Nonlinear Model Reference Adaptive Control (NMRAC) for First Order Systems

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***Abstract*—...**

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I. Introduction

Model reference control is a technique that imposes a behavior on a system, by equating the the output of the model reference with the output of the controlled system. This way, the gains of a pre-defined controller structure can be found.

However, this only works if the system's dynamics are known exactly, which is often far from reality. If the system's dynamics are not fully known, the controller gains can be found adaptively by a technique called *model reference adaptive control* (MRAC). The adaptation law can be chosen in two ways, firstly a gradient descent-based update law as first proposed by Whitaker et al. [1] and applied in for example [2], [3], and secondly, a Lyapunov-based update law as proposed by Shackcloth et al. [4]. The latter enforces that in every update step, we have a stabilizing controller, which is not guaranteed when using the former. In this work, we make use of a Lyapunov-based update law, to guarantee stability during the learning process.

Neural networks are ideal candidate functions for control laws, since they can be considered to be universal approximators [5]. Since at least the 90s, research has been conducted on NN controllers [6], and it has been shown that they can learn a desired behavior, for example in [2], [7]–[9]. However, standard NNs are nonlinear, through the use of nonlinear activation functions, which makes their the formal verification of challenging. They often rely on post-training stability verification, as done in [10], [11], which does not support any type of online learning for imporivng the control strategy, while guaranteeing a stabilizing contoller.

The algorithm, proposed in this paper, is derived from a Lyapunov-based, linear MRAC, as for example in [12]–[14], which defines a stable learning strategy for first-order systems controlled by NNs.

II. Literature Review

Model Reference Adaptive Control (MRAC) was first introduced in the early 1970s by Whitaker et al. [15]. Originally, it was thought to deal with process uncertainties and disturbance dynamics, however, the proposed techniques were also used in different contexts, including but not limited to auto-tuning, automatic construction of gain schedules, and adaptive filtering [16], [17].

The adaptation mechanism of the controller parameters follows two main approaches, namely 1) the MIT method or gradient descent-based method, and 2) the Lyapunov method [14]. The MIT method does not come with stability guarantees [18], whereas the Lyapunov method is based on an adaptation rule derived from Lyapunov's second method [4]. It imposes stability since the adaptation rule is chosen in a way such that the decrease condition on the Lyapunov function is always satisfied, thus, implying system convergence.

Generally, MRACs can be split into three categories, firstly direct and secondly indirect MRAC. The former aims to adapt controller parameters directly and the latter aims to update the model parameters. The third category is a hybrid approach called *Combined MRAC*, first proposed by Duarte et al. in [19] for first-order systems. The approach is based on estimating two parts with two separate adaptation rules, namely 1) unmatched model uncertainty and 2) system parameters. Their approach showed in simulations to be more robust than both direct and indirect adaptive control if considered individually [20], however, this has yet to be proven. Combined MRACs are generalizable for n-th order linear systems, as proposed by Lavretsky and Tao et al. [12], [21].

Additionally, Lavretsky proposes *Combined/Composite Model Reference Adaptive Control*, which combines direct and indirect MRAC. He suggests using RBFs to perform system identification and estimate the unmatched uncertainties model, with proven stability guarantees, which enables us to apply this method to a larger class of nonlinear systems [12].

III. Contributions

The method proposed in this work is based on the logic of the perviously mentioned works and the novelty of this apporach lies in the ability of learning a stabilizing controller, despite of a nonlinear NN being present in the closed-loop system. Additionally, the propsed algorithm is validated in simulation, which show the convergence of the NN parameters to their desired values.

IV. Definitions and nomenclature

A. Systems and Stability

In order to define stable learning algorithms, it is imperal to firstly define, first-order systems and stability, which is taken from [22].

Definition 1 (First-Order System)**.** A continuous-time first-order system is defined by the following map

$$f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}, (x(t), u(t)) \mapsto \dot{x}(t) = f(x(t), u(t)). \quad (1)$$

Additionally, if $u(t)$ is a function of x, the system is autonomous, with closed-loop dynamics $f(x(t))$. The trajectory of the system is defined by the evolution of $x(t)$. Furthermore, the system has an equilibrium point at $f(x(t)) = 0$.

