# Homework 1: Density and Distribution

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1. Random variable $X$ has the values $x_1 = 1, x_2 = 3, x_3 = -1, x_4 = 1.5$.
   $P(X = -1) = P(X = 1) = 0.25$
   $P(X = 1.5) = 0.12$
   $P(X = 3) = 0.38$

   a. Find and plot distribution function.
   b. Find and plot density function.
   c. Find the probability that rv $X$ is less than 2.
   d. Find the probability that rv $X$ is less than -1.

   **Solution:**

   Given the probabilities and knowing that $X$ is a discrete r.v. then we can use the following equation:

$$F_X(x) = \sum_{k=-\infty}^{\infty} P(x_k)u(x - x_k) \tag{1}$$

   then

$$F_X(x) = 0.25u(x + 1) + 0.25u(x - 1) + 0.12u(x - 1.5) + 0.38u(x - 3) \tag{2}$$

   and for the PDF we know:

$$f_X(x) = \sum_{k=-\infty}^{\infty} P(x_k)\delta(x - x_k) \tag{3}$$

   then

$$f_X(x) = 0.25\delta(x + 1) + 0.25\delta(x - 1) + 0.12\delta(x - 1.5) + 0.38\delta(x - 3) \tag{4}$$

   To compute the probability that $X < 2$ we can directly evaluate $x = 1.5$ in the distribution function or compute the following integral:

$$P\{X < 2\} = \int_{-\infty}^{1.5} f_X(x)dx = 0.25 + 0.25 + 0.12 = 0.68 \tag{5}$$

   and for $X < -1$ the probability is zero because before -1 we do not have another value, or simply evaluating in the distribution function.

$$P\{X < -1\} = 0 \tag{6}$$

   in Figure 1 we can see the plots for this rv.

2. Random variable $X$ is uniform in the interval $[-1, 3]$.

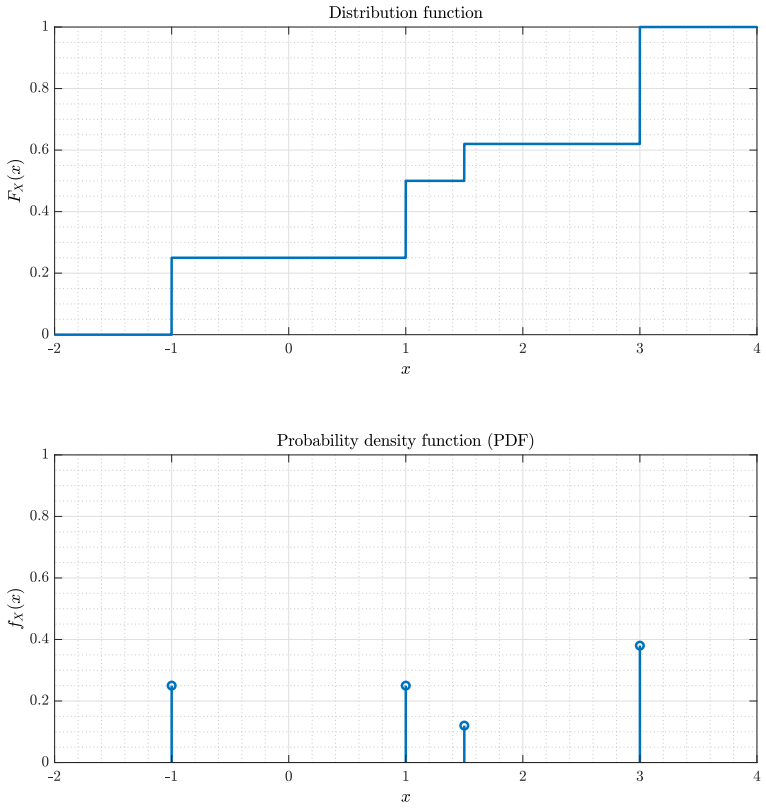


Figure 1: Plot of Distribution function and PDF.

a. Find and plot density and distribution.
b. Find the probability that rv $X$ is less than 2.
c. Find the probability that rv $X$ is equal to 0.

