Dr. Benkouider – 5-D system with 3 positive LEs and MLE about 12

System Dynamics:

$$\begin{cases} \dot{z}_1 = a\left(z_2 - z_1\right) + z_2 z_3 + z_4 \\ \dot{z}_2 = z_1\left(b - z_3\right) + cz_4 \\ \dot{z}_3 = z_1^2 + z_1 z_2 - dz_3 \\ \dot{z}_4 = -z_2 + z_5 \\ \dot{z}_5 = -z_4 \end{cases} \quad (1)$$

Initial Conditions: $z_1(0)=1,\ z_2(0)=1,\ z_3(0)=1,\ z_4(0)=1,\ z_5(0)=1.$ (2)

System Parameters (Hyperchaotic Case):

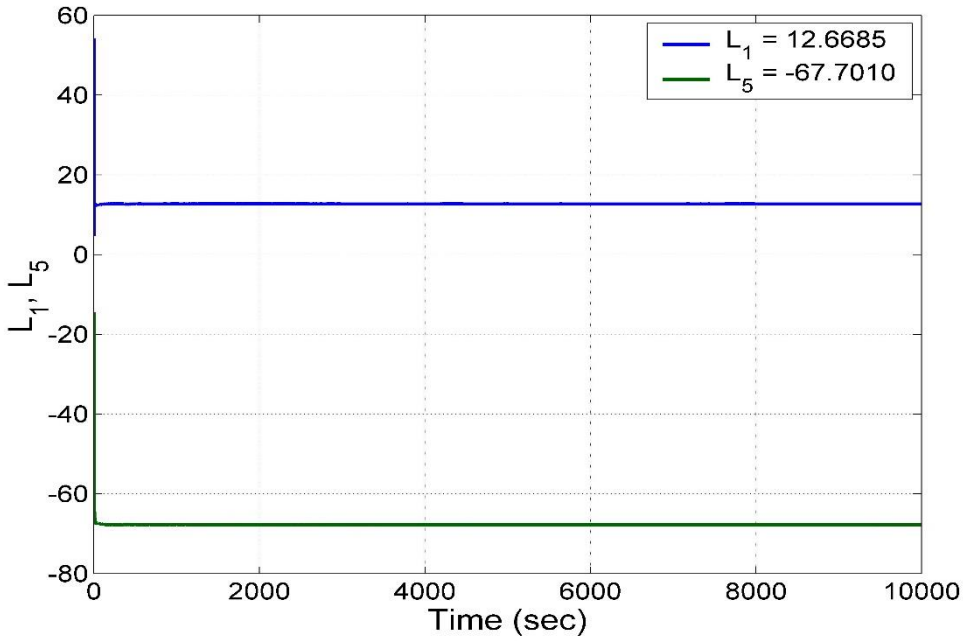
$$a=40,\ b=90,\ c=16,\ d=15 \quad (3)$$

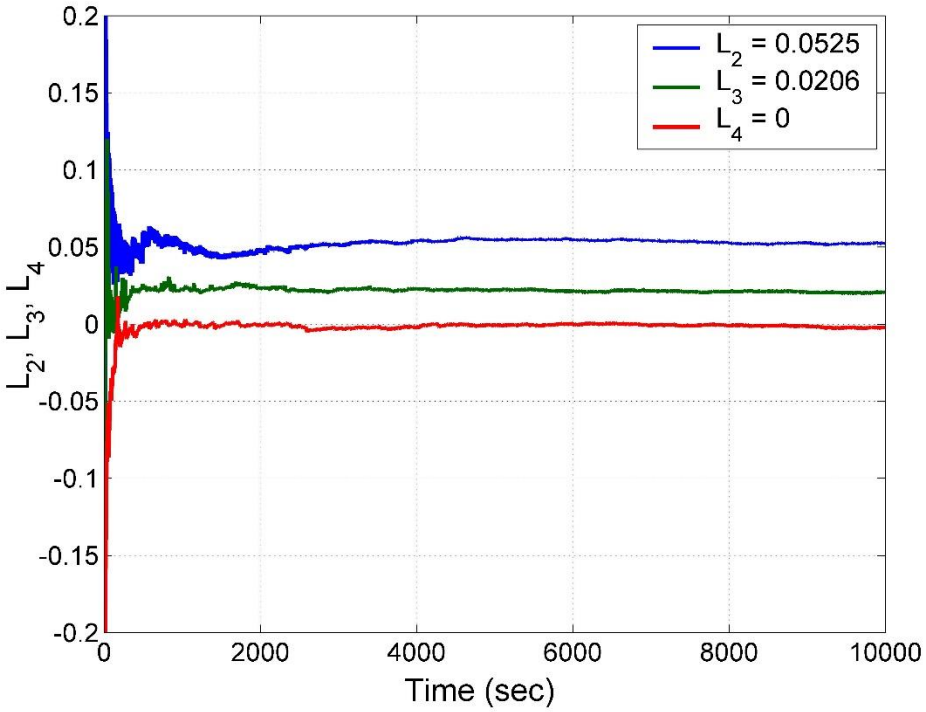
Lyapunov exponents:

For calculating the Lyapunov exponents for system (1), we consider the initial point as in (2) constants as in (3) and T=10 000 sec (Wolf algorithm). Using Matlab, we derived the Lyapunov exponents as given below:

$$L_1=12.6685,\ L_2=0.0525,\ L_3=0.006,\ L_4=0,\ L_5=-67.7010 \quad (4)$$

Figure 1 depicts the Lyapunov exponents spectrum of the 5-D hyperchaotic system (1) for initial conditions as in (2) and parameters as in (3).





Equilibrium Points:

We solve the equations

$$a(z_2 - z_1) + z_2 z_3 + z_4 = 0 \tag{5a}$$

$$z_1(b - z_3) + cz_4 = 0 \tag{5b}$$

$$z_1^2 + z_1 z_2 - dz_3 = 0 \tag{5c}$$

$$-z_2 + z_5 = 0 \tag{5d}$$

$$-z_4 = 0 \tag{5e}$$

Thus, there is three equilibrium points for the system (1):

$$E_1 = (0,0,0,0,0)$$

$$E_{2,3} = \left(\pm\lambda\sqrt{\frac{bd}{\lambda^2+\lambda}},\ \pm\sqrt{\frac{bd}{\lambda^2+\lambda}},\ b,\ 0,\ \pm\lambda\sqrt{\frac{bd}{\lambda^2+\lambda}} \right) \tag{6}$$

where $\lambda = \dfrac{a+b}{a}$.

Suppose we take the parameters as in the hyperchaotic case (3), i.e.

$$a = 40,\ b = 90,\ c = 16,\ d = 15$$

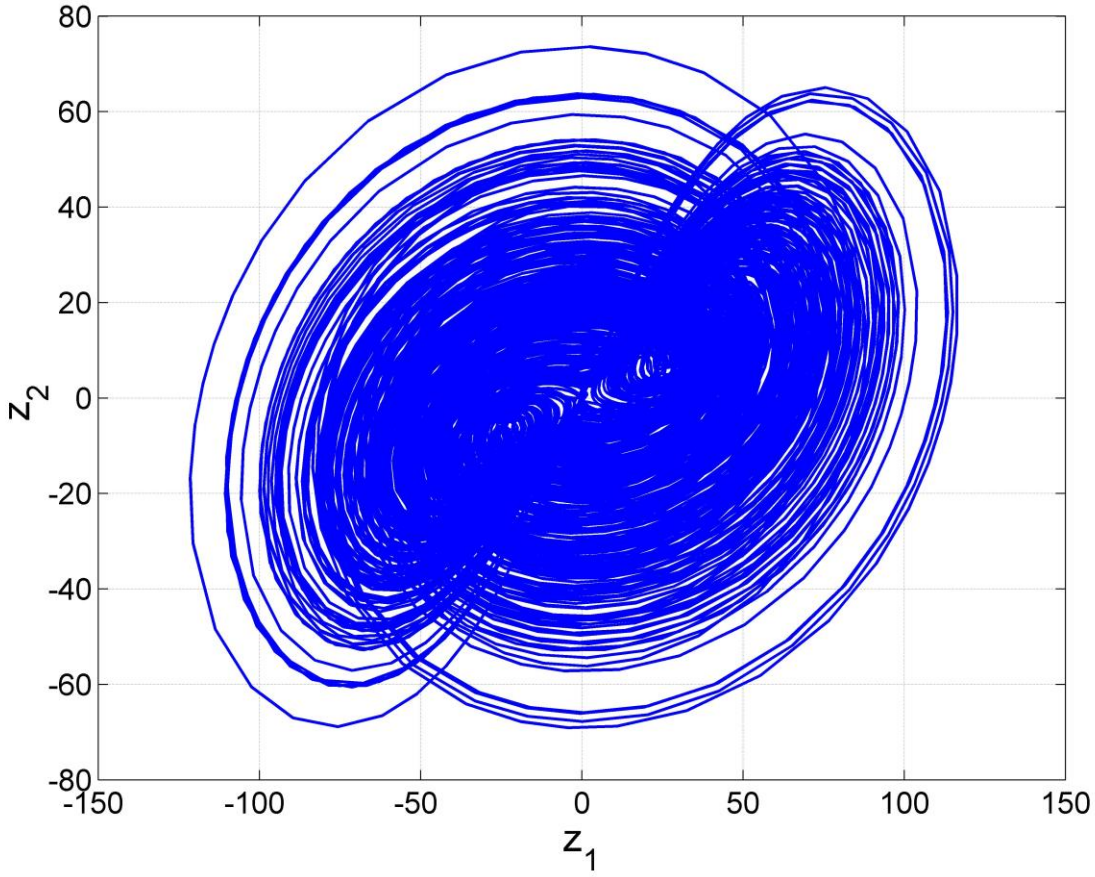
For this case, $\lambda = \dfrac{a+b}{a} = 3.25$. Also, the equilibrium points are obtained as follows:

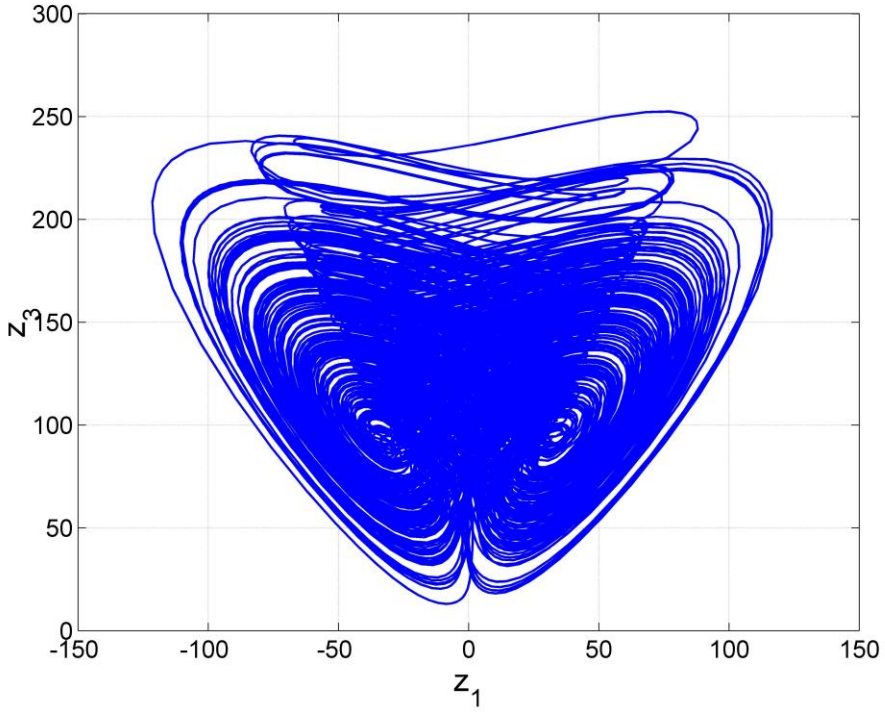
$$E_1 = (0,0,0,0,0)$$

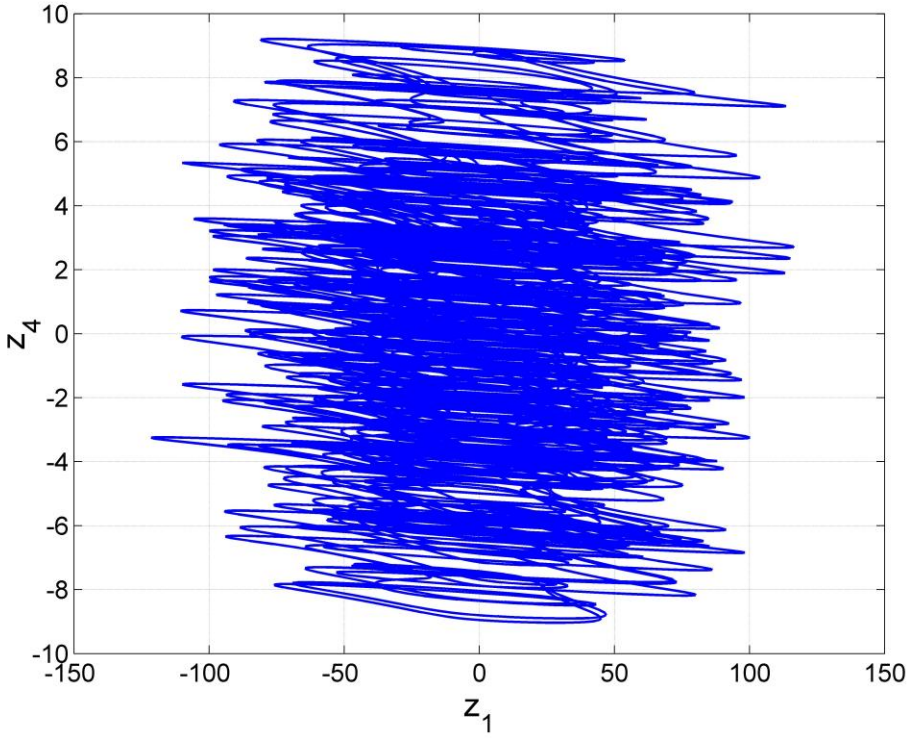
$$E_2 = (32.1302, 9.8862, 90, 0, 9.8862)$$
$$E_3 = (-32.1302, -9.8862, 90, 0, -9.8862) \qquad (7)$$