Definition 2 (Stability)**.** Consider the equilibrium point $x = 0$ of (1). Then the system is...

- *stable*, if for each $\epsilon > 0$, there is a $\delta > 0$, sich that

$$||x(0)|| < \delta \Rightarrow ||x(t)|| < \epsilon, \quad \forall t \geq 0$$

- *unstable*, if not stable, and
- *asymptotically stable* if it is stable and δ can be chosen such that

$$||x(0)|| < \delta \Rightarrow \lim_{t\rightarrow\infty} x(t) = 0$$

From here henceworth, the dependency of the dynamics on time is considered to be implicit and will be neglected in the notation.

Definition 3 (Lyapunov function)**.** A Lyapunov function is a continuous function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ with the following properties:

(a) positive definitness: $V(x) > 0, \quad \forall x \in \mathbb{R}^n \setminus \{0\}$ and $V(0) = 0$,
(b) decrease condition: $\dot{V}(x) \leq 0, \quad \forall x \in \mathbb{R}$.

Definition 4 (Stability in the sense of Lyapunov)**.** If there exists a continuous function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ such that

(a) $V(x)$ is positive definite, and
(b) $\dot{V}(x) \leq -l(x)$, for some positive semidefinite function $l(x)$,

then the system is considered to be stable. Additionally, if $l(x)$ is positive definite, then the system is asymptotically stable.

The ultimate goal of this research is to develop stable update laws for neural network (NN) controllers. Hence, the designed control law will be considered as a NN. Hence, NNs are defined as follows throughout this work.

Definition 5 (Neural network)**.** A neural network (NN) $\phi : \mathbb{R}^n \rightarrow \mathbb{R}^p$ is defined as:

$$\begin{aligned} &\phi(x) = (L_1 \circ \varphi_1 \cdots \circ L_{H-1} \circ \varphi_{H-1} \circ L_H \circ \varphi_H)(x) \\ &L_i(x) = \theta_i x + b_i \quad \forall i \in \{1, ..., H\}, \end{aligned} \quad (2)$$

where $\varphi_i(\cdot)$ are called activation functions, θ_i and b_i are the weight matrix and bias of layer i, respectively.

Whenever, a bias is not mentioned it is assumed to be zero.

NNs usually make use of nonlinear activation functions, which enable them to approximate nonlinear functions. Typically, functions such as the hyperbolic tangent, sigmoid, or ReLU are used in machine learning. This work will utilize an actiavtion function that is designed to model a smooth saturation function, as used in [2], [8] and defined in (3). Note, that it activation function saturates at $\pm\frac{2}{a}$.

$$\sigma(x) = \frac{2(1 - e^{-ax})}{a(1 + e^{-ax})} \quad (3)$$

B. Model Reference Adaptive Control

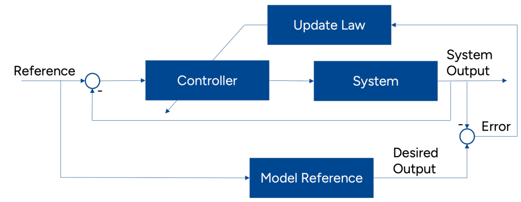


Figure 1: MRAC for first-oder systems controlled by simple NNCs

V. Nonlinear Model Reference Adaptive Control (NMRAC)

One of the goal of this work is to analyse stability properties of nonlinear NNCs. To this end, we propose stable update scheme. We commence by looking at the simpler case of a first-order system. More, specifically, we define a problem, where we show that the even in the nonlinear case, we are able to learn a parameter with an algorithm that is based on the logic of a linear MRAC. We call this nonlinear Model Reference Adaptive Control (NMRAC).

We desire to control a system of the form, as described in equation (4), where $\phi : \mathbb{R} \rightarrow \mathbb{R}$ is a nonlinear function, $e_x = x_r - x$,and θ is the parameter we aim to learn. Here, we would like to point that θ can be seen as a weight, and ϕ the activation function of a NN of the form of a sigmoidial saturation function, as described in equation (3).

$$\dot{x} = -ax + b\phi(\theta e_x) \quad (4)$$

Next, we define a stable reference model, as defined in equation (5), where $e_m = x_r - x_m$, and $a_m > 0$. Note, that we want ϕ_m to be of the same form as ϕ.

$$\dot{x}_m = -a_m x_m + b_m \phi_m(\theta_m e_m) \quad (5)$$

Since we aim for the system to follow the reference model, we define the error dynamics as in equation (6). Note, that from this definition it follows that $e = e_x - e_m$. We will use this alternative definition later for the stability analysis of the update law.

$$e = x_m - x \quad (6)$$

Following the error dynamics, we define the Lyapunov candidate to be defined as in equation (7). The factor $c > 0$, is used to accelerate or decelerate the learning process. The norm of the error captures the distance between the internal states of the reference model and the system, and the term $||\alpha||_2^2$ captures the distance between the NNC and the desired NNC, which can be seen as the difference in dynamics between the controlled system and the model reference. Note, that both e and α should be 0, when our system follows the reference model and the desired parameters are learned. Futhermore, this property renders our Lyapunov candidate to be positive definite.

$$V(e, \alpha) = ||e||_2^2 + ||c\alpha||_2^2 \tag{7}$$

To ensure that the controlled system is stable, we require the time derivative of the Lyapunov function to be negative. Therefore, we analyze the behavior of the resulting time derivative of the Lyapunov function. The resulting equation is defined in (8).

$$\dot{V}(e, \alpha) = 2e\dot{e} + 2\alpha\dot{\alpha}c \tag{8}$$

We now construct the time derivative of the error dynamics, which is defined by equation (9). We extend the equation by $\pm(a_m x b_m \phi_m(\theta_m e_x))$, to construct the term α, which is now dependent on e_x, and θ. We are then left with two other terms, namely, $-a_m e$, and $\gamma_m(e_m, e_x)$.

$$\begin{aligned}
\dot{e} =& \dot{x}_m - \dot{x} \\
=& - a_m x_m + b_m \phi_m(\theta_m e_m) - (-ax + b\phi(\theta e_x)) \\
& \pm (a_m x + b_m \phi_m(\theta_m e_x)) \\
=& - a_m \underbrace{(x_m - x)}_{=e} \\
& + \underbrace{(a - a_m)x + b_m \phi_m(\theta_m e_x) - b\phi(\theta e_x)}_{=\alpha(e_x, \theta)} \\
& + \underbrace{b_m(\phi_m(\theta_m e_m) - \phi_m(\theta_m e_x))}_{=\gamma_m(e_m, e_x)}
\end{aligned} \tag{9}$$

$$\begin{aligned}
\dot{V}(e, \alpha) =& 2e(-a_m e + \alpha(e_x, \theta) + \gamma_m(e_m, e_x)) + 2\alpha c\dot{\alpha}(e_x, \theta) \\
=& - 2a_m e^2 \\
& + 2e\gamma_m(e_m, e_x) \\
& + 2\alpha(e_x, \theta)(e + c\dot{\alpha}(e_x, \theta))
\end{aligned} \tag{10}$$

By substituting equation (9) into equation (8), we get equation (10).

Note, that the first term $-2a_m e^2$ is always negative, since $a_m > 0$, and $e^2 > 0$. The second term will always be negative, due to the property of $e = e_x - e_m$. It follows that if $e < 0 \Rightarrow \gamma_m(e_m, e_x) > 0$, which implies that the second term is negative, and if $e > 0 \Rightarrow \gamma_m(e_m, e_x) < 0$, which again implies that the second term is negative. Hence, the second term reamins negative.

