Feynman Rules for Scalar QED

N.B. coupling to A_μ is through $\partial_\mu \hat{\phi}$ not $\hat{\phi}$

$$\hat{\mathcal{L}}_{INT,1} = -ie\hat{A}_\mu \left[\hat{\phi}^\dagger \partial^\mu \hat{\phi} - (\partial^\mu \hat{\phi}^\dagger)\hat{\phi} \right]$$

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3} \Big[\underbrace{\hat{a}(\vec{k})}_{\text{annihilates } e^-} e^{-ik\cdot x} + \underbrace{\hat{b}^\dagger(\vec{k})}_{\text{creates } e^+} e^{+ik\cdot x} \Big]$$

$$\hat{\phi}^\dagger(x) = \int \frac{d^3k}{(2\pi)^3} \Big[\underbrace{\hat{a}^\dagger(\vec{k})}_{\text{creates } e^-} e^{+ik\cdot x} + \underbrace{\hat{b}(\vec{k})}_{\text{annihilates } e^+} e^{-ik\cdot x} \Big]$$

$$\hat{A}^\mu(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E(\vec{k})}} \sum_{j=1}^{2} \Big[\epsilon^\mu_j(k) \underbrace{\hat{a}_j(\vec{k})}_{\text{annihilates } \gamma} e^{-ik\cdot x} + \epsilon^{*\mu}_j(k) \underbrace{\hat{a}^\dagger_j(\vec{k})}_{\text{creates } \gamma} e^{+ik\cdot x} \Big]$$

$$\partial^\mu \hat{\phi} = \eta^{\mu\nu} \partial_\nu \hat{\phi} \longrightarrow \eta^{\mu\nu}(\pm i k_\nu) = \mp i k^\mu$$

annihilation ($-$), creation ($+$)

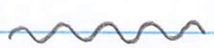
$$\partial^\mu \hat{\phi}^\dagger = \eta^{\mu\nu} \partial_\nu \hat{\phi}^\dagger \longrightarrow \eta^{\mu\nu}(\mp k_\nu) = \mp i k^\mu$$

annihilation ($-$), creation ($+$)

$A^\mu \longrightarrow \epsilon^\mu_j$ photon annihilation

$\longrightarrow \epsilon^{*\mu}_j$ photon creation

$$\frac{i}{k^2 - m^2 + i\epsilon}$$

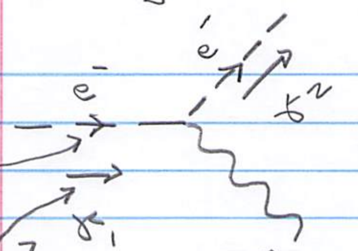


$$\frac{-i}{k^2 + i\epsilon} \left[\eta_{\mu\nu} - (1-\xi) \frac{k_\mu k_\nu}{k^2} \right]$$

$-(\partial_\mu \hat\phi^\dagger)\hat\phi$ creates e^- and gives $-(+i\gamma^2_\mu)$

from $\partial_\mu\hat\phi$ — from $\partial_\mu\hat\phi^\dagger$ — ②

e^- SCATTERING



particle flow arrow

momentum flow arrow

$+\hat\phi^\dagger\partial_\mu\hat\phi$ annihilates e^- and gives $+(-i\gamma^1_\mu)$

A^μ creates γ with $\epsilon^{*\mu}_j$

$$i(-ie)(-i\gamma^1_\mu - i\gamma^2_\mu)\epsilon^{*\mu}_j$$

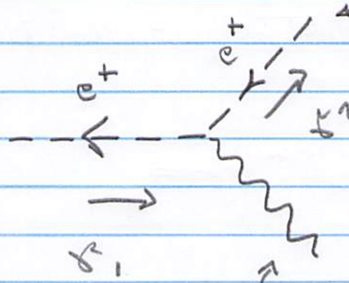
from $e^{i\int d^4x\, \hat{\mathcal{L}}_{INT}}$

