

Basic Concepts of Probability

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Puzzle

Stacy has twelve black socks and twelve white socks in her drawer. In complete darkness, and without looking, how many socks must she take from the drawer in order to be sure to get a pair that match?

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- ▶ First sock: White

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In complete darkness, and without looking, how many socks must she take from the drawer in order to be sure to get a pair that match?

- ▶ First sock: White
- ▶ Second sock: Black

Puzzle

Stacy has twelve black socks and twelve white socks in her drawer.

In complete darkness, and without looking, how many socks must she take from the drawer in order to be sure to get a pair that match?

- ▶ First sock: White
- ▶ Second sock: Black
- ▶ Third sock: White or Black

Puzzle

Stacy has twelve black socks and twelve white socks in her drawer.

In complete darkness, and without looking, how many socks must she take from the drawer in order to be sure to get a pair that match?

- ▶ First sock: White
- ▶ Second sock: Black
- ▶ Third sock: White or Black

Answer: 3 socks

What is Probability?

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- ▶ It is the branch of mathematics that deals with uncertainty.

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- ▶ It is the branch of mathematics that deals with uncertainty.
- ▶ It is a measure or estimation of how likely it is that an event will occur.

What is an Experiment?

It is an activity which can be repeated over and over again and which have well-defined results.

Examples of Experiments



Flipping a Coin

Examples of Experiments



Picking a Card from a Standard Deck of
Cards without Looking

Examples of Experiments



Throwing Dice

What is an Outcome?

It is a result of an experiment.

