Coral Patterns:

A Study on Coral Growth using Diffusion-Limited Aggregation

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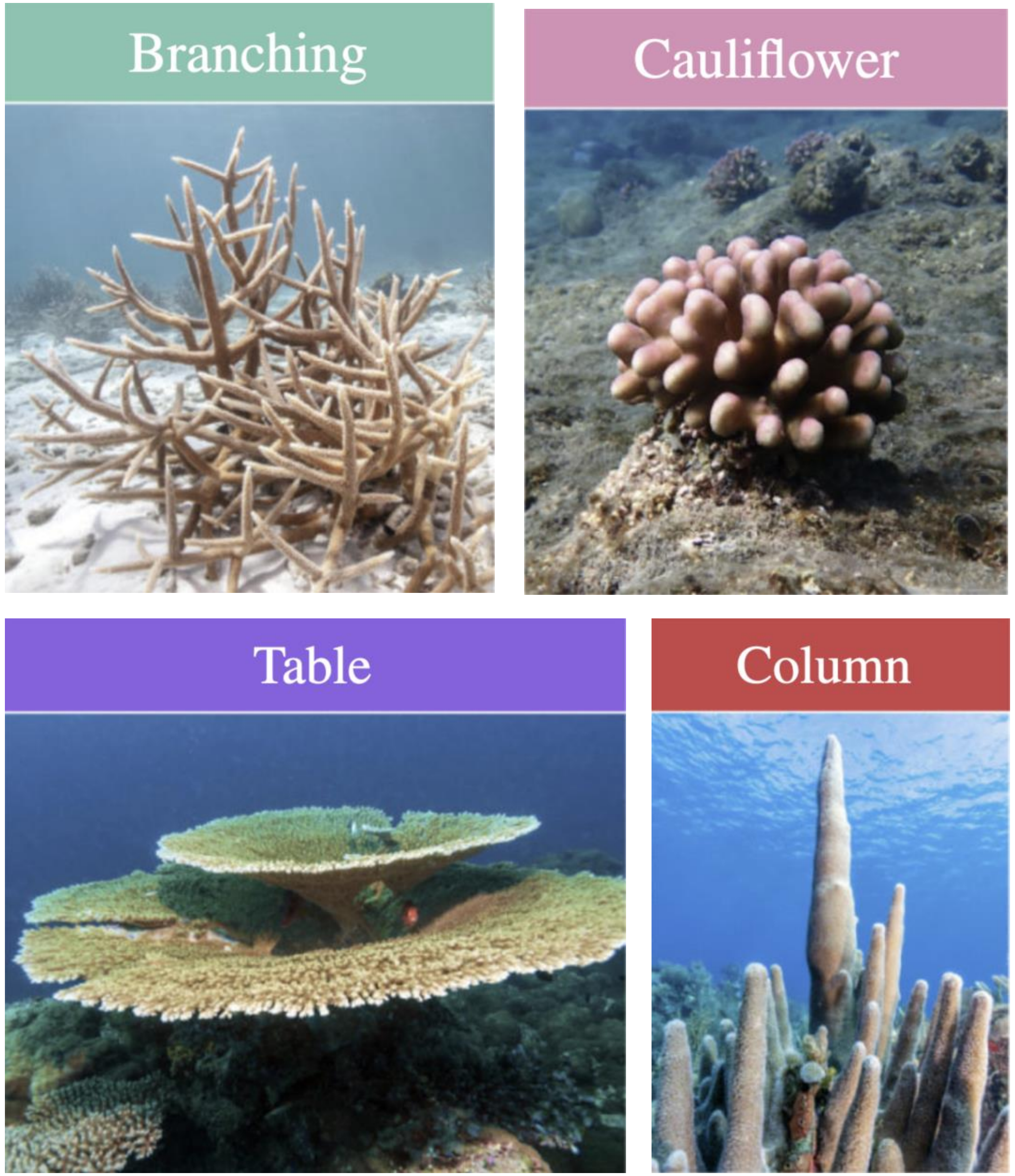
Coral Patterns

- **Coral:** Colony of Polyps
- **Polyps:** small animal
- **Emergent Behavior:** Construction of different shapes
- **DLA**: simulate coral growth

➡ Corals are complex systems



🔍 Research questions and hypotheses

1) **Growth Parameters:** Do they cause phase transitions?

H1: Growth parameters induce phase transitions between coral structures.

2) **Multifractality**: Is the fractal dimension the same at all scales?

H2: Different components of the coral structure exhibit different fractal dimensionality.

🔍 Research questions and hypotheses

3) **Fractal Dimension**: How does it change during growth?

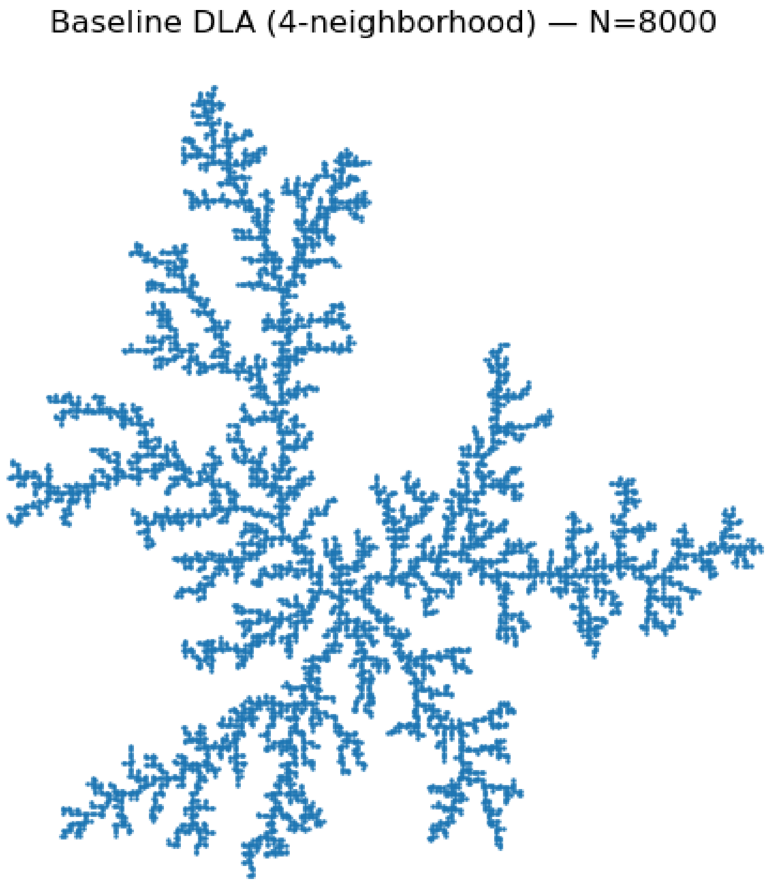
H3: Fractal Dimension follows a power law and is governed by fractal scaling.

4) **Mass vs. Radius**: Does the growth exhibit power law relations between mass and radius?

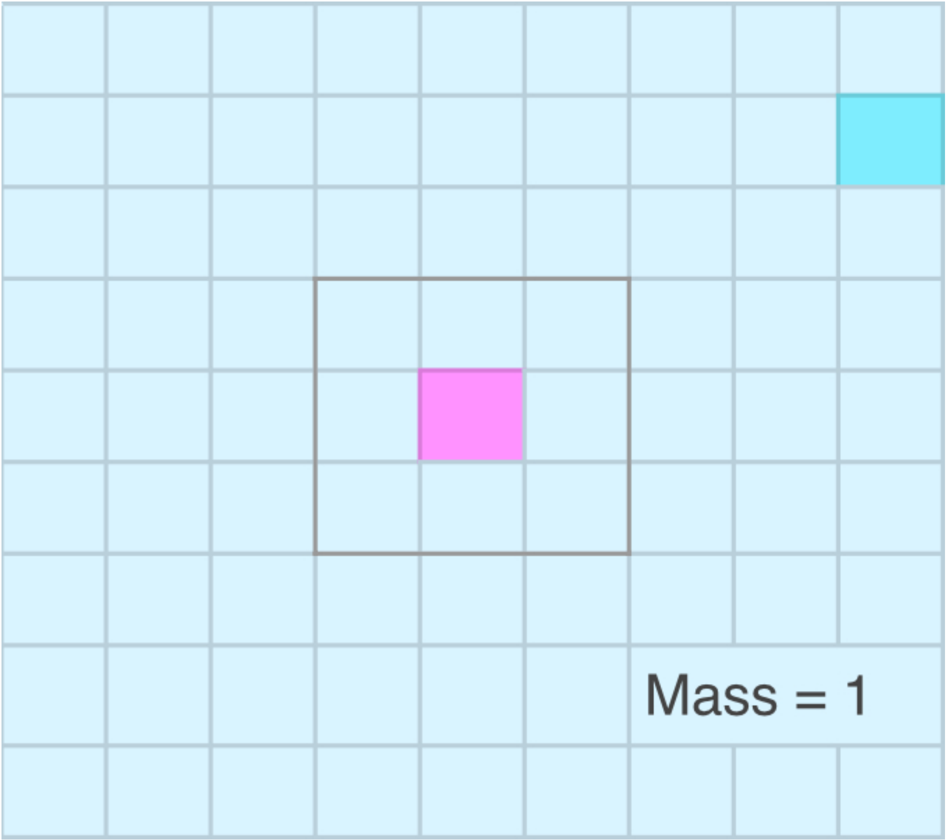
H4: Mass vs. Radius: scales as a power law.

