Fake news detection using discourse segment structure analysis

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Abstract— Online news platforms greatly influence our society and culture in both positive and negative ways. As online media becomes more dependent for source of information, a lot of fake news is posted online, that widespread with people following it without any prior or complete information of event authenticity. Such misinformation has the potential to manipulate public opinions. The exponential growth of fake news propagation have become a great threat to public for news trustworthiness. It has become a compelling issue for which discovering, examining and dealing with fake news has increased in demand. However, with the limited availability of literature on the issue of uncovering fake news, a number of potential methodologies and techniques remains unexplored. The primary aim of this paper is to review existing methodologies, to propose and implement a method for automated deception detection. The proposed methodology uses deep learning in discourse-level structure analysis to formulate the structure that differentiates fake and real news. The baseline model achieved 74% accuracy.

Keywords—fake news detection, news content model; social context model, discouse parsing fake news detection

I. INTRODUCTION

Social media platforms greatly influence our society and culture in both positive and negative ways [1]. As online media becomes more dependent for source of information, we tend to get a lot of fake news that widespread with people following it without any prior or complete information of the event authenticity. The increasing growth of fake news propagation has become a great threat to public trust for news trustworthiness. So the need of discovering, examining and dealing with fake news has increased in demand. However, it faces many challenges such as limited literature and dataset availability. This paper proposes to inculcate a method for detecting fake news centered on the idea of a news articles' structural representation. The main idea is to detect fake news by justifying some structure-related properties that explains the structure formulated for different type of news. Most of the existing methods rely heavily upon existence of a detailed corpus to segregate fake and real news, which is very difficult to define for fake news.

Existing methodologies might be biased to underestimate the characteristics of the way in which content is presented in a news article. Use of discourse-level structure analysis for style-based detection would help in automatic fake news detection.

The rest of the paper is structured as follows: in section II, background study is carried out on the broad spectrum of fake news detection approaches. In section III, literature survey is done on most of the latest work done for detecting fake news. Section IV gives a formal definition of the problem statement for proposed methodology and deep leaning approach for structure parsing. The results obtained from the experimental setup is explained in section V and section VI gives future insights to this research.

II. BACKGROUND STUDY

There has not been a specific definition of fake news since a news is considered fake depending on the way it is perceived. There are different categories into which a news article may fall:

Table 1: Categories of Fake news [2]

	Authenticity	Intention	News?
Maliciously false			
News	FALSE	Bad	Yes
Fake News	FALSE	Unknown	Yes
Satire News	Unknown	Not Bad	Yes
Disinformation	FALSE	Bad	Unknown
Misinformation	FALSE	Unknown	Unknown
Rumor	Unknown	Unknown	Unknown

Fake news can be simply termed as false news. It can be considered as deliberate misinformation that spreads through the traditional or online news media in order to mislead public, cause damage of any kind or for financial gains. Fake news generally has two main features: intent and authenticity that can classify categories of fake news as shown in table 1.

Fake news detection is done based on the perspective of fake news via main models:

- 1) News Content Model
- a) Knowledge based detection The informative content presented in the news article
- b) Style based detection Writing style and linguistic pattern in the content
 - 2) Social Context Model
 - a) The propagation pattern of the news
- b) The credibility of the creators and the propagators of the news

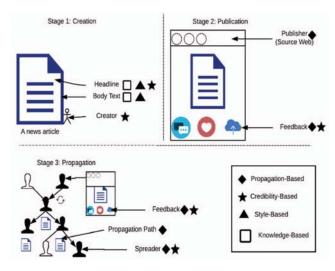


Figure 1: Fake news detection perspectives in its news life cycle [2]

Fake news can be detected via the content presented within the article or by the social media metadata it carries when it is published and propagated online. Multiple techniques can be applied to any of the above-mentioned models for detection. These models are further explained below.

A. Knowledge based detection

This style of fake news detection sounds mostly manual and deals with true fact checking of the news conveyed in the article. The primary purpose of knowledge based detection is fact-proving in two ways – traditional fact checking and automated fact checking. This aims for assessing the authenticity of news by comparing the knowledge extracted from news article to be verified with actual events.

