A new 4-D hyperchaotic system with a unique saddle rest point, its bifurcation analysis, circuit design and an application to secure communication

(Joint Paper – New IJCAT Submission, 2022)

System Dynamics:

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + d y_4 \\ \dot{y}_2 &= -y_1 + b y_2 - y_1 y_3 + d y_4 \\ \dot{y}_3 &= y_2^2 - c y_3 \\ \dot{y}_4 &= -y_2 \end{aligned} \tag{1}$$

Parameter values for hyperchaoticity:

$$a = 40,\ b = 28,\ c = 4,\ d = 7 \tag{2}$$

Lyapunov exponents:

$$\tau_1 = 3.1274,\quad \tau_2 = 0.1287,\ \ \tau_3 = 0,\ \ \tau_4 = -19.2675 \tag{3}$$

Kaplan-Yorke Dimension:

$$D_{KY} = 3.1690 \tag{4}$$

Special Properties:

1) The hyperchaotic system (1) has rotation symmetry about the y_3 − axis in R^4.
2) The hyperchaotic system (1) has a self-excited, dissipative, attractor with an unstable rest point. (We can show that $y = 0$ is the unique rest point, which is a saddle point.)

Equilibrium Analysis:

We solve the equations

$$a(y_2 - y_1) + y_2 y_3 + d y_4 = 0 \tag{5a}$$

$$-y_1 + b y_2 - y_1 y_3 + d y_4 = 0 \tag{5b}$$

$$y_2^2 - c y_3 = 0 \tag{5c}$$

$$-y_2 = 0 \tag{5d}$$

Solving by back-substitution gives: $y_2 = 0, y_3 = 0, y_4 = 0, y_1 = 0.$

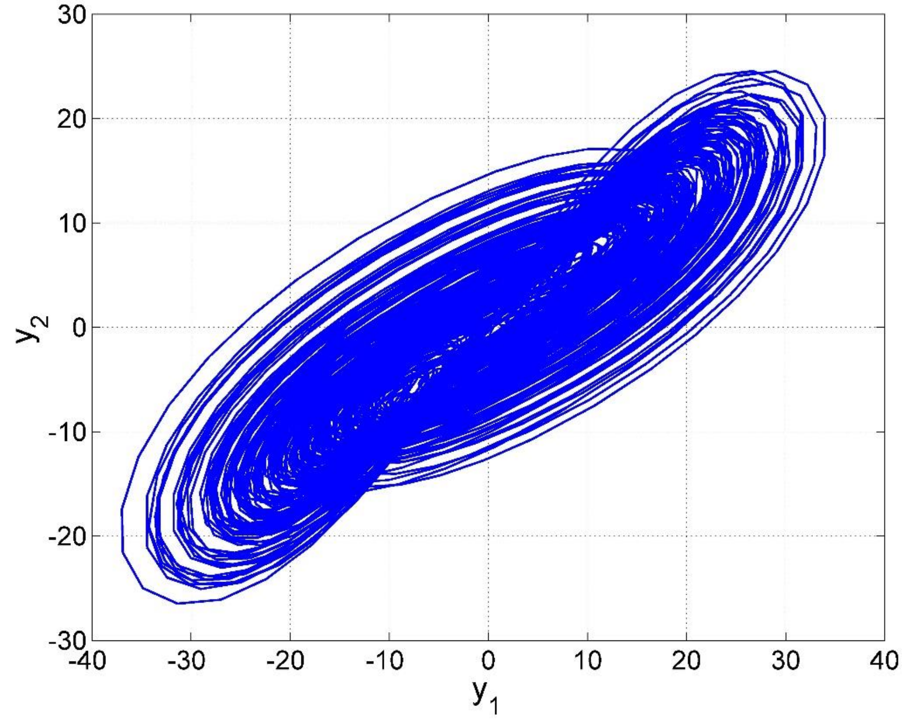
Thus, $E_0 = (0,0,0,0)$ is the unique rest point of the hyperchaotic system (1). Furthermore, linearizing the system (1) at E_0 for the parametric values (2), we get the Jacobian matrix of the system as

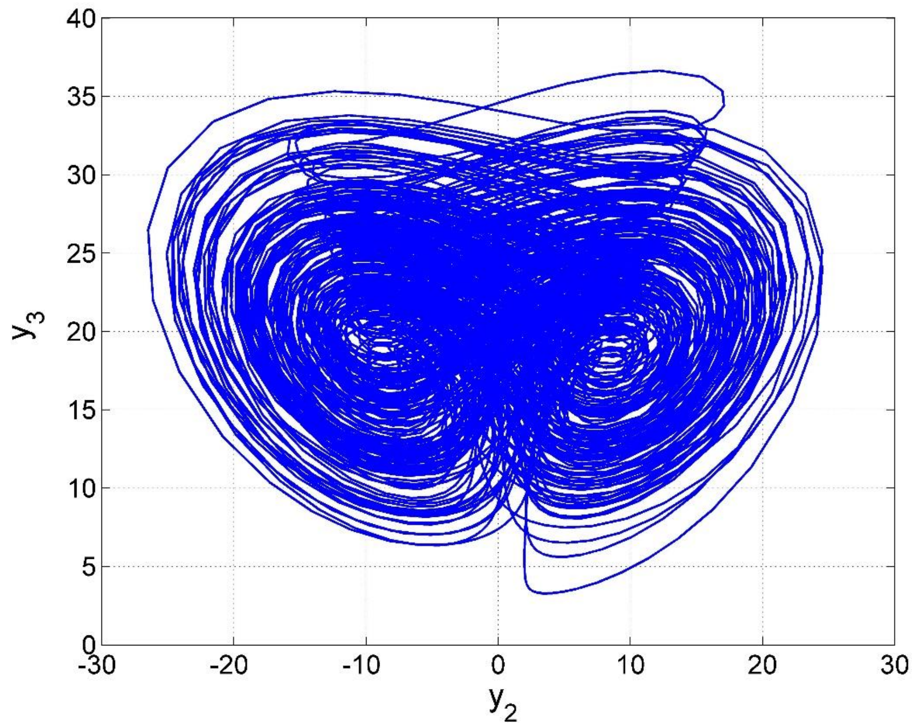
$$J_0 = \begin{bmatrix} -40 & 40 & 0 & 7 \\ -1 & 28 & 0 & 7 \\ 0 & 0 & -4 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \tag{6}$$