❄️ Baseline Diffusion-Limited-Aggregation

- A single occupied seed
- Launch a random walker
- Unbiased walk in four directions
- Two little implementation tricks

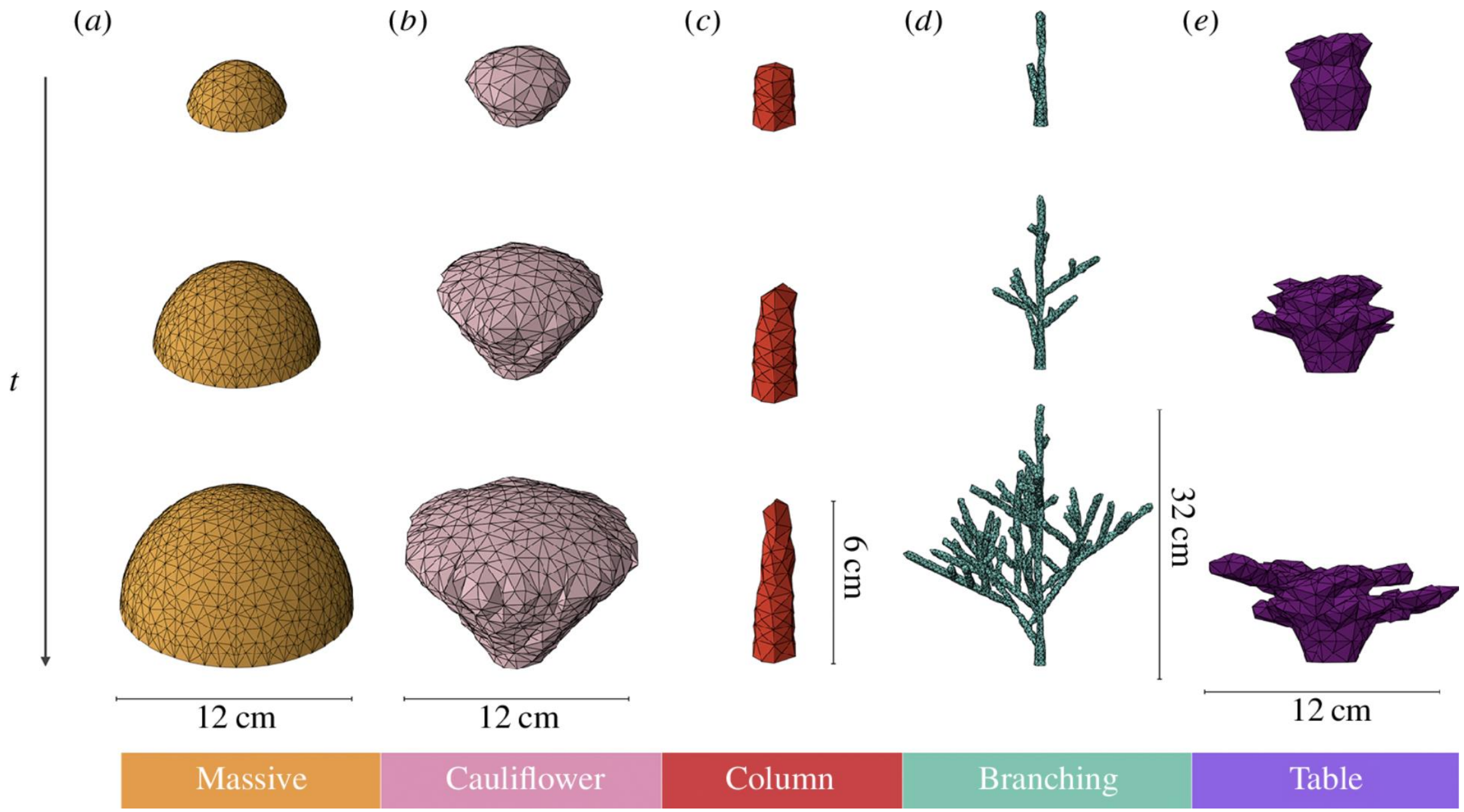


👇 How the DLA works



🕵️‍♀️ Agent-based coral growth

- Modelling corals as triangular mesh grids
- 5 parameters:
  1) **Growth mode:** Does it like to grow horizontally or vertically?
  2) **Elongation rate:** How fast do the triangles grow (distance between vertices)?
  3) **Subdivision distance:** How big do the triangles get before a new vertex grows?
  4) **Inter branching distance:** How far apart are the branches?
  5) **Branching angle:** At what angle do the branches grow?

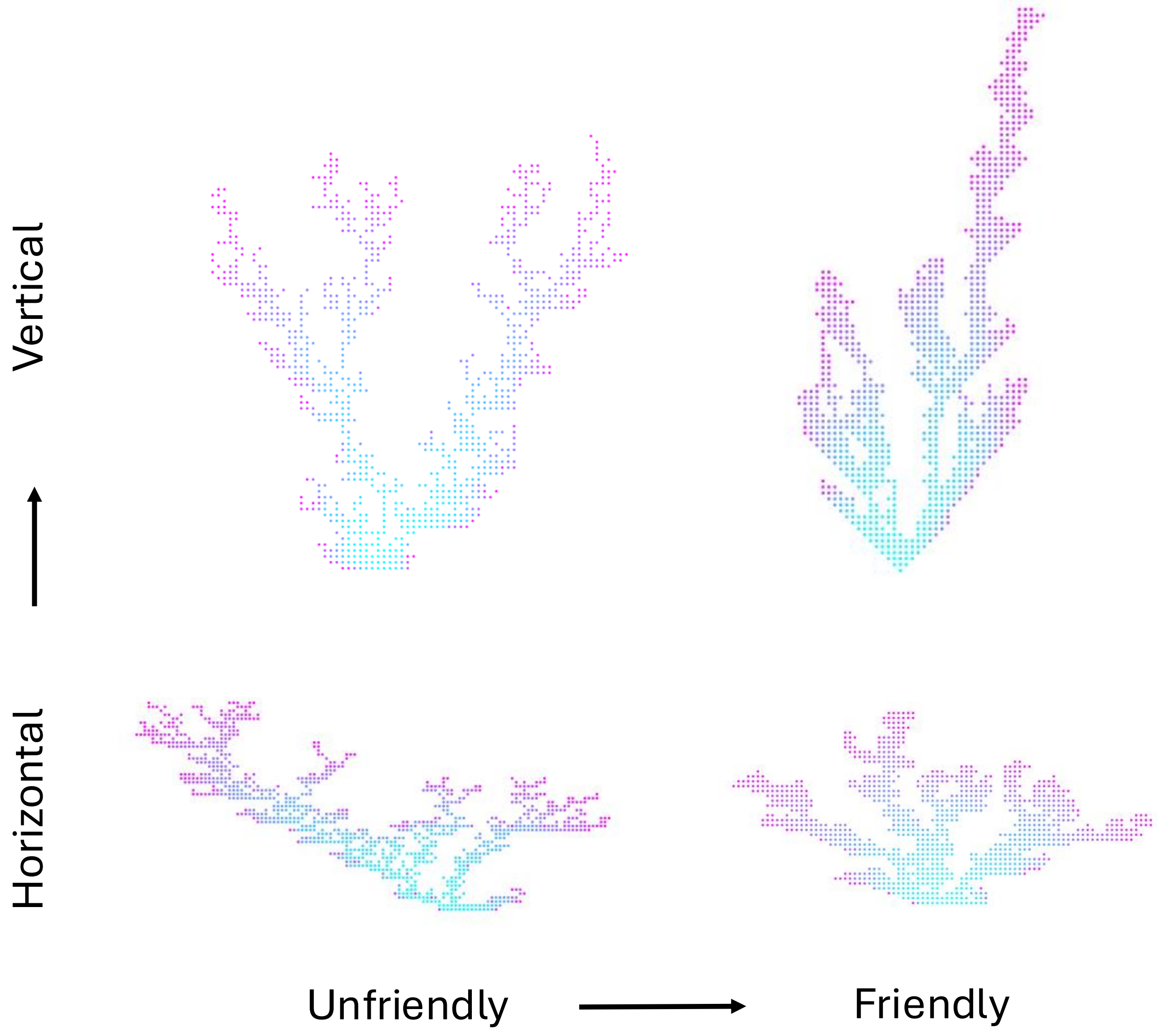


Llabrés et al. 2024

Translating Llabrés parameters to our model

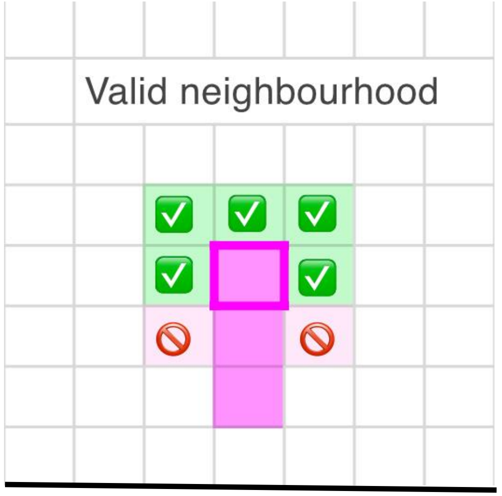
- Parameters:
  - **Growth mode:** Grow vertically or horizontally?
  - **Friendliness:** Grow closer to or farther from neighbours?

