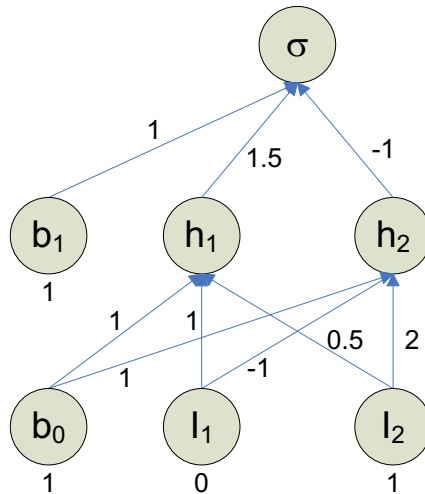


Multi-layer Perceptron Learning: one sample

Below is a snapshot of a neural network during training. There are two input units, two hidden layer perceptrons, and a single output unit. Input l_1 has a value of 0; input l_2 has a value of 1; all bias have value 1. Edges are labeled with their corresponding weights. Learning factor $\eta = 0.5$. Target value is 1.



Step 1: Feed the inputs forward

Use the formula, output = $\frac{1}{1 + e^{-\sigma}}$ where $\sigma = \sum_i w_i x_i + bias$

$$h_1 = (1 \cdot 0) + (0.5 \cdot 1) + (1 \cdot 1) = 1.5 \Rightarrow 1/(1+e^{-1.5}) = 0.818$$

$$h_2 = (-1 \cdot 0) + (2 \cdot 1) + (1 \cdot 1) = 3 \Rightarrow 1/(1+e^{-3}) = 0.953$$

$$y = (1.5 \cdot 0.818) + (-1 \cdot 0.953) + (1 \cdot 1) = 1.274 \Rightarrow 1/(1+e^{-1.274}) = 0.781$$

Calculate total error in network, $E = \frac{1}{2}(t - y)^2$

$$E = \frac{1}{2}(1 - 0.781)^2 = 0.024$$

Step 2: Backpropagate the errors

a) Calculate the error for the output unit y ,

Use the formula, $E_y = y(1 - y)(t - y)$

$$E_y = (0.781)(1 - 0.781)(1 - 0.781) = 0.037$$

b) Calculate the error for each hidden unit h_i

Use the formula, $E_{h_i} = h_i(1 - h_i)(w_{h_i,y} \cdot E_y)$

$$E_{h1} = (0.818)(1 - 0.818)(1.5 \cdot 0.037) = 0.008$$

$$E_{h2} = (0.953)(1 - 0.953)(-1 \cdot 0.037) = -0.002$$

Step 3: Learn

a) Update network weights proportionately

Use the formula, $w_{i,j} = w_{i,j} + \eta E_j z_i$ where z_i is value of i

$$w_{h1,y} = 1.5 + (0.5)(0.037)(0.818) = 1.515$$

$$w_{h2,y} = -1 + (0.5)(0.037)(0.953) = -0.982$$

$$w_{b1,y} = 1 + (0.5)(0.037)(1) = 1.019$$

$$w_{l1,h1} = 1 + (0.5)(0.008)(0) = 1$$

$$w_{l2,h1} = 0.5 + (0.5)(0.008)(1) = 0.504$$

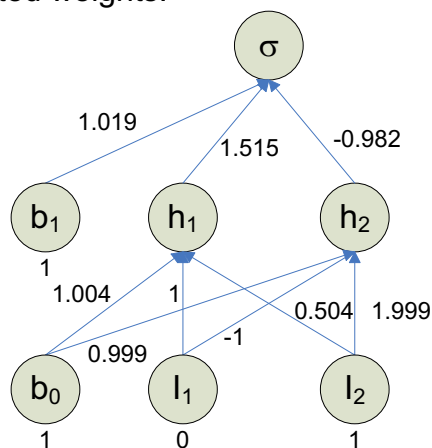
$$w_{b0,h1} = 1 + (0.5)(0.008)(1) = 1.004$$

$$w_{l1,h2} = -1 + (0.5)(-0.002)(0) = -1$$

$$w_{l2,h2} = 2 + (0.5)(-0.002)(1) = 1.999$$

$$w_{b0,h2} = 1 + (0.5)(-0.002)(1) = 0.999$$

Label network with updated weights:



Repeat for all training samples \Rightarrow one epoch.