Quantization of the Massless Spin-1 Field

All of the work has been done at the level of the classical field, and we can simply write

$$\hat{A}^{\mu}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E(\vec{p})}} \sum_{j=1}^{2} \left[\epsilon^{\mu}_{j}(p)\, \hat{a}(\vec{p}, j) e^{-ip\cdot x} + \epsilon^{*\mu}_{j}(p)\, \hat{a}^{\dagger}(\vec{p}, j) e^{ip\cdot x} \right]$$

with

$$\sqrt{2E(\vec{p})}\, \hat{a}^{\dagger}(\vec{p}, j) |0\rangle = |\vec{p}, j\rangle$$

For

$$p^{\mu} = (E, 0, 0, E)$$

we can choose the following physical polarization basis

$$\epsilon^{\mu}_{1}(p) = (0, 1, 0, 0)$$

$$\epsilon^{\mu}_{2}(p) = (0, 0, 1, 0)$$

The photon propagator is

$$\langle 0 | T\{ \hat{A}^{\mu}(x) \hat{A}^{\nu}(y) \} | 0 \rangle \equiv G_F^{\mu\nu}(x-y)$$

We know that $G_F^{\mu\nu}(x-y)$ is the Green's function:

$$\Box G_F^{\mu\nu}(x-y) = \eta^{\mu\nu}\delta^{(4)}(x-y)$$

For

$$\Box A^{\mu} = J^{\mu}$$

the solution is

$$A^{\mu}(x) = \int \frac{d^4y}{(2\pi)^4} J_{\nu}(y)\, G_F^{\mu\nu}(x-y)$$

To obtain

$$\tilde{G}_F^{\mu\nu}(p)$$

which we will need for our Feynman rules, take the Fourier transform of both sides of the equation for $G_F^{\mu\nu}(x-y)$:

$$\underbrace{\int \frac{d^4x}{(2\pi)^4} e^{-ip\cdot(x-y)}\, \Box G_F^{\mu\nu}(x-y)}_{-p^2 \tilde{G}_F^{\mu\nu}(p)} = \eta^{\mu\nu} \underbrace{\int \frac{d^4x}{(2\pi)^4} e^{-ip\cdot(x-y)}\, \delta^{(4)}(x-y)}_{e^{-ip\cdot(0)} = 1}$$

with

$$\tilde{G}^{\mu\nu}_F(k) \equiv \int \frac{d^4x}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, G^{\mu\nu}_F(x-y)$$

Solving for $\tilde{G}^{\mu\nu}_F(k)$

$$\tilde{G}^{\mu\nu}_F(k) = \frac{-\eta^{\mu\nu}}{k^2 + i\epsilon}$$