Manual or traditional fact checking can be done via two sources by expert checking and via crowd sourced fact checking. Expert checks are mostly done via small group of trustworthy fact checkers such as PolitiFact [3], Factcheck [4] and many others that give accurate results. These are costly and non -scalable for handling great volume of incoming news traffic. On the other hand, crowd sourced fact checking relies on participation of large number of individuals acting as fact checkers. Crowd sourced fact checking happens to be more scalable than expert fact checking but it is less credible and accurate due to bias of fact checkers to one side or another.

Automated fact checking has the ability to deal with huge volume of to be verified news and relies heavily on information retrieval, natural language processing and to some extent on network theory as well. Automated fact checking is divided into two steps knowledge base construction and knowledge comparison. Here, knowledge means representation of information in a set of attributes extracted from the knowledge.

B. Style based detection

In this technique, a news is denoted as a set of machine learning features that are able to identify fake news and separate it from real news. It aims to focus on the issue that a fake news article is deceptive and this deception needs to be detected in order to separate it from real news. It captures the disorientations in the writing style in the content of the news article. This form of detection methodology can be categorized into objectivity oriented and deception oriented.

Approaches related to capture the objective orientation of a news article indicate the non-neutrality and partial opinion of an event to mislead the readers such as biased writings and yellow journalism. Biased writings present a greatly favored opinion towards a particular group, which can be detected using linguistic features that are even able to differentiate satire from mainstream as well as from hyperpartisan [5]. Yellow journalism highlights display of not so well examined news but acts as clickbait with eye catchy headlines to raise curiosity and hostility, a sudden urge to make sensitive readers anxious and magnify their fear of danger to their beliefs. Content style that aims to deceive the readers with exaggerated expressions and strong annotations is different from that of a truly factual news.

C. Social Context Model for fake news detection

The networking and sharing features of social media platforms give extra information like social engagements in analysis thus making fake news detection perceivable a multiple different angles. Again, social engagements that can deliver the metadata about reception of the news by the readers such as comments, can be identified by majorly two methods propagation based and credibility based.

Propagation based techniques aim to find out the relations between several news propagated on the social media and track it to the original news. Credibility based techniques use relationships between news articles and other components such as users, publishers and posts. For example, news published by unreliable websites or forwarded by unreliable users has a very high chance to be fake than the news originating from respected and credible users. Fake news can be identified using credibility for the following components of a propagated news: headlines, comments, source and propagators.

III. LITERATURE SURVEY

Table 2 below highlights the recent journal articles, conference proceedings and research papers published relating to various methodologies for fake news detection. The search results are restricted to be published from or after the year 2016. The table summarizes the technique adopted and its performance measure in success of distinguishing fake news from real news for every paper. Most of these methods are related to use of sentiment analysis and NLP while others consider some other features like discourse and pragmatic analysis of the text. It was observed that the success of the classification was also dependent on the machine learning model chosen/formulated and trained for the purpose.

Table 2: Literature Survey, papers taken after 2016

AUTHOR	OUTCOME	CONCLUSION/ TECHNIQUE USED
Monu Waskale, Prof. Pritesh Jain, 2019 [6]	GRU-2 can identify bits of gossip with exactness 83.9% for twitter within 12-hours.	Rational investigation of large portion of existing techniques identified from the earlier bits of gossip using GRU-2 and Tanh-RNN classifier for comparison.
Cody Buntain, Jennifer Golbeck, 2018 [7]	66.93% and 70.28% in PHEME and CREDBANK	Automated system for distinguishing fake news in famous Twitter threads.
Kursuncu et al., 2018 [8]	Analysis resulting in 73% F score. Supervised learning F score of 88.2 and ROC of 94.3.	Discussed Sentiment predictive analysis by time series in different domains.
Jiawei Zhang, Bowen Dong, Philip S. Yu, 2019 [9]	Comparing with existing systems, Bi-Class Inference Results in 14.5% higher and 40% higher in multi-Class Inference.	A deep diffusive network model to learn the depictions of news articles, subjects and creators.
Ruchansky et al., 2017 [10]	Positive correlation of 0.867 for Weibo, 0.631 for Twitter.	Propose the CSI model which consists of three modules.
Monther Aldwairi, Ali Alwahedi ,2018 [11]	99.4% accuracy	Clickbait Detection using Logistic Classifier
Traylor et al. ,2019 [12]	Classifier Accuracy: 0.69 Overall Classifier Error:0.31.	ML-classification for large fake news documents with one extraction feature.
Oshikawa et al. ,2018 [13]	Maximum accuracy in 94.4% using GCN model. Series of methods	Comparison of existing fake news detection models
Gurav et al. ,2019 [14]	accomplished by Machine Leaning	Pure NLP perspective towards false news detection