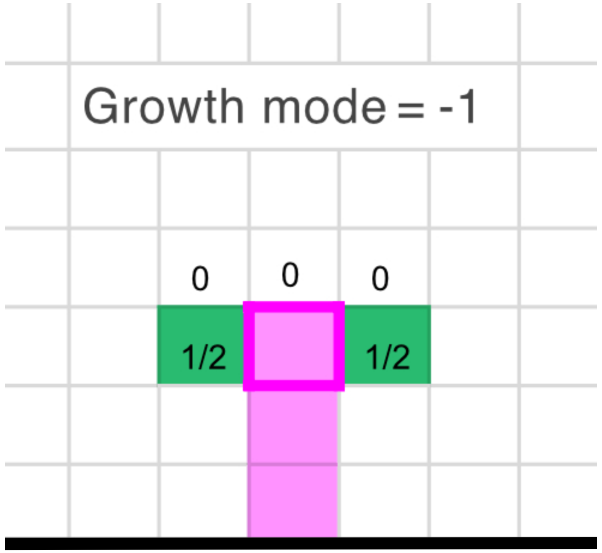
- These 2 new parameters affect:
  - ✅ P1 Growth direction
  - ✅ P2 & P3 Branch thickness
  - ✅ P4 Branch length
  - ✅ P5 Branch angle

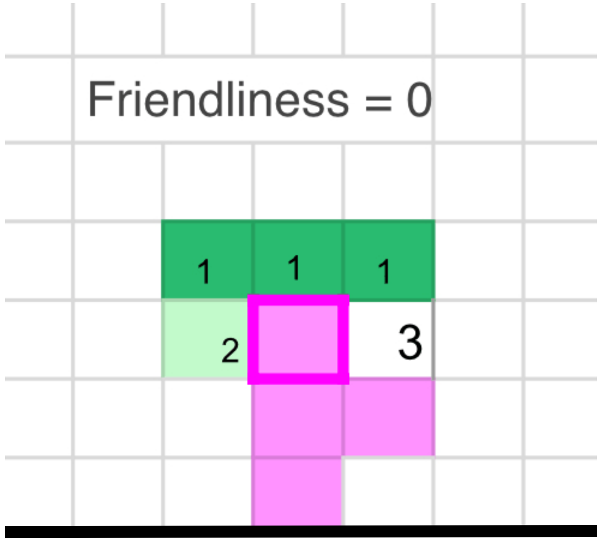


👇 How it works

- Modify probability of attaching to a site
  - i.e. "growing from a polyp"
- Other modifications
  - Seed grows from "ground"
  - No downwards growth
- Linear combination of probs