Examples of Outcomes



Examples of Outcomes



Experiment: Rolling a die

Examples of Outcomes



Experiment: Rolling a die
Possible outcomes: 1, 2, 3, 4, 5, 6

Examples of Outcomes



Examples of Outcomes



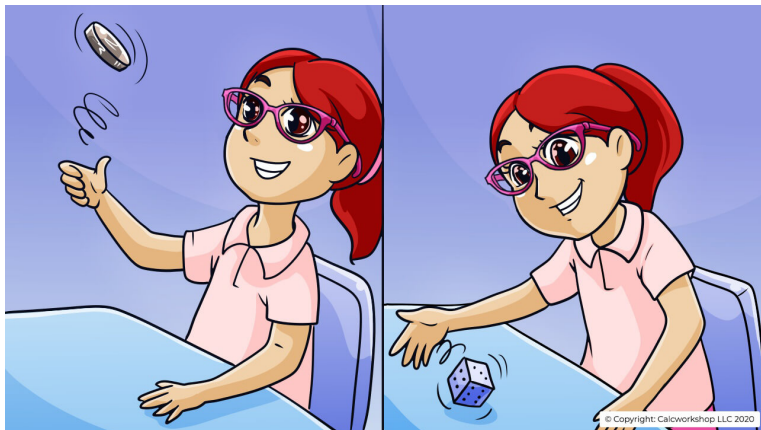
Experiment: Flipping two coins

Examples of Outcomes

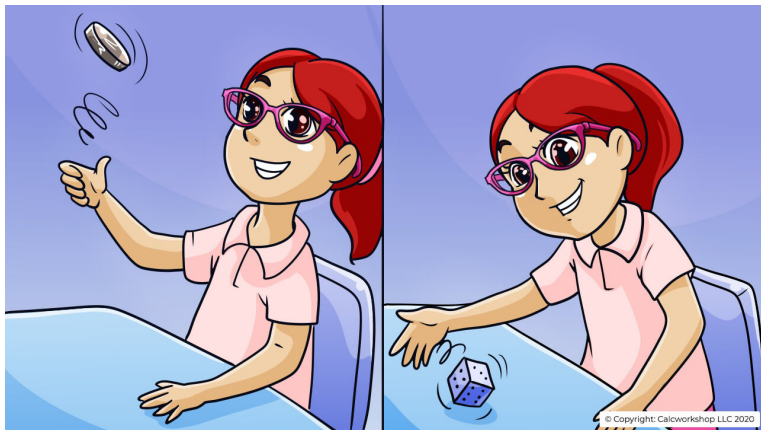


Experiment: Flipping two coins
Possible outcomes: HH, HT, TH, TT

Examples of Outcomes

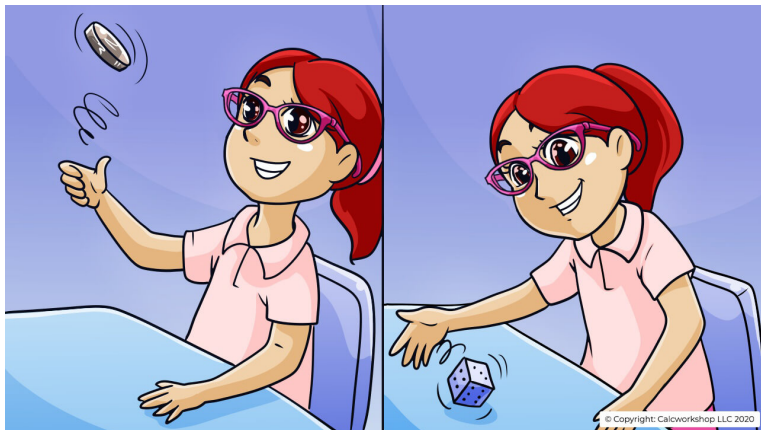


Examples of Outcomes



Experiment: Rolling a coin and a die simultaneously

Examples of Outcomes



Experiment: Rolling a coin and a die
simultaneously

Possible outcomes: H1, H2, H3, H4, H5, H6,
T1, T2, T3, T4, T5, T6

Examples of Outcomes



Examples of Outcomes



Experiment: Drawing a card from a deck of 52 cards

Examples of Outcomes



Experiment: Drawing a card from a deck of 52 cards

Possible outcomes: 13 Diamonds \diamond , 13 Hearts \heartsuit , 13 Spades \spadesuit , 13 Clubs \clubsuit (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King)

What is a Sample Space or Probability Space?

It is the set of all possible outcomes of an experiment. Each individual outcome is a sample point.

Examples of Sample Space



Examples of Sample Space



Experiment: Rolling a die

Examples of Sample Space



Experiment: Rolling a die
Sample Space = $\{1, 2, 3, 4, 5, 6\}$

Examples of Sample Space



Experiment: Rolling a die
Sample Space = $\{1, 2, 3, 4, 5, 6\}$
Sample Point: 4

Examples of Sample Space



Examples of Sample Space



Experiment: Flipping two coins

Examples of Sample Space



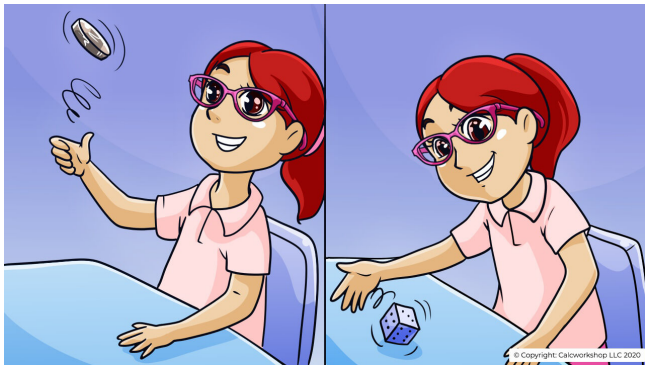
Experiment: Flipping two coins
Sample Space = $\{HH, HT, TH, TT\}$