from A^μ

only for an external γ

$$= ie(-\gamma^1_\mu - \gamma^2_\mu)\epsilon^{*\mu}_j$$

e^+ SCATTERING



$+\hat\phi^\dagger\partial_\mu\hat\phi$ creates e^+ and gives $+(+i\gamma^2_\mu)$

$$i(-ie)(i\gamma^1_\mu + i\gamma^2_\mu)\epsilon^{*\mu}_j$$

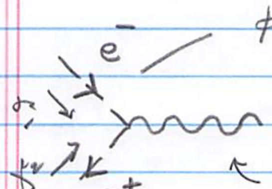
$$= ie(\gamma^1_\mu + \gamma^2_\mu)\epsilon^{*\mu}_j$$

$-(\partial_\mu\hat\phi^\dagger)\hat\phi$ annihilates e^+ and gives $-(-i\gamma^1_\mu)$

A^μ creates γ with $\epsilon^{*\mu}_j$

so, a SED vertex gives $-ie$ times sum of momenta aligned with particle flow arrows minus the sum of momenta antialigned with their particle flow arrows

e^+e^- ANNIHILATION



$\hat\phi^\dagger\partial_\mu\hat\phi$ annihilates e^- and gives $+(-i\gamma^1_\mu)$

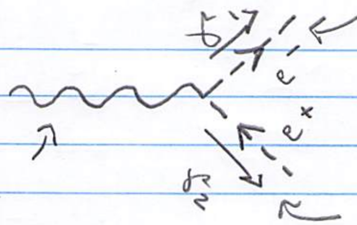
$$i(-ie)(-i\gamma^1_\mu + i\gamma^2_\mu)\epsilon^{*\mu}_j$$

A^μ creates γ with $\epsilon^{*\mu}_j$

$$= ie(-\gamma^1_\mu + \gamma^2_\mu)\epsilon^{*\mu}_j$$

$-(\partial_\mu\hat\phi^\dagger)\hat\phi$ annihilates e^+ and gives $-(-i\gamma^2_\mu)$

e^+e^- CREATION



$-(\partial_\mu \hat{\phi}^\dagger)\hat{\phi}$ creates e^- and gives $-(+p^1_\mu)$

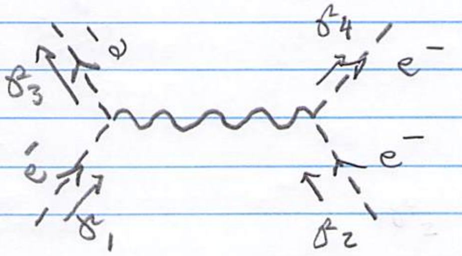
A^μ annihilates γ with ϵ^μ_γ

$+\hat{\phi}^\dagger \partial_\mu \hat{\phi}$ creates e^+ and gives $+(+p^2_\mu)$

$$i(-ie)(-ip^1_\mu + ip^2_\mu)\,\epsilon^\mu_\gamma$$

$$= ie(-p^1_\mu + p^2_\mu)\,\epsilon^\mu_\gamma$$

Let's look at Moller Scattering in Scalar QED



$$\mathcal{M} = i\,\tfrac{1}{2}(-ie)(-ip_1^\mu - ip_3^\mu)\,\frac{-i\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]}{k^2}\,(-ie)(-ip_2^\nu - ip_4^\nu)$$

$$= \underbrace{i(-i)(-i)(-i)(-i)(-i)}_{-(i)^6 = -(-1)^3 = +1}\,\tfrac{1}{2}e^2(p_1^\mu + p_3^\mu)\,\frac{\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}}{k^2}\,(p_2^\nu + p_4^\nu)$$