**Solution:**

Knowing that $X$ is a uniform rv then we can use the following equation:

$$f_X(x) = \begin{cases} \frac{1}{x_2 - x_1} & \text{if} \quad x_1 \leq x \leq x_2 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

then in our problem:

$$f_X(x) = \begin{cases} \frac{1}{4} & \text{if} \quad -1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

then we can find the distribution function as follow:

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{4} dx = \frac{1}{4} x \Big|_{-1}^{x} = \frac{1}{4}(x-1) \qquad \text{for} \qquad -1 \leq x \leq 3 \tag{9}$$

then

$$F_X(x) = \begin{cases} 1 & \text{if} \quad x > 3 \\ \frac{1}{4}(x-1) & \text{if} \quad -1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The probability that $X < 2$ can be obtain computing the integral or simply evaluating in $f_X(x)$ with $x = 2$, then:

$$P\{X < 2\} = F_X(2) = \frac{3}{4} = 0.75 \tag{11}$$

this result is consistent if we integrate the PDF function from $-\infty$ to 2.

The probability that $X = 0$ is zero, because for a continuous rv the probability that takes a particular value is zero. If we want to confirm this we can do the following:

$$P\{0 < X \leq 0\} = P\{X = 0\} = F_X(0) - F_X(0) = 0 \tag{12}$$

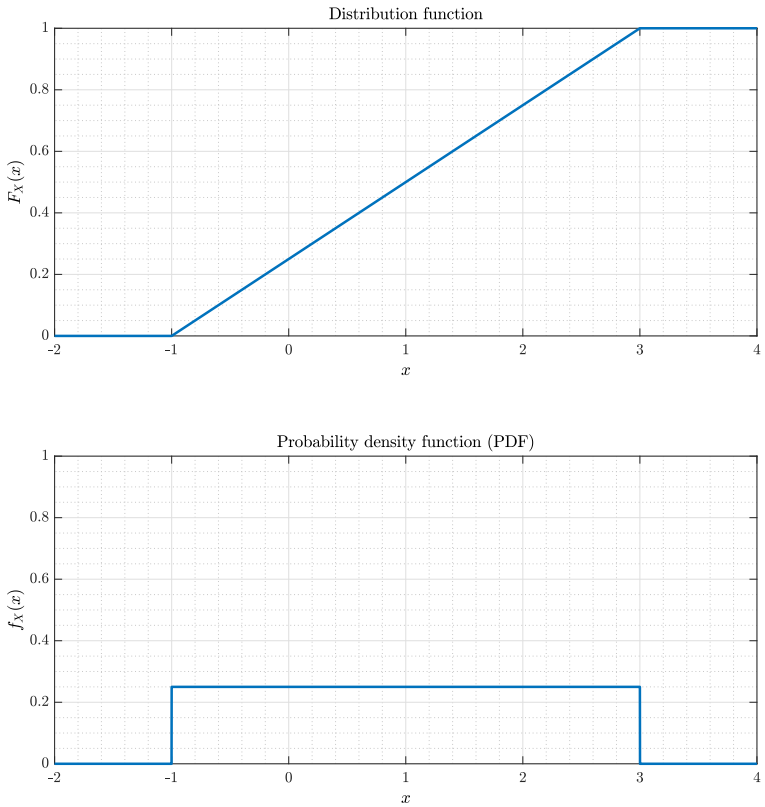


Figure 2: Plot of Distribution function and PDF.

In Figure 2 we have the plots.

3. Generate a uniform rv $X$ in the interval $[0, 1]$ using the **rand.m** function.

   a. Find the density and distribution of the rv $X$.

   b. Plot histogram, probabilities in cell, and estimated PDF taking $N = 10000$ values and 20 cells.

   c. Plot estimated distribution.

**Solution:**

$$f_X(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

$$F_X(x) = \begin{cases} 1 & \text{if} \quad x > 1 \\ x & \text{if} \quad 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

