NUMBER SYSTEMS AND DATA REPRESENTATION

There are 10 kinds of people in the world, those who use binary, and those who don't.

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Base 10 (Decimal numbers)

What does 157 mean?

$157 = 1 \times 100 + 5 \times 10 + 7 \times 1$

$= 1 \times 10^2 + 5 \times 10^1 + 7 \times 10^0$

Decimal Numbering System

The Arabic numbering system uses ten symbols and is a base ten numbering system. In any numbering system the base is raised to consecutive powers to determine positional (place) values.

10^3	10^2	10^1	10^0	$\text{base}^{\text{powers}}$
1000	100	10	1	place values
	2	3	1	
	4	1	3	
7	8	6	0	

$$231 = 2 * 10^2 + 3 * 10^1 + 1 * 10^0$$

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Computer Representation
Binary System

At the hardware level computers must store numbers using either the presence or absence of a voltage. Then there are only two states (symbols) available for number representation. So base 2 (binary) is a suitable system for computers.

2^6	2^5	2^4	2^3	2^2	2^1	2^0	base$^{\text{powers}}$
64	**32**	**16**	**8**	**4**	**2**	**1**	place values

Positional Systems

What value does 1110 represent?

Undetermined until the numbering system base is stated.

Base 10 place values:	10,000	1,000	100	10	1
Base 2 place values:	16	8	4	2	1
		1	1	1	0

base		**base 10**
10?	1*1000 + 1*100 + 1*10 + 0*1	1,110
2?	1*8 + 1*4 + 1*2 + 0*1	14

Base 10 vs Base 2

Base 10

157

$157 = 1 \times 100 + 5 \times 10 + 7 \times 1$

$= 1 \times 10^2 + 5 \times 10^1 + 7 \times 10^0$

Base 2

$1011 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$1011 = 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$

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Binary to Base Ten

01010010

0	1	0	1	0	0	1	0
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

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Binary to Base Ten

01010010

0	1	0	1	0	0	1	0
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$$

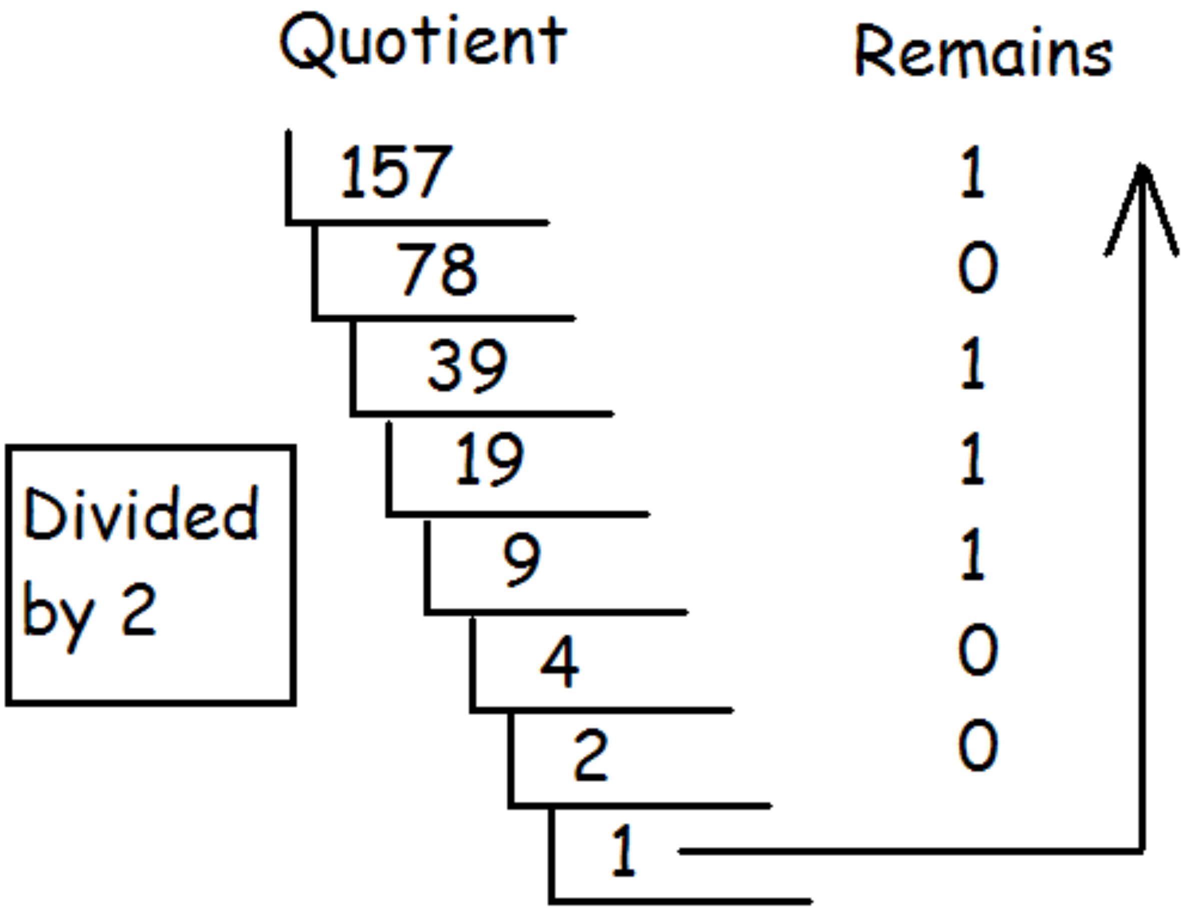
$$0 \cdot 128 + 1 \cdot 64 + 0 \cdot 32 + 1 \cdot 16 + 0 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 0 \cdot 1$$