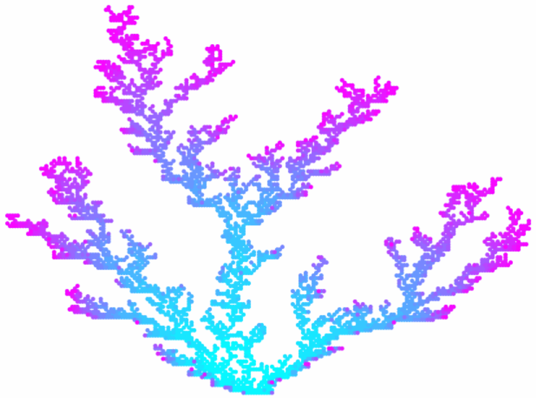




👾 DLA animated

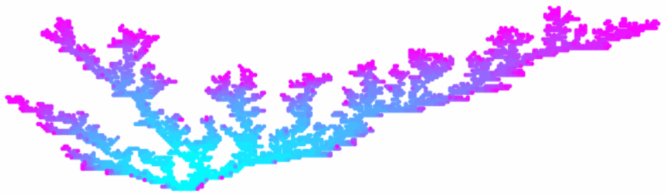
🤝 Friendliness

Parameterized DLA (8-neighborhood) — N=1000
friendliness=0.000



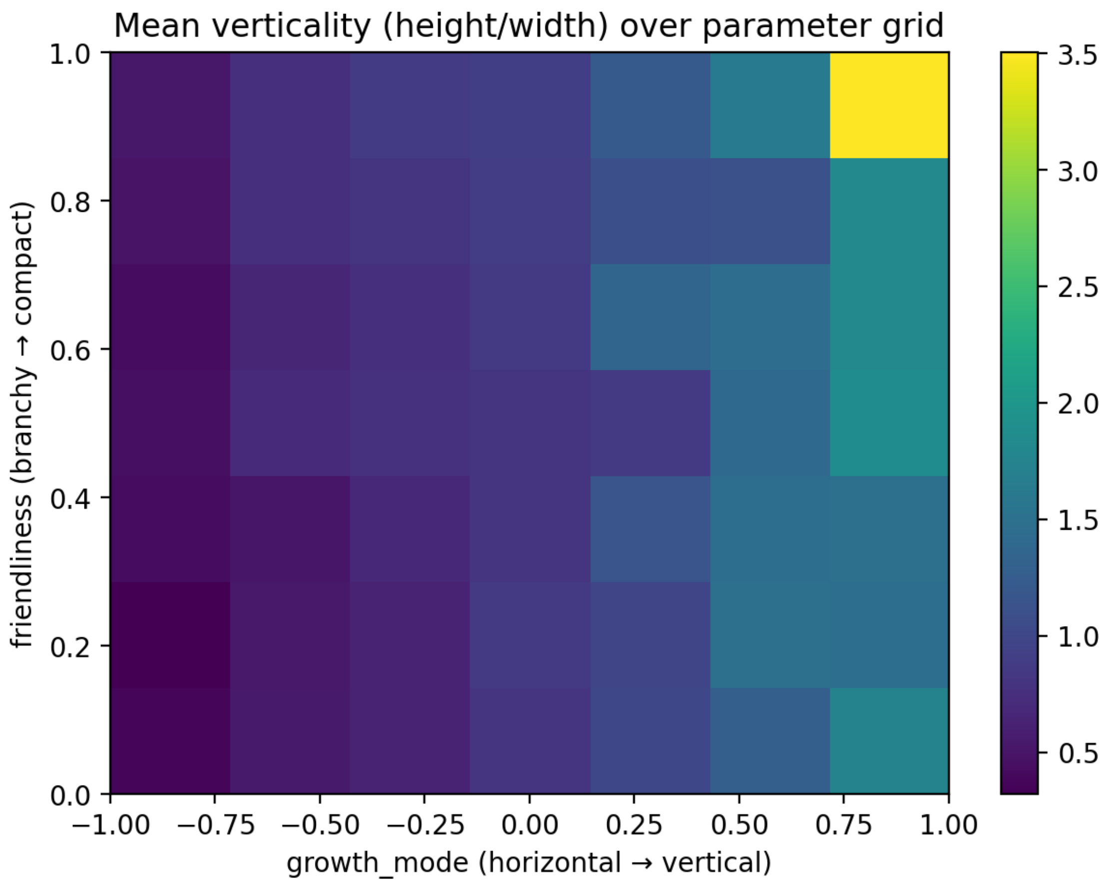
🌴 Growth mode

Parameterized DLA (8-neighborhood) — N=1000
growth_mode=-1.000



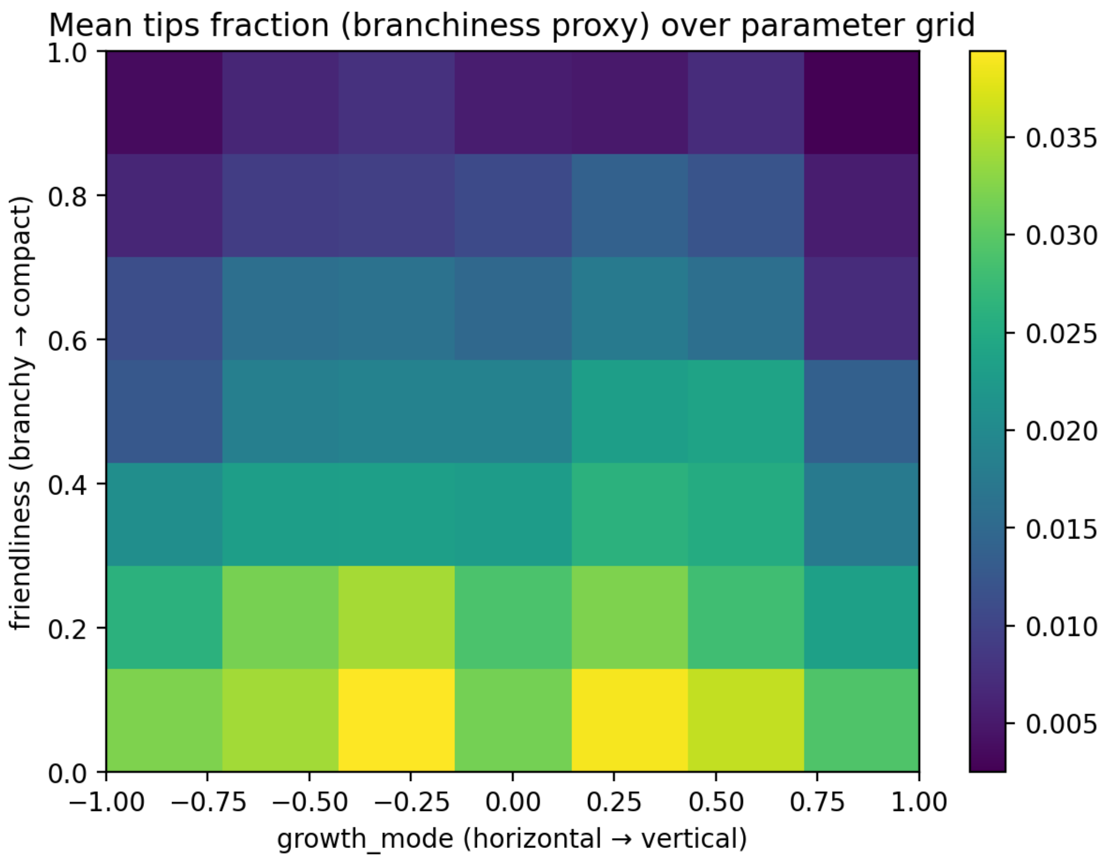
🥵 Heatmap: Verticality

- Easiest one to read
- Growth mode main parameter
- Growth mode increases => the cluster gets taller
- Extreme right = more column-friendly behaviour
- Friendliness is a secondary parameter
- Continuous phase transition



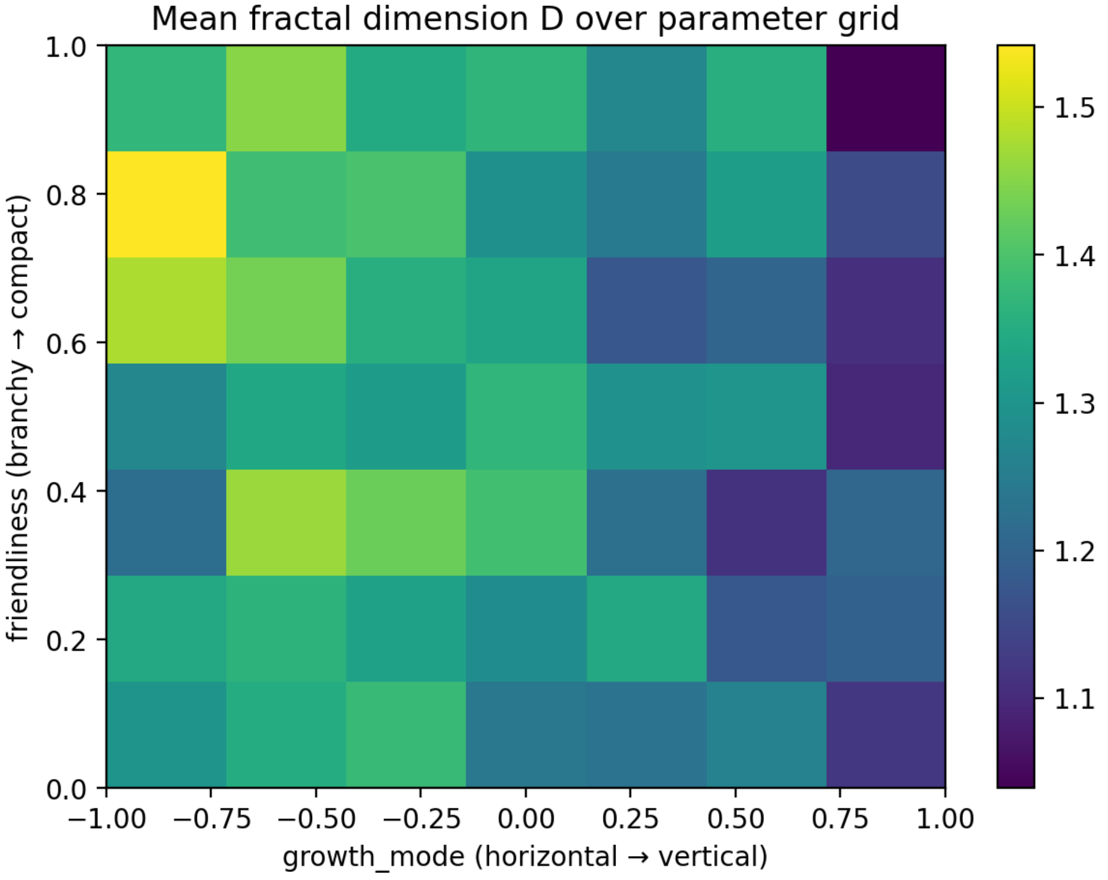
Heatmap: Branchiness

- How branchy is it?
- Friendliness = main parameter
- Low friendliness => ramified
- High friendliness => compact because tip fraction decreases
- Bottom to top = branchy to compact
- Continuous phase transition

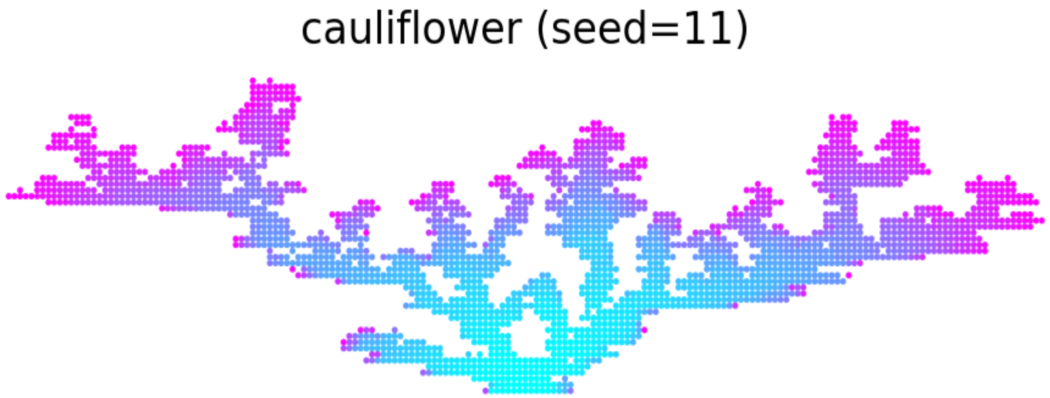


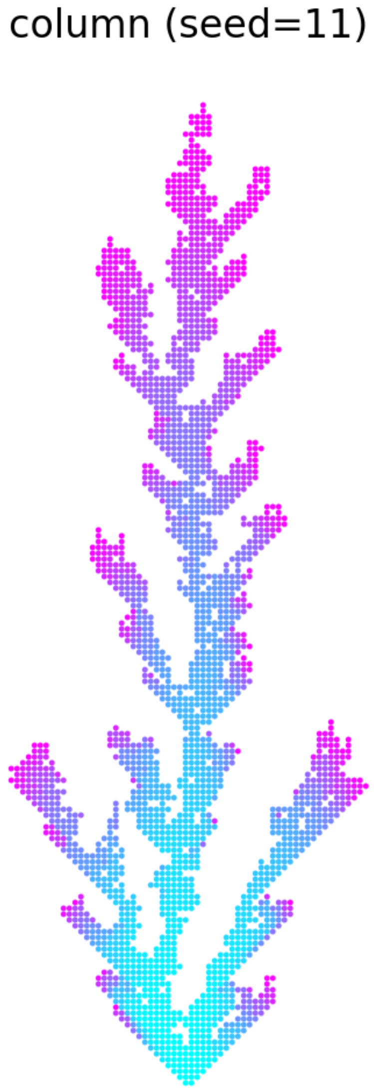
🤒 Heatmap: Fractal Dimension

- D is our summary number
- D = how space-filling is the growth?
- ⬆️ High D = fills efficiently, compact structure
- ⬇️ Low D = thin and branchy structures
- Continuous phase transition



