# Goodness of fit for ellipses.

This Python program implements a goodness of fit measure for ellipses that is independent of the number of sample points and that is scaled to the size of the ellipse to allow comparison of the fitting errors for ellipses of different sizes. The program calculates the angle of each point relative to the center of the ellipse, and then finds the intersection point of a line at that angle on the ellipse.

Calculating the error distance

$$c = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$

$$c^2 = a^2 + b^2$$

Pythagorean Theorem
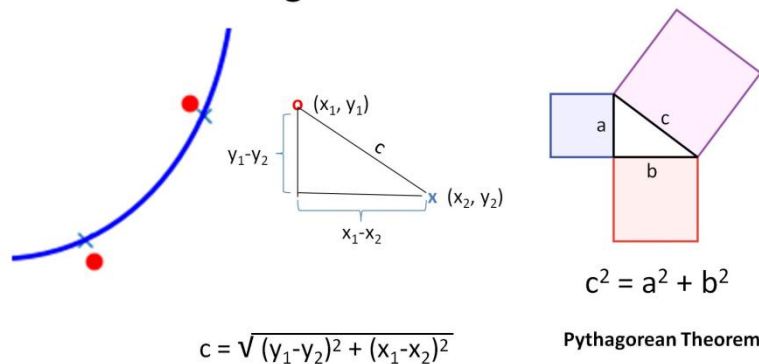
The program adjusts the values obtained from the trigonometric functions depending on the quadrant relative to the center of the ellipse. As you can see in this example, the angle obtained by the **arctan** function for a point in the third quadrant is the same as the angle for the first quadrant. To get the correct value, we add 180 degrees. The calculation is different for the **atan2** function, which gives negative values for the third and fourth quadrant.
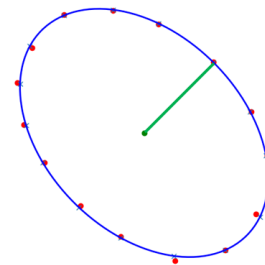

$\tan(\theta) = y/x$
$\theta = \arctan(y/x)$

$x = -2, y = -2$
$\arctan(-2/-2) = 45°$
$+ 180° \Rightarrow 225°$
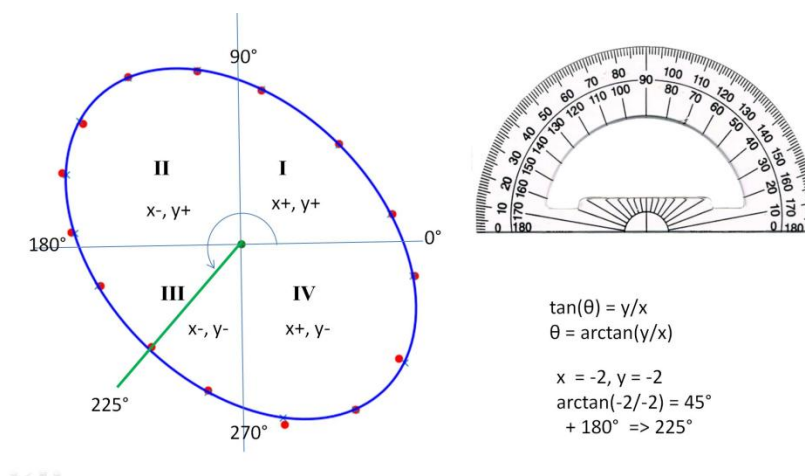
Defining the Fitting Error

$$\text{error\_distance} = c = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$
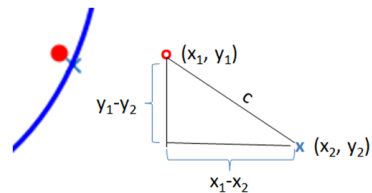
$$\text{average\_error} = \frac{1}{n} \sum_{i=1}^{n} c_i$$

$$\text{fitting\_error} = \frac{\text{average\_error} \times 100}{\text{semiminor\_axis}}$$

semiminor axis
of the ellipse

Standard measures of fitness, like the Mean Squared Error, do not provide meaningful comparisons for ellipses of different sizes. The error measure has to be independent of the number of sample points and the size of the ellipse. The error distance is the distance between the observed point and a point on the ellipse. The average error is the sum of all the error distances divided by the number of points. The fitting error is the percentage of the average error relative to the semiminor axis of the ellipse, which is half the width of the ellipse. Dividing by the semiminor axis scales the average error to the ellipse to allow comparisons of the fitting errors for ellipses of different sizes.

Results for Example #2:

Number of points = 21
 center (x,y): 318.991, 483.750
 semimajor axis = 535.6987, semiminor axis = 389.9497
 angle of rotation phi = 0.8176 (46.845 deg.)
 Average error distance = 8.2523
 Average error distance relative to semiminor axis = 2.1163%

Number of points: 21
Average error distance relative to semiminor axis: 2.116%