Nonlinear Model Reference Adaptive Control (NMRAC) for First Order Systems

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***Abstract*—Lyapunov-based Model Reference Adaptive Control (MRAC) is a well known and widely spread control strategy, that by design comes with stability guarantees. However, it does not allow for nonlinearities being present in the closed-loop system, which limits its application to linear systems with linear feedback controllers. In this work, a Lyapunov-based update mechanism for first-order systems controlled by Neural Network controllers (NNC) is proposed. The update mechanism is validated in simulation for a constant reference signal, a sinusodial reference signal, and a smooth pulse signal. The numerical results show that the propsed algorithm converges towards the desired values and that the speed of convergence can be tuned by manipulating the learning rate. Additionally, this work identifies numerical instabilities as a potential issue, which can be reduced by reducing the sampling time.**

***Keywords*—Neural Networks, Model Reference Adaptive Control, Lyapunov Method, Nonlinear Control, Stability, Adaptive Control, Robotics**

I. Introduction

Model reference control is a technique that imposes a behavior on a system, by equating the the output of the model reference with the output of the controlled system. This way, the gains of a pre-defined controller structure can be found.

However, this only works if the system's dynamics are known exactly, which is often far from reality. If the system's dynamics are not fully known, the controller gains can be found adaptively by a technique called *model reference adaptive control* (MRAC). The adaptation law can be chosen in two ways, firstly a gradient descent-based update law as first proposed by Whitaker et al. [1] and applied in for example [2], [3], and secondly, a Lyapunov-based update law as proposed by Shackcloth et al. [4]. The latter enforces that in every update step, the control law is stabilizing, which is not guaranteed when using the former. In this work, a Lyapunov-based update law is used, to guarantee stability during the learning process.

Neural networks are ideal candidate functions for control laws, since they can be considered to be universal approximators [5]. Since at least the 90s, research has been conducted on NN controllers [6], and it has been shown that they can learn a desired behavior, for example in [2], [7]–[9]. However, standard NNs are nonlinear, through the use of nonlinear activation functions, which makes their the formal verification challenging. They often rely on post-training stability verification, as done in [10], [11], which does not support any type of online learning for imporivng the control strategy, while guaranteeing a stabilizing contoller.

The algorithm, proposed in this paper, is derived from a Lyapunov-based, linear MRAC, as for example in [12]–[14], and defines a stable learning strategy for first-order systems controlled by NNs.

This work is organized as follows. The following two subsections highlight where this work can be positioned within existing literature in Section I-A, and clearly define its contributions in Section I-B. Subsequently, required definitions are introduced in Section II. This will be followed the introduction of the proposed framework in Section III, with numerical results in Section IV, and finally, the conclusion in Section V.

A. Literature Review

Model Reference Adaptive Control (MRAC) was first introduced in the early 1970s by Whitaker et al. [15]. Originally, it was thought to deal with process uncertainties and disturbance dynamics, however, the proposed techniques were also used in different contexts, including but not limited to auto-tuning, automatic construction of gain schedules, and adaptive filtering [16], [17].

The adaptation mechanism of the controller parameters follows two main approaches, namely 1) the MIT method or gradient descent-based method, and 2) the Lyapunov method [14]. The MIT method does not come with stability guarantees [18], whereas the Lyapunov method is based on an adaptation rule derived from Lyapunov's second method [4]. It imposes stability since the adaptation rule is chosen in a way such that the decrease condition on the Lyapunov function is always satisfied, thus, implying system convergence.

Generally, MRACs can be split into three categories, firstly direct and secondly indirect MRAC. The former aims to adapt controller parameters directly and the latter aims to update the model parameters. The third category is a hybrid approach called *Combined/Composite MRAC*, first proposed by Duarte et al. in [19] for first-order systems. The approach is based on estimating two parts with two separate adaptation rules, namely 1) unmatched model uncertainty and 2) system parameters. Their approach showed in simulations to be more robust than both direct and indirect adaptive control if considered individually [20], however, this has yet to be proven. Combined MRACs are generalizable for n-th order linear systems, as proposed by Lavretsky and Tao et al. [21]. Additionally, Lavretsky proposes Combined MRACs using RBF NNs to perform system identification and estimate the unmatched uncertainties model, with proven stability guarantees, which enables us to apply this method to a larger class of nonlinear systems [12].

The porposed algorithm is a direct MRAC method, utilizing a Lyapunov-based learning mechanism, that includes nonlinearities in the form of a simple feedforward NN.

B. Contributions

The method proposed in this work is based on the logic of the perviously mentioned works and the novelty of this apporach lies in the ability of learning a stabilizing controller, despite of a nonlinear NN being present in the closed-loop system. Additionally, the propsed algorithm is validated in simulation, which show the convergence of the NN parameters to their desired values.