🌱 Growth structures : Emergent behaviours

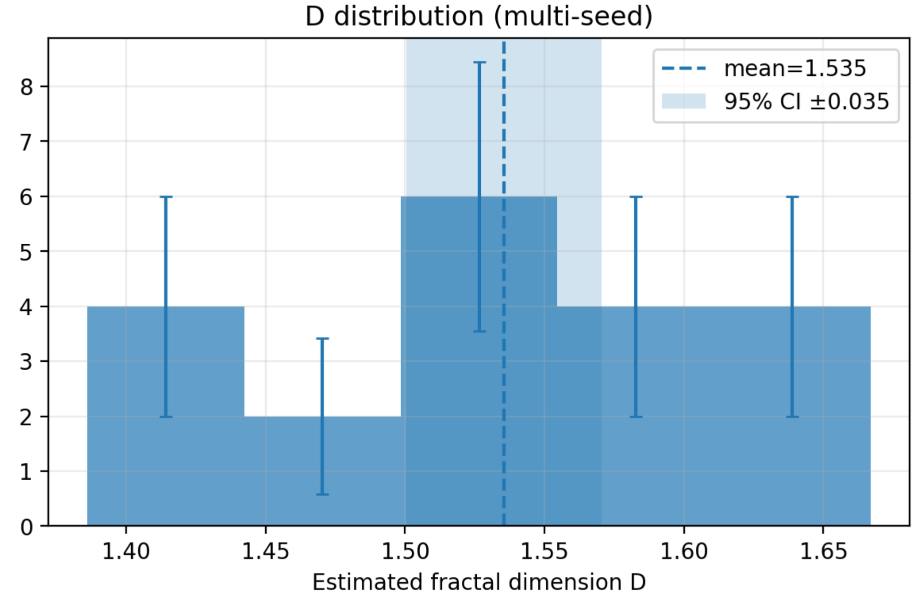


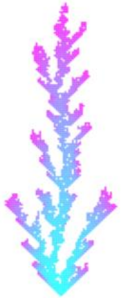


Distribution of fractal dimensions

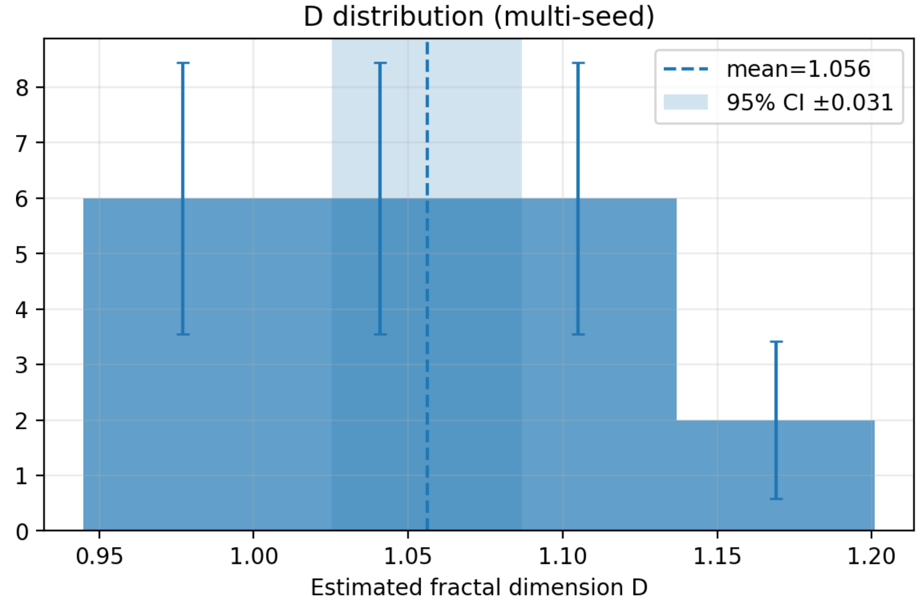
- **Cauliflower:** more filling => So D is higher
- **Column:** growth more constrained => So D is lower

