1. Multivariate Gaussian Distribution

1.1 Affine Transformation of Gaussian Random Variables

Two random variables y_1 and y_2 are generated from independent normal distributed random variables $x_1 \sim \mathcal{N}(0, 1)$ and $x_2 \sim \mathcal{N}(0, 1)$ by

$$\begin{aligned} y_1 &= x_1 - 2x_2 \\ y_2 &= x_1 + x_2. \end{aligned}$$

a) Determine the probability density function (PDF) of the random variable $\boldsymbol{y} = [y_1,\ y_2]^{\mathrm{T}}$.

b) The random variable $\boldsymbol{z}$ is generated from $\boldsymbol{x}$ by $\boldsymbol{z} = \boldsymbol{B}\boldsymbol{D}\boldsymbol{x}$, where

$$\boldsymbol{B} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix},$$

and $\alpha \in [0, 2\pi[$ is a fixed and given angle. Determine the PDF of the random variable $\boldsymbol{z}$ and compare it to the PDF of $\boldsymbol{y}$.

In the following, let $x_1 \sim \mathcal{N}(3, 1)$ and $x_2 \sim \mathcal{N}(1, 2)$ with x_1 and x_2 being independent.

c) Draw contour lines of the PDFs of $\boldsymbol{x}$ and $\boldsymbol{y} = \boldsymbol{B}\boldsymbol{x}$.

d) Determine the conditional PDF of y_1 given $y_2 = a$.