

Model Predictive Control - Assignment 1

Question 1: See code astrobee.py line 56/57

Question 2:

Computed with casadi_c2d:

```
Ad = [[1.  0.1]
      [0.  1. ]]
Bd = [[0.00024417]
      [0.00478469]]
Cd = [1, 0]
Poles: [1.+0.j 1.+0.j]
Zeros: [-0.9595789+0.j]
```

Analytically discretized:

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_c} x + \underbrace{\begin{pmatrix} 0 \\ \frac{1}{m_g} \end{pmatrix}}_{B_c} u$$

$$A = e^{A_c h} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & h \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$$

$$B = \int_0^h e^{A_c s} ds B_c = \int_0^h \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} ds B_c$$

$$= \left[\begin{bmatrix} s & \frac{1}{2}s^2 \\ 0 & s \end{bmatrix} \right]_0^h B_c = \begin{bmatrix} h & \frac{1}{2}h^2 \\ 0 & h \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{m_g} \end{bmatrix}$$

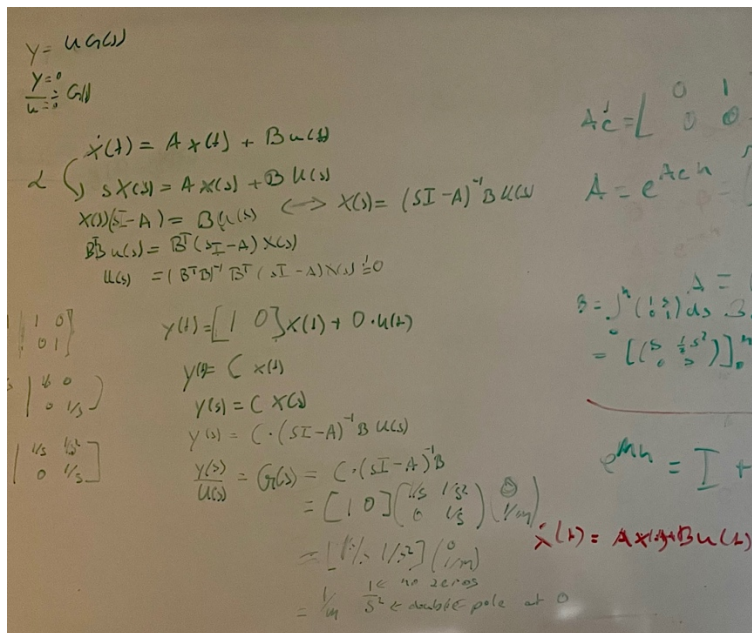
$$= \begin{bmatrix} \frac{1}{2}h^2 \\ h \end{bmatrix} \cdot \frac{1}{m_g}$$

$$\Rightarrow A = \begin{pmatrix} 1 & 0.1 \\ 0 & 1 \end{pmatrix} \text{ and } B = \frac{1}{m_g} \begin{pmatrix} h^2/2 \\ h \end{pmatrix} = \frac{1}{20.9} \begin{pmatrix} 0.005 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.000239 \\ 0.004784 \end{pmatrix},$$

with $h = dt = 0.1$ and $m_g = 20.9$

As we can see the values for A and B are approximately the same.

Question 3:



Handwritten solution for Question 3:

$$Y = U G(s)$$

$$\frac{Y}{U} = G(s)$$

$$\dot{x}(t) = A x(t) + B u(t)$$

$$sX(s) = A X(s) + B U(s)$$

$$X(s)(sI - A) = B U(s) \rightarrow X(s) = (sI - A)^{-1} B U(s)$$

$$Y(s) = C X(s) = C (sI - A)^{-1} B U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = C (sI - A)^{-1} B$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} (sI - A)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$\det(sI - A) = s^2$$

$$(sI - A)^{-1} = \frac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{s^2}$$

The system has two poles at $s=0$ and no zeros.

The system has two poles and no zeros. The poles are both located at $s=0$.

We expected no zeros and two poles. We expect the poles to be at 1, due to the following relationship: $\lambda = e^{\lambda_c} = e^0 = 1$, with $\lambda_c = 0$.

Our intuition was not entirely correct. The poles were at 1, as expected. However, there is one zero at about -0.96. This is due to the sample-and-hold operation.

Question 4: See code in file controller.py in function control_law().

We bound the control input signal, so that $|u_{\max}| = 0.85$. We chose one pole to be closer to the origin, to achieve a faster response. We chose to put the second pole close to the unit circle, to get rid of the overshoot.

The poles $p_1 = 0.6$ and $p_2 = 0.96$, to achieve convergence to the reference position after about 10 seconds.

Question 5:

With the pole placement from Q4, we get a steady-state error of approximately $\epsilon_{ss} = 0.01$, due to the disturbance. After implementing the integral action, we drive the steady-state error down to $\epsilon_{ss} = 0.0002$.