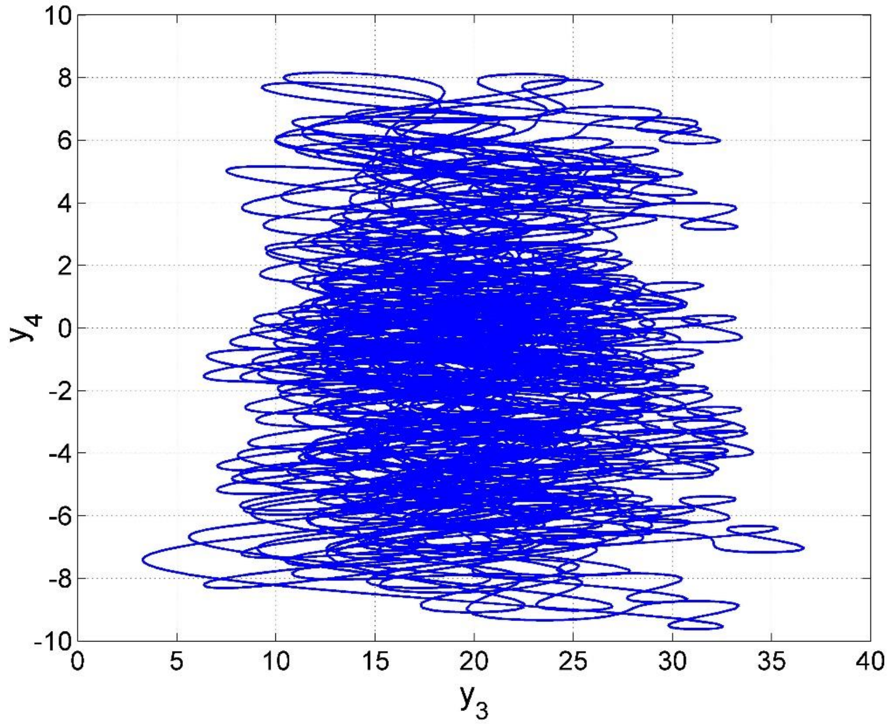
The matrix J_0 has the eigen values:

$$\lambda_1 = -4,\ \lambda_2 = -39.4055,\ \lambda_3 = 0.2552,\ \lambda_4 = 27.1503 \tag{7}$$

Thus, the system (1) has a unique saddle rest point at the origin, E_0.


y_2
y_1


y_3
y_2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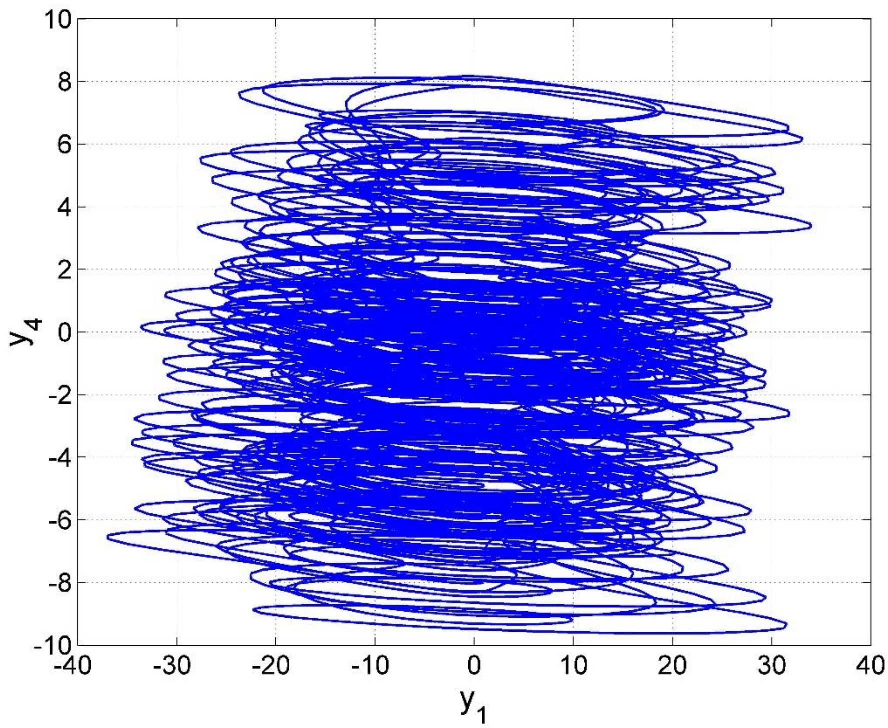
y_4
y_3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y_4
y_1