A 11	A	A 1 11 1 1
Agarwalla	Accuracy in Naïve	An algorithm have been
et al. ,2019	Bayes classifier	explored that can
[15]	with lid stone	distinguish the difference
	smoothing is 83%	between the fake and true
	and in Naïve	news
	Bayes (without lid	
	stone smoothing)	
	is 74%.	
Zellers et	Grover-Large	Investigated the threats
al. ,2019	yields 78%	posed by adversaries
[16]	accuracy, 92%	seeking to spread
	when dataset is	disinformation and the
	increased.	possibilities of machine
		generated fake news.
Sivasangar	Precision: 0.86	Rumor Detection by lever
i V et al.	F1-score: 0.86	maturing the setting
,2018 [17]		going before a tweet with
		a consecutive classifier.
O'Brien et	Accuracy: 93.5%	Deep neural networks to
al. ,2018	± 0.2.	capture steady
[18]		differences in the
[10]		language of fake and real
		news: signatures of
		exaggeration and other
		forms of rhetoric.
Shu et al.	Studying existing	Datasets, evaluation
,2017 [19]	literature in two	metrics, and promising
,2017 [13]	segments:	future ways in fake news
	detection and	detection discussed.
	characterization.	detection discussed.
Silva et al.	ANN achieving	An amalgamation of
,2019 [20]	75% accuracy.	classic techniques with
,2017 [20]	7370 accuracy.	neural network.
		neural network.
Dong et al.	Detect fake news	Deep semi-supervised
,2019 [21]	from PHEME	learning model by
	datasets using	constructing two-path
	labeled data and	CNN
	unlabeled data.	
Yang et al.	88.063% accuracy	Soft labels are used to
,2019 [22]		fine-tune NLI models,
, []		BERT, and the
		Decomposable Attention
		model. NLI models are
		trained independently
		and ensemble with a
		BERT model to define
		the soft labels.
		the cott lebels

Monti et al. ,2019 [23] Ajao et al. ,2018 [24]	More than 93% accuracy achieved by automatic fake news detection model built on geometric deep learning. 82% accuracy via both text and images by automatic	.The underlying core algorithms allow for a fusion of dissimilar data such as content, profile, activity of a user, social graph and propagation pattern of news, which is achieved by generalizing CNN to graphs. A hybrid deep learning model of LSTM and CNN models is used.
	identification of features within Twitter posts.	
Thota et al. ,2018 [25]	94.21% accuracy. The Dense Neural Network beats existing model architectures by 2.5%.	A finely tuned TF-IDF Dense neural network architecture to predict the stance between a given pair of headline and article body.
Helmstette r, Heiko Paulheim ,2018 [26]	F1 score of 0.9 achieved	Trustworthy or untrustworthy source are used to automatically label the data during collection, and train a classifier on this dataset.
Atodiresei et al. ,2018 [27]	Tweet score can be 1000, -500 or in [-50,100] User score can be in [0,12]	Credibility score from hashtag sentiments, emoji sentiments, text sentiments and named- entity recognition. Higher the credibility score, higher the trust
Hamid Karimi, Jiliang Tang ,2019 [28]	82% accuracy	Hierarchical Discourse level structural data analysis for fake news detection. A structure is trained on the dataset, quantifiable properties of which are used for classification process in the model.
Shuo Yang et al., 2019 [29]	Graphical model is built taking into account reliability of the news and credibility score	Unsupervised method is investigated. Opinion is extracted from hierarchy social engagement information acquired

	of the user. 75.9%	from social media users
	max. Accuracy	Reality and credibility is
	accomplished on	considered by an efficient
	LIAR dataset.	Gibbs-sampling method.
Zhou et al.	Models so far	Simply looking into
,2018 [30]	possess a greater	Linguistic aspects is not
	possibility to	enough for fake news
	misclassify fake	detection.
	news that tampers	
	with facts as well	
	as under-written	
	real news articles.	
Álvaro and	93% accuracy	BERT, LSTM and
Lara,2019	with superior	Convolutional Neural
[31]	metrics compared	Network models are
	to other deep	trained based merely on
	learning models	textual features.

