Categorizing Systems of Linear Equations in Two Variables

Jonathan R. Bacolod

Sauyo High School

How to Categorize a System of Linear Equations?

To identify the kind of system of linear equations in two variables like

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

get the ratios of $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

How to Categorize a System of Linear Equations?

Ratios	**Kind of System**

How to Categorize a System of Linear Equations?

Ratios	**Kind of System**
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Consistent-independent

How to Categorize a System of Linear Equations?

Ratios	**Kind of System**
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Consistent-independent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Inconsistent

How to Categorize a System of Linear Equations?

Ratios	**Kind of System**
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Consistent-independent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Inconsistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Consistent-dependent

Example 1

Identify the kind of system and describe the graph of the following system of linear equations using ratios.

$$\begin{cases} 2x + y = 3 \\ 2x - 2y = 3 \end{cases}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\qquad$ $b_1 = 1$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\qquad$ $b_1 = 1$ $\qquad$ $c_1 = 3$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\qquad$ $b_1 = 1$ $\qquad$ $c_1 = 3$

$a_2 = 2$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\qquad$ $b_1 = 1$ $\qquad$ $c_1 = 3$

$a_2 = 2$ $\qquad$ $b_2 = -2$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ $\quad \frac{b_1}{b_2}$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{1}{-2}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2} \qquad \frac{c_1}{c_2}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2} \qquad \frac{c_1}{c_2} = \frac{3}{3}$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2} \qquad \frac{c_1}{c_2} = \frac{3}{3} = 1$$

Example 1

$$\begin{cases} 2x + \quad y = 3 \\ 2x - 2y = 3 \end{cases}$$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = 3$

$a_2 = 2$ $\quad b_2 = -2$ $\quad c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2} \qquad \frac{c_1}{c_2} = \frac{3}{3} = 1$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system is Consistent-independent and its graphs are intersecting.

Example 2

Identify the kind of system and describe the graph of the following system of linear equations using ratios.

$$\begin{cases} 2x - y - 2 = 0 \\ \quad y \quad\quad = 2x + 1 \end{cases}$$

Example 2

Change the equations to standard form.

Example 2

Change the equations to standard form.

Original system

Example 2

Change the equations to standard form.

Original system $\begin{cases} 2x - y - 2 = 0 \\ y = 2x + 1 \end{cases}$

Example 2

Change the equations to standard form.

Original system $\begin{cases} 2x - y - 2 = 0 \\ y = 2x + 1 \end{cases}$

Use APE

Example 2

Change the equations to standard form.

Original system $\begin{cases} 2x - y - 2 = 0 \\ \quad y \quad = 2x + 1 \end{cases}$

Use APE $\begin{cases} \quad 2x - y = 2 \\ -2x + y = 1 \end{cases}$

Example 2

Change the equations to standard form.

Original system $\begin{cases} 2x - y - 2 = 0 \\ y = 2x + 1 \end{cases}$

Use APE $\begin{cases} 2x - y = 2 \\ -2x + y = 1 \end{cases}$

Use MPE

Example 2

Change the equations to standard form.

Original system

$$\begin{cases} 2x - y - 2 = 0 \\ \quad y \quad\quad = 2x + 1 \end{cases}$$

Use APE

$$\begin{cases} \quad 2x - y = 2 \\ -2x + y = 1 \end{cases}$$

Use MPE

$$\begin{cases} 2x - y & = & 2 \\ -1(-2x + y) & = & -1(1) \end{cases}$$

Example 2

Change the equations to standard form.

Original system $\begin{cases} 2x - y - 2 = 0 \\ y = 2x + 1 \end{cases}$

Use APE $\begin{cases} 2x - y = 2 \\ -2x + y = 1 \end{cases}$

Use MPE $\begin{cases} 2x - y = 2 \\ -1(-2x + y) = -1(1) \end{cases}$

Simplify

Example 2

Change the equations to standard form.

Original system
$$\begin{cases} 2x - y - 2 = 0 \\ \quad y \quad\quad = 2x + 1 \end{cases}$$

Use APE
$$\begin{cases} \quad 2x - y = 2 \\ -2x + y = 1 \end{cases}$$

Use MPE
$$\begin{cases} 2x - y & = & 2 \\ -1(-2x + y) & = & -1(1) \end{cases}$$

Simplify
$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\qquad$ $b_1 = -1$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad$ $b_1 = -1$ $\quad$ $c_1 = 2$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $b_1 = -1$ $c_1 = 2$

$a_2 = 2$ $b_2 = -1$ $c_2 = -1$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $b_1 = -1$ $c_1 = 2$

$a_2 = 2$ $b_2 = -1$ $c_2 = -1$

$\frac{a_1}{a_2}$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$$\frac{a_1}{a_2} = \frac{2}{2}$$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1$$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ $\quad \frac{b_1}{b_2}$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ $\quad \frac{b_1}{b_2} = \frac{-1}{-1}$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{-1}{-1} = 1$$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ $\quad \frac{b_1}{b_2} = \frac{-1}{-1} = 1$ $\quad \frac{c_1}{c_2}$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ $\quad \frac{b_1}{b_2} = \frac{-1}{-1} = 1$ $\quad \frac{c_1}{c_2} = \frac{2}{-1}$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ $\quad \frac{b_1}{b_2} = \frac{-1}{-1} = 1$ $\quad \frac{c_1}{c_2} = \frac{2}{-1} = -2$

Example 2

$$\begin{cases} 2x - y = 2 \\ 2x - y = -1 \end{cases}$$

$a_1 = 2$ $\quad b_1 = -1$ $\quad c_1 = 2$

$a_2 = 2$ $\quad b_2 = -1$ $\quad c_2 = -1$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1 \qquad \frac{b_1}{b_2} = \frac{-1}{-1} = 1 \qquad \frac{c_1}{c_2} = \frac{2}{-1} = -2$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system is Inconsistent and its graphs are parallel.

