Fully Dynamic de Bruijn Graphs

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Introduction

2 Data Structure

Implementation

Genome assembly: De-Bruijn graph

De-Bruijn graph:Reconstructing a string from a set of its k-mers

- Data structure method on genome assembly.
- 2 Consist of multiple K-mers which generated by genome sequence.
- 3 Same vertices, K-1 mers are glue together in the final step.



Fig. 1. The de Bruijn graph constructed from string ATGGAAGTCGATGGAAG, with k=7.

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- Next-generation sequencing (NGS) data usually comes with large volume and short size, which has large amount of repetitive regions.
- Compared to 'overlap-consensus-layout' method, De Bruijn graph-based assembly approach handles the assembly of repetitive regions better

Application problem

- Using de Bruijn graph in practice is the high memory occupation for certain organisms
- Human genome encoded in a de Bruijn graph with a k-mer size of 27 requires 15GB to store the node sequences
- Bulges and whirls occur because of sequencing errors or repeats in the genome, so would like to be able to efficiently add and remove edges from graph

How to efficiently update graph while maintaining memory space efficiency?

Fully Dynamic de Bruijn Graphs(Belazzougui et al. (2016))

Introduces compact, dynamic representation of De Bruijn graph

- Nodes and edges can be inserted and deleted efficiently
- k-mers are represented by integers using a combination of Karp-Rabin hashing and minimal perfect hashing.
- A partition of the graph into a forest allows efficient membership queries with no error.

Project

- Implement the data structure from the paper.
- Evaluate our data structure on graphs built from real sequencing data.
- Ompare our data structure with alternative approaches for De Bruijn graphs

Static hash function

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- De Bruijn graph and forest

Hash function f

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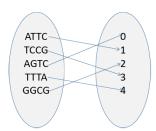
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- **2 Karp-Rabin**: Given a prime P and base r, a Rabin-Karp hash function f is a function defined over the space of all strings of length k such that $f(x_1...x_k) = (\sum_{i=1}^n x_i r^i) \bmod P$.

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Given a static set N of n k-tuples over an alphabet Σ of size σ , with high probability in O(kn) expected time we can build a function $f: \Sigma^k \longrightarrow \{0, \cdots, n-1\}$ with the following properties:

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- given u and v, such that suffix of u of length k-1 is the prefix of v of length, or vice versa, we can compute f(v) in O(1) time if we already computed f(u).

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- Similarly, "TTCG" can be computed by $(4^4 \cdot 3 + 4^3 \cdot 3 + 4^2 \cdot 1 + 4^1 \cdot 2) \mod 13$.
- Suppose we only have two k-mers, one qualified minimal perfect hashing function f needs to ensure that f("ATTC") = 0 and f("TTCG") = 1 or f("ATTC") = 1 and f("TTCG") = 0.

Lemma

If N is dynamic then we can maintain a function f as described in 1 except that:

• the range of f becomes $\{0, \dots, 3n-1\}$.

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- lacktriangledown insertions and deletions take O(k) amortized expected time.
- \odot the data structure may work incorrectly with very low probability (inverse polynomial in n).

Representation of edges in de Bruijn graph

- The edges (E) of G are stored in two binary matrices, IN and OUT, each of size $n \times |\Sigma|$.
- These two matrices are used to maintained the IN and OUT edge of each vertex. We can move each vertex forward and backward using this information.

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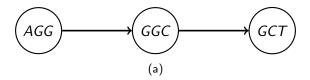
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- These two matrices are used to maintained the IN and OUT edge of each vertex. We can move each vertex forward and backward using this information.
- 3 The IN and OUT matrices can be constructed as:

$$(u = ba_1 \dots a_{k-1}, v = a_1 a_2 \dots a_{k-1} c) \in E$$

 $\iff OUT(f(u), c) = 1, IN(f(v), b) = 1.$

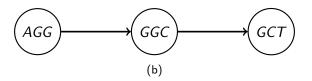
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Suppose f(AGG) = 0, f(GGC) = 1, f(GCT) = 2, the *IN* and *OUT* matrices can be initialized as:

Example of IN and OUT matrices

IN	Α	G	С	Т
0(AGG)	0	0	0	0
1(GGC)	1	0	0	0
2(GCT)	0	1	0	0

OUT	А	G	С	T
0(AGG)	0	0	1	0
1(GGC)	0	0	0	1
2(GCT)	0	0	0	0

(a)

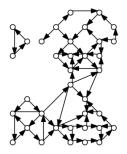
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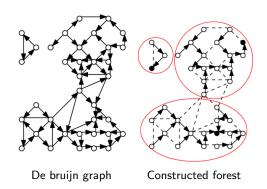
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- 2 How can we do membership queries maintaining a reasonable memory usage?
- Our strategy is to sample a subset of nodes for which we store the plain-text k-tuple and connect all the un-sampled nodes to the sampled ones.
- Given a start point, we can move forward and backward using IN and OUT matrices. Once we reached the root, we can check if the resulting k-mer matches with root k-mer.

