MergeSort

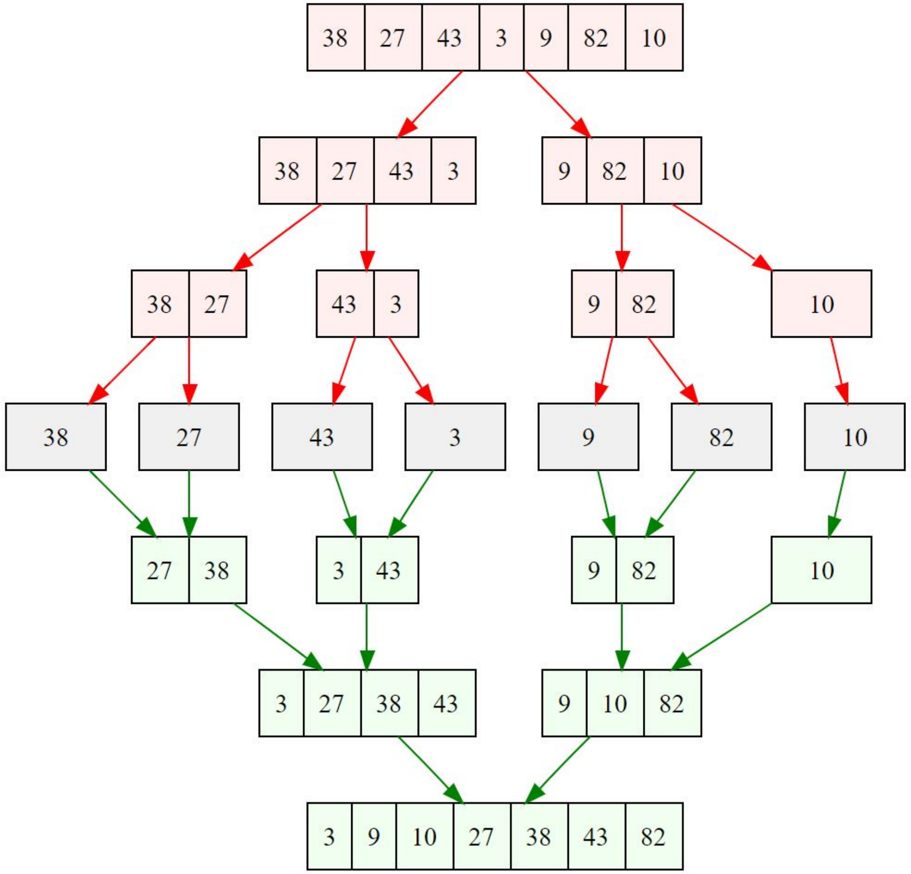
Divide and Conquer Algorithms

- A technique for designing algorithms where a solution is found by breaking the problem into smaller (similar) subproblems
- The subproblems solutions are combined to form the solution to the original problem.
- Often implemented using *recursion*

Divide and Conquer: Sorting

- Break the array to be sorted into smaller pieces,
- Process (sort) the pieces, and then
- Put them back together
- This is the idea behind *Mergesort*

Mergesort



38 27 43 3 9 82 10

38 27 43 3 | 9 82 10

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27 38 | 3 43 | 9 82 | 10

3 27 38 43 | 9 10 82

3 9 10 27 38 43 82

Mergesort

- Pseudo code

```
public static Array mergesort(Array inlist) {
  if (inlist.length <= 1){
    return inlist;
  }
  Array L1 = half of the items from inlist;
  Array L2 = other half of the items from inlist;
  return merge(mergesort(L1), mergesort(L2));
}
```

Merge function

- Combines two pre-sorted lists into a sorted whole.
- The hardest step about Mergesort
- Algorithm:
    - Examine the first record of each sublist and picks the smaller value as the smallest record overall
    - The smaller value is removed from its sublist and placed into the output list
    - Merging continues in this way, comparing the front records of the sublists and continually appending the smaller to the output list until no more input records remain

Merge function implementation

```
public static int[] merge(int[] A, int[] B){
  int[] tmp = new int[A.length + B.length];
  int i1 = 0; // will iterate through A
  int i2 = 0; // will iterate through B
  for (int i = 0; i < tmp.length; i++){
    if (i1 >= A.length){ // A exhausted
      tmp[i] = B[i2++]; }
    else if (i2 >= B.length) { // B exhausted
      tmp[i] = A[i1++];}
    else if (A[i1] <= B[i2]) { // Get smaller value
      tmp[i] = A[i1++];}
    else{
      tmp[i] = B[i2++];}
  }
  return tmp; //the sorted array
}
```

Mergesort implementation

- Problem: avoid having each merge operation to create a new array
- Solution: Use an auxiliary array
- The initial call
  - `mergesort(arrayToSort, auxiliaryArray, 0, n-1)`