IV. PROPOSED METHODOLOGY

Discourse structure is the connection between surface features and document-level properties, which remains unseen and underestimated most of the times [32]. This paper proposes a plan to make use of discourse level analysis for deception detection of news documents to make a hierarchical structure that will have distinguishable properties associated with fake and real news articles. These properties can be used to differentiate between the two.

Given a news article β consisting of various creator features that describe the content features containing a set of attributes that represent the news item, the job of fake news detector is to predict whether the news article β is fake or not, i.e.,

$$\tau(\beta) = \{1, \text{ if } \beta \text{ is fake news article,} \\ 0, \text{ elsewise}\} \qquad \dots (1)$$

Where τ is the representation of the prediction function in Eq.(1). The model would take news event by means of an input and based on the sequence of information provided in the news article predicts news being fake or real.

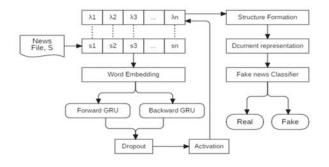


Figure 2: Framework for Fake news detection using document structural analysis

Evaluation of properties learnt from classification and analysis, will lead to describing news existing to be fake or real. Discourse level structure parsing of the news article is preferred for better understanding of how fake news articles are structures since the meaning of a sentence depends on the sentence that precedes it and may influence the meaning of the sentence succeeding it. For this, discourse dependency parsing approach is used to create a dependency tree structure similar to (Liu and Lapata, 2018 [33]).

Forward and backward Gated Recurrent Unit (GRU) (Kyunghyun Cho et al, 2014 [34]) is used to create Bidirectional GRU and get the representation of a sentential clause in a news document. Next, making use of inter-sentential matrix to get discourse level relationships in automated manner, a dependency tree is completed (Karimi and Tang, 2019 [28]). This dependency tree is going to be the structural representation of the news document (Liu and Lapata, 2018 [33]) It is done by computing parent child properties of each sentence using activation function.

The classifier model attained after achieving document level representation of the news document can be trained for binary classification using Eq.(2) and Eq.(3):

$$\chi i = - (y^{i} \log(p_{i}^{f}) + (1 - y^{i}) \log(p_{i}^{f})) \qquad \dots (2)$$
$$\Lambda(\Theta) = \sum_{\forall m \in \mathbb{N}} (\chi i) \qquad \dots (3)$$

 χj is the cross entropy loss, y^j represents the ground label for a text, p_i^f and p_i^f are the probabilities of a text to be fake or real obtained from structural level document representation. n_i represent the news article in the corpus N. Λ is the total loss

V. EXPERIMENT

In the experiment, the aim is to find out the performance of proposed methodology on fake and real news classification, and the degree of association of the structure related properties to their respective fake or real class news documents.

Dataset: Thee following five datasets combined is collected and combined in order to avoid any biased results towards any particular kind of news limited to a certain climate. The datasets are collected by kaggle.com, (Khan et al., 2019 [35]) and (Shu et al., 2017 [36]) which includes articles from Buzzfeed and politifact [3].

Simulation: The dataset gathered was already preprocessed with all non-English characters and stop words removed and converted to lowercase. With 6398 data documents for training, 188 were reserved for testing on unseen data. We used Google's pre trained word2vec [37] embedding and ELU as the nonlinear activation function. The resulting and final simulation is run for 300 steps with mini-batch size of 50 documents with initial learning rate of 0.01, dropout rate to 0.0 and L2 regularization set to 0.01 .The hidden unit size was set to 100. ADAM optimizer [38] was used for hyperparameter tuning to implement our method. We performed our simulations on Google Colab GPU hardware accelerator.

Results: The proposed model was able to attain 74.62% accuracy on unseen data with F1-score of 0.76 on detecting

fake news articles. The training error and step wise accuracy is shown in Figure 3(a) and 3(b) respectively.

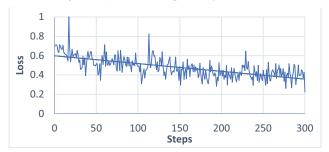


Figure 3(a): Training error



Figure 3(b): Accuracy with step learning

The structural analysis on the three general attributes obtained from particular tree data structure supports our argument on the associated properties with the contextual analysis of news documents. (Karimi and Tang, 2019 [28]) have also observed some of these properties in their model. The below-mentioned properties have been attained as a normalized sum over the length of inter-sentential matrix (number of sentences in an article):-

Preorder traversal difference: - The difference between preorder traversal of a discourse dependency tree and its normal sequential order.

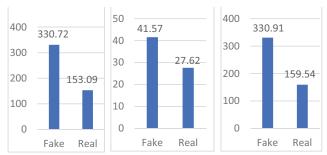


Figure 4: Avg. preorder positional diff, Avg. postorder positional diff and Avg. parent-child distance respectively.

Postorder traversal difference: - The difference between postorder traversal of a discourse dependency tree and its normal sequential order.

Distance between a parent and a child node: - The distance difference between child and parent unit in a discourse dependency tree in their original sequential order.

VI. CONCLUSION AND FUTURE WORK

Automatic fake news detection is a thought-provoking problem in deception detection, and it has incredible real-world political & social influences. In this paper, Deep Learning (GRU) was used for discourse segment analysis and constructing the dependency tree that offered distinguishable features for real and fake news. The model achieved 0.76 F1 score and 74.62% accuracy. This paper can be carried out further to various other directions. There can be many other structures like dependencytrees with more advanced structure related properties that can be considered. In addition, using ensemble learning and other structure breeding techniques is also an area of research in order to improve the accuracy of our presented model. This model can be further developed and applied for real time detection using Intra document and Inter document linking such as relation between events, participants, credibility of sources and positive responders. At last, testing the proposed model on datasets limited to certain period of news climate may also offer some interesting insights. This refers to training and testing political/economic/miscellaneous news only considered over restricted period and region.

REFERENCES

- [1] M. G. Hunt Allcott, "Social media and Fake News in the 2016 Election," Journal of Economic Perspectives, vol. 31, no. 2, pp. 211-236, 2017.
- [2] R. Z. Xinyi Zhou, "Fake News: A Survey of Research, Detection Methods, and Opportunities," arXiv:1812.00315v1.
- [3] "Fact-checking U.S. politics | PolitiFact," Poynter Institute, [Online]. Available: https://www.politifact.com/. [Accessed 2019].
- [4] "FACTCHECK.ORG," The Annenberg Public Policy Center of the University of Pennsylvania, [Online]. Available: https://www.factcheck.org/.
- [5] J. K. K. R. J. B. B. S. Martin Potthast, "A Stylometric Inquiry into Hyperpartisan and Fake News," arXiv, 2017.
- [6] P. P. J. Monu Waskale, "A Review Rumors Detection on Twitter Using Machine Learning Techniques," *International Journal of Scientific Research & Engineering Trends*, vol. 05, no. 03, 2019.
- [7] J. G. Cody Buntain, "Automatically Identifying Fake News in Popular Twitter Threads," arXiv, 2018.
- [8] M. G. U. L. K. T. Ugur Kursuncu, "Predictive Analysis on Twitter: Techniques and Applications," arXiv, 2018.
- [9] B. D. P. S. Y. Jiawei Zhang, "FAKEDETECTOR: Effective Fake News Detection," arXiv, 2019.
- [10] S. S. Y. L. Natali Ruchansky, "CSI: A Hybrid Deep Model for Fake News Detection," in CIKM 2017 International Conference on Information and Knowledge management, Singapore, 2017.
- [11] A. A. Monther Aldwairi, "Detecting Fake News in Social Media Networks," in *The 9th International Conference on Emerging Ubiquitous Systems and Pervasive Networks*, Leuven, Belgium, 2018.
- [12] J. S. G. N. S. Terry Traylor, "Classifying Fake News Articles Using Natural Language Processing to Identify In-Article Attribution as a Supervised Learning Estimator," in *IEEE 13th International Conference* on Semantic Computing, Newport Beach, California, 2019.
- [13] J. Q. W. Y. Ray Oshikawa, "A Survey on Natural Language Processing for Fake News Detection," arXiv, 2018.
- [14] S. S. S. S. P. W. S. H. Subhadra Gurav, "Survey on Automated System for Fake News Detection using NLP & Machine Learning Approach," *International Research Journal of Engineering and Technology*, vol. 06 Issue, no. 01, 2019.
- [15] S. N. V. A. N. D. D. H. Kushal Agarwalla, "Fake News Detection using Machine Learning and Natural Language Processing," *International Journal of Recent Technology and Engineering*, vol. 07, no. 06, 2019.

- [16] A. H. H. R. Y. B. F. F. R. Y. C. Rowan Zellers, "Defending Against Neural Fake News," arXiv, 2019.
- [17] V. A. P. S. R. Sivasangari V, "A Modern approach to identify the fake news using machine learning," *International Journal of Pure and Applied Mathematics*, vol. 118, no. 20, 2018.
- [18] N. L. S. E. G. B. X. O'Brien, "The Language of Fake News: Opening the Black-Box of Deep Learning Based Detectors," in "Al for Social Good", NIPS 2018, Montreal, Canada, 2018.
- [19] A. S. S. W. J. T. H. L. Kai Shu, "Fake News Detection on Social Media: A Data Mining Perspective," ACM SIGKDD Explorations Newsletter, pp. 22-36, 2017.
- [20] R. V. d. C. A. A. C. B. G. Fernando Cardoso Durier da Silva, "Can Machines Learn to Detect Fake News? A Survey Focused on Social Media," in 52nd Hawaii International Conference on System Sciences, Grand Wailea, Maui, 2019.
- [21] U. V. S. C. L. Q. Xishuang Dong, "Deep Two-path Semi-supervised Learning for Fake News Detection," arXiv, 2019.
- [22] T. N. H.-Y. K. Kai-Chou Yang, "Fake News Detection as Natural Language Inference," arXiv, 2019.
- [23] F. F. D. E. D. M. M. M. B. Federico Monti, "Fake News Detection on Social Media using Geometric Deep learning," arXiv, 2019.
- [24] D. B. S. Z. Oluwaseun Ajao, "Fake News Identification on Twitter with Hybrid CNN and RNN Models," in SMSociety '18 Proceedings of the 9th International Conference on Social Media and Society, Copenhagen, Denmark, 2018.
- [25] P. T. S. A. N. L. Aswini Thota, "Fake News Detection: A Deep Learning Approach," SMU Data Science Review, 2018.
- [26] H. P. Stefan Helmstetter, "Weakly Supervised Learning for Fake News," in IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining (ASONAM), Barcelona, Spain, 2018.
- [27] A. A. I. Costel-Sergiu Atodiresei, "Identifying Fake News and Fake Users on Twitter," *Procedia Computer Science*, pp. 451-461, 2018.
- [28] J. T. Hamid Karimi, "Learning Hierarchical Discourse-level Structure for Fake News Detection," arXiv, 2019.
- [29] K. S. S. W. R. G. F. W. H. L. Shuo Yang, "Unsupervised Fake News Detection on Social Media: A Generative Approach," in AAAI Conference on Artificial Intelligence, Honolulu, Hawaii, 2019.
- [30] H. G. M. M. B. J. H. Zhixuan Zhou, "Fake News Detection via NLP is Vulnerable to Adversarial Attacks," arXiv, 2019.
- [31] L. L. I. Álvaro Ibrain Rodríguez, "Fake news detection using Deep Learning," arXiv:1910.03496v2, 2019.
- [32] E. F. M. S. Joan Bachenko, "Verification and implementation of language-based deception indicators in civil and criminal narratives," in COLING '08 Proceedings of the 22nd International Conference on Computational Linguistics, Manchester, UK, 2008.
- [33] M. L. Yang Liu, "Learning Structured Text Representations," Transactions of the Association for Computational Linguistics, vol. 6, pp. 63-75, 2018.
- [34] B. v. M. C. G. D. B. F. B. H. S. Y. B. Kyunghyun Cho, "Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation," in *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNL*, Doha, Qatar, 2014.
- [35] M. T. I. K. A. I. S. A. Junaed Younus Khan, "A Benchmark Study on Machine Learning Methods for Fake News Detection," arXiv:1905.04749v1, 2019.
- [36] S. W. a. H. L. Kai Shu, "Ex-ploiting tri-relationship for fake news detection.," arXiv preprint arXiv:1712.07709, 2017.
- [37] K. C. G. C. J. D. Tomas Mikolov, "Efficient Estimation of Word Representations in Vector Space," arXiv preprintarXiv:1301.3781, 2013.
- [38] J. B. Diederik P Kingma, "Adam: A method for stochastic optimization," arXivpreprint arXiv:1412.6980, 2014.