3. Behavioural Modelling

David Pereira José Proença

MScCCSE 2021/22

CISTER – ISEP
Porto, Portugal

https://cister-labs.github.io/ramde2122

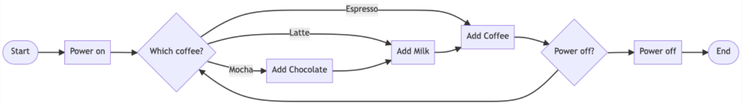
Overview

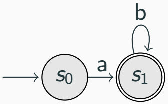
So far

- Class diagrams, boolean and 1st order logic, …

Next

- Look at UML behaviour diagrams
- Use a domain with a precise semantics
  - Non-deterministic finite automata (NFA)
  - Simple language for processes
  - Encode processes $\rightarrow$ NFA
  - Equivalence of processes





What are formal methods?

Formal methods are techniques to

model complex systems using

rigorous mathematical models

Specification

Define part of the system using a modelling language

Verification

Prove properties.

Show correctness.

Find bugs.

Implementation

Generate correct code.

All formal models are wrong

All formal models are wrong

... but some of them are usefull!

Syllabus

- High-level overview or requirements and associated processes
- Mathematical Preliminaries
  - Basic mathematical notations
  - Set theory
  - PropositionalLogic
  - First Order Logic
  - The Z3 automatic theorem prover
- Behavioural modelling
  - Single component
    - State diagrams and Flow charts
    - Formal modelling: Automata, Process Algebra in mCRL2
  - Many components
    - Communication diagrams and Sequence diagrams
    - Formal modelling: Process algebra with interactions
  - Equivalences
  - Verification

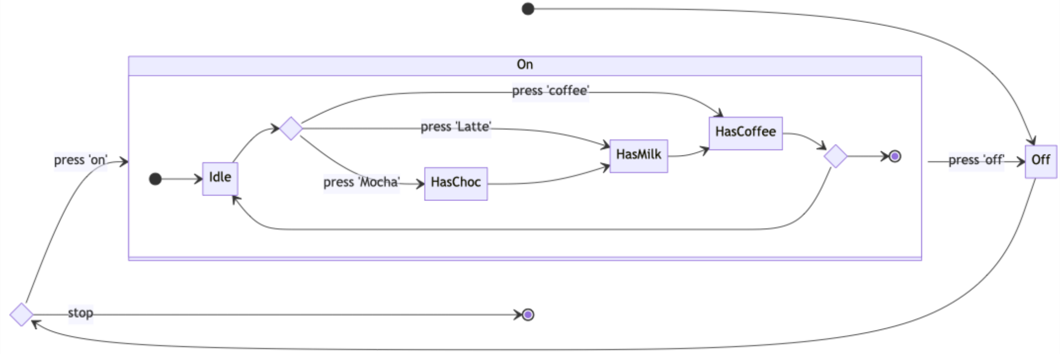
UML behaviour diagrams

UML behaviour diagrams

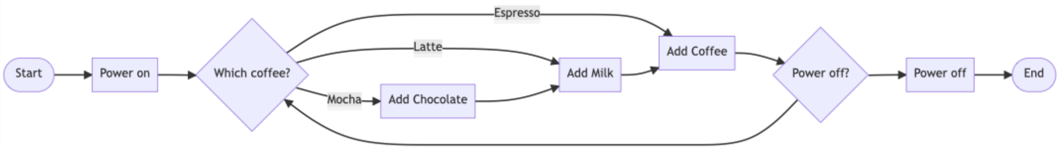
Describe the state of a component, what actions it can do, and how it evolves during its life cycle.

- State Diagram focus on states
- Flowchart focus on actions (also known as *activity diagrams*)

Coffee State Diagram



Coffee Flowchart



Used symbols: *processes*, *decisions*, and *start/end*

Other symbols include: data (or input/output), documents, connectors, comments

Automata – Basic definitions

Sequential and Reactive systems

Sequential systems

Meaning is defined by the results of finite computations

We start here...

Reactive systems

Meaning is determined by interaction and mobility of non-terminating processes, evolving concurrently

then we go reactive

Non-Deterministic Finite Automata (NFA)

Definition

A NFA over a set N of names is a tuple $\langle S, I, \downarrow, N, \longrightarrow\rangle$ where

- $S = \{s_0, s_1, s_2, \ldots\}$ is a set of states
- $I \subseteq S$ is the set of initial states
- $\downarrow \subseteq S$ is the set of terminating or final states

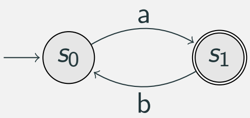
$$\downarrow s \;\equiv\; s \in \downarrow$$

- $\longrightarrow \;\subseteq\; S \times N \times S$ is the transition relation, often given as an N-indexed family of binary relations

$$s \xrightarrow{a} s' \;\equiv\; \langle s, a, s'\rangle \in \longrightarrow$$

Example

Example of an automaton



s_0 is an initial state

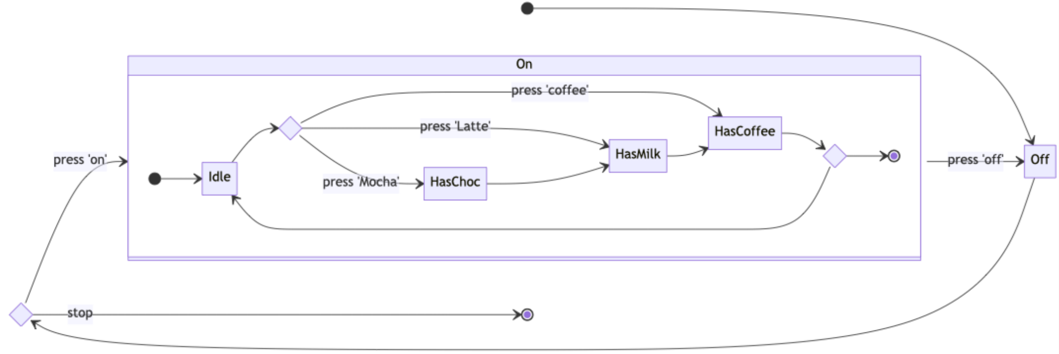
s_1 is a final state

(Formalise this automata)

Ex. 3.1: Formalise these automata as $\langle S, I, \downarrow, N, \longrightarrow\rangle$

Homework deadline: Exercises from this week should be submitted in a single PDF file until Sunday next week.

Exercise



Ex. 3.2: Draw LTS

(suggestion: by hand on a paper, and take a photo of it.)

Labelled Transition System

More generally, a NFA $\langle S, I, \downarrow, N, \longrightarrow\rangle$ is a labelled transition system (LTS) $\langle S, N, \longrightarrow\rangle$, where each state $s \in S$ determines a system over all states reachable from s and the corresponding restriction of $\longrightarrow$.

LTS classification

- deterministic
- non deterministic
- finite
- finitely branching
- image finite
- ...

Reachability

Definition

The reachability relation, $\longrightarrow^* \subseteq S \times N^* \times S$, is defined inductively

- $s \xrightarrow{\epsilon}^* s$ for each $s \in S$, where $\epsilon \in N^*$ denotes the empty word;
- if $s \xrightarrow{a} s''$ and $s'' \xrightarrow{\sigma}^* s'$ then $s \xrightarrow{a\sigma}^* s'$, for $a \in N, \sigma \in N^*$

Reachable state

$t \in S$ is reachable from $s \in S$ iff there is a word $\sigma \in N^*$ st $s \xrightarrow{\sigma}^* t$

Language of an Automaton

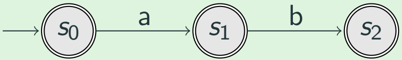
Language

A word σ is in the language L_A of an automata $A = \langle S, I, \downarrow, N, \longrightarrow \rangle$

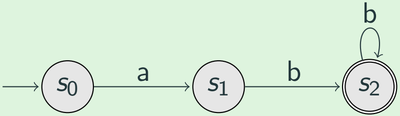
iff

there are states $s \in I$, $s' \in \downarrow$ such that $s \xrightarrow{\sigma}^* s'$.

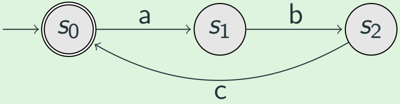
Ex. 3.3: What is the language of this automata?



Ex. 3.4: What is the language of this automata?



Ex. 3.5: What is the language of this automata?



Extra: Regular Expressions

Regular Expressions – syntax

- $w_1 w_2$: word w_1 followed by word w_2
- $w_1 + w_2$: word w_1 or word w_2
- a^*: 0 or more a's
- a^+: 1 or more a's
- ϵ: empty word

Examples

- $ab + c$: (a followed by b) or c
- $(ab)^*b$: b or abb or $ababb$ or ...
- $c((ab)^*b)^+$: cb or $cabb$ or $cababb$ or ...

Extra: Regular Expressions

Regular Expressions – syntax

- $w_1 w_2$: word w_1 followed by word w_2
- $w_1 + w_2$: word w_1 or word w_2
- a^*: 0 or more a's
- a^+: 1 or more a's
- ϵ: empty word

Examples

- $ab + c$: (a followed by b) or c
- $(ab)^* b$: b or abb or $ababb$ or ...
- $c((ab)^* b)^+$: cb or $cabb$ or $cababb$ or ...

NFA vs. Reg. Expr.

Word w expressible by a NFA $\Leftrightarrow$ w expressible by a Reg. Expr.

Process algebra

Sequential CCS - Syntax

$$\mathcal{P} \ni P, Q ::= K \mid \alpha.P \mid P + Q \mid \mathbf{0} \mid P[f] \mid P \backslash L \mid P|Q$$

where

- $\alpha \in N \cup \{\tau\}$ is an action
- K s a collection of process names or process constants
- I is an indexing set
- $L \subseteq N$ is a set of labels
- f is a function that renames actions s.t. $f(\tau) = \tau$
- notation:

  $[f] = [a_1 \mapsto b_1, \ldots, a_n \mapsto b_n]$

Process algebras

Syntax

$$\mathcal{P} \ni P, Q ::= K \mid \alpha.P \mid P + Q \mid \mathbf{0} \mid P[f] \mid P \backslash L \mid P|Q$$

Ex. 3.6: Which are NOT syntactically correct? Why?

$$a.b.A + B \qquad (1)$$

$$(a.\mathbf{0} + b.A) \backslash \{a, b, c\} \qquad (2)$$

$$(a.\mathbf{0} + b.A) \backslash \{a, \tau\} \qquad (3)$$

$$a.B + [b \mapsto a] \qquad (4)$$

$$\tau.\tau.B + \mathbf{0} \qquad (5)$$

$$a.(a + b).A \qquad (6)$$

$$(a.B + b.B)[a \mapsto a, \tau \mapsto b] \qquad (7)$$

$$(a.B + \tau.B)[b \mapsto a, a \mapsto a] \qquad (8)$$

$$(a.b.A + b.\mathbf{0}).B \qquad (9)$$

$$(a.b.A + b.\mathbf{0}) + B \qquad (10)$$

CCS semantics - building a NFA

$$\text{(act)}\quad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \text{(sum-1)}\quad \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \xrightarrow{\alpha} P_1'} \qquad \text{(sum-2)}\quad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1 + P_2 \xrightarrow{\alpha} P_2'}$$

$$\text{(res)}\quad \frac{P \xrightarrow{\alpha} P'}{P\backslash L \xrightarrow{\alpha} P'\backslash L}\ \alpha \notin L \qquad \text{(rel)}\quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

- Initial states: the process being translated
- Final states: all states are final
- Language: possible sequence of actions of a process

CCS semantics - building a NFA

$$\text{(act)}\quad \frac{}{\alpha.P \xrightarrow{\alpha} P}$$

$$\text{(sum-1)}\quad \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \xrightarrow{\alpha} P_1'}$$

$$\text{(sum-2)}\quad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1 + P_2 \xrightarrow{\alpha} P_2'}$$

$$\text{(res)}\quad \frac{P \xrightarrow{\alpha} P'}{P \backslash L \xrightarrow{\alpha} P' \backslash L}\ \alpha \notin L$$

$$\text{(rel)}\quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

Ex. 3.7: Build a derivation tree to prove the transitions below

1. $(a.A + b.B) \xrightarrow{b} B$
2. $(a.b.A + (b.a.B + c.a.C)) \xrightarrow{b} a.B$
3. $((a.B + b.A)[a \mapsto c])\backslash\{a, b\} \xrightarrow{c} B$

Exercise

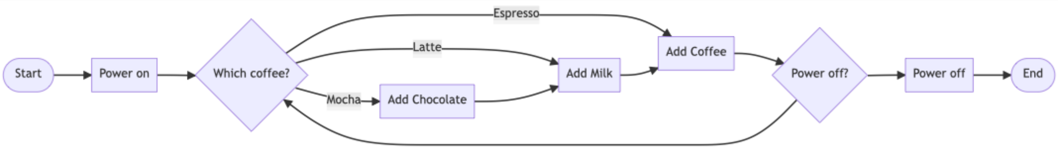
Ex. 3.8: Draw the automata

$$CM = \text{coin.coffee.}CM$$
$$CS = \text{pub.(coin.coffee.CS} + \text{coin.tea.CS)}$$

Ex. 3.9: What is the language of this process?

$$A = \text{goLeft.A} + \text{goRight.B}$$
$$B = \text{rest.}\mathbf{0}$$

Exercise



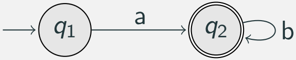
Ex. 3.10: Write the process of the flowchart above

$$
\begin{aligned}
P &= \mathsf{powerOn}.Q \\
Q &= \mathsf{selMocha}.\mathsf{addChocolate}.Mk + \mathsf{selLatte}.Mk + \ldots \\
Mk &= \mathsf{addMilk}\ldots
\end{aligned}
$$

Concurrent Process algebra

Overview

Recall

1. Non-deterministic Finite Automata: 

2. (Sequential) Process algebra: $P = a.Q \qquad Q = b.Q$
3. Meaning of (2) using (1)

Still missing

- **Interaction** between processes
- *Interaction diagrams* vs. *interacting processes*
- Enrich (2) and (3)

CCS - Updated Syntax

$$\mathcal{P} \ni P, Q ::= K \mid \alpha.P \mid P + Q \mid \mathbf{0} \mid P[f] \mid P\backslash L \mid P|Q$$

where

- $\alpha \in N \cup \overline{N} \cup \{\tau\}$ is an action
- K s a collection of process names or process constants
- I is an indexing set
- $L \subseteq N$ is a set of labels
- f is a function that renames actions s.t. $f(\tau) = \tau$ and $f(\bar{a}) = \overline{f(a)}$
- notation:

$$[f] = [a_1 \mapsto b_1, \ldots, a_n \mapsto b_n] \quad \text{where } a_i, b_i \in N \cup \{\tau\}$$

Syntax

$$\mathcal{P} \ni P, Q ::= K \mid \alpha.P \mid P + Q \mid \mathbf{0} \mid P[f] \mid P\backslash L \mid P|Q$$

Ex. 3.11: Which are syntactically correct?

$$a.\overline{b}.A + B \quad (11)$$
$$(a.\mathbf{0} + \overline{a}.A)\backslash \{\overline{a}, b\} \quad (12)$$
$$(a.\mathbf{0} + \overline{a}.A)\backslash \{a, \tau\} \quad (13)$$
$$(a.\mathbf{0} + \overline{\tau}.A)\backslash \{a\} \quad (14)$$
$$\tau.\tau.B + \overline{a}.\mathbf{0} \quad (15)$$
$$(\mathbf{0}|\mathbf{0}) + \mathbf{0} \quad (16)$$
$$(a.B + b.B)[a \mapsto a, \tau \mapsto b] \quad (17)$$
$$(a.B + \tau.B)[b \mapsto a, b \mapsto a] \quad (18)$$
$$(a.B + b.B)[a \mapsto b, b \mapsto \overline{a}] \quad (19)$$
$$(a.b.A + \overline{a}.\mathbf{0})|B \quad (20)$$
$$(a.b.A + \overline{a}.\mathbf{0}).B \quad (21)$$
$$(a.b.A + \overline{a}.\mathbf{0}) + B \quad (22)$$

CCS semantics - building an NFA

$$\text{(act)}\quad \frac{}{\alpha.P \xrightarrow{\alpha} P}$$

$$\text{(sum-1)}\quad \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \xrightarrow{\alpha} P_1'}$$

$$\text{(sum-2)}\quad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1 + P_2 \xrightarrow{\alpha} P_2'}$$

$$\text{(res)}\quad \frac{P \xrightarrow{\alpha} P'}{P\backslash L \xrightarrow{\alpha} P'\backslash L}\ \alpha \notin L$$

$$\text{(rel)}\quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

$$\text{(com1)}\quad \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$$

$$\text{(com2)}\quad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$$

$$\text{(com3)}\quad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

CCS semantics - building an NFA

$$\text{(act)}\quad \frac{}{\alpha.P \xrightarrow{\alpha} P}$$

$$\text{(sum-1)}\quad \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \xrightarrow{\alpha} P_1'}$$

$$\text{(sum-2)}\quad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1 + P_2 \xrightarrow{\alpha} P_2'}$$

$$\text{(res)}\quad \frac{P \xrightarrow{\alpha} P'}{P\backslash L \xrightarrow{\alpha} P'\backslash L}\ \alpha \notin L$$

$$\text{(rel)}\quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

$$\text{(com1)}\quad \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$$

$$\text{(com2)}\quad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$$

$$\text{(com3)}\quad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

Ex. 3.12: Draw the NFAs

$$CM = \text{coin}.\overline{\text{coffee}}.CM$$

$$CS = \text{pub}.\overline{\text{coin}}.\text{coffee}.CS$$

$$SmUni = (CM|CS)\backslash\{\text{coin}, \text{coffee}\}$$

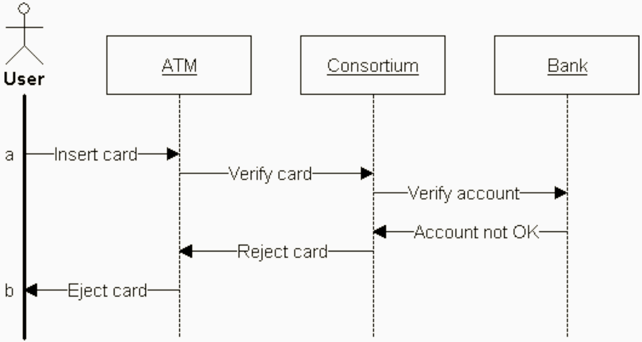
Exercises

Ex. 3.13: Let $A = b.a.B$. Show that:

1. $(A \mid \overline{b}.\mathbf{0})\backslash\{b\} \xrightarrow{\tau} (a.B \mid \mathbf{0})\backslash\{b\}$
2. $(A \mid b.a.B) + (b.A)[a/b] \xrightarrow{a} A[a/b]$

Sequence Diagrams vs. Interactive Processes

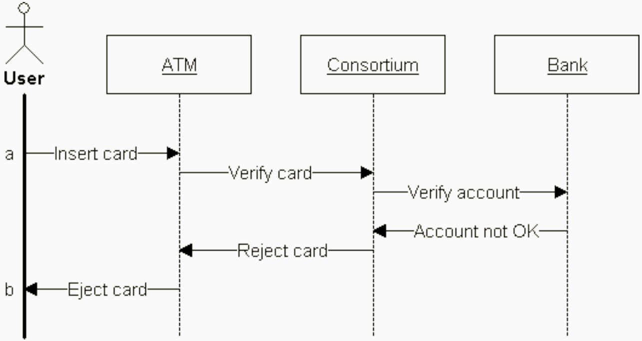
Sequence Diagrams as Interactive Processes



- Objects as Processes (e.g.,processes U, A, C, B)
- Send actions (e.g., $insertCard$)
- Reveive actions (e.g., $\overline{insertCard}$)

- Unique action for each object pair
- Do not write $(\ldots + \mathbf{0})$

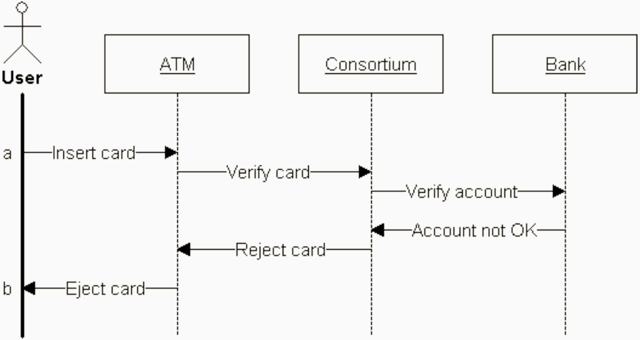
Language of Sequence Diagrams, Informally



This example has only 1 word and its prefixes

$$\mathrm{Tr}(sd) = \textit{insertCard} \cdot \textit{verifyCard} \cdot \textit{verifyAccount} \cdot \textit{accountNotOK} \cdot \textit{rejectedCard} \cdot \textit{ejectCard}$$

Sequence Diagrams as Interactive Processes



Ex. 3.14: Write an interactive processes that acts as above

$$Sys = (U|A|C|E)\backslash \ldots$$
$$U = insertCard.\overline{ejectCart}.\mathbf{0}$$
$$A = \ldots$$
$$C = \ldots$$
$$E = \ldots$$