Cauliflower

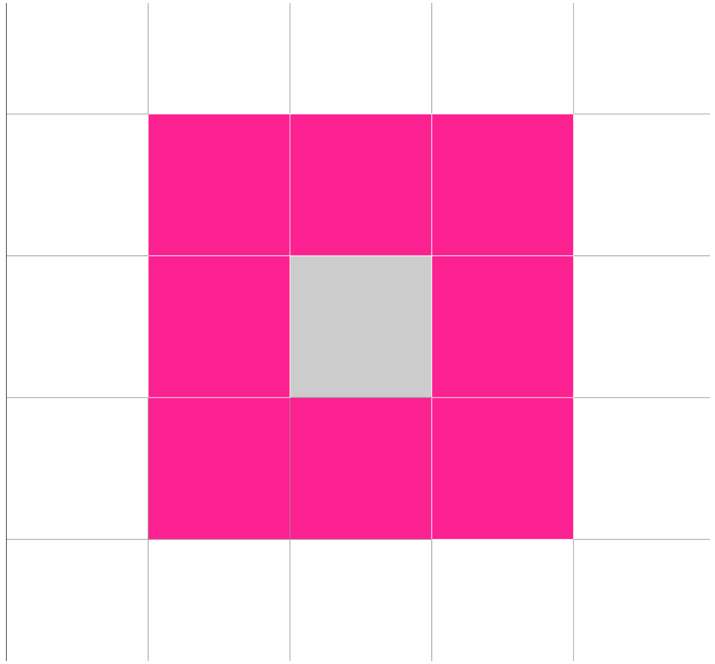




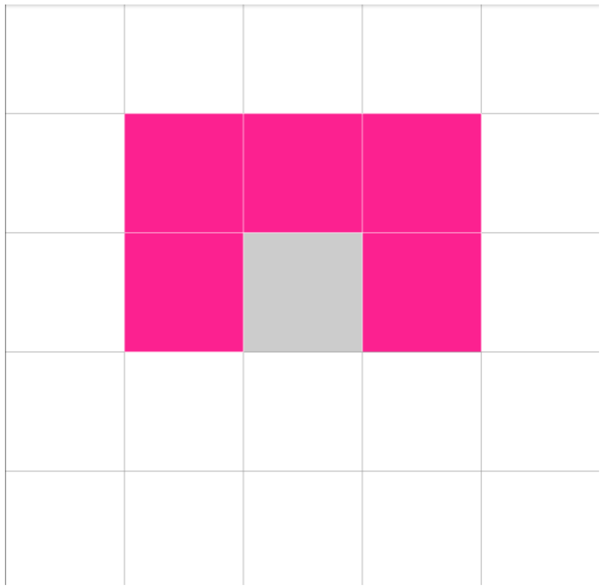
Column



5-Neighborhood



Moore Neighborhood

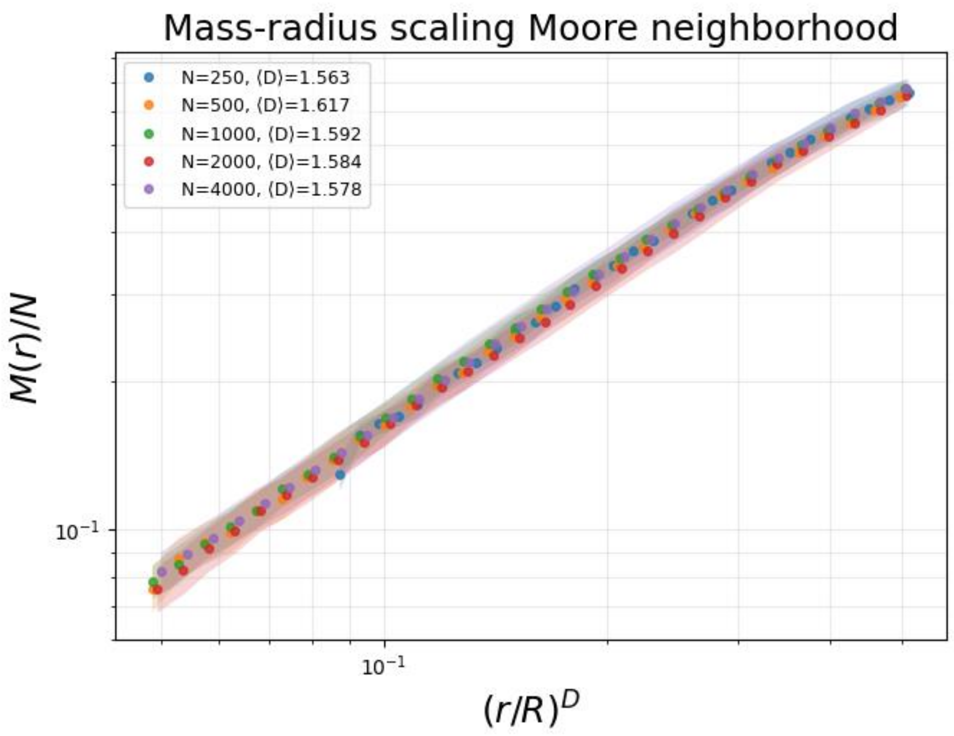


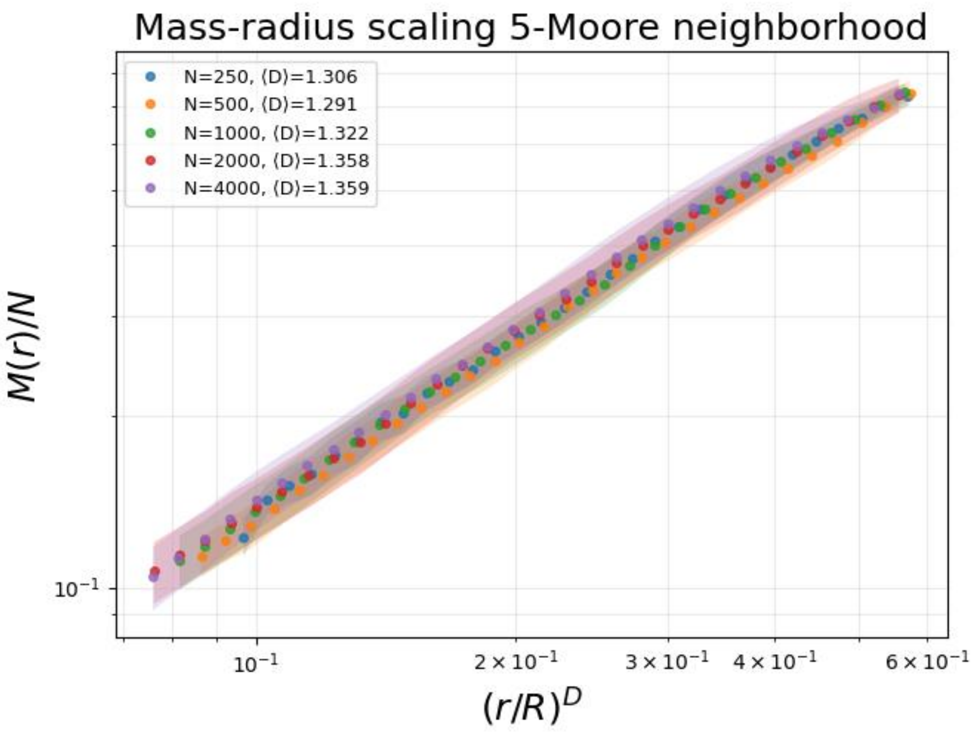
5-Neighborhood

Power law scaling

Average case

- Fractal dimension Moore neighborhood ≈ 1.6
- Fractal dimension 5-Moore neighborhood ≈ 1.3
- Power law: $M(r) \propto r^D$

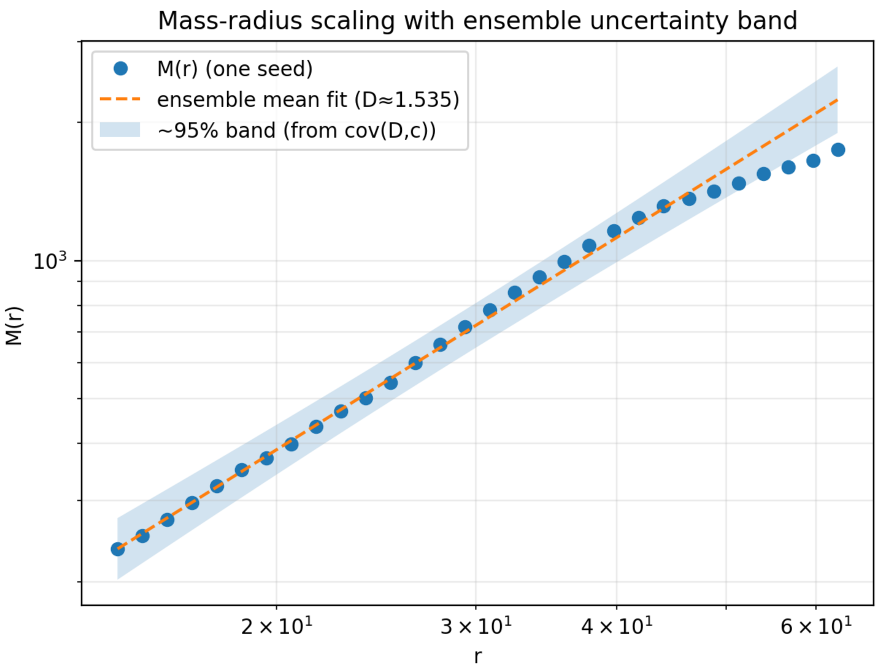




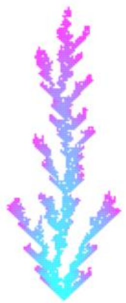
Power law Scaling:
Mass vs Radius

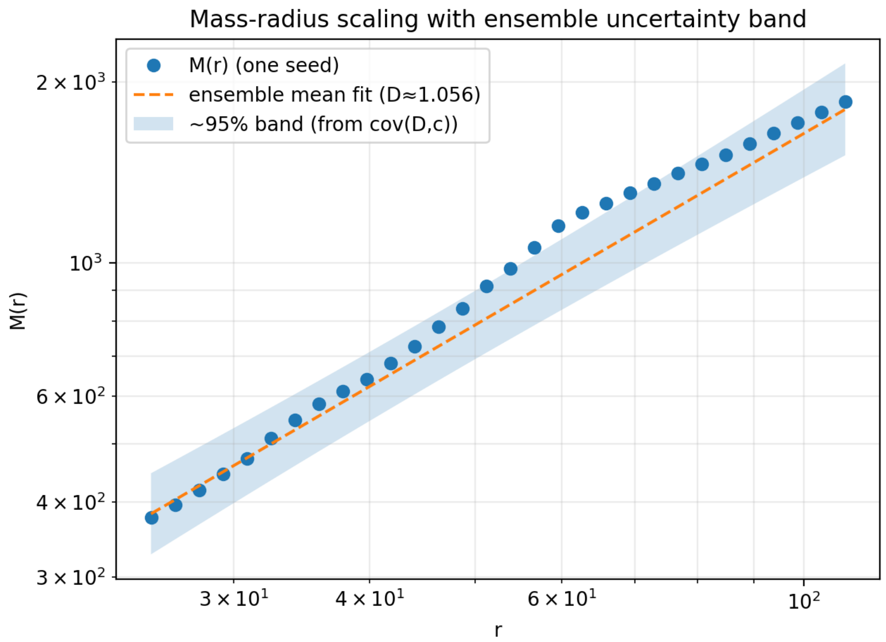
- **Cauliflower :** Power law, with a D around 1.535 => structure relatively space-filling and compact
- **Column:** Power law, with a lower D, thinner, and more line-like growth
- Both follow the mean trend and remains within the uncertainty band!