Examples of Sample Space

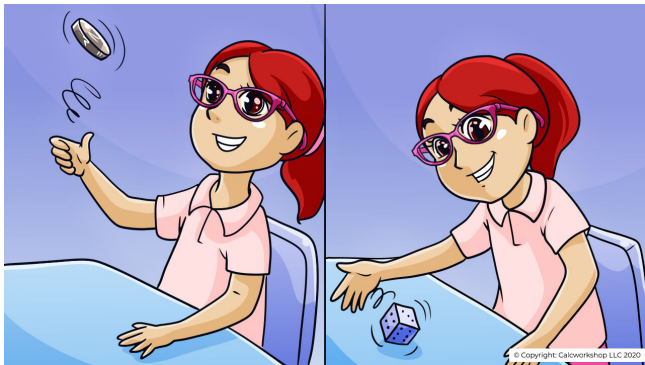


Experiment: Flipping two coins
Sample Space = $\{HH, HT, TH, TT\}$
Sample Point: TH

Examples of Sample Space

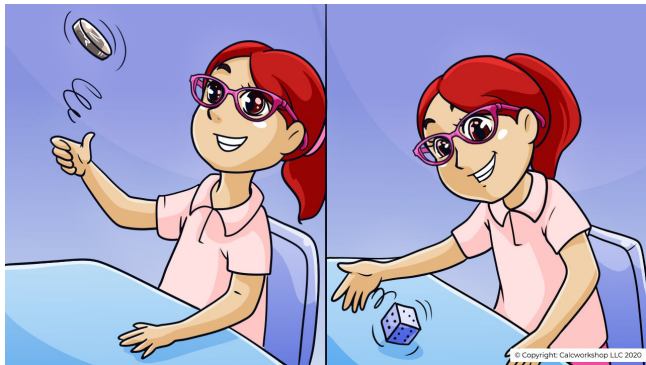


Examples of Sample Space



Experiment: Rolling a coin and a die simultaneously

Examples of Sample Space

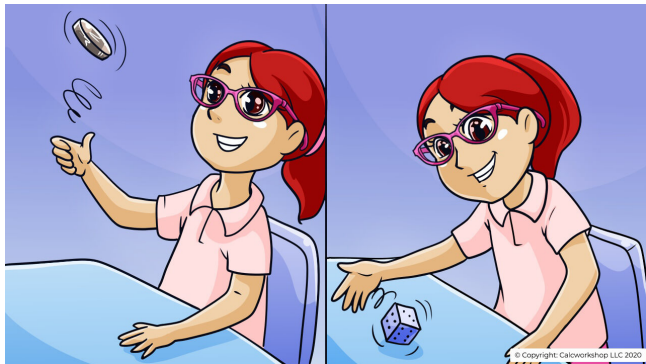


Experiment: Rolling a coin and a die
simultaneously

Sample Space

$$= \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

Examples of Sample Space



Experiment: Rolling a coin and a die
simultaneously

Sample Space

$= \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

Sample Point: T3

Examples of Sample Space



Examples of Sample Space



Experiment: Drawing a card from a deck of 52 cards

Examples of Sample Space



Experiment: Drawing a card from a deck of 52 cards

Sample Space: 13 Diamonds \diamond , 13 Hearts \heartsuit , 13 Spades \spadesuit , 13 Clubs \clubsuit (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King)

Examples of Sample Space



Experiment: Drawing a card from a deck of 52 cards

Sample Space: 13 Diamonds \diamond , 13 Hearts \heartsuit , 13 Spades \spadesuit , 13 Clubs \clubsuit (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King)

Sample Point: Queen of hearts

What is an Event?

It is any combination of outcomes.

