GEORGIA INSTITUTE OF TECHNOLOGY, COLLEGE OF COMPUTING

CX4640: Homework 2

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PROBLEM 2.7

"I don't wanna work, I don't wanna work; I just want to bang on this mug all day!" – Michael Scott

PART A

$$det(A) = 1-(1+\epsilon)(1-\epsilon) \tag{0.1}$$
$$= \epsilon^2 \tag{0.2}$$

PART B

So for the determinant to be greater than 0,

$$0 \le \epsilon < \epsilon_{mach} \tag{0.3}$$

PART C

$$A = \begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix}$$

$$M_1 A = \begin{bmatrix} 1 & 1+\epsilon \\ 0 & \epsilon^2 \end{bmatrix} = U$$

$$M_1 = \begin{bmatrix} 1 & 0 \\ -1+\epsilon & 1 \end{bmatrix}$$

$$M_1^{-1} = \begin{bmatrix} 1 & 0 \\ 1-\epsilon & 1 \end{bmatrix} = L$$

0.1 Part D

$$\begin{aligned} det(U) &= \epsilon^2 - 0 \\ &= \epsilon^2 \end{aligned}$$

$$0 \le \epsilon < \sqrt{\epsilon_{mach}}$$

Problem 2.21

$$\begin{aligned} \vec{x} &= B^{-1}(2A+I)(C^{-1}+A)\vec{b} \\ B\vec{x} &= 2AC^{-1}\vec{b} + 2A^2\vec{b} + C^{-1}\vec{b} + A\vec{b} \end{aligned}$$

From this conclusion, we can solve two linear systems since $x' = C^{-1}\vec{b}$ can be solved for with $Cx' = \vec{b}$, without computing any inverses. The implementation for this question can be found in the included file “q2.m”.

Problem 2.26

Part A

Because the columns of uv^T are multiples of each other, the only thing we need to worry about is annihilating one of the diagonals, and therefore degrees of freedom of the matrix. Ergo, for an $n \times n$ matrix

$$u_i v_i \ne 1; 0 \le i < n-1$$

Part B

By the Sherman-Morrison formula,

$$(A - \vec{u}\vec{v}^T)^{-1} = A^{-1} + A^{-1}\vec{u}(1 - v^T A^{-1} u)^{-1} v^T A^{-1}$$

$$(I - \vec{u}\vec{v}^T)^{-1} = I^{-1} + I^{-1}\vec{u}(1 - v^T I^{-1} u)^{-1} v^T I^{-1}$$

$$(I - \vec{u}\vec{v}^T)^{-1} = I + \vec{u}(1 - v^T u)^{-1} v^T$$

And since $1 - v^T u$ is a scalar,

$$(I - \vec{u}\vec{v}^T)^{-1} = I + \sigma\vec{u}v^T$$

where $\sigma = (1 - v^T u)^{-1}$.
So if we want to write $(I - \vec{u}\vec{v}^T)^{-1} = I - \sigma\vec{u}v^T$, we factor the negative into σ:

$$\sigma = (v^T u - 1)^{-1}$$

.

Part C

Yes, an elementary elimination matrix is, in fact, elementary because it differs from the identity matrix by only one column. Because of this, the "perturbation matrix" (the difference between the identity and the elimination matrix) is rank one since all of its columns are multiples of each other (linearly dependent).
Because of this, the elementary elimination matrix can be represented as:

$$M_k = I - \vec{m}\vec{e}_k^T$$

where

$$m = (0, \ldots, m_{k+1}, \ldots, m_n)^T$$

M_k eliminates all numbers below the kth row in a vector.

Problem 4

Using the 1-norm, both results produced low relative errors on estimating the condition number – with the random method working better on the first matrix A_1 and the iterative, deterministic method from part B working better on the second test matrix, A_2.
This is evidenced in the program output:
» q4
A1 rel error method 1: 0.486764
A1 rel error method 2: 0.644651
A2 rel error method 1: 0.201978
A2 rel error method 2: 0.482083
» q4
A1 rel error method 1: 0.486764

A1 rel error method 2: 0.066616
A2 rel error method 1: 0.201978
A2 rel error method 2: 0.451165
» q4 A1 rel error method 1: 0.486764
A1 rel error method 2: 0.410768
A2 rel error method 1: 0.201978
A2 rel error method 2: 0.296503

The code that runs these tests can be found in “q4.m”.