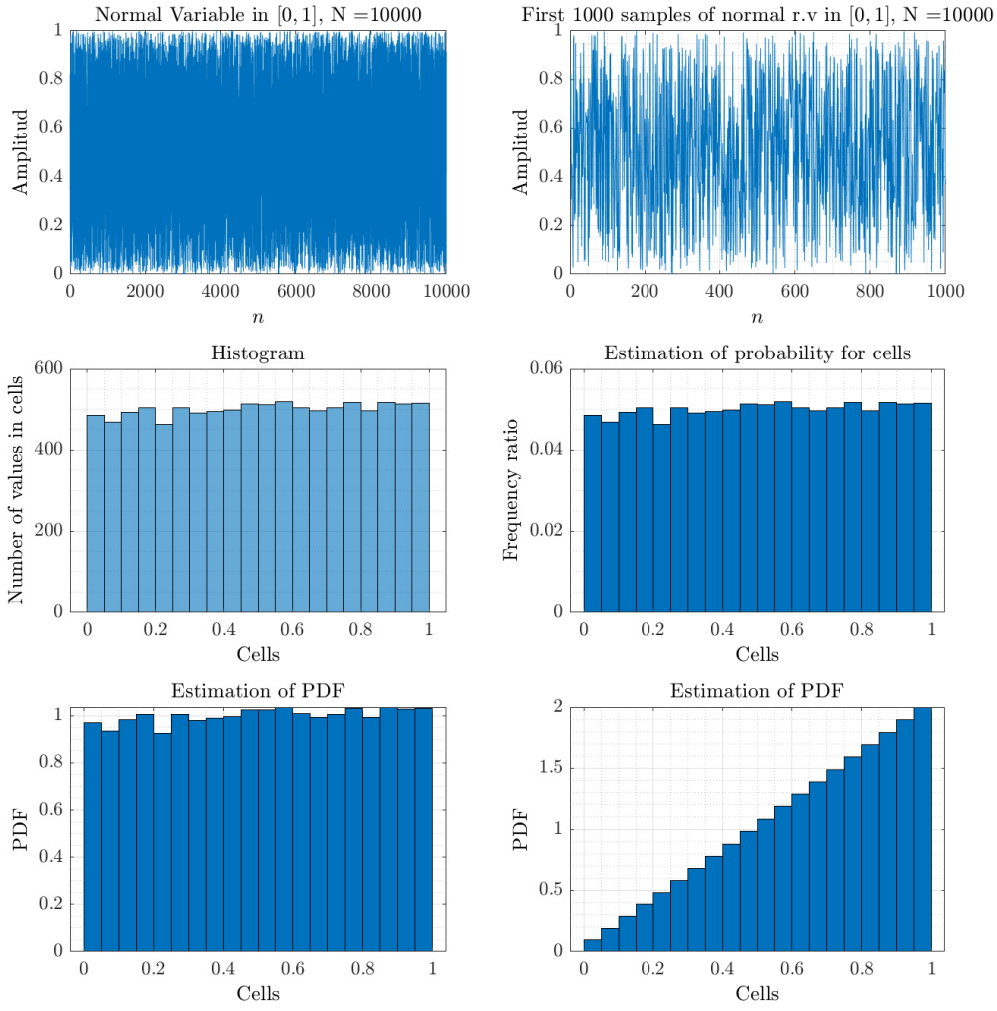


Figure 3: Complete analysis of normal rv.

```
clear;
close;
clc;

x = -2:0.001:4;
d = 0.25*d_step(x+1) + 0.25*d_step(x-1)...
    + 0.12*d_step(x-1.5) + 0.38*d_step(x-3);

x_pdf = [-1 1 1.5 3];
pdf = 0.25*d_delta(x_pdf+1) + 0.25*d_delta(x_pdf-1)...
    + 0.12*d_delta(x_pdf-1.5) + 0.38*d_delta(x_pdf-3);

subplot(2,1,1);
plot(x,d,'LineWidth',2);
grid on;
grid minor;
axis([-2 4 0 1]);
title('Distribution function','Interpreter','latex','FontSize',14);
xlabel('$x$','Interpreter','latex','Color','black','FontSize',12);
ylabel('$F_{X}(x)$','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

subplot(2,1,2);
stem(x_pdf,pdf,'LineWidth',2);
grid on;
grid minor;
axis([-2 4 0 1]);
title('Probability density function (PDF)','Interpreter','latex','FontSize',14);
xlabel('$x$','Interpreter','latex','Color','black','FontSize',12);
ylabel('$f_{X}(x)$','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

function R = d_step(n)
    R = zeros(size(n));
    for i = 1:length(n)
        if n(i) >= 0
            R(i) = 1;
        else
            R(i) = 0;
        end
    end
end


function R = d_delta(n)
    R = zeros(size(n));
    for i = 1:length(n)
        if n(i) == 0
            R(i) = 1;
        else
            R(i) = 0;
        end
    end
end
```