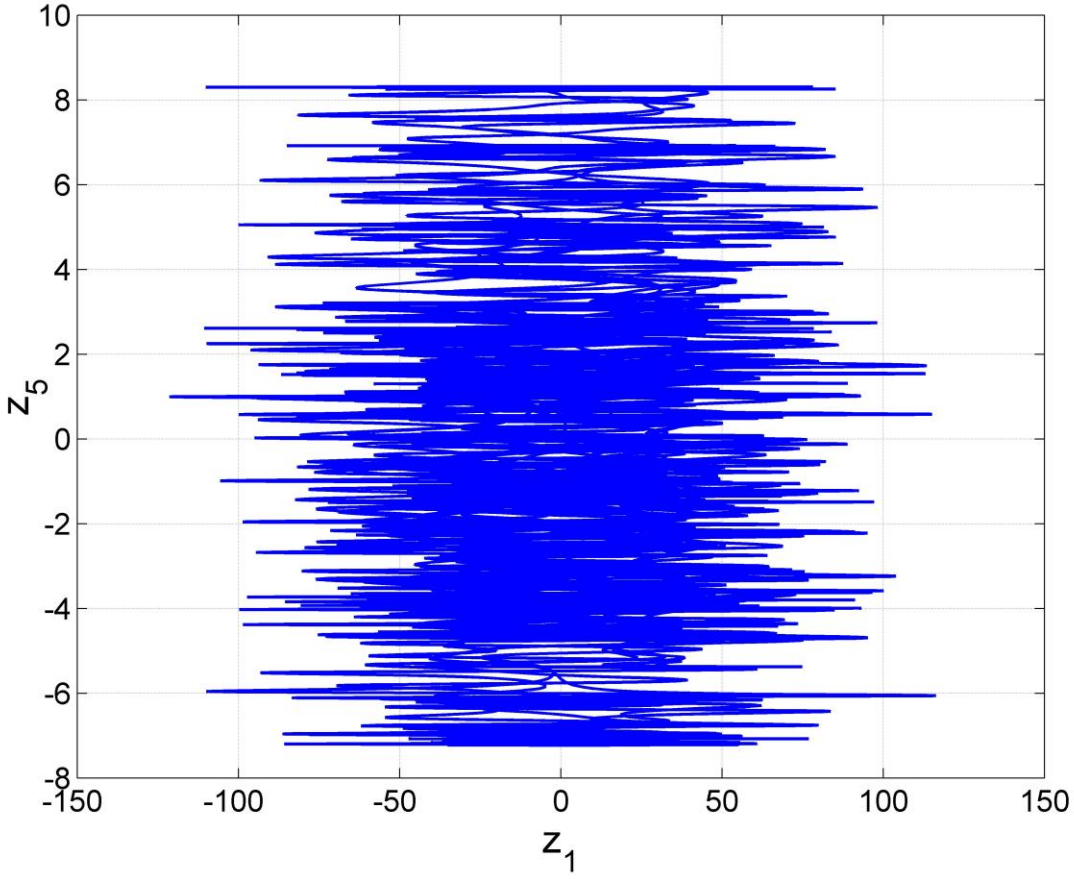
The 5-D hyperchaotic system (1) has rotation symmetry about the $z_3 -$ axis. (This is observed in the equilibrium points as shown in the equations (6) and (7).)

Figure 2 depicts the Matlab plots of the 5-D hyperchaotic system (1) for initial conditions as in (2) and parameters as in (3).




300
250
200
150
100
50
0
-150
-100
-50
0
50
100
150
z3
z1


10
8
6
4
2
0
-2
-4
-6
-8
-10
-150
-100
-50
0
50
100
150
z4
z1


10
8
6
4
2
0
-2
-4
-6
-8
-150
-100
-50
0
50
100
150
z_5
z_1