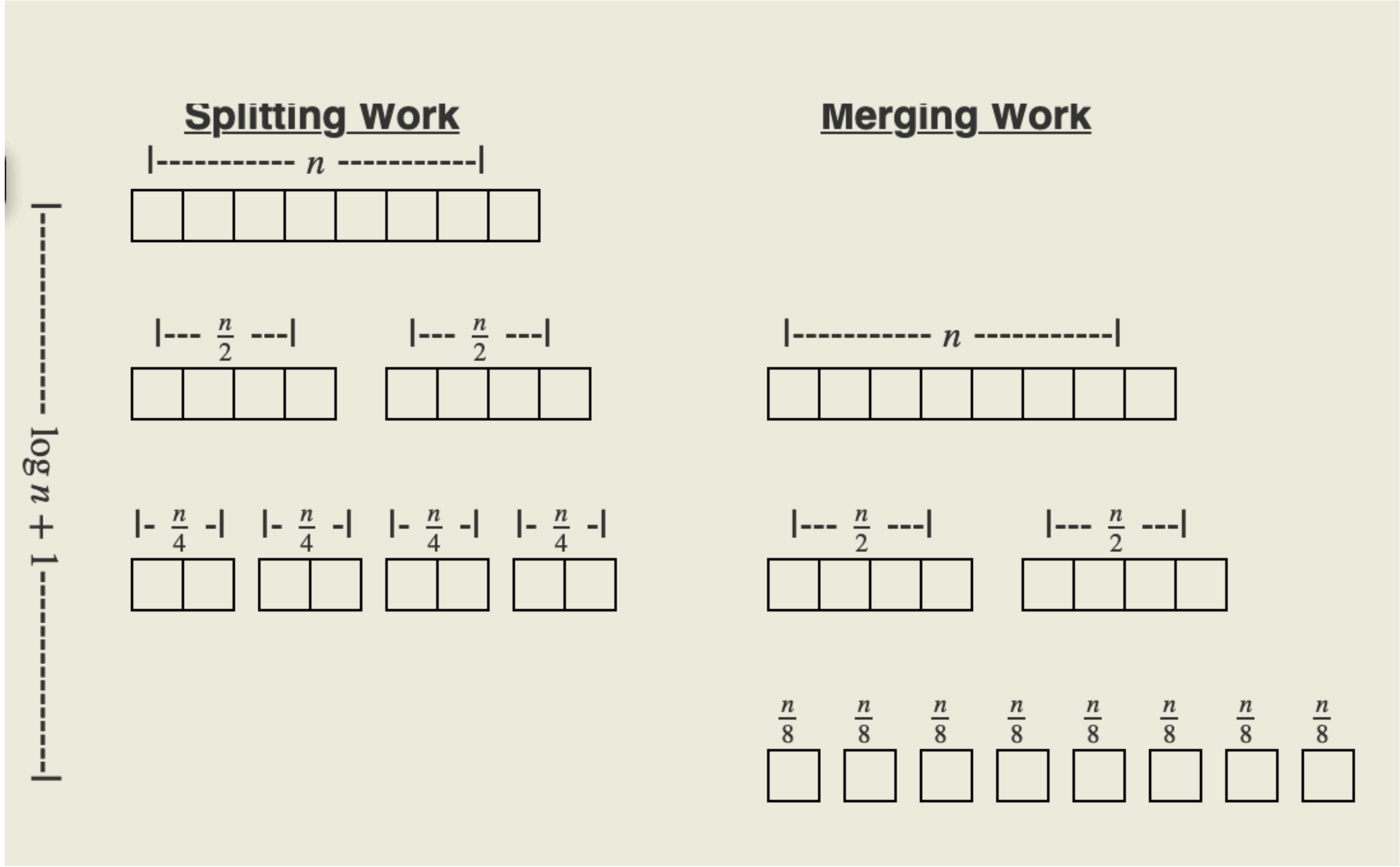
  (n = arrayToSort.length)

Mergesort implementation

```
static void mergesort(Comparable[] A, Comparable[] temp, int left, int right) {
  if (left == right) return; // List has one record
  int mid = (left+right)/2; // Select midpoint
  mergesort(A, temp, left, mid); // Mergesort first half
  mergesort(A, temp, mid+1, right); // Mergesort second half
  for (int i=left; i<=right; i++) // Copy subarray to temp
    temp[i] = A[i];
  // Do the merge operation back to A
  int i1 = left;
  int i2 = mid + 1;
  for (int curr = left; curr <= right; curr++) {
    if (i1 == mid+1) // Left sublist exhausted
      A[curr] = temp[i2++];
    else if (i2 > right) // Right sublist exhausted
      A[curr] = temp[i1++];
    else if (temp[i1].compareTo(temp[i2]) <= 0) // Get smaller value
      A[curr] = temp[i1++];
    else
      A[curr] = temp[i2++];
    }
}
```

Mergesort running time

- Mergesort runs in O(n log n)


Splitting Work
Merging Work
|----------- n -----------|
|--- $\frac{n}{2}$ ---|
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|----------- $\log n + 1$ -----------|