Code 1: Solve problem 1.

```
clear;
close;
clc;

x = -2:0.001:4;
d = Fx(x);

pdf = fx(x);

subplot(2,1,1);
plot(x,d,'LineWidth',2);
grid on;
grid minor;
axis([-2 4 0 1]);
title('Distribution function','Interpreter','latex','FontSize',14);
xlabel('$x$','Interpreter','latex','Color','black','FontSize',12);
ylabel('$F_{X}(x)$','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);


subplot(2,1,2);
plot(x,pdf,'LineWidth',2);
grid on;
grid minor;
axis([-2 4 0 1]);
title('Probability density function (PDF)','Interpreter','latex','FontSize',14);
xlabel('$x$','Interpreter','latex','Color','black','FontSize',12);
ylabel('$f_{X}(x)$','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

function R = fx(x)
    R = zeros(size(x));
```

```
    for i = 1:length(x)
        if x(i) >= -1 && x(i) <= 3
            R(i) = 1/4;
        else
            R(i) = 0;
        end
    end
end

function R = Fx(x)
    R = zeros(size(x));
    for i = 1:length(x)
        if x(i) >= -1 && x(i) <= 3
            R(i) = (1/4)*(x(i)+1);
        elseif x(i) > 3
            R(i) = 1;
        else
            R(i) = 0;
        end
    end
end
```

Code 2: Solve problem 2.

```
%%
clear;
close;
clc;

N = 10000;
X = rand(1,N);

% Uniform r.v [0,1] all graph
subplot(3,2,1);
plot(X);
grid on;
grid minor;
str1 = strcat( 'Normal Variable in $[0,1]$, N = ',num2str(N) );
title(str1,'Interpreter','latex','FontSize',14);
xlabel('$n$','Interpreter','latex','Color','black','FontSize',12);
ylabel('Amplitud','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

% Uniform r.v [0,1] first 1000
subplot(3,2,2);
plot(X(1:1000));
grid on;
grid minor;
str2 = strcat( 'First 1000 samples of normal r.v in $[0,1]$, N = ',num2str(N) );
title(str2,'Interpreter','latex','FontSize',14);
xlabel('$n$','Interpreter','latex','Color','black','FontSize',12);
ylabel('Amplitud','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

% Histogram
subplot(3,2,3);
M = 20;                         % Number of cells
h = histogram(X,M);
NN = h.Values;                  % Values Ni
lim = h.BinLimits;              % BinEdges
width = h.BinWidth;
temp = h.BinEdges;
x = temp(1:end-1) + width/2;
grid on;
grid minor;
title('Histogram','Interpreter','latex','FontSize',14);
xlabel('Cells','Interpreter','latex','Color','black','FontSize',12);
ylabel('Number of values in cells','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

% Frequency ratio
subplot(3,2,4);
freq_ratio = NN./N;
bar(x,freq_ratio,1);
grid on;
grid minor;
title('Estimation of probability for cells','Interpreter','latex','FontSize',14);
xlabel('Cells','Interpreter','latex','Color','black','FontSize',12);
ylabel('Frequency ratio','Interpreter','latex','Color','black','FontSize',12);
set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);

% Estimation of PDF
subplot(3,2,5);
pdf = freq_ratio/width;
bar(x,pdf,1);
grid on;
grid minor;
title('Estimation of PDF','Interpreter','latex','FontSize',14);
xlabel('Cells','Interpreter','latex','Color','black','FontSize',12);
ylabel('PDF','Interpreter','latex','Color','black','FontSize',12);
```

```
67 set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);
68
69 % Estimation of distribution
70 subplot(3,2,6);
71 d = cumsum(pdf)*0.1;
72 bar(x,d,1);
73 grid on;
74 grid minor;
75 title('Estimation of PDF','Interpreter','latex','FontSize',14);
76 xlabel('Cells','Interpreter','latex','Color','black','FontSize',12);
77 ylabel('PDF','Interpreter','latex','Color','black','FontSize',12);
78 set(gca,'TickLabelInterpreter','latex', 'FontSize', 12);
```

Code 3: Solve PDF and distribution.

# References

[1] Random Signals and Processes Primer with MATLAB. Gordana Jocanovic Dolecek.