Look at

$$(p_1^\mu + p_3^\mu)\,\eta_{\mu\nu}\,(p_2^\nu + p_4^\nu)$$

$$= (p_1^\mu + p_3^\mu)(p^2_\mu + p^4_\mu)$$

and

$$(p_1^\mu + p_3^\mu)\,k_\mu$$

$$= (p_1^\mu + p_3^\mu)(p^1_\mu - p^3_\mu)$$

$$= p_1^\mu p^1_\mu - p_3^\mu p^3_\mu$$

$$= m^2 - m^2$$

$$= 0$$

So, we are left with

$$\mathcal{M} = \frac{1}{2} e^2 \frac{(p_1^\mu + p_3^\mu)(p^2_\mu + p^4_\mu)}{(p_1 - p_3)^2} = \frac{e^2 (p_1^\mu + p_3^\mu)(p^2_\mu + p^4_\mu)}{2t}$$

Finally,

$$(p_1^\mu + p_3^\mu)(p^2_\mu + p^4_\mu) = \ldots$$

$$\gamma_1^\mu \gamma_\mu^2 + \gamma_1^\mu \gamma_\mu^4 + \underbrace{\gamma_3^\mu \gamma_\mu^2 + \gamma_3^\mu \gamma_\mu^4}$$

Look at $\qquad (\gamma_1^\mu + \gamma_2^\mu - \gamma_4^\mu)\gamma_\mu^2 + (\gamma_1^\mu + \gamma_2^\mu - \gamma_4^\mu)\gamma_\mu^4$

$$= \gamma_1^\mu \gamma_\mu^2 + \gamma_2^\mu \gamma_\mu^2 - \gamma_4^\mu \gamma_\mu^2 + \gamma_1^\mu \gamma_\mu^4 + \gamma_2^\mu \gamma_\mu^4 - \gamma_4^\mu \gamma_\mu^4$$

$s - u$

$$= \gamma_1^\mu \gamma_\mu^2 + \gamma_1^\mu \gamma_\mu^4$$

$$= (\gamma_1 + \gamma_2)^2 - (\gamma_1 - \gamma_4)^2$$

$$= (\gamma_1^\mu + \gamma_2^\mu)(\gamma_\mu^1 + \gamma_\mu^2) - (\gamma_1^\mu - \gamma_4^\mu)(\gamma_\mu^1 - \gamma_\mu^4)$$

$$= \gamma_1^\mu \gamma_\mu^1 + \gamma_1^\mu \gamma_\mu^2 + \gamma_2^\mu \gamma_\mu^1 + \gamma_2^\mu \gamma_\mu^2$$

$$- \gamma_1^\mu \gamma_\mu^1 + \gamma_1^\mu \gamma_\mu^4 + \gamma_4^\mu \gamma_\mu^1 - \gamma_4^\mu \gamma_\mu^4$$

$$= 2(\gamma_1^\mu \gamma_\mu^2 + \gamma_1^\mu \gamma_\mu^4)$$

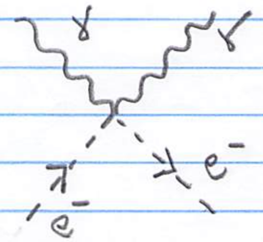
Then

$$\eta = \frac{e^2 (s - u)}{2t}$$

Now look at

$$\hat{\mathcal{L}}_{INT, 2} = e^2 \hat{A}_\mu \hat{A}^\mu \hat{\phi}^\dagger \hat{\phi}$$

There is only 1 Feynman diagram associated with this, known as the Seagull Vertex



$$= ie^2 \epsilon_{\nu,j} \epsilon^{*\nu}_{j} \qquad \epsilon^\mu_1 = (0,1,0,0)$$

$$= ie^2 \eta_{\mu\nu} \epsilon^\mu_j \epsilon^{*\nu}_j \qquad \epsilon^\mu_2 = (0,0,1,0)$$

$$= ie^2 \left(\eta_{11} \epsilon^1_1 \epsilon^{*1}_1 + \eta_{22} \epsilon^2_2 \epsilon^2_2 \right)$$

$$= -2ie^2$$