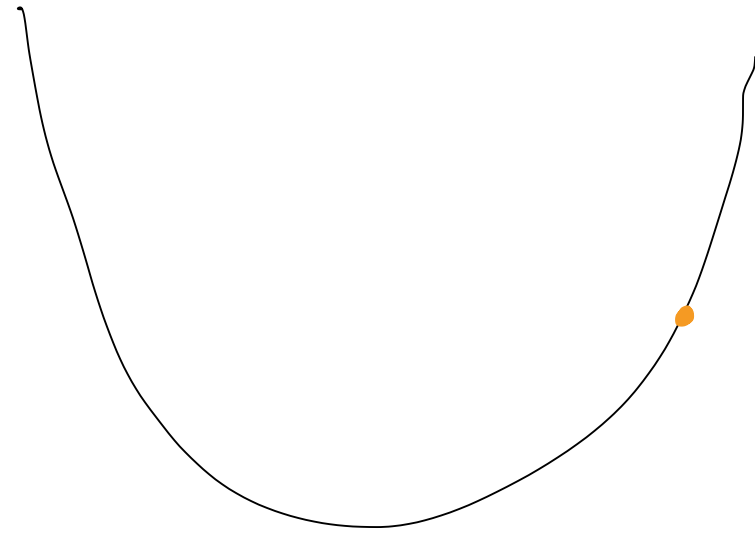


01/31/2023

$$f(\vec{v}) = v_1^2 + 2v_2 + 6v_1v_2$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial v_1} \\ \frac{\partial f}{\partial v_2} \end{bmatrix}$$



norms

$$\|a\|_2 = \sqrt{\sum (a_i^2)} = \sqrt{\langle a, a \rangle}$$

$$\|a\|_1 = \sum |a_i|$$

trans
after

Substron

valerate

$$f(x) = 4x_1 + 3x_2 - 6x_1x_2 + 8x_3$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \frac{\partial f}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 4 - 6x_2 \\ 3 - 6x_1 \\ 8 \end{bmatrix}$$

$g(\vec{u})$

gradient descent

$$\vec{u}^{(t+1)} = \vec{u}^{(t)} - \alpha \nabla g(\vec{u}^{(t)})$$

$$\left\| \vec{u}^{(t+1)} - \vec{u}^{(t)} \right\|$$

$$l(\vec{a}, b, \lambda) = \left[\frac{1}{N} \sum_{i=1}^N \max(0, 1 - y_i (\vec{a}^T x_i + b)) \right] + \lambda \frac{\|\vec{a}\|^2}{2}$$

$$l(\vec{u}) = \left[\frac{1}{N} \sum_{i=1}^N l_i(\vec{u}) \right] + l_0(\vec{u})$$

$$-\nabla l(\vec{u}) = - \left(\underbrace{\left[\frac{1}{N} \sum_{i=1}^N \nabla l_i(\vec{u}) \right]}_{\text{over all elements in dataset}} + \nabla l_0(\vec{u}) \right) \vec{u} = \begin{bmatrix} b \\ \uparrow \\ a \\ \downarrow \end{bmatrix}$$

over all elements in dataset

initialize \vec{u} randomly

repeat until convergence:

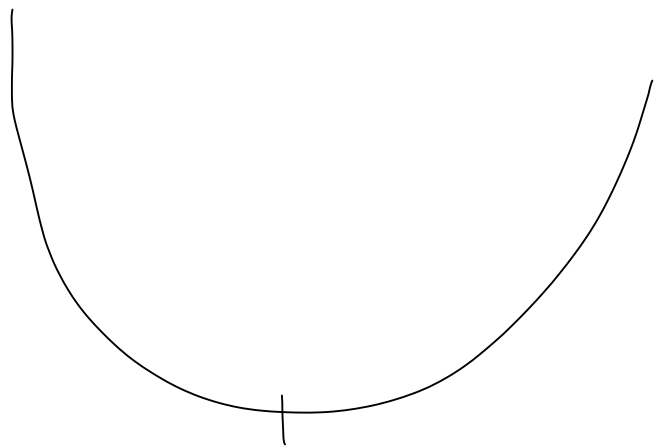
randomly shuffle samples

loop through samples and update weights:

$$\vec{u}^{(t+1)} = \vec{u}^{(t)} - \alpha \nabla l(\vec{u}^{(t)})$$

alpha
constant

gradient



Mini-batch gradient descent

choose fixed:

$N_b = \#$ of data points per batch

repeat until convergence

for N/N_b

grab random batch \mathcal{D} of size N_b

update \vec{u}

$$\vec{u}^{(t+1)} = \vec{u}^{(t)} - \alpha \left[\frac{1}{N_b} \sum_{i \in \mathcal{D}} \nabla \ell_i(\vec{u}^{(t)}) \right] + \lambda \nabla \ell_0(\vec{u}^{(t)})$$