II. Definitions and nomenclature

A. Systems and Stability

In order to define stable learning algorithms, it is imperal to firstly define, first-order systems and stability, which is taken from [22].

Definition 1 (First-Order System)**.** A continuous-time first-order system is defined by the following map

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, (x(t), u(t)) \mapsto \dot{x}(t) = f(x(t), u(t)). \quad (1)$$

Additionally, if $u(t)$ is a function of x, the system is autonomous, with closed-loop dynamics $f(x(t))$. The trajectory of the system is defined by the evolution of $x(t)$ over time. Furthermore, the system has an equilibrium point at $f(x(t)) = 0$.

Definition 2 (Stability)**.** Consider the equilibrium point $x = 0$ of (1). Then the system is...

- *stable*, if for each $\epsilon > 0$, there is a $\delta > 0$, such that
$$||x(0)|| < \delta \Rightarrow ||x(t)|| < \epsilon, \quad \forall t \geq 0$$
- *unstable*, if not stable, and
- *asymptotically stable* if it is stable and δ can be chosen such that
$$||x(0)|| < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0$$

From here henceworth, the dependency of the dynamics on time is considered to be implicit and will be neglected in the notation.

Definition 3 (Lyapunov function)**.** A Lyapunov function is a continuous function $V : \mathbb{R}^n \to \mathbb{R}$ with the following properties:

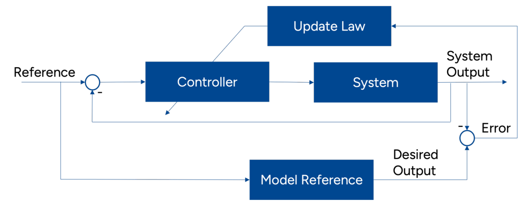


Figure 1: MRAC for first-oder systems controlled by simple NNCs

(a) positive definitness: $V(x) > 0, \quad \forall x \in \mathbb{R}^n \setminus \{0\}$ and $V(0) = 0$,
(b) decrease condition: $\dot{V}(x) \leq 0, \quad \forall x \in \mathbb{R}$.

Definition 4 (Stability in the sense of Lyapunov)**.** If there exists a continuous function $V : \mathbb{R}^n \to \mathbb{R}$ such that

(a) $V(x)$ is positive definite, and
(b) $\dot{V}(x) \leq -l(x)$, for some positive semidefinite function $l(x)$,

then the system is considered to be stable. Additionally, if $l(x)$ is positive definite, then the system is asymptotically stable.

The ultimate goal of this work is to develop stable update laws for NNCs. Hence, the designed control law will be considered as a NN. Therefore, throughout this work NNs are defined as follows.

Definition 5 (Neural network)**.** A neural network (NN) $\phi : \mathbb{R}^n \to \mathbb{R}^p$ is defined as:

$$\begin{aligned} \phi(x) &= (L_1 \circ \varphi_1 \cdots \circ L_{H-1} \circ \varphi_{H-1} \circ L_H \circ \varphi_H)(x) \\ L_i(x) &= \theta_i x + b_i \quad \forall i \in \{1, ..., H\}, \end{aligned} \quad (2)$$

where $\varphi_i(\cdot)$ are called activation functions, θ_i and b_i are the weight matrix and bias of layer i, respectively. Whenever, a bias is not mentioned it is assumed to be zero.

NNs usually make use of nonlinear activation functions, which enable them to approximate nonlinear functions. Typically, functions such as the hyperbolic tangent, sigmoid, or ReLU are used in machine learning. This work will utilize an actiavtion function that is designed to model a smooth saturation function, as used in [2], [8] and defined in (3). Note, that it activation function saturates at $\pm\frac{2}{a}$.

$$\sigma(x) = \frac{2(1 - e^{-ax})}{a(1 + e^{-ax})} \quad (3)$$

III. Nonlinear Model Reference Adaptive Control (NMRAC)

Direct MRAC is charaterized by the controller parameters being direclty updated through an update mechanism, which takes into acount the error of the system with respect to the desired output, as shown in Fig. 1. The porposed approach is a Lyapunov-based approach that defines the update mechanism based on a Lyapunov candidate. In this Section, the NMRAC problem for first-order systems is defined.