• More specifically, partition an undirected graph G into a forest \mathcal{F} where each $T \in \mathcal{F}$ with $\alpha \leq h(T) \leq 3\alpha$, where h(T) is the height of tree T, $\alpha = k \log \sigma$.

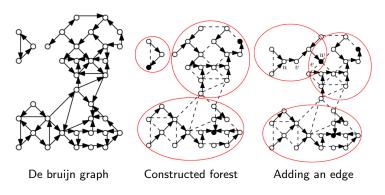


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De Bruijn graph and forests properties

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- ② checking whether a node is in G takes $O(klog\sigma)$ time.
- 3 listing the edges incident to a node we are visiting takes $O(\sigma)$ time, and crossing an edge takes O(1) time.

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- K-mer representation
- Hash function implementation
- the BitArray class
- IN, OUT, and forest implementation
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- Dynamic edges (Partially complete)

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- "Semi-dynamic De Bruijn Graph"

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- Example:

Zeros
$$2K$$
 bits $0000 \cdots 11100100 = TGCA$



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- Even though P and powers can be stored (barely) in 64-bit integer, computation requires slow 128-bit arithmetic, for which we used Boost² library type.
- For K = 27 on *E. coli*, lower bound for prime is

16584693176107222092.

Max value in unsigned 64-bit integer:

18446744073709551615.



• Recall: A minimal perfect hash function on a set A of size n is a hash function that maps elements of A injectively to the set $\{0, ..., n-1\}$.



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- That completes the generation of the hash function.



Construction of Hash function

```
1: procedure GENERATEHASH
    InputS, a set of n k-tuples over an alphabet \sum of size \sigma
       R = \max(\sigma, kn^2)
 2:
     P = getPrime(R)
 3:
      r = \text{randomNumber}(0, P - 1)
 4:
       f = rabinHash(r, P)
 5:
       while isInjective(f, S) is FALSE do
 6:
           r = \text{randomNumber}(0, P - 1)
 7:
           f = rabinHash(r, P)
8:
       end while
g.
       g = minimalPerfectHash(f(S));
10:
       return g \circ f;
11:
12: end procedure
```

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• What is the problem with naively implementing this update?

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- Solution to second problem: precompute $r^{-1} \mod P$ and store it. (Requires generalized Euclidean algorithm)
- Solution to first problem? Hint: What is P mod P?
- Add $(4P first \cdot r)$ to f(m), as this will always be nonnegative.

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- Each bit of an integer is treated as an element in the array and can be set, returned as a bool, etc..
- This will slow data access down since multiple operations are required for each access, but much more memory efficient.

$$\underbrace{010\cdots}_{\text{int }0}\underbrace{101\cdots}_{\text{}}$$

• Example with two ints (each containing 32 bits).

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 - Next compute which bit in this int: $33 = 1 \pmod{32}$
 - To get value of this bit, shift it to right-most position and do bitwise AND with 00...1.
- With this class, only waste at most 31 bits in memory per instance of the class.

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- This is how we will do membership queries (explained more later).

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- We break the graph up into trees in the desired height range as we go along.



Before this point, the forest has been initialized to be a BitArray of the correct size and an empty map of root kmers.

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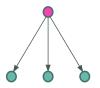


- First, we choose a kmer in our De Bruijn graph that has not yet been explored.
- Hash it to find its place in the forest's BitArray.
- Store it as being a root, its IN/OUT and parent bits are left alone since it has no parent.
- Add its hash and kmer to the map.

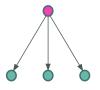


De Bruijn breadth first search

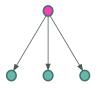
 We use IN and OUT to find all the kmers of neighbors in the De Bruijn graph.



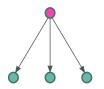
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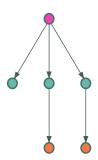


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- Set the root bit to false, and don't store these kmers.

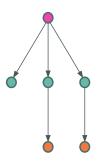


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- Keep doing that until we get to a kmer that is of height $\alpha{+}1$ from the root (orange)
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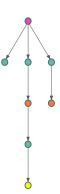


- Keep doing that until we get to a kmer that is of height $\alpha{+}1$ from the root (orange)
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- If we get to a height over the maximum allowed, we can break off at this root and be sure the remaining tree's height is still above the minimum allowed.



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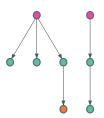


- Once we get to a kmer that is of height $3\alpha + 1$ from the root (yellow), we need a new tree.
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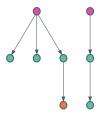


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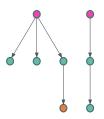
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- Continue on until the entire De Bruijn graph has been visited.



The Entire Data Structure

 We have now constructed the hash function, IN and OUT, and the forest.

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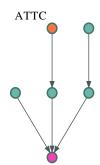
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- Suppose we have a nucleotide sequence of length k and we want to find if it exists
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- How do we check the membership of that kmer?
- We travel up the tree to the root ...

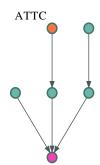
Membership of "ATTC"

• First, hash the kmer to get i in $\{0, ..., n-1\}$



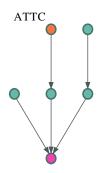
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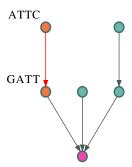
Membership of "ATTC"

- First, hash the kmer to get i in $\{0, ..., n-1\}$
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- Note:
 Even if the kmer is not
 in our De Bruijn graph,
 we can still hash it and
 get a place in the forest.

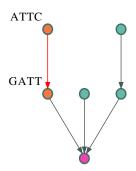


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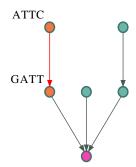
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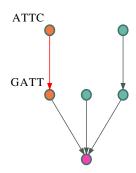
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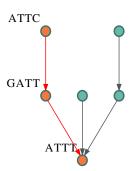


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- It's possible that the hash for "GATT" doesn't have an OUT edge for "C", therefore "ATTC" can't possibly be an actual kmer.

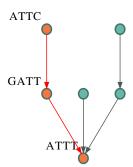


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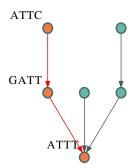
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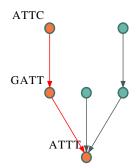
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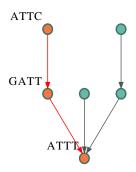
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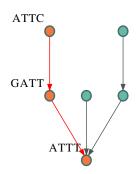
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- We rely on the library BBHash for our minimal perfect hash function, but we need a dynamic perfect hash function.

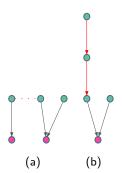
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- This would affect the forest if the edge is between two graph components where at least one was not big enough to have a tree above the minimum height.
- We may be able to combine the trees so that we have trees with heights in the desired range.

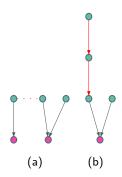
Adding an Edge

 Suppose that the edge (dotted) is between two trees in the forest below the minimum height, shown in (a).



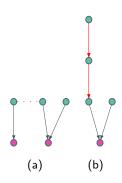
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- We can combine the trees into one tree of height less than the maximum.
- You have to add another edge to the forest, reverse the direction of some forest edges, then get rid of one of the tree's roots.



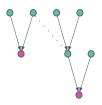
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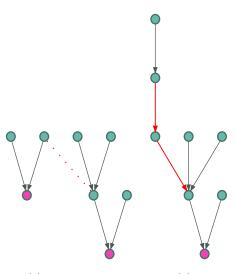


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- Now suppose that only one of the trees are below the desired minimum height and we add an edge (dotted).
- What we do depends on the height of the node in the edge from the taller tree.
- If the height is less than α , we can change the trees similar to before.



Adding Edges



Adding an Edge

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- Now suppose that the height is greater than or equal to α
- We can't necessarily just combine the trees like before, we could end up with a tree greater than the maximum height.

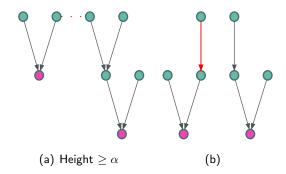


Adding an Edge

- Now suppose that the height is greater than or equal to α
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- We break off some part of the bigger tree into the smaller.



Adding Edges



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- We need to find a new root for one of the trees
- If a tree is too short, use similar techniques to above to produce a taller tree.

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- We would need to implement our own dynamic perfect hash function.

Experiments

Datasets:

Dataset	Chromosomes	Read count	Read length	Size
S288C ⁴	17	-	-	12M
E. coli	1	$27 \cdot 10^6$	101	3.3G

Platform:

 Intel(R) Xeon(R) CPU E5-2697 v4 @ 2.30GHz (18 cores) with 396 GB RAM

The results are tested by one run, which could be affected by computational resource conflicts.

Live Demo

- We have developed a web-based demo to allow users do multiple types of comparisons, which makes the evaluation easier.
- The link is here: http://128.227.162.189:9999/. Visit and play with it!

K value:	
20	
# of queries:	
1M	
Dataset:	
yeast	

Comparative Algorithms

- Bloom Filter: Standard Bloom filter
- **KBF1:** One-sided Bloom filter improves false postive rate three fold without using any additional storage.
- KBF2: Two-sided Bloom filter improves FPR by an order of magnitude while using very little additional memory⁵.



Query k-mer Generation

• We generate query *k*-mers with three different number of queries:

-	1	2	3
number of queries (million)	0.5	1	10

- We use two random ways to generate query k-mers
 - Muting one base of randomly extracted from the input k-mers
 - Purely random *k* mer generation. However, it does not result in obvious difference compared with the first one.

FDBG Data Structure Information

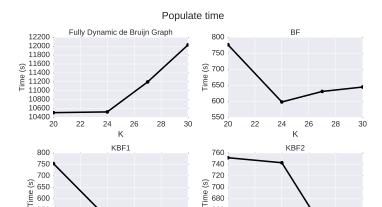
Table: The FDBG data structure information on E coli.

k	<i>k</i> -mers	RAM (MB)	Trees	Avg. height
20	770,956,037	1,485	5,492,320	48.14
24	784,990,222	1,519	6,412,386	50.45
27	783,739,686	1,517	6,532,142	47.28
30	776,321,600	1,505	6,752,622	48.46

- RAM = IN and OUT matrices + forest + minimal perfect hashing
- An example when

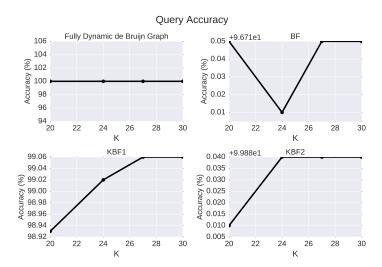
$$k = 20, \dot{N} = 770956037, R = 5492320, \sigma = 4:$$
 $N \times \sigma \times 2 + (N \times 4 + R * 64) + mph$
 $= (735 + (367 + 335) + 48)MB \times (bit/MB)$

Populate Time w.r.t k

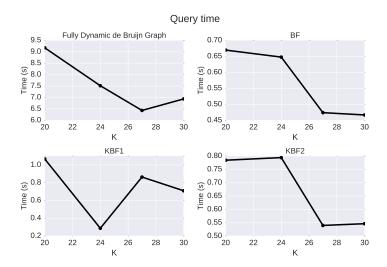
 

Κ

Query Accuracy w.r.t k



Query Time w.r.t k



Query Time w.r.t Tree Height

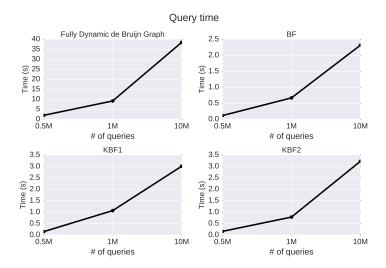
Table: Query time w.r.t the tree height.

k	Avg. height	Query time (s)
20	48.14	6.69 ⁶
24	50.45	7.51
27	47.28	6.43
30	48.46	6.93

Lower tree height gives rive to lower query time, because it needs fewer steps to trace the tree.

⁶Averaged by two runs.

Query Time w.r.t Number of Queries



Conclusion

- Have implemented static De Bruijn graph data structure and will complete edge insertion and deletion.
- We have tested the algorithms on two gene datasets. Compared with all Bloom filter based methods, ours has exactly 100% accuracy.
- We have compared with three different methods.
- We have made a website and a live demo.

End

Thanks and Questions?