Cauliflower



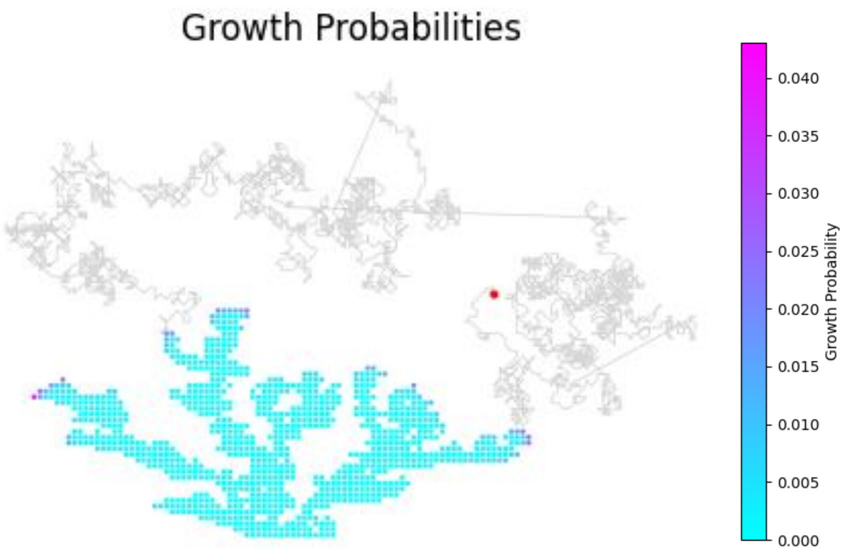
Column

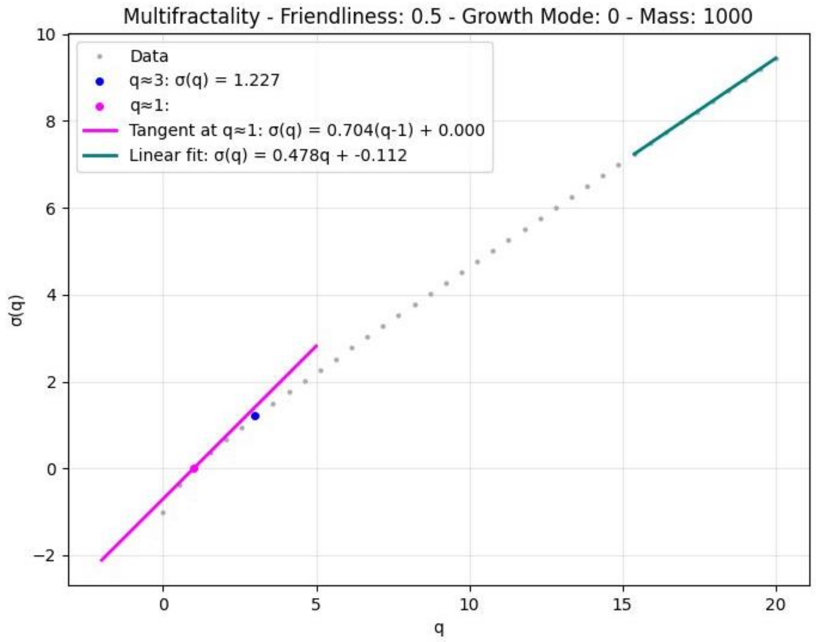




Multifractality

- Probability of growing from each site?
- Different fractal dimension at "branches" vs the "fjords"
- Sum probabilities to power of q
- $$\sum_{i=1}^{n} p_i^q = n^{-\sigma(q)}$$
  - p_i = growth probability
  - q = 1, 2, 3 … ∞
  - n = number of pixels in coral
    - *n = 1000 in this experiment*
  - σ(q) is non-trivial (i.e. a function)

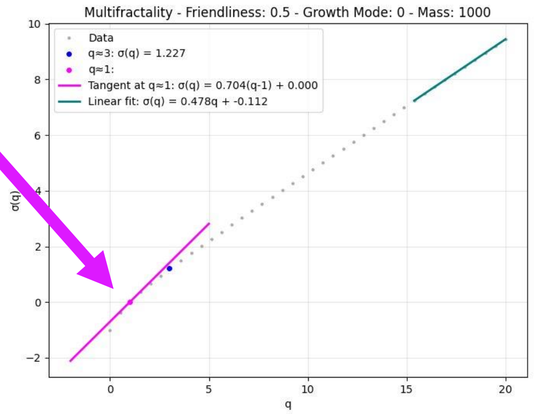


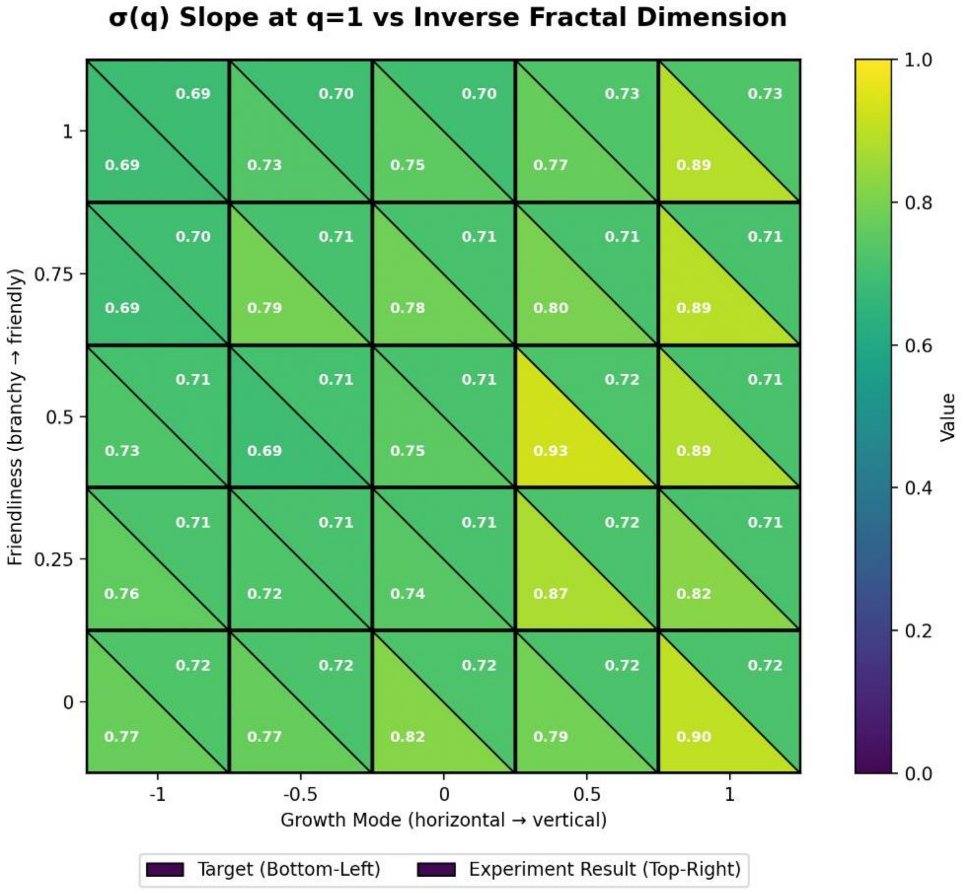


Halsey TC. 2000.

Scaling relation #1: Behaviour around q=1

- For q=1 it's just a straight up sum of probabilities
- $$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}q}\right|_{q=1} = D^{-1},$$
  - Slope of σ(q) around q=1 is 1/D
  - (D = fractal dimension)
- Averaged over 5 seeds



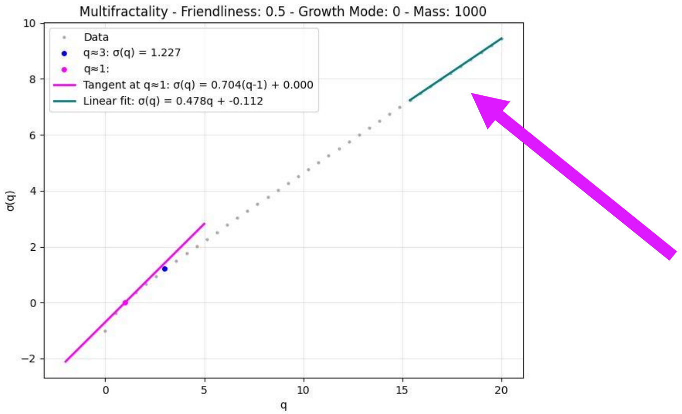


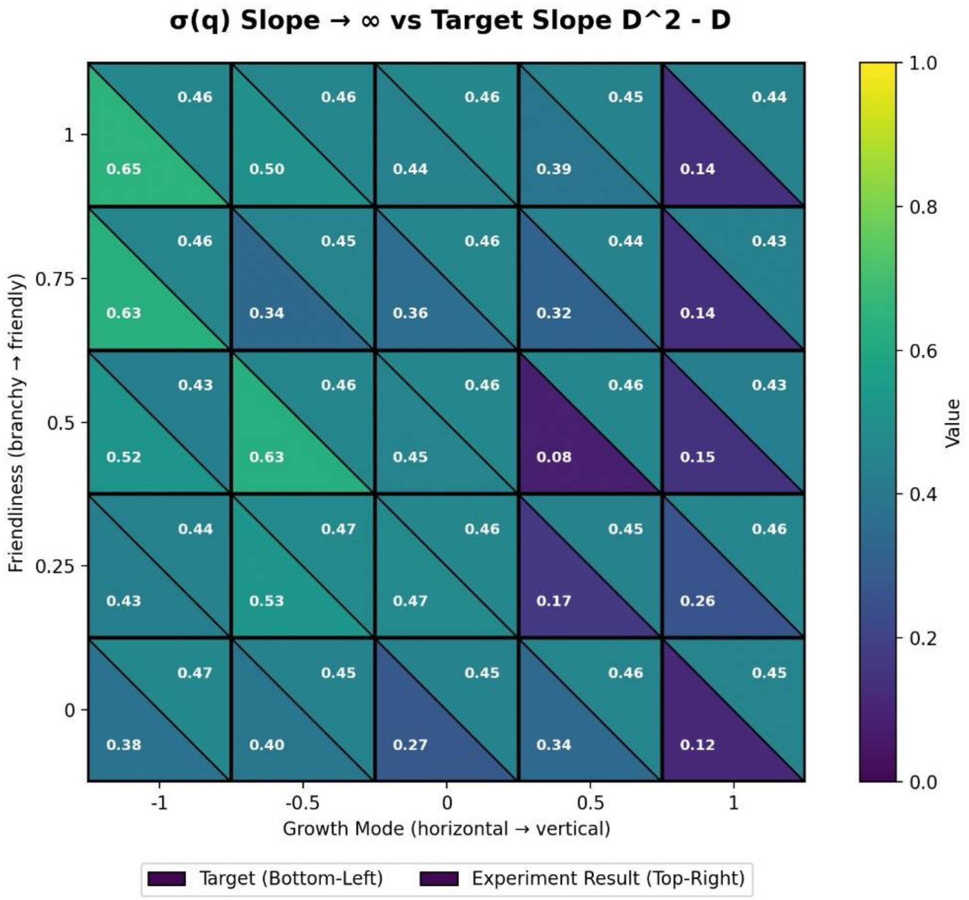
Scaling Relation #2: Behaviour for large q