The system to control is of the form, as described in equation (4), where $\phi : \mathbb{R} \rightarrow \mathbb{R}$ is a nonlinear function, $e_x = x_r - x$, and θ is an adaptable parameter. It can be seen as a weight of NN, with actiavtion function ϕ, as defined in (3).

$$\dot{x} = -ax + b\phi(\theta e_x) \tag{4}$$

Next, a stable model reference is defined, as per equation (5), where $e_m = x_r - x_m$, and $a_m > 0$.

$$\dot{x}_m = -a_m x_m + b_m \phi_m(\theta_m e_m) \tag{5}$$

Since the goal is, that the system learns the behavior of the model reference, error dynamics are defined as follows.

$$e = x_m - x \tag{6}$$

Note, that from this definition it follows that $e = e_x - e_m$, which is used as an alternative definition for the stability analysis of the update law later on in this work.

Following the error dynamics, a Lyapunov candidate is defined, as in equation (7).

$$V(e, \alpha) = ||e||_2^2 + ||c\alpha||_2^2 \tag{7}$$

The factor $c > 0$ can be considered to be the learning rate, which is used to accelerate or decelerate the learning process. The norm of the error captures the distance between the internal states of the reference model and the system, and the term $||\alpha||_2^2$ captures the distance between the NNC and the desired NNC, which can be seen as the difference in dynamics between the controlled system and the model reference. Note, that both e and α should be 0, when the system follows the model reference and the desired parameters are learned. Futhermore, this property renders our Lyapunov candidate to be positive definite.

To ensure that the controlled system is stable, a negative time derivative of the Lyapunov function is required. Therefore, we analyze the behavior of the resulting time derivative of the Lyapunov function. The resulting equation is defined by:

$$\dot{V}(e, \alpha) = 2e\dot{e} + 2\alpha\dot{\alpha}c \tag{8}$$

The time derivative of the error dynamics are defined by (9). The equation is extended by $\pm(a_m x b_m \phi_m(\theta_m e_x))$, to construct the term α, which is now dependent on e_x, and θ. Two term remain, namely, $-a_m e$, and $\gamma_m(e_m, e_x)$.

$$\begin{aligned}\dot{e} =& \dot{x}_m - \dot{x} \\ =& -a_m x_m + b_m \phi_m(\theta_m e_m) - (-ax + b\phi(\theta e_x)) \\ & \pm (a_m x + b_m \phi_m(\theta_m e_x)) \\ =& -a_m \underbrace{(x_m - x)}_{=e} \\ & + \underbrace{(a - a_m)x + b_m \phi_m(\theta_m e_x) - b\phi(\theta e_x)}_{=\alpha(e_x, \theta)} \\ & + \underbrace{b_m(\phi_m(\theta_m e_m) - \phi_m(\theta_m e_x))}_{=\gamma_m(e_m, e_x)}\end{aligned} \tag{9}$$

Table I: Simulation summary

	Constant	Sinusoidal	Smooth pulse
Amplitude	-	1	0.25
Simulation time $[s]$	10	30	5
θ_m	4	4	4
Initial θ	0	0	0
Learning rate c	0.05	0.15	0.005
Convergence time $[s]$	5	25	2

Through substitution of (9) into (8), equation (10) is obtained.

$$\begin{aligned}\dot{V}(e, \alpha) =& 2e(-a_m e + \alpha(e_x, \theta) + \gamma_m(e_m, e_x)) + 2\alpha c\dot{\alpha}(e_x, \theta) \\ =& -2a_m e^2 \\ & + 2e\gamma_m(e_m, e_x) \\ & + 2\alpha(e_x, \theta)(e + c\dot{\alpha}(e_x, \theta))\end{aligned} \tag{10}$$

Note, that the first term $-2a_m e^2$ is always negative, since $a_m > 0$, and $e^2 > 0$. The second term will always be negative, due to the property of $e = e_x - e_m$. It follows that if $e < 0 \Rightarrow \gamma_m(e_m, e_x) > 0$, which implies that the second term is negative, and if $e > 0 \Rightarrow \gamma_m(e_m, e_x) < 0$, which again implies that the second term is negative. Hence, the second term always stays negative.

It remains to show that the last term does not influence equation (10) such that it changes sign. Since θ is dependent on time, this implies $\dot{\alpha}(e_x, \theta)$ will include a $\dot{\theta}$ term. Therefore, $\dot{\theta}$ is chosen, such that the third term is nullified. Note, that due to the way the update law is constructed, stability is guaranteed, since the Lyapunov function is positive definite and simultaneously the decrease condition is satisfied. The resulting update law is constructed as follows.