Example 3

Identify the kind of system and describe the graph of the following system of linear equations using ratios.

$$\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$$

Example 3

Change the equations to standard form.

Example 3

Change the equations to standard form.

Original system

Example 3

Change the equations to standard form.

Original system

$$\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$$

Example 3

Change the equations to standard form.

Original system $\begin{cases} 4x + 4y + 4 = 0 \\ -3y = 3x + 3 \end{cases}$

Use APE

Example 3

Change the equations to standard form.

Original system

$$\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$$

Use APE

$$\begin{cases} \quad 4x + 4y = -4 \\ -3x - 3y = 3 \end{cases}$$

Example 3

Change the equations to standard form.

Original system $\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$

Use APE $\begin{cases} \quad 4x + 4y = -4 \\ -3x - 3y = 3 \end{cases}$

Use MPE

Example 3

Change the equations to standard form.

Original system

$$\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$$

Use APE

$$\begin{cases} \quad 4x + 4y = -4 \\ -3x - 3y = 3 \end{cases}$$

Use MPE

$$\begin{cases} 4x + 4y & = & -4 \\ -1(-3x - 3y) & = & -1(3) \end{cases}$$

Example 3

Change the equations to standard form.

Original system

$$\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$$

Use APE

$$\begin{cases} \quad 4x + 4y = -4 \\ -3x - 3y = 3 \end{cases}$$

Use MPE

$$\begin{cases} 4x + 4y & = & -4 \\ -1(-3x - 3y) & = & -1(3) \end{cases}$$

Simplify

Example 3

Change the equations to standard form.

Original system

$$\begin{cases} 4x + 4y + 4 = 0 \\ \quad -3y \quad\quad = 3x + 3 \end{cases}$$

Use APE

$$\begin{cases} \quad 4x + 4y = -4 \\ -3x - 3y = 3 \end{cases}$$

Use MPE

$$\begin{cases} 4x + 4y \quad\quad\quad = \quad -4 \\ -1(-3x - 3y) \quad = \quad -1(3) \end{cases}$$

Simplify

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4 \qquad b_1 = 4$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$

$a_2 = 3$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4$ $\quad$ $b_1 = 4$ $\quad$ $c_1 = -4$

$a_2 = 3$ $\quad$ $b_2 = 3$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4$ $\quad$ $b_1 = 4$ $\quad$ $c_1 = -4$

$a_2 = 3$ $\quad$ $b_2 = 3$ $\quad$ $c_2 = -3$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4$ $\quad$ $b_1 = 4$ $\quad$ $c_1 = -4$

$a_2 = 3$ $\quad$ $b_2 = 3$ $\quad$ $c_2 = -3$

$$\frac{a_1}{a_2}$$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$

$a_2 = 3 \qquad b_2 = 3 \qquad c_2 = -3$

$$\frac{a_1}{a_2} = \frac{4}{3}$$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4$ $\quad$ $b_1 = 4$ $\quad$ $c_1 = -4$

$a_2 = 3$ $\quad$ $b_2 = 3$ $\quad$ $c_2 = -3$

$\frac{a_1}{a_2} = \frac{4}{3}$ $\quad$ $\frac{b_1}{b_2}$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$

$a_2 = 3 \qquad b_2 = 3 \qquad c_2 = -3$

$\frac{a_1}{a_2} = \frac{4}{3} \qquad \frac{b_1}{b_2} = \frac{4}{3}$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$

$a_2 = 3 \qquad b_2 = 3 \qquad c_2 = -3$

$\dfrac{a_1}{a_2} = \dfrac{4}{3} \qquad \dfrac{b_1}{b_2} = \dfrac{4}{3} \qquad \dfrac{c_1}{c_2}$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$$

$$a_2 = 3 \qquad b_2 = 3 \qquad c_2 = -3$$

$$\frac{a_1}{a_2} = \frac{4}{3} \qquad \frac{b_1}{b_2} = \frac{4}{3} \qquad \frac{c_1}{c_2} = \frac{-4}{-3}$$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$$

$$a_2 = 3 \qquad b_2 = 3 \qquad c_2 = -3$$

$$\frac{a_1}{a_2} = \frac{4}{3} \qquad \frac{b_1}{b_2} = \frac{4}{3} \qquad \frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$$

Example 3

$$\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$$

$$a_1 = 4 \qquad b_1 = 4 \qquad c_1 = -4$$

$$a_2 = 3 \qquad b_2 = 3 \qquad c_2 = -3$$

$$\frac{a_1}{a_2} = \frac{4}{3} \qquad \frac{b_1}{b_2} = \frac{4}{3} \qquad \frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the system is Consistent-dependent and its graphs are coinciding.

Thank you for watching.