Individual Exercises
Set Theory, Propositional Logic, and First Order Logic

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RAMDE – 2021/2022

Rationale

This exercises sheet is made of three questions, each covering one of the topics taught during the first module of RAMDE. For each of the questions, you are asked to select a subset of its exercises for evaluation (the precise number of exercises for you to select is specified in the question)..

What to Submit

A PDF/word report (preferable PDF) containing your answers to the evaluation exercises. You can do the exercises with pen and paper and then scan/take a photo, and include them in the report file. The name of your file must have the following pattern:

RAMDE-individual-*<student number>*.*<file extension>*

Deadline of Submission

The file with your answers must be submitted until 23h59PM of the 28th of November, 2021. You can do the submission either by sending an email to `drp@isep.ipp.pt` or upload the file (with the format described abobe) in RAMDE's Teams environment, in the `"Individual Exercises"` folder.

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1 Set Theory

Exercise 1. Consider the following sets: $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, and $D = \{2, 4, 6, 8\}$. The goal of this exercise is for you to calculate the sets resulting from performing the defined set operations. **Select four** of the questions below for evaluation.

(a) $(A \cup B) \cap C$

(b) $A \cup (B \cap C)$

(c) $(A \cap B) \cup (C \cap D)$

(d) $(A \cup B) \cap (C \cup D)$

(e) $(A \cup B) \setminus C$

(f) $A \cup (B \setminus C)$

(g) $(A \setminus B) \setminus C$

(h) $A \setminus (B \setminus C)$

(i) $(B \setminus C) \setminus D$

(j) $D \setminus (B \setminus C)$

2   Propositional Logic

Exercise 2. Provide the proofs, using natural deduction for Propositional Logic, of the following valid formulas. **Select three** of the exercises for evaluation.

(a) $(\delta \land \neg\varphi) \rightarrow \psi, \neg\psi, \delta \vdash \varphi$

(b) $\vdash (\delta \rightarrow \psi) \rightarrow ((\neg\delta \rightarrow \neg\psi) \rightarrow (\varphi \rightarrow \psi))$

(c) $\vdash (\delta \rightarrow (\varphi \rightarrow \psi)) \rightarrow (\varphi \rightarrow (\delta \rightarrow \psi))$

(d) $\vdash \delta \rightarrow (\neg\varphi \rightarrow \neg(\delta \rightarrow \varphi))$

(e) $\vdash (\delta \rightarrow \varphi) \rightarrow (\neg\delta \lor \varphi)$

(f) $(\psi \rightarrow \varphi) \lor (\delta \rightarrow \gamma) \vdash (\psi \rightarrow \gamma) \lor (\delta \rightarrow \varphi)$

(g) $\psi \vdash (\psi \land \varphi) \lor (\psi \land \neg\varphi)$

3   First Order Logic

Exercise 3. Provide the proofs, using natural deduction for First Order Logic, of the following valid formulas. **Select two** of the exercises for evaluation.

(a) $\forall x \forall y R(x, y) \vdash \forall y \forall x R(x, y)$

(b) $\exists x \forall y P(x, y) \vdash \forall y \exists x P(x, y)$

(c) $\forall x \forall y (x = y \rightarrow f(x) = f(y))$

(d) $\exists x (P(x) \rightarrow R(x)) \rightarrow (\forall x P(x) \rightarrow \exists x R(x))$

(e) $\forall x (S(x) \rightarrow R(x)) \rightarrow (\exists x \neg R(x) \rightarrow \exists x \neg S(x))$

(f) $\forall x (\neg P(x) \rightarrow S(x)) \rightarrow (\exists x \neg S(x) \rightarrow \exists x P(x))$