Table I: Simulation parameters

	Constant	Sinusoidal	Smooth pulse
Learning rate γ	0.05	0.15	0.005
Amplitude	-	1	0.25
Sim. time	10	30	
θ_m	4	4	4

Since, we can already ensure that the first two terms of equation (10) are negative, it remains to show that the last term is always negative or equal to zero at all times. Since θ is dependent on time, this implies $\dot{\alpha}(e_x, \theta)$ will have a term that includes $\dot{\theta}$. Therefore, we choose $\dot{\theta}$, such that the third term in equation (10) is nullified. Note, that due to the way we construct the update law, stability is guaranteed, since the Lyapunov function is positive definite and simultaneously the decrease condition is satisfied.

$$0 = 2\alpha(e_x, \theta)(e + \dot{\alpha}(e_x, \theta)) \tag{11}$$

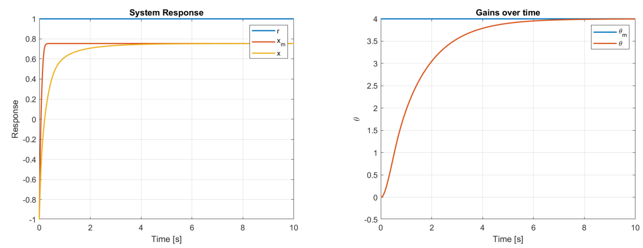
$$\begin{aligned}
\dot{\theta} =& \frac{1}{e_x b \frac{\partial \phi(\tilde{x})}{\partial \tilde{x}}|_{\tilde{x}=\theta e_x}} \\
& (\frac{1}{c}e + (a - a_m)\dot{x} + b_m \frac{\partial \phi_m(\tilde{x})}{\partial \tilde{x}}|_{\tilde{x}=\theta_m e_x} \theta_m \dot{e}_x) \\
& - \frac{\theta}{e_x}\dot{e}_x
\end{aligned} \tag{12}$$

When taking a closer look at the update rule in equation (12), it follows that we require to implement an update hold, when $e_x = 0$. This is reasonable, since whenever the state of our reference system is equal to the reference signal, we will not be able to extract information on how to change the weights in order for our system to converge towards the reference signal. In practice this means that we implement a threshold ε, where we do not update the weights.

VI. Numerical Results

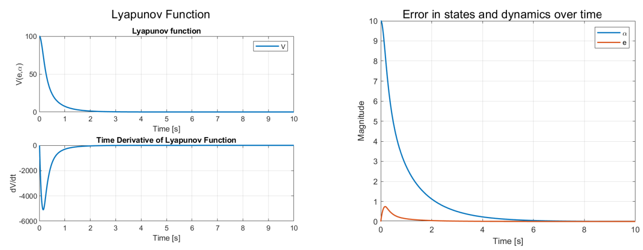
In this section, the simulation results of NMRAC for a first-order system are presented. The example is constructed, to show that the parameters of the NN converge to the desired values, using the proposed algorithm. The NN consists of one neuron, with an nonlinear actiavtion function, as defined in (3). Hence, the goal of the simulation is to show that $\theta \rightarrow \theta_m$, while keeping all other parameters constant and equal to one another.

The dynamics are chosen to be $a_m = a = 10$, $b_m = b = 10$, $\theta_m = 4$, and the activation function saturates at a value of 1. The precision threshold ϵ is chosen to be computer precision in all simulations. We compute the weight update as the analytical solution for each time instance. The sampling time for the simulations is $T_s = 0.01$, and the learning rate differs in each simulation to showcase faster and slower convergence. Different simulations for different reference signals are provided, including a constant signal, a sinusodial signal, and a smooth pulse signal. The simluation parameters can be taken from table I.



(a) System response (b) Evolution of gain update

Figure 2: Simulation results of nonlinear MRAC with a constant input



(a) Lyapunov function and its time derivate (b) Error in internal states and dynamics of the system

Figure 3: Lyapunov function and convergence of the system w.r.t. internal states and dynamics