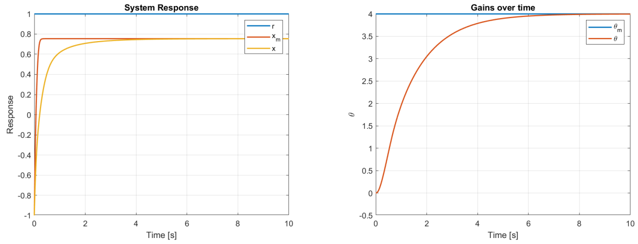
$$\dot{\theta} = \frac{(\frac{1}{c}e + (a - a_m)\dot{x} + b_m \frac{\partial \phi_m(\tilde{x})}{\partial \tilde{x}}|_{\tilde{x}=\theta_m e_x} \theta_m \dot{e}_x)}{e_x b \frac{\partial \phi(\tilde{x})}{\partial \tilde{x}}|_{\tilde{x}=\theta e_x}} - \frac{\theta}{e_x}\dot{e}_x \tag{11}$$

When taking a closer look at the update rule in equation (11), it follows that an update hold is rerquired, when $e_x = 0$. In practice this means that we implement a threshold ε, where the weights are not updated.

IV. Numerical Results

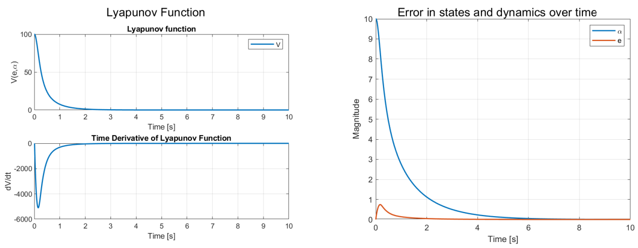
In this section, the numerical results of NMRAC for first-order systems are presented. The example is constructed to show that the parameters of the NN converge to the desired values, using the proposed algorithm. The NN consists of one neuron, with a nonlinear actiavtion function, as defined in (3). Hence, the goal of the simulation is to show that θ converges to θ_m.

The dyanmics of the system are charaterized by $a_m = a = 10$, $b_m = b = 10$, $\theta_m = 4$, and the activation function saturates at a value of 1. The precision threshold ϵ is chosen to be computer precision. The weight update is computed as the analytical solution for each time instance. The sampling time for the simulations is $T_s = 0.01$, and the learning rate differs in each simulation to showcase faster



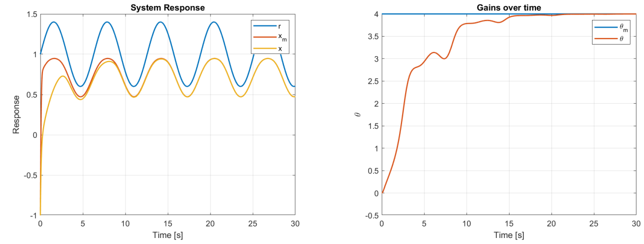
(a) System response (b) Evolution of gain update

Figure 2: Simulation results of nonlinear MRAC with a constant input



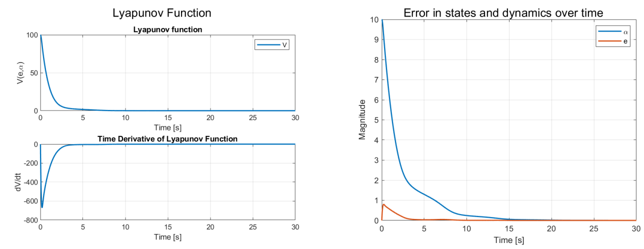
(a) Lyapunov function and its time derivate (b) Error in internal states and dynamics of the system

Figure 3: Lyapunov function and convergence of the system w.r.t. internal states and dynamics



(a) System response (b) Evolution of gain update

Figure 4: Simulation results of nonlinear MRAC with a sine input



(a) Lyapunov function and its time derivate (b) Error in internal states and dynamics of the system

Figure 5: Lyapunov function and convergence of the system w.r.t. internal states and dynamics

and slower convergence. Different simulations for different reference signals are provided, including a constant signal, a sinusodial signal, and a smooth pulse signal. A summary of the important simulation parameters and results can be taken from Table I.

A. Constant Reference Signal

Firstly, simulations for a constant reference signal are presented. The system's initial conditions are chosen to be 0. The value of the constant reference signal is 1. This will result in a steady state error for the model reference since only a feedback controller is taken into account. However, this steady state error is ignored, since the learning ability of the controller is of interest and charaterized by the response of the model reference. Additionally, $\theta_m = 4$ is chosen and the initial value of the weight $\theta = 0$. Finally, the system response is simulated for 5 seconds and the learning rate is chosen to be 0.05.

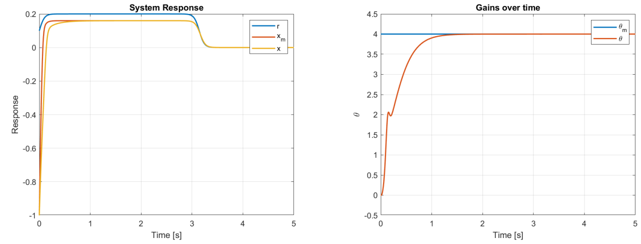
In Fig. 2a, a steady state error can be observed, which is at around 0.2. As aforementioned, this will be disregard, since the learning ability of the NNC is of interest. After 5 seconds system convergence is achieved. Furthermore, Fig. 2b depicts the convergence of the weight θ and the error e. Additionally, in Fig. 3a, the Lyapunov function is shown and it clearly converges to zero over time. Additionally, the time derivate of the Lyapunov function is negative and only reaches zero when the Lyapunov function itself converges to zero. This indicates that the proposed learning schematic is stable for the desired reference signal, and the desired parameter converges.

B. Sine Signal

The second reference is a sinusoid signal, with an amplitude of 0.25, to stay in a region that is not affected too much by the saturation function. As before, the initial θ and θ_m are chosen to be 0 and 4, respectively. The initial conditions of both the system and the reference model to be at -1. Finally, the system response is simulated for 30 seconds and the learning rate is chosen to be 0.15, which corresponds to a less aggressive learning strategy compared to the constant input signal.

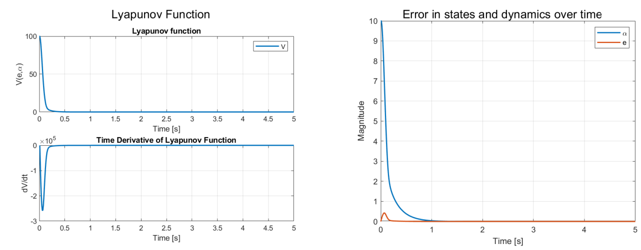
Fig. 4a shows the response of the system. It can be observed that the system state, and model reference state both converge towards the desired model reference within around 25 seconds. Simultaneously, the parameter θ converges to towards the desired value, which can be seen in Fig. 4b. Even though the parameter converges, oscillations can be observed. This behavior is expected, due to the update law. Since the update law is dependent on $\dot{e}_x$, it is dependent on $\dot{r}$ and $\dot{x}$. Whenever $\dot{e}_x$ switches signs and simultaneously e is small, $\dot{\theta}$ changes sign, and hence the parameter starts oscillating. This behavior can be influenced thourgh adapting the learning rate. A more aggressive learning strategy will be showcased in the next example.

In Fig. 5a the Lyapunov function is shown, and it converges to zero over time. Additionally, the derivate of the Lyapunov function is negative and only reaches zero when the Lyapunov function itself converges to zero. Again, the results of this simulation indicate again that the proposed learning algorithm is stable for the desired reference signal.



(a) System response (b) Evolution of gain update

Figure 6: Simulation results of nonlinear MRAC with a smooth pulse input



(a) Lyapunov function and its time derivate (b) Error in internal states and dynamics of the system

Figure 7: Lyapunov function and convergence of the system w.r.t. internal states and dynamics

C. Smooth Pulse Signal

The third and last input signal is a pulse signal. More specifically, a smooth pulse signal is used, since the update law requires a continuous time derivative of the reference signal. The smooth pulse signal is constructed, by using a sine function and applying a smooth saturation function, as defined in equation (3). The signal is chosen to have an amplitude of 0.25 and a bias such that its minimum value touches upon 0. As before, the initial conditions of the system and plant are -1. The response is simulated for 5 seconds and a learning rate of 0.005 is chosen.

Fig. 6a shows the response of the system. It can be observed that the system converges towards the model reference within just under 1 second. The parameter θ converges to towards the desired value within 2 seconds, which can be seen in Fig. 6b. Even though the parameter converges, there are oscillations present. They are present due to the same argument as for the sine signal.

The Lyapunov function and its derivative, as shown in Fig. 7a indicate the convergence of the system and the parameters, since V converges to 0 and its time derivative is always negative and converges to zero when V converges.

V. Conclusion

The simulation results of the scalar system strongly indicate that the proposed update law successfully enables the system to learn the desired parameters across different reference signals. The speed of convergence can be influenced by adjusting the learning rate c, with smaller values leading to faster convergence and larger values leading to slower convergence. Another observation that does not become evident in the presented simulations, is that the proposed update is sensitive to numerical instabilities. However, this instability can be reduced by lowering the sampling time, thus enhancing the precision of integral and derivative approximations, improving overall simulation accuracy. Future work will focus on tackling these numerical instabilities and developing update laws for more complex systems.

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