GEORGIA INSTITUTE OF TECHNOLOGY, COLLEGE OF COMPUTING

CX4640: Homework 2

Nathan Korzekwa

October 2, 2013

PROBLEM 3.5

"The kind of man who would achieve that end, is at the end of achieving his own ends." – Some Pretentious Moron

PART A

The answer here is C. We can find this without computing the residual because a few quick multiplications reveals that $\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ is orthogonal to $span(A)$.

PROBLEM 3.17

$$\vec{v} = \vec{a} - \alpha\vec{e}_1$$
$$\alpha = -sign(v_1)\|\vec{v}\|$$
$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

This can be verified in MATLAB.

Problem 3.18

Part A

3, since there are 3 non-diagonal columns, and Householder makes each column a "diagonal" column by zeroing lower entries.

Part B

The first column of A becomes $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Part C

This column is untouched by the transformation since after that row has been zeroed, we work on the $n-1 \times n-1$ submatrix A'.

Part D

There are 6 Givens rotations required because each rotation zeros one position in the matrix, and there are 6 sub-diagonal non-zero entries in this matrix.

Problem 3.28

Part A

The product $(I - P_k)(I - P_{k-1})(I - P_{k-2})...$ can be rewritten as $I - P_k - P_{k-1} - P_{k-2} + P_k P_{k-1} + P_{k-1}P_{k-2} + P_k P_{k-1} P_{k-2}....$ Since the matrices $P_j for 0 < j \leq k$ are all rank-one matrices with each column being orthogonal to each other, $P^m P^n = 0 for m \neq n$, reducing the expression to $(I - P_k - ... - P_1)$.

Part B

This is equivalent to the classical Gram-Schmidt procedure because it can be rewritten as:

$$a_k - P_1 a_k - P_2 a_k - ... - P_{k-1} a_k$$

This clearly follows the logic behind Gram-Schmidt: you expand to the next dimension an orthogonal basis by subtracting the projection of a on to the current vectors in the basis.