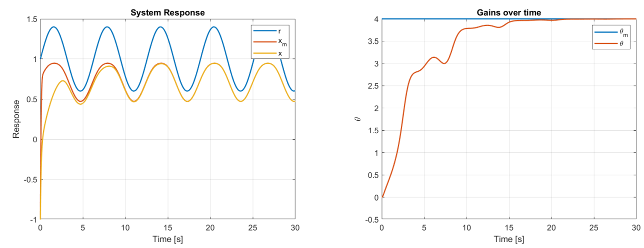
A. Constant Reference Signal

We commence by using a constant reference signal. We choose the system's initial conditions to be at 0. The value of the constant reference signal is 1. This will result in a steady state error for the reference model since we only take a feedback controller into account. However, this steady state error is ignored, since the learning ability of the controller is of interest. Additionally, we set $\theta_m = 4$ and the initial value of the weight $\theta = 0$. Finally, we simulate the system response for 5 seconds and choose the constant parameter $c = 0.05$.

In Figure 2a, we observe the steady state error, which is at around 0.2. As aforementioned, we will disregard this error, since we are interested in how well our NNC can learn the behavior of our reference model. To this end, we can observe that the system converges to the reference model within 5 seconds. Furthermore, in Figure 2b, we can see the weights of the NNC and the error e are equal to their desired values, $\theta = 4$ and $e = 0$ respectively. In Figure 3a, we can see that the Lyapunov function converges to zero over time. Additionally, we can see that the derivate of the Lyapunov function is negative and only reaches zero when the Lyapunov function itself converges to zero. This indicates that the proposed learning schematic is stable for the desired reference signal, and we can learn the desired parameter.

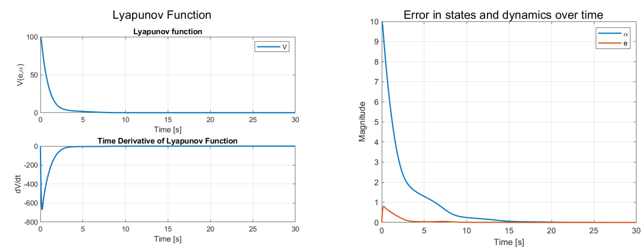
B. Sine Signal

The second reference is a sinusoid signal, with an amplitude of 0.25, to stay in a region that is not affected too much by the saturation function. As before, we choose the initial $\theta = 0$,



(a) System response (b) Evolution of gain update

Figure 4: Simulation results of nonlinear MRAC with a sine input



(a) Lyapunov function and its time derivate (b) Error in internal states and dynamics of the system

Figure 5: Lyapunov function and convergence of the system w.r.t. internal states and dynamics

$\theta_m = 4$, and the initial conditions of both the system and the reference model to be at -1. We choose this specific initial condition only to be able to see the convergence in the plots. Finally, we simulate the system response for 30 seconds and choose the constant parameter $c = 0.15$, which corresponds to a less aggressive learning strategy compared to the constant input signal.

Figure 4a shows the response of the system. We observe that the system state, and reference model state both converge towards the desired reference model within around 25 seconds. Simultaneously, the parameter θ converges to towards the desired value, which can be seen in Figure 4b. Even though the parameter converges, we observe oscillations. This behavior is expected, due to the update law. Since the update law is dependent on $\dot{e}_x$, it is dependent on $\dot{r}$ and $\dot{x}$. We observe that whenever $\dot{e}_x$ switches signs and simultaneously e is small, $\dot{\theta}$ changes sign, and hence the parameter starts oscillating. This behavior can be changed thourgh adapting the learning rate. A more aggressive learning strategy will be showcased in the next example.

In Figure 5a we can see that the Lyapunov function converges to zero over time. Additionally, we can see that the derivate of the Lyapunov function is negative and only reaches zero when the Lyapunov function itself converges to zero. The results of this simulation indicate again that the proposed learning schematic is stable for the desired reference signal, and we can learn the desired parameter.

VII. Conclusion

The simulation results of the scalar system indicate that the proposed update law successfully enables the system to learn the desired parameters across different reference signals. The speed of convergence can be controlled by adjusting the factor c, with smaller values leading to faster convergence. Additionally, this work identifies numerical instability as a potential issue. This instability can be reduced by lowering the sampling time, thus enhancing the precision of integral and derivative approximations, improving overall simulation accuracy.

Acknowledgment

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