Examples of Events



Examples of Events



Experiment: Rolling a die

Examples of Events



Experiment: Rolling a die
Sample Space = $\{1, 2, 3, 4, 5, 6\}$

Examples of Events



Experiment: Rolling a die

Sample Space = $\{1, 2, 3, 4, 5, 6\}$

Event: $A = \{\text{Getting an even number}\}$

Examples of Events



Experiment: Rolling a die

Sample Space = $\{1, 2, 3, 4, 5, 6\}$

Event: $A = \{\text{Getting an even number}\}$

$A = \{2, 4, 6\}$

Examples of Events



Examples of Events



Experiment: Flipping two coins

Examples of Events



Experiment: Flipping two coins
Sample Space = $\{HH, HT, TH, TT\}$

Examples of Events



Experiment: Flipping two coins

Sample Space = $\{HH, HT, TH, TT\}$

Event: $B = \{\text{Getting a head and a tail}\}$

Examples of Events



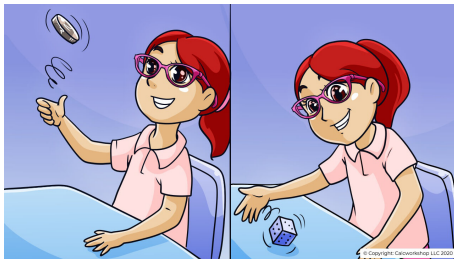
Experiment: Flipping two coins

Sample Space = $\{HH, HT, TH, TT\}$

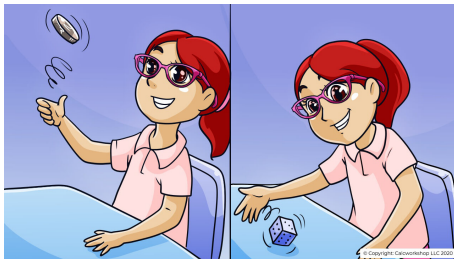
Event: $B = \{\text{Getting a head and a tail}\}$

$$B = \{HT, TH\}$$

Examples of Events

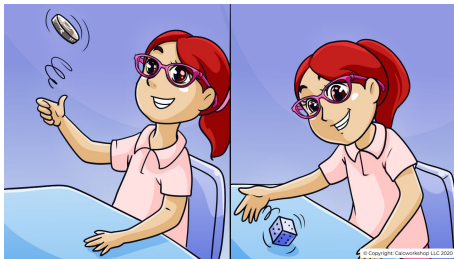


Examples of Events



Experiment: Rolling a coin and a die simultaneously

Examples of Events

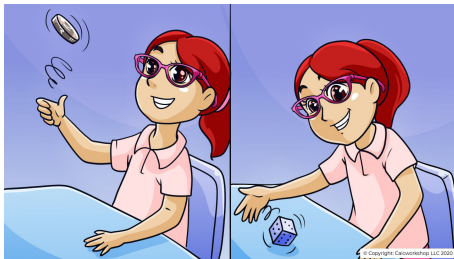


Experiment: Rolling a coin and a die
simultaneously

Sample Space

$$= \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

Examples of Events



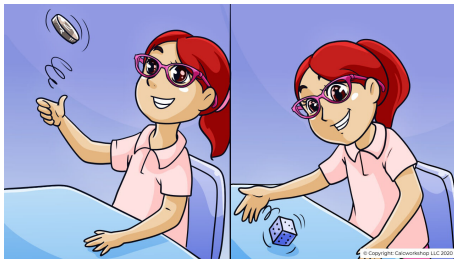
Experiment: Rolling a coin and a die
simultaneously

Sample Space

$= \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

Event: $C = \{\text{Getting a tail and an odd number}\}$

Examples of Events



Experiment: Rolling a coin and a die
simultaneously

Sample Space

$= \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

Event: $C = \{\text{Getting a tail and an odd number}\}$

$C = \{T1, T3, T5\}$

Examples of Events



Examples of Events



Experiment: Drawing a card

Examples of Events



Experiment: Drawing a card

Sample Space: 13 Diamonds \diamond , 13 Hearts \heartsuit ,
13 Spades \spadesuit , 13 Clubs \clubsuit (Ace, 2, 3, 4, 5, 6, 7,
8, 9, 10, Jack, Queen, King)

Examples of Events



Experiment: Drawing a card

Sample Space: 13 Diamonds \diamond , 13 Hearts \heartsuit ,
13 Spades \spadesuit , 13 Clubs \clubsuit (Ace, 2, 3, 4, 5, 6, 7,
8, 9, 10, Jack, Queen, King)

Event: $D = \{\text{Drawing a red face card}\}$

Examples of Events



Experiment: Drawing a card

Sample Space: 13 Diamonds \diamond , 13 Hearts \heartsuit ,
13 Spades \spadesuit , 13 Clubs \clubsuit (Ace, 2, 3, 4, 5, 6, 7,
8, 9, 10, Jack, Queen, King)

Event: $D = \{\text{Drawing a red face card}\}$

$D = \{\text{Jack } \diamond, \text{Jack } \heartsuit, \text{Queen } \diamond, \text{Queen } \heartsuit, \\ \text{King } \diamond, \text{King } \heartsuit\}$

Example 1

Write the sample space for each experiment.

1. A coin tossed three times.

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Write the sample space for each experiment.

1. A coin tossed three times.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Example 1

Write the sample space for each experiment.

2. A vowel of the English alphabet picked at random from a box.

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Write the sample space for each experiment.

2. A vowel of the English alphabet picked at random from a box.

$$S = \{a, e, i, o, u\}$$

Example 1

Write the sample space for each experiment.

3. A roll of a die and tossing a coin.

Example 1

Write the sample space for each experiment.

3. A roll of a die and tossing a coin.

$$S = \{1H, 2H, 3H, 4H, 5H, 6H, \\ 1T, 2T, 3T, 4T, 5T, 6T\}$$

Example 1

Write the sample space for each experiment.

4. A day of the week picked at random from a box.

Example 1

Write the sample space for each experiment.

4. A day of the week picked at random from a box.

$S = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

1. $A = \{\text{tail and an even number}\}$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

1. $A = \{\text{tail and an even number}\}$

$A = \{T2, T4, T6\}$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

2. $B = \{\text{an outcome with a number less than 4}\}$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

2. $B = \{\text{an outcome with a number less than 4}\}$

$$B = \{H1, H2, H3, T1, T2, T3\}$$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

3. $C = \{\text{head and a number less than 5}\}$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

$$3. C = \{\text{head and a number less than 5}\}$$
$$C = \{H1, H2, H3, H4\}$$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

4. $D = \{\text{an outcome with a number at most 4}\}$

Example 2

Let a coin and a die be tossed. Give the elements of the following events.

4. $D = \{\text{an outcome with a number at most 4}\}$

$$D = \{H1, H2, H3, H4, T1, T2, T3, T4\}$$

Example 2

A pair of dice is rolled. Write the elements of each event.

1. $E = \{\text{the pair of numbers have a sum of 5}\}$

Example 2

A pair of dice is rolled. Write the elements of each event.

1. $E = \{\text{the pair of numbers have a sum of 5}\}$
 $E = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

Example 2

A pair of dice is rolled. Write the elements of each event.

2. $F = \{\text{the pair of numbers whose sum is 6 but none of the two is even}\}$

Example 2

A pair of dice is rolled. Write the elements of each event.

2. $F = \{\text{the pair of numbers whose sum is 6 but none of the two is even}\}$

$$F = \{(1, 5), (3, 3), (5, 1)\}$$

Example 2

A pair of dice is rolled. Write the elements of each event.

3. $G = \{\text{the two numbers whose sum is at most 5}\}$

Example 2

A pair of dice is rolled. Write the elements of each event.

3. $G = \{\text{the two numbers whose sum is at most 5}\}$

$$G = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$$

Example 2

A pair of dice is rolled. Write the elements of each event.

4. $H = \{\text{the pair of numbers whose sum is 6 but none of the two is odd}\}$

Example 2

A pair of dice is rolled. Write the elements of each event.

4. $H = \{\text{the pair of numbers whose sum is 6 but none of the two is odd}\}$

$$H = \{(2, 4), (4, 2)\}$$

**Thank you for attending
the virtual class.**