fa25-hw01

Probability

Fall 2025

DUE: As upload to Gradescope via Canvas by Friday, SEP-04 11:59pm.

Directions: In the problems below do not just write an answer to the question. You should also include an *explanation* of why the answer is what you gave. Answers without explanations may receive *no credit.*

In **1 thru 9** below consult pages 25-27 of the typed notes `lec-total-edit6`.

1. Problem 1

2. Problem 2

3. Problem 5

4. Problem 9

5. Problem 10
Assume all 165 players can play every position.

6. Problem 12

7. Problem 14

8. Problem 15

9. Problem 19

10.
(a) You toss a fair coin two times and note the outcome on each flip. How many distinct outcomes are possible? What's the probability of getting exactly one head?
(b) Another experiment: Now you toss the coin two times, and then hand the coin to a friend who also tosses it two times. How many outcomes are possible? What's the probability you each toss exactly one head? What's the probability you each toss the same number of heads?

11.
We have three urns labeled 1,2,3. In urn 1 are 1 red, 1 blue, 1 yellow ball. In urn 2 are 1 blue and 1 yellow ball. In urn 3 are 1 red and 1 blue ball.
(a) If one ball will be chosen from each urn, how many distinct outcomes are possible? Why is the basic counting principle applicable here?
(b) Treat the sample points as ordered 3-tuples (x_1, x_2, x_3), where x_i is the color (r, b, y) drawn from urn i. Write out the sample space Ω. Remember: Ω is a set!
(c) Assume that the balls are drawn uniformly at random from each urn. From the Ω you constructed in part (b), compute the probability that you draw exactly two balls of the same color. Why are we allowed to use the

classical probabiliy law here.

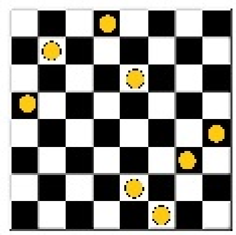
12. An urn contains five red, five black, and five white balls. If three balls are chosen without replacement at random, what is the probability that they are of exactly two different colors?

13. The letters in the word FULL are rearranged at random. What is the probability that it still spells FULL?

14. (Coincidence). Three families, each with three members, are lined up at random for a picture. What is the probability that members of each family happen to be together (that is, not separated by someone from another family) in the picture?

In a completely dark room with ten chairs, six people come and occupy six chairs at random. What is the probability that at least one of three specific chairs gets occupied?

15. In how many ways can eight pennies be placed on eight random squares of a (8×8 grid) chess board? What is the probability that no two pennies are in the same row or the same column? It may help to see such a configuration:



------------ for honors students only ---------------

h.1. Please recall the pigeon-hole principle from discrete math.
(a) The population of Danville is 20,000, and every person in this population has three initials. Can it be said with certainty that there must be two or more people in Danville with exactly the same three initials? Briefly explain.
(b) There are 10 people in a room whose ages are integers between 1 and 60 inclusive (of course, there can be repeated ages). Prove that there must exist two *disjoint* non-empty subsets of people in this room whose sums of ages are the same.
To illustrate (b) suppose the ages of the 10 people are 1,6,12,13,22,48,50,53,58,60. Then, for example, notice that the two non-empty disjoint subsets of people with ages $\{6, 53\}$ and $\{1, 58\}$ both have their ages summing to 59. Also, the two disjoint sets of people $\{6, 12, 13, 22\}$ and $\{53\}$ have their ages summing to 53.