0.1 Part C

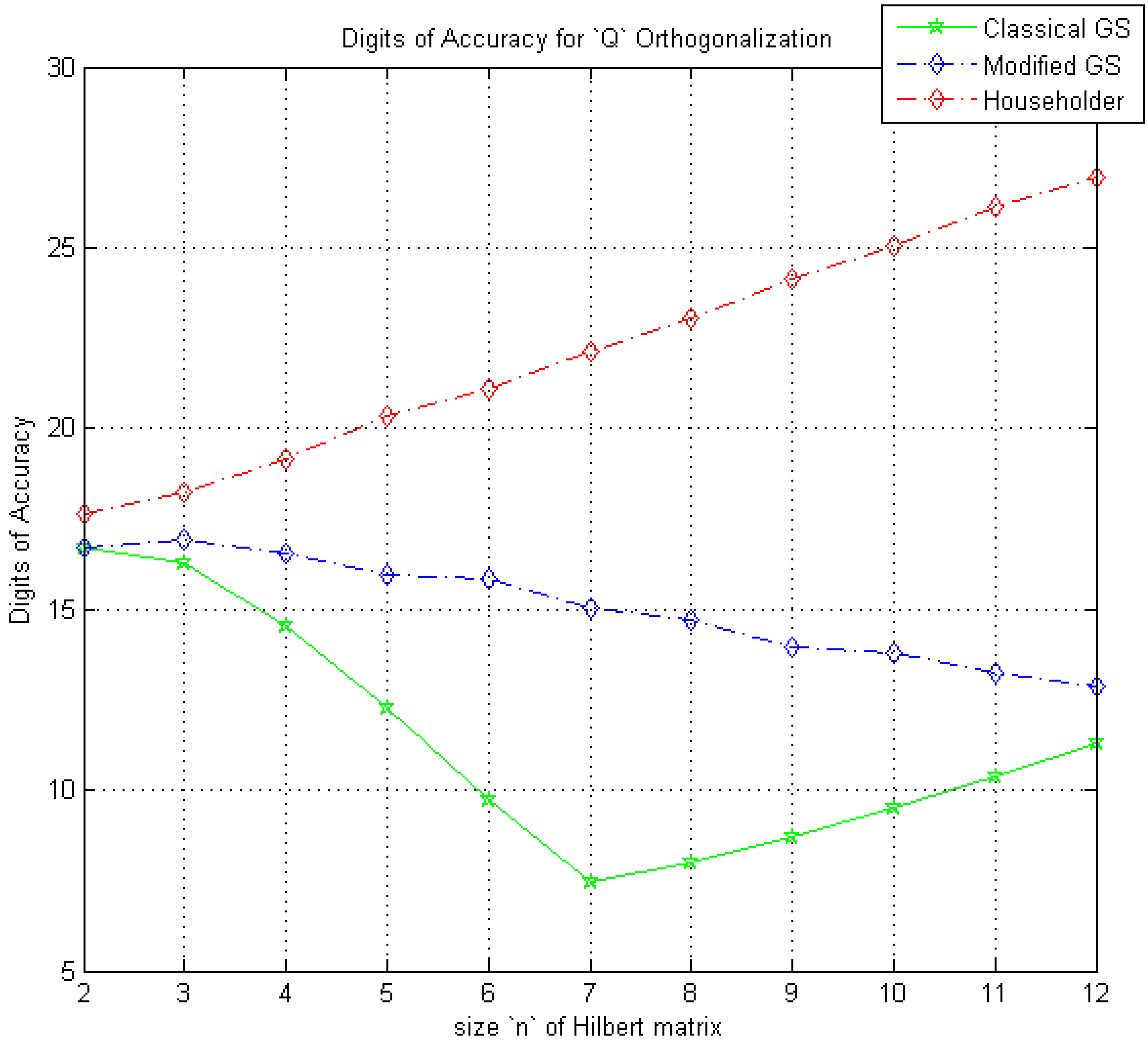
First, we note that $(I - P)$ yields a projection matrix onto a space orthogonal to the space P projects onto. From this, it is easy to see why it represents the Modified Gram-Schmidt process. In the MGS process, when you build the orthogonal basis of dimension n with n linearly independent vectors, when you add a vector to the basis, you first subtract the component of the vector being added from the remaining vectors to be added. So if we start at a_1, no vectors have been added yet, so a_1 remains unperturbed. But by the time we get to a_2, it has already had the component of a_1 subtracted off. Interestingly, subtracting off the component of a_1 from a_n for any n, is equivalent to projecting the vector a_n onto a space orthogonal to the space spanned by a_1. This can be computed with $(I - P_1)$. So by the time you start to add a_k, it has already been projected onto spaces orthogonal to a_1 thru a_{k-1} respectively, thus yielding: $(I - P_1)...(I - P_{k-1})a_k$.

0.2 Part D

The transformation in Part C is equivalent to the transformation in Part B, as shown in Part A. The formulation shown here is Equivalent to the ones in Part B and C because after one transformation by $(I - P_1 - P_2...P_n)$ produces a vector that is orthogonal to ALL of the spaces projected onto by P_1 thru P_n, and as such, transforming an already transformed vector $\vec{v}$ will result in $P_k\vec{v} = 0$ for all k, leaving the only nonzero term in the original formulation to be I. As a result, any powers of $m > 1$ are equivalent.

PROBLEM 3.12

PART A AND B



In observing the above Graph, it is clear that the accuracy of Householder is the best, followed by the Modified Gram-Schmidt process, followed by the Classical Gram-Schmidt Process. Computationally, they are all bound by the number of columns in the matrix. The Classical and Modified Gram Schmidt processes are equal in run-time since they simply do things in a different order. However, the Modified process allows you to modify the A matrix in-place as it goes along, eliminating the need for separate storage for Q.

Finally, we have householder – householder is $O(n^3)$ like G-S, and it also works on sets recursively smaller in size, so the runtime is more or less the same. However, it should be noted that it is more similar to the Modified G-S process in that it can modify the matrix in-place, eliminating the need for extra space for Q.