Higgs
$e^-, e^+, \mu^-, \mu^+, \tau^-, \tau^+$
$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$
u, d, s, c, t, b
γ, W^+, W^-, Z^0, g

The Big Picture : In Words

Associate a free quantum field w/ every free elementary particle in nature.

Particles are their respective field's quanta.

– Spin 0 particles first. Example : Higgs Boson

$\updownarrow$

Free Scalar Fields
(Real, Complex)

simple model for particle/antiparticle pairs

Natural extension of particle quantum mechanics :

classical particles $\longrightarrow$ classical fields

$\downarrow$ $\downarrow$

canonical quantization $\longrightarrow$ canonical field quantization

natural that there is a quantum counterpart to the classical field

Causality Restored! $\Delta(x,y) = [\phi(x), \phi(y)] = 0$
for $(x-y)^2 < 0$

Free Particles $\longrightarrow$ Interacting Particles

$\updownarrow$ $\updownarrow$

Free Fields $\longrightarrow$ Interacting Fields

Locality! Interactions happen at a spacetime point.

Interactions mediated by particles!

Change in momentum of each of two interacting particles associated with the emission of the particle mediating the interaction (e.g., the photon for electromagnetic interactions) by one of the two interacting particles and absorption of the particle mediating the interaction by the other of the two interacting particles.

Since particles are field quanta, when we talk about interacting particles we must also talk about interacting fields.

We have always been doing QM!

When talking about interacting particles/fields, we start out with an initial <u>state</u> of our system.

system ≡ our physical system – i.e., all of our fields and their quanta

state ≡ the state of our physical system – i.e., the state of our fields – i.e., the number of quanta in each mode

<u>EXAMPLES</u>

Pion Decay in Scalar Yukawa Theory

- System : three fields, one for nucleons, one for antinucleons, and one for pions, along with their quanta (excitations)

- State (Initial) : one pion in mode $\vec{k}$ (i.e., in a momentum eigenstate – so its wavefunction is just a plane wave) and the vacuum state (ground state) for both the nucleon and antinucleon fields

- State (Final) : one nucleon in mode $\vec{p}$ and one antinucleon in mode $\vec{q}$ (so both of their wavefunctions are plane waves) and the ground state for the pion field

The specification of the initial and final states depend on the setup (experiment) and the questions being asked.

Examples

1. Pion Decay (One-Particle Initial State)

$$|i\rangle = |\vec{k}\rangle$$

This is perfect for a single particle that has a well-defined momentum ($\vec{k}$). It is in an eigenstate of momentum and has wavefunction

$$\langle\vec{x}|\vec{k}\rangle = \psi(\vec{x}) = e^{i\vec{k}\cdot\vec{x}}$$

Of course, since the momentum is defined, the position is not, and

$$|\psi(\vec{x})|^2 = 1$$

Then

$$\int_{-\infty}^{\infty} d^3x \, |\psi(\vec{x})|^2 = \infty$$

But in reality, we integrate only over a finite volume such that

$$\int_V d^3x \, |\psi(\vec{x})|^2 = 1$$

and instead write

$$\psi(\vec{x}) = \frac{1}{\sqrt{V}} e^{i \vec{k} \cdot \vec{x}}$$

The probability of finding the particle is uniform across V.

(2) Two-Nucleon Scattering (Two-Particle Initial State)

$$|i\rangle = |\vec{k}_1 \vec{k}_2\rangle$$

That is, we have two nucleons with distinct momenta $\vec{k}_1$ and $\vec{k}_2$.

Here, the wavefunction would be

$$\langle \vec{x}_1 \vec{x}_2 | \vec{k}_1 \vec{k}_2 \rangle = \psi(\vec{x}_1, \vec{x}_2) = \psi_{\vec{k}_1}(\vec{x}_1)\, \psi_{\vec{k}_2}(\vec{x}_2) \qquad \text{DISTINGUISHABLE PARTICLES}$$

$$\text{"} = \psi(\vec{x}_1, \vec{x}_2) = \frac{1}{\sqrt{2}}\left[\psi_{\vec{k}_1}(\vec{x}_1)\, \psi_{\vec{k}_2}(\vec{x}_2) + \psi_{\vec{k}_1}(\vec{x}_2)\, \psi_{\vec{k}_2}(\vec{x}_1)\right] \qquad \text{IDENTICAL BOSONS}$$

$$\text{"} = \psi(\vec{x}_1, \vec{x}_2) = \frac{1}{\sqrt{2}}\left[\psi_{\vec{k}_1}(\vec{x}_1)\, \psi_{\vec{k}_2}(\vec{x}_2) - \psi_{\vec{k}_1}(\vec{x}_2)\, \psi_{\vec{k}_2}(\vec{x}_1)\right] \qquad \text{IDENTICAL FERMIONS}$$

with

$$\psi_{\vec{k}}(\vec{x}) \equiv \frac{1}{\sqrt{V}} e^{i \vec{k} \cdot \vec{x}}$$

(3) Multiparticle Initial State of One Species

$$|i\rangle = |n_1\, n_2 \cdots n_{\vec{k}} \cdots n_\infty\rangle$$

where $n_{\vec{k}}$ is the number of particles/quanta in mode $\vec{k}$ of the field.

This is where the number representation rather than the momentum representation is best.

For example,

$$|\vec{k}_1\, \vec{k}_2 \cdots \vec{k}_N\rangle$$

implies that our N particles have distinct momenta. But we could have

$$|\underbrace{\vec{k}_1\, \vec{k}_1 \cdots \vec{k}_1}_{n_{k_1}}\, \underbrace{\vec{k}_2\, \vec{k}_2 \cdots \vec{k}_2}_{n_{k_2}} \cdots\rangle$$

This could be

$$|\vec{k}_1\, \vec{k}_1\, \vec{k}_2\, \vec{k}_3\, \vec{k}_1\, \vec{k}_2\, \vec{k}_2 \cdots\rangle$$

and it would be better to use the n representation.

If we know how $|i\rangle$ evolves, we can ask what is the amplitude for ending up in state $|f\rangle$. This is

$$\langle f|\hat{S}|i\rangle$$

and we evaluate this in perturbation theory, each order of perturbation theory corresponding to a set of possible processes that can occur when the quantum fields interact. But in the and include only those terms that connect our prepared initial state to our assumed final state.

Once we know

$\langle f \,|\, \hat{S} \,|\, i \rangle$

we can compute decay rates, differential scattering cross sections, etc.