$$0 + 64 + 0 + 16 + 0 + 0 + 2 + 0$$

$$82$$

Base 10 to Binary

What is 157_{10} in binary?



Answer: 10011101

Binary mathematics

0+0=0

1+0=1

1+1=10

$$
\begin{array}{r}
11001011 \\
+\;\; 11100110 \\
\hline
110110001
\end{array}
$$

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Representing Negative Integers

TOY words are 16 bits each.

- We could use 16 bits to represent 0 to $2^{16} - 1$.
- We want negative integers too.
- Reserving half the possible bit-patterns for negative seems fair.

Highly desirable property. If x is an integer, then the representation of -x, when added to x, is zero.

```
       x        0 0 1 1 0 1 0 0
   +(-x)      + ? ? ? ? ? ? ? ?
   -----      -----------------
       0        0 0 0 0 0 0 0 0
```

```
       x        0 0 1 1 0 1 0 0
              + 1 1 0 0 1 0 1 1
              -----------------
   +(-x)        1 1 1 1 1 1 1 1
              +               1
   -----      -----------------
       0        0 0 0 0 0 0 0 0
```

flip bits and add 1

Two's Complement Integers

To compute -x from x:

- Start with x.

leading bit

+4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

- Flip bits.

-5	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1

- Add one.

-4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

Two's Complement Integers

dec	hex	binary															
		15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
+32767	7FFF	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
									· · ·								
+4	0004	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
+3	0003	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
+2	0002	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
+1	0001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
+0	0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	FFFF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-2	FFFE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
-3	FFFD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
-4	FFFC	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
									· · ·								
-32768	8000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Hexadecimal (base 16)

Binary code is too long in representation.
Hex is much shorter.

Converting a binary number to a Hex number is relatively easy

- Every 4 bits can convert to a single Hex value

Problem: we are short of numbers

- A=10 B=11 C=12 D=13 E=14 F=15

Hexadecimal (Base 16) Representation

Binary	HEX
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	HEX
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Example

$$\underbrace{1\ 0\ 1\ 1}\ \underbrace{1\ 1\ 1\ 1}\ \underbrace{0\ 0\ 1\ 0}\ \underbrace{1\ 1\ 0\ 0}$$

Dec	11	15	2	12
Hex	B	F	2	C

Result BF2C

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Bits, Bytes, and Words

Information in the computer are stored as a groups of binary digits. An individual digit is called a **bit**.

Bits are grouped into 8-bit collections called **bytes**. Memory is normally measured in terms of bytes.

- 256 possible values from 0-255 base 10 or
- 00000000 to 11111111 base 2

Bytes are further grouped into 4 or more byte groupings to make up a computer **word**. The most common word sizes in modern computers is 32 bits (4 8-bit bytes) or 64-bits

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