- At large q, highest probabilities (tips) dominate
- Inequality:

$$\left.\frac{d\sigma}{dq}\right|_{q\to\infty} \leq D^2 - D$$

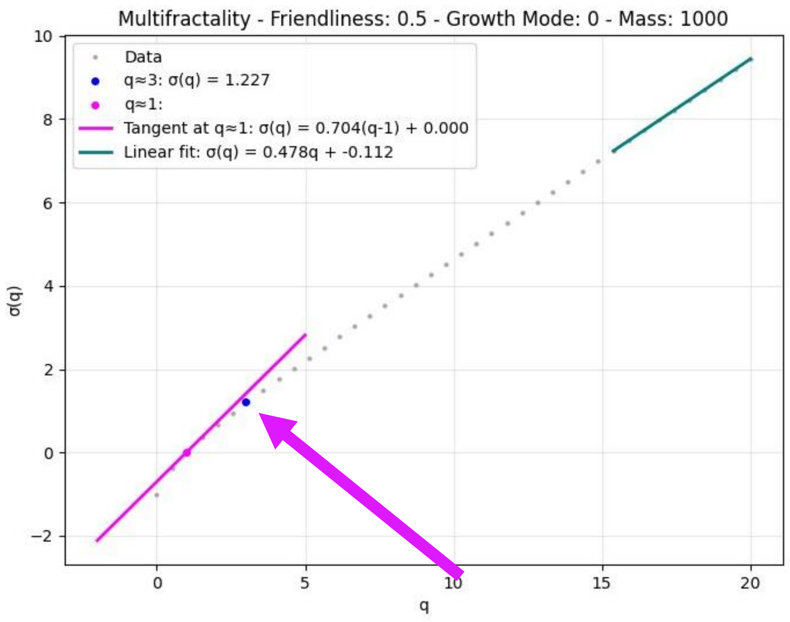
- Averaged over 5 seeds

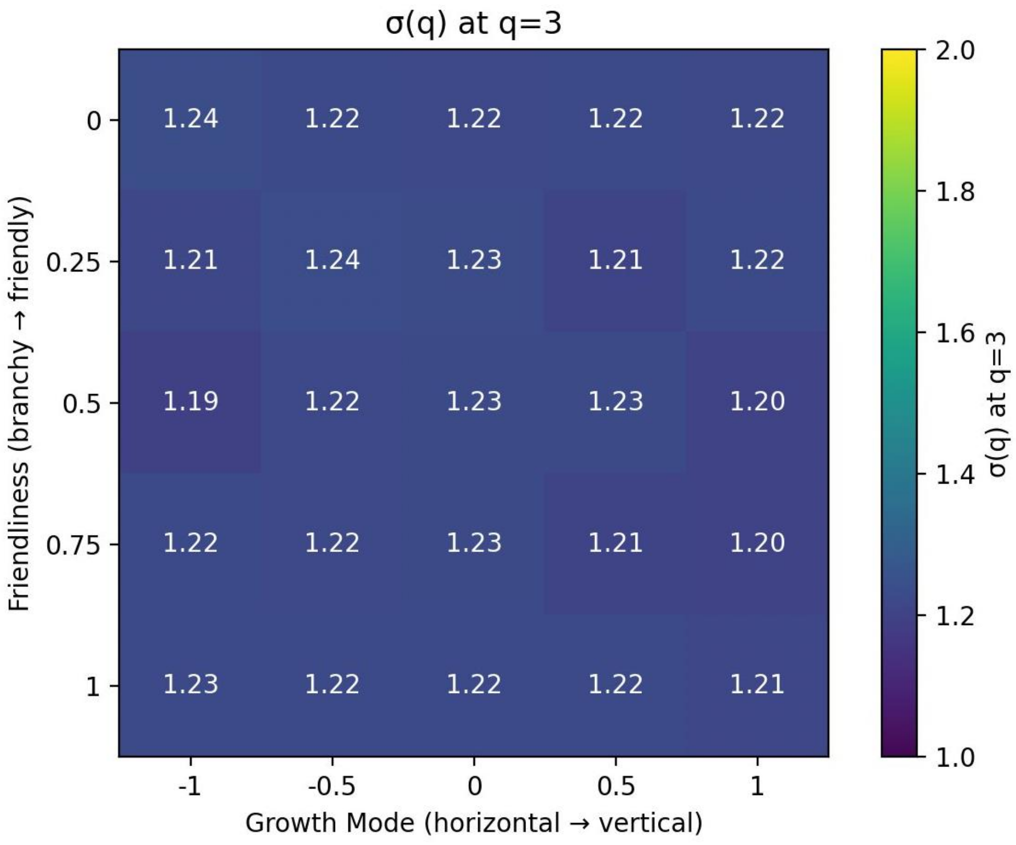




Scaling Relation # 3: Value at q=3

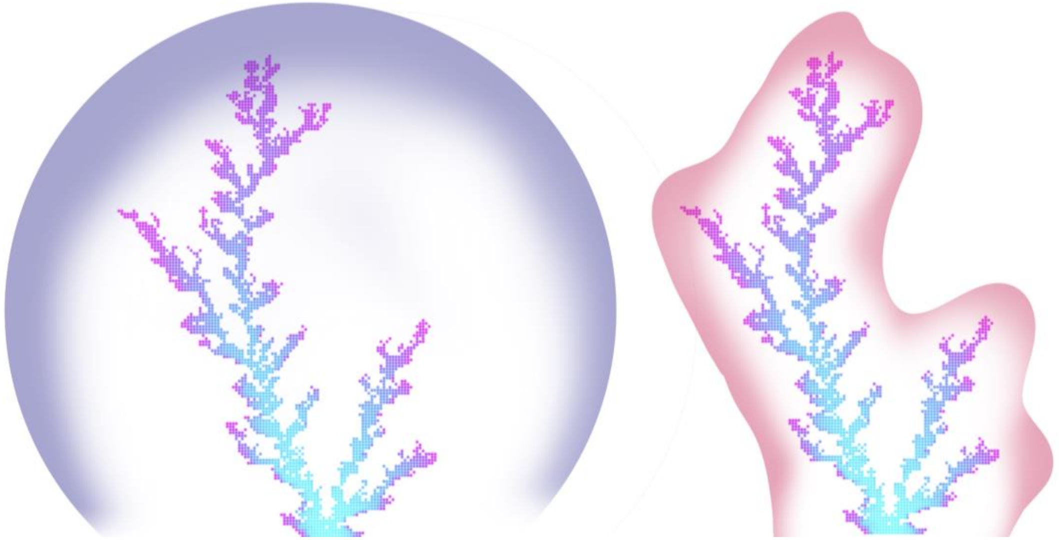
- related to electrostatics
- $\sigma(3) = 1$
- Averaged over 5 seeds





❌ Limitations

1) Only 2D simulations of 3D objects
2) More equal launch parameters, scaled to the cluster
3) Structure is highly dependent on the seed
4) No "blob" like growths
   - (outwards-in growth instead of inwards-out)



💡 Conclusion

1) Continuous phase transitions between shapes
2) Multifractality
3) Power law and scale-free
4) Mass vs. Radius scales as a power law

📅 Future Work

- **Coral bleaching:** stress parameter affecting growth probability
- **Resilience:** Are some structures more resistant to stress?

📚 References

- Llabrés E, Re E, Pluma N, Sintes T, Duarte CM. 2024. A generalized numerical model for clonal growth in scleractinian coral colonies. Proceedings. Biological sciences. 291(2030):20241327. doi:10.1098/rspb.2024.1327. http://dx.doi.org/10.1098/rspb.2024.1327.
- Halsey TC. 2000. Diffusion-Limited Aggregation: A Model for Pattern Formation. Physics Today. 53(11):36–41. doi:10.1063/1.1333284. http://dx.doi.org/10.1063/1.1333284.