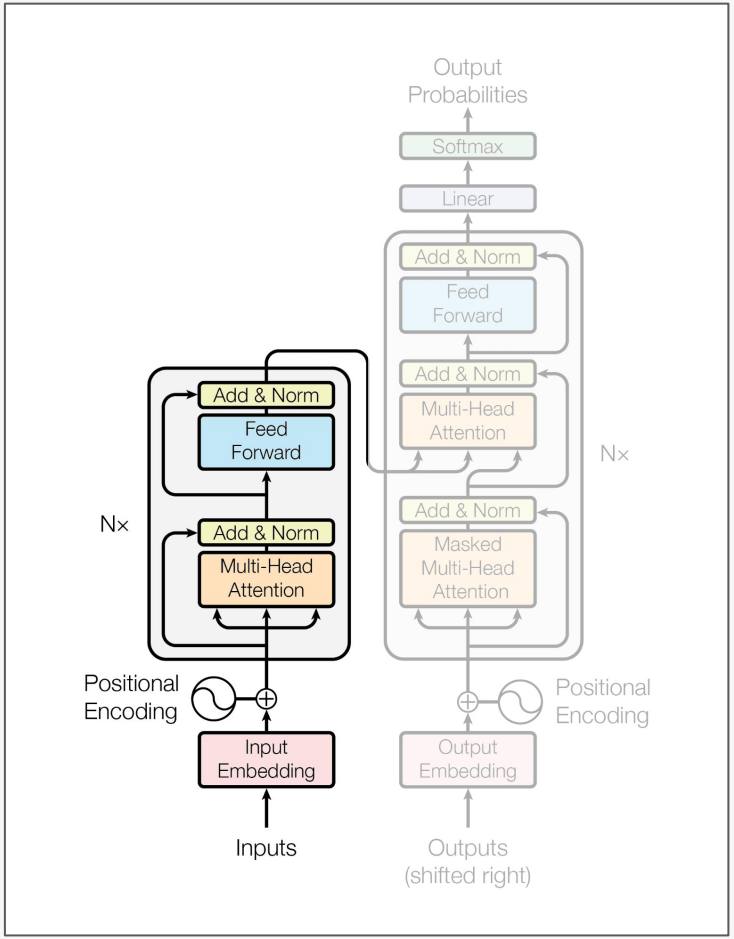
- Attention fixes the problem of one single “thought vector” dropping details on long inputs.
- The decoder now computes a weighted look-back over all encoder states.
- Benefit: soft alignments, better long sentences, partial interpretability.



Transformer encoder

- Builds global, order-aware token states (not one summary), fixing long-range context.
- Each word looks at the whole sentence so nothing important is forgotten.

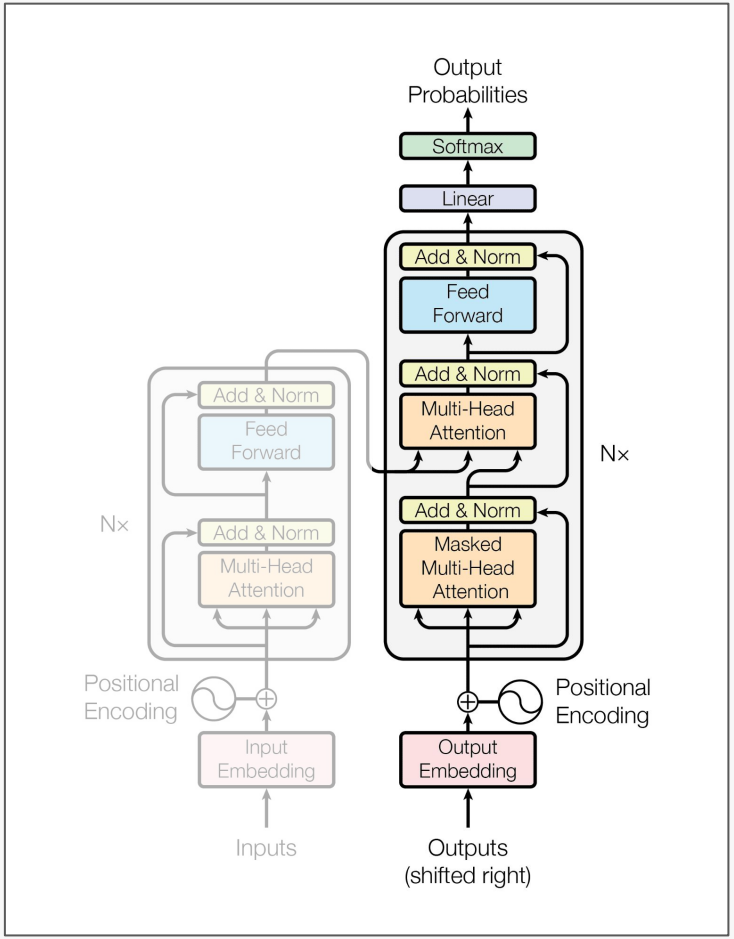
- Input embeddings + positional info.
- Multi-head self-attention (bidirectional).
- Feed-forward network (per token).
- Residual connections and LayerNorm.
- **Output:** contextual token representations.



Transformer decoder

- Generates causally while retrieving source info via cross-attention for faithful outputs.
- It writes the next word and checks the original input so the answer stays on topic.

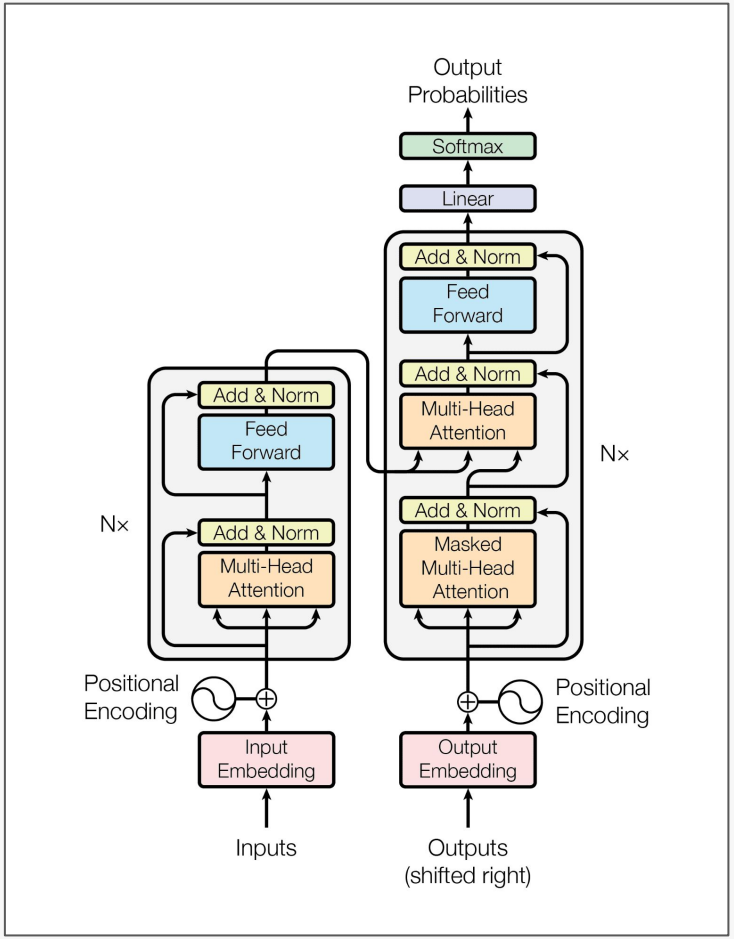
- Masked self-attention over generated tokens.
- Cross-attention to encoder outputs.
- Feed-forward network.
- Residuals and LayerNorm.
- **Output:** next-token logits for autoregressive generation.



Putting it all together

- Encoder builds contextual representations; decoder generates with masked self-attention and cross-attention, enabling parallel training and strong long-range modeling.

- Encoder stack produces source representations.
- Decoder stack generates targets using masked self-attention + cross-attention.
- Parallelizable training, strong long-range modeling.
- Families: encoder-only (BERT), decoder-only (GPT), encoder-decoder (T5).



Predicting sequences

Session-based recs

- **Use case:** personalized recommendations for b2c ecommerce websites.
- **Hard constraints:**
  - 40–50% of all shoppers leave a website after few interactions,
  - 10% of users come back more than 2 times in a year.
  - Less than 10% of users are logged in.
- Collaborative filtering cannot handle the cold-start problem and building a user profile is out of the question. So what do we do?

Hard constraints

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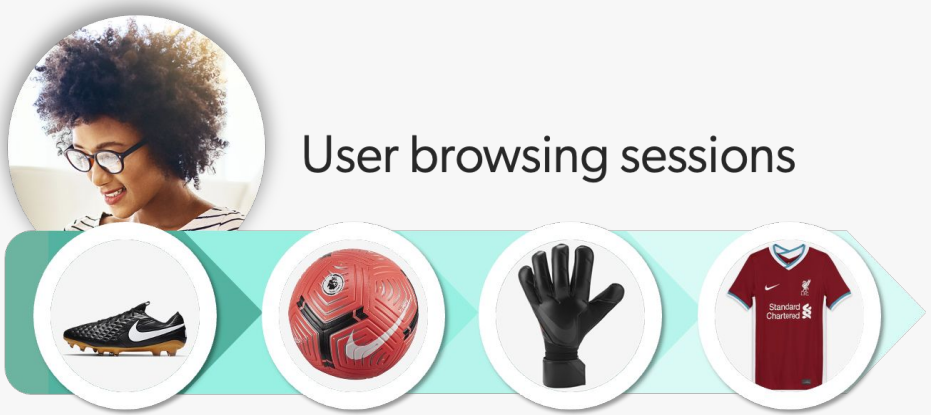
Shopping sessions as a source of information

- Shopping sessions are sequences of products.
- We can turn shopping sessions into vectors.
- Similar products will occur in similar contexts (that is in sessions that are similar).



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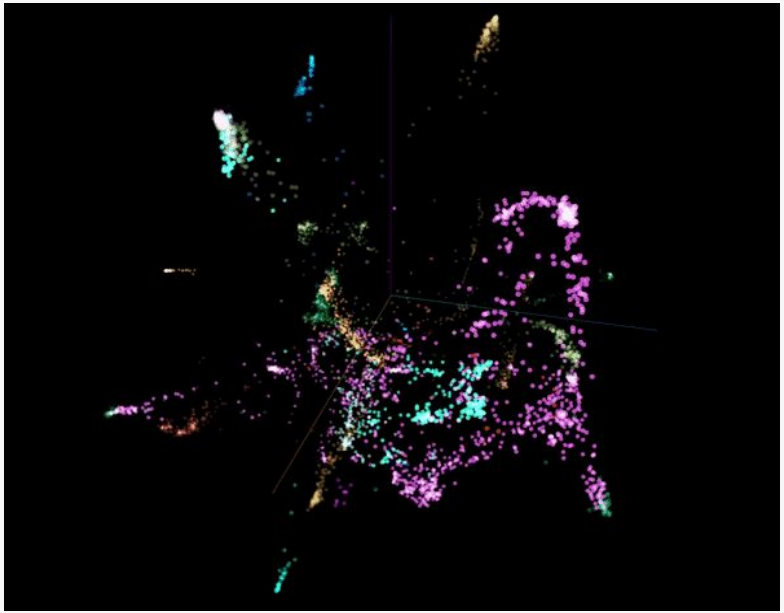
Prod2Vec

- When a prod2vec model is trained, similar products appear to be closer in the embedding space.
- The 3D T-SNE projection shows how products related to the same sport activity cluster together (for a catalog in the sport apparel vertical).



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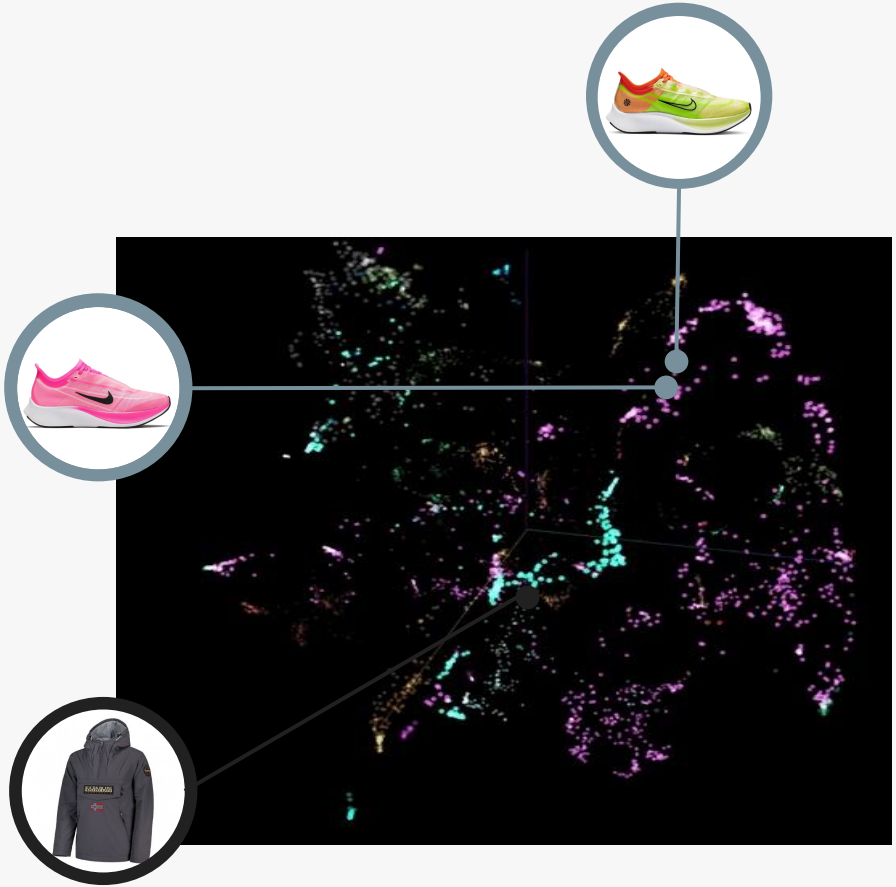
Prod2Vec

- Embedding space: products which are “semantically” similar will end up close in the embedding space.
- Prediction: when shopper is visiting item A, retrieve embedding for A and do KNN to retrieve similar items.
- Training: skip-gram (same as word2vec).



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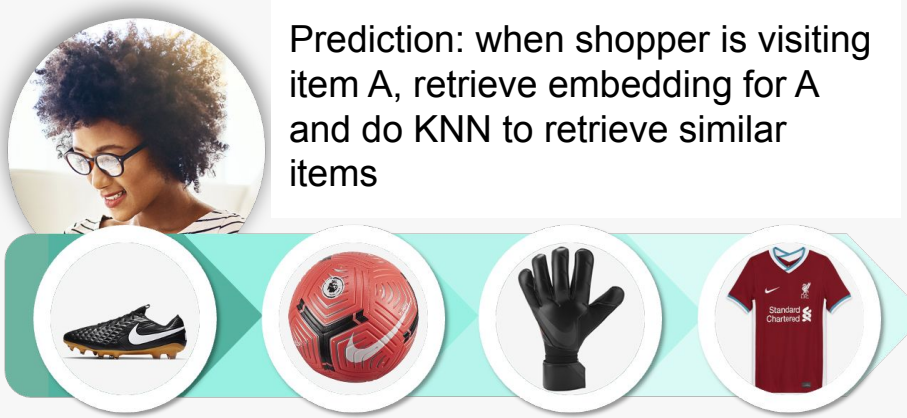
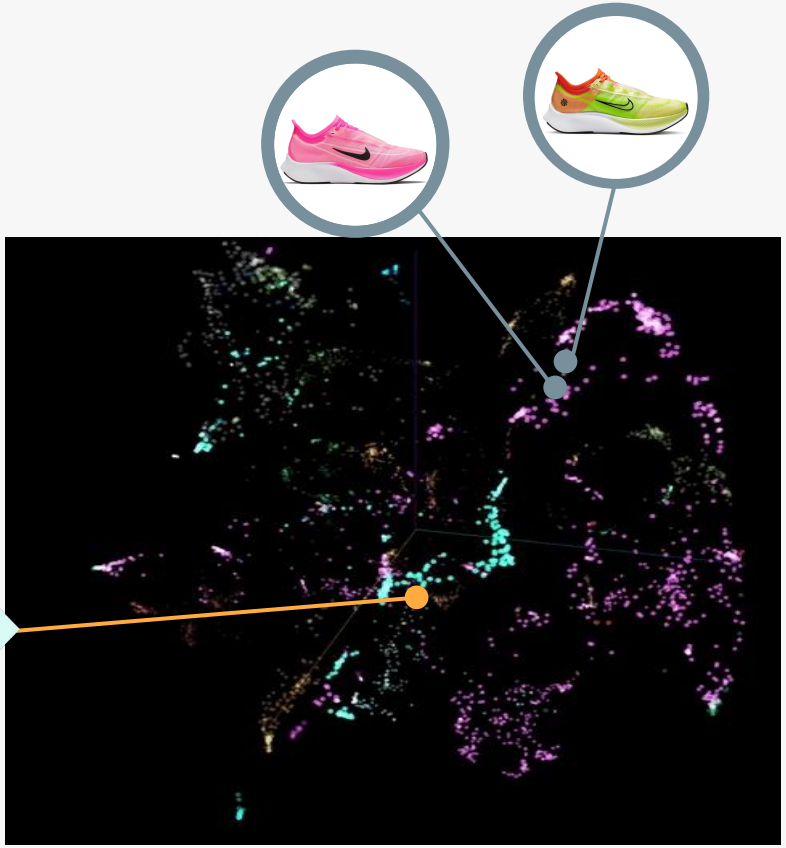
Session-base RS



Prediction: when shopper is visiting item A, retrieve embedding for A and do KNN to retrieve similar items



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