Probability of an Event

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What is the Probability of an Event?

If an event E has n(E) equally likely outcomes and its sample space S has n(S) likely outcomes, then the probability of the event E is:

$$P(E) = \frac{\text{number of elements in } E}{\text{number of elements in } S} = \frac{n(E)}{n(S)}$$

$$S = \{HH, HT, TH, TT\}$$

$$S = \{HH, HT, TH, TT\}$$

 $E = \{HH\}$

$$S = \{HH, HT, TH, TT\}$$
 $n(S) = 4$
 $E = \{HH\}$

$$S = \{HH, HT, TH, TT\}$$
 $n(S) = 4$
 $E = \{HH\}$ $n(E) = 1$

$$S = \{HH, HT, TH, TT\}$$
 $n(S) = 4$
 $E = \{HH\}$ $n(E) = 1$

$$P(E) = \frac{n(E)}{n(S)}$$

$$S = \{HH, HT, TH, TT\}$$

$$E = \{HH\}$$

$$n(S) = 4$$

$$n(E) = 1$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{4}$$

$$S = \{HH, HT, TH, TT\}$$
 $n(S) = 4$
 $E = \{HH\}$ $n(E) = 1$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{4} \text{ or } 25\%$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{2, 3, 5\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2, 3, 5\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2, 3, 5\}$ $n(E) = 3$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2, 3, 5\}$ $n(E) = 3$
 $P(E) = \frac{n(E)}{n(S)}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2, 3, 5\}$ $n(E) = 3$
 $P(E) = \frac{n(E)}{n(S)}$
 $P(E) = \frac{3}{6}$

A die is rolled. What is the probability of getting a prime number?

n(S) = 6n(E) = 3

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{2, 3, 5\}$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{6}$$

$$P(E) = \frac{1}{2}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2, 3, 5\}$ $n(E) = 3$
 $P(E) = \frac{n(E)}{n(S)}$
 $P(E) = \frac{3}{6}$
 $P(E) = \frac{1}{2}$ or 50%

$$S = \{A\heartsuit, \ldots, K\heartsuit, \}$$

$$\mathcal{S} = \{A\heartsuit, \dots, K\heartsuit, A\diamondsuit, \dots, K\diamondsuit, A\clubsuit, \dots, K\clubsuit, \dots$$

$$\mathcal{S} = \{A\heartsuit, \dots, K\heartsuit, A\diamondsuit, \dots, K\diamondsuit, A\clubsuit, \dots, K\clubsuit, A\spadesuit, \dots, K\spadesuit\}$$

$$S = \{A\heartsuit, \dots, K\heartsuit, A\diamondsuit, \dots, K\diamondsuit, A\clubsuit, \dots, K\clubsuit, A\spadesuit, \dots, K\spadesuit\}$$
 $E = \{A\clubsuit, A\spadesuit\}$

$$S = \{A\heartsuit, \dots, K\heartsuit, \qquad n(S) = 52$$

 $A\diamondsuit, \dots, K\diamondsuit, \qquad A\clubsuit, \dots, K\clubsuit, \qquad A\spadesuit, \dots, K\spadesuit\}$
 $E = \{A\clubsuit, A\spadesuit\}$

$$S = \{A\heartsuit, \dots, K\heartsuit, \qquad n(S) = 52$$

 $A\diamondsuit, \dots, K\diamondsuit, \qquad n(E) = 2$
 $A\clubsuit, \dots, K\clubsuit, \qquad A\spadesuit, \dots, K\spadesuit\}$
 $E = \{A\clubsuit, A\spadesuit\}$

$$S = \{A\heartsuit, \dots, K\heartsuit, \qquad n(S) = 52$$

 $A\diamondsuit, \dots, K\diamondsuit, \qquad n(E) = 2$
 $A\clubsuit, \dots, K\clubsuit, \qquad A\spadesuit, \dots, K\spadesuