

# Domain and Range of Functions

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# What is the Domain of a Function?

Domain: the set of all permissible values of  $x$  that give real values for  $y$

# What is Function Notation?

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- ▶ If  $f$  is a function, the symbol  $f(x)$  is used to denote the value of the function  $f$  at a given value of  $x$ .

# What is Function Notation?

- ▶ The  $f(x)$  notation can also be used to define a function.
- ▶ If  $f$  is a function, the symbol  $f(x)$  is used to denote the value of the function  $f$  at a given value of  $x$ .
- ▶  $f(x)$  denotes the  $y$ -value that the function  $f$  associates with  $x$ -value.

# Example 1

Using the function notation, how do you write the rule of function  $f$  such that  $y = -2x + 1$ ?

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$$f(x) = -2x + 1$$



# Example 2

Using the function notation, how do you write the rule of function  $g$  such that  $y = 5x - 2$ ?

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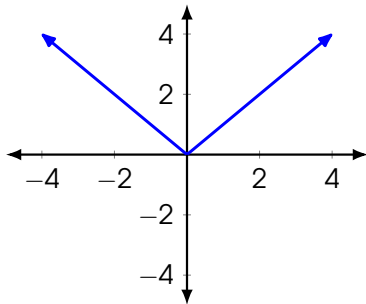
$$g(x) = 5x - 2$$

# What is the Range of a Function?

Range: the set of permissible values for  $y$  or  $f(x)$  that give the values of  $x$  real numbers

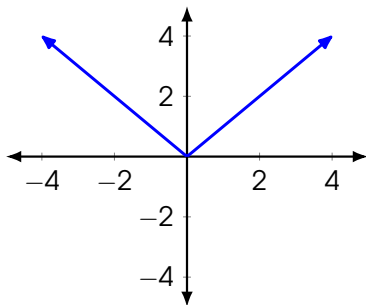
# Example 1

Determine the domain and the range of the function described in this graph.



# Example 1

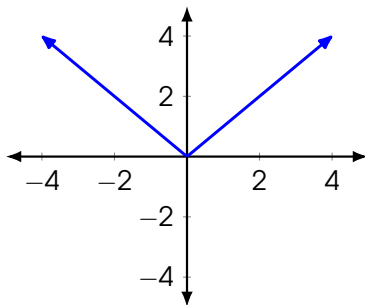
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\}$$

# Example 1

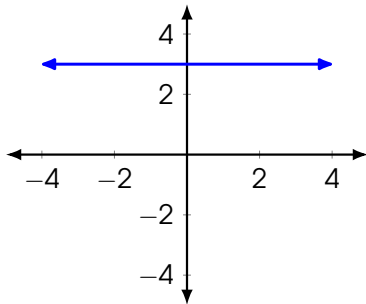
Determine the domain and the range of the function described in this graph.



$$D = \{x|x \in \mathbb{R}\} , R = \{y|y \in \mathbb{R}, y \geq 0\}$$

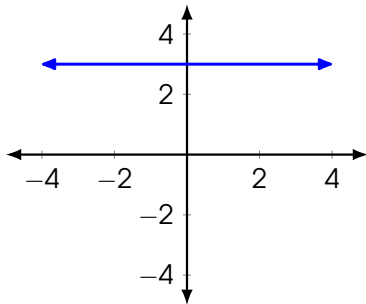
## Example 2

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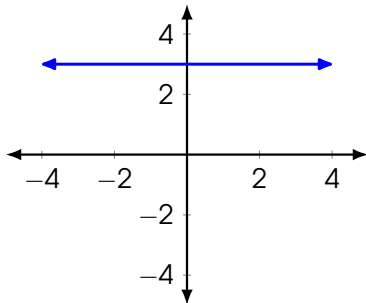


$$D = \{x | x \in \mathbb{R}\}$$



## Example 2

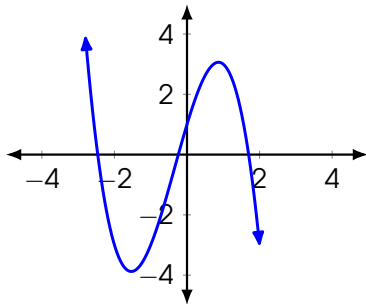
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\} , R = \{y | y = 3\}$$

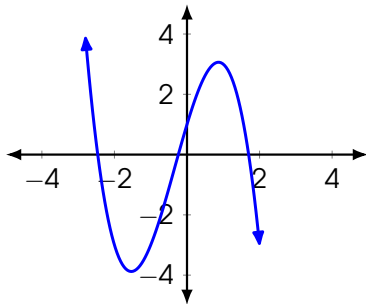
# Example 3

Determine the domain and the range of the function described in this graph.



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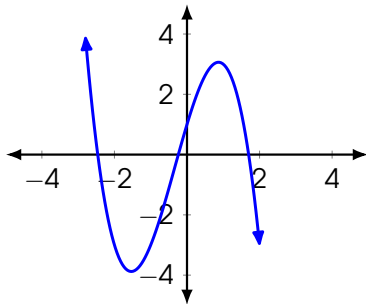
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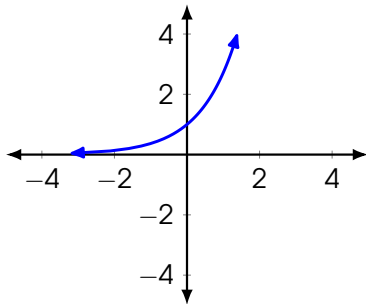
Determine the domain and the range of the function described in this graph.



$$D = \{x|x \in \mathbb{R}\} , R = \{y|y \in \mathbb{R}\}$$

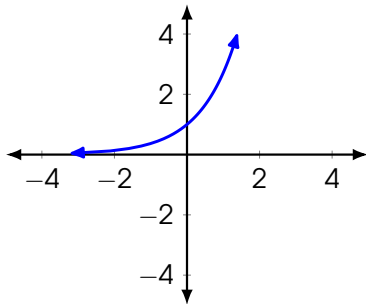
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Determine the domain and the range of the function described in this graph.



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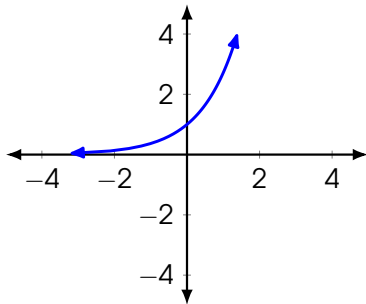
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\}$$

# Example 4

Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\} , R = \{y | y \in \mathbb{R}, y > 0\}$$

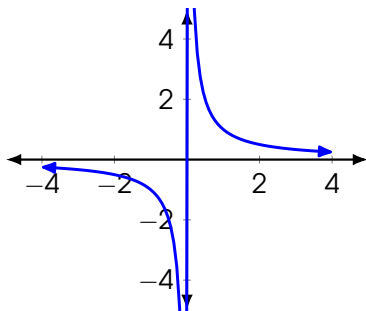
# What is an Asymptote?

Asymptote: a line that the graph of a function approaches, but never intersects



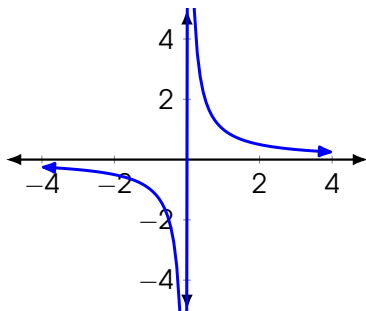
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Determine the domain and the range of the function described in this graph.



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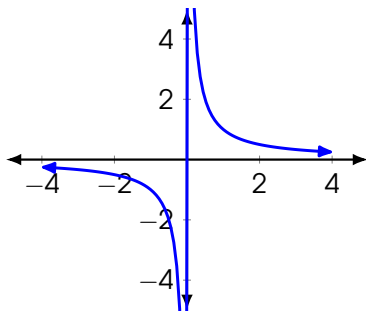
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \neq 0\}$$

# Example 5

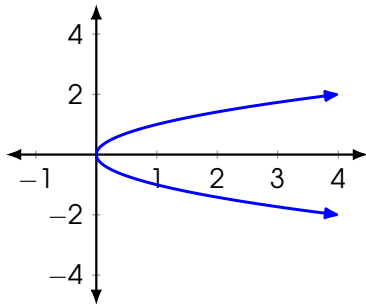
Determine the domain and the range of the function described in this graph.



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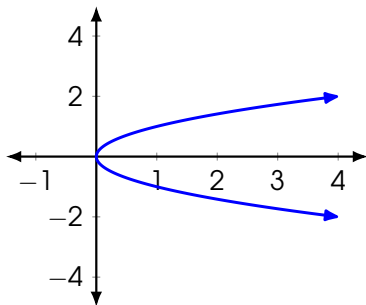
# Example 6

Determine the domain and the range of the function described in this graph.



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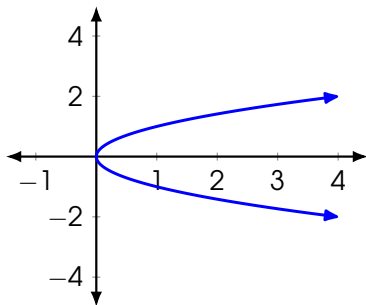
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \geq 0\}$$

# Example 6

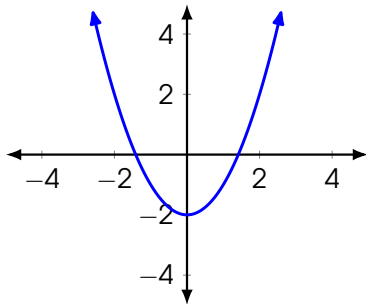
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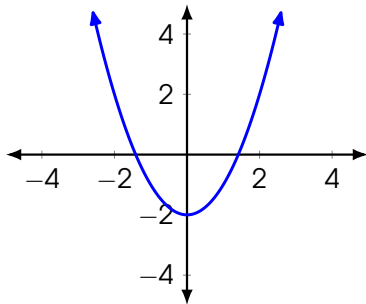
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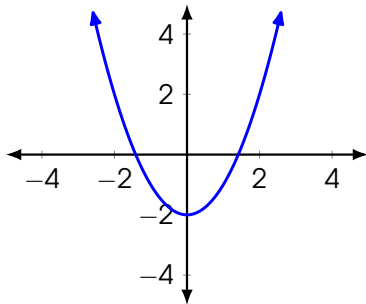


$$D = \{x | x \in \mathbb{R}\}$$



# Example 7

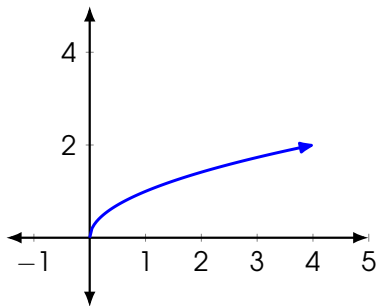
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\} , R = \{y | y \in \mathbb{R}, y \geq -2\}$$

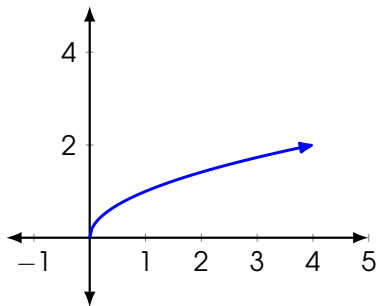
# Example 8

Determine the domain and the range of the function described in this graph.



# Example 8

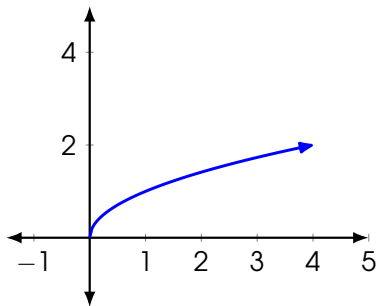
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \geq 0\}$$

# Example 8

Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \geq 0\} , R = \{y | y \in \mathbb{R}, y \geq 0\}$$

# Example 9

Determine the domain and the range of the following function.

$$f(x) = 3x$$

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# Example 10

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# Take Note!

The value of the function will not be a real number if it is an imaginary number ( $\sqrt{-1}$ ) or undefined (division by 0).

# How to Determine the Domain of a Function with Square Root?

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- ▶ Write the radicand as an expression greater than or equal to zero.

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- ▶ Write the radicand as an expression greater than or equal to zero.
- ▶ Solve for  $x$ .

# How to Determine the Domain of a Rational Function?

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- ▶ Write the denominator as an expression not equal to zero.



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- ▶ Write the denominator as an expression not equal to zero.
- ▶ Solve for  $x$ .

# Example 11

Determine the domain of the following function.

$$f(x) = \sqrt{x - 2}$$

# Example 11

Step 1: Write the radicand as an expression greater than or equal to zero.

$$f(x) = \sqrt{x - 2}$$

# Example 11

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

# Example 11

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{x - 2}$$

Radicand

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Use Addition Prop.

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Step 2: Solve for  $x$ .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

$$x - 2 + 2 \geq 0 + 2$$

# Example 11

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

$$x - 2 + 2 \geq 0 + 2$$

Simplify

# Example 11

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{x - 2}$$

Radicand

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Simplify

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Radicand

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Simplify

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Domain:

# Example 11

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$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

$$x - 2 + 2 \geq 0 + 2$$

Simplify

$$x \geq 2$$

Domain:

$$D = \{x | x \in \mathbb{R}, x \geq 2\}$$

# Example 12

Determine the domain of the following function.

$$f(x) = \frac{x+1}{x-1}$$

# Example 12

Step 1: Write the denominator as an expression not equal to zero.

$$f(x) = \frac{x+1}{x-1}$$

# Example 12

Step 2: Solve for  $x$ .

$$f(x) = \frac{x+1}{x-1}$$

Denominator

$$x - 1 \neq 0$$

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Use Addition Prop.

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Simplify

$$x \neq 1$$

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$$f(x) = \frac{x+1}{x-1}$$

Denominator

$$x - 1 \neq 0$$

Use Addition Prop.

$$x - 1 + 1 \neq 0 + 1$$

Simplify

$$x \neq 1$$

Domain:

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Step 2: Solve for  $x$ .

$$f(x) = \frac{x+1}{x-1}$$

Denominator

$$x - 1 \neq 0$$

Use Addition Prop.

$$x - 1 + 1 \neq 0 + 1$$

Simplify

$$x \neq 1$$

Domain:

$$D = \{x | x \in \mathbb{R}, x \neq 1\}$$

# Example 13

Determine the domain of the following function.

$$f(x) = \sqrt{2x + 3}$$

# Example 13

Step 1: Write the radicand as an expression greater than or equal to zero.

$$f(x) = \sqrt{2x + 3}$$

# Example 13

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

# Example 13

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

# Example 13

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$



# Example 13

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

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Use Subtraction Prop.

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Simplify

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Step 2: Solve for  $x$ .

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Radicand

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Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

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Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

# Example 13

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

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Step 2: Solve for  $x$ .

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Radicand

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Simplify

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$$\frac{2x}{2} \geq \frac{-3}{2}$$

Simplify

$$x \geq -\frac{3}{2}$$

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Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

Simplify

$$x \geq -\frac{3}{2}$$

Domain:

# Example 13

Step 2: Solve for  $x$ .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

Simplify

$$x \geq -\frac{3}{2}$$

Domain:

$$D = \left\{ x \mid x \in \mathbb{R}, x \geq -\frac{3}{2} \right\}$$



# Example 14

Determine the domain of the following function.

$$f(x) = \frac{x - 3}{3x + 2}$$

# Example 14

Step 1: Write the denominator as an expression not equal to zero.

$$f(x) = \frac{x - 3}{3x + 2}$$

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator  $3x + 2 \neq 0$

Use Subtraction Prop.  $3x + 2 - 2 \neq 0 - 2$

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator  $3x + 2 \neq 0$

Use Subtraction Prop.  $3x + 2 - 2 \neq 0 - 2$

Simplify

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator	$3x + 2 \neq 0$
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Use Subtraction Prop.	$3x + 2 - 2 \neq 0 - 2$
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Simplify	$3x \neq -2$
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Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

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Use Division Prop.	
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Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator  $3x + 2 \neq 0$

Use Subtraction Prop.  $3x + 2 - 2 \neq 0 - 2$

Simplify  $3x \neq -2$

Use Division Prop.  $\frac{3x}{3} \neq \frac{-2}{3}$

Simplify

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Simplify

$$x \neq -\frac{2}{3}$$

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Step 2: Solve for  $x$ .

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Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Simplify

$$x \neq -\frac{2}{3}$$

Domain:

# Example 14

Step 2: Solve for  $x$ .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Simplify

$$x \neq -\frac{2}{3}$$

Domain:

$$D = \left\{ x \mid x \in \mathbb{R}, x \neq -\frac{2}{3} \right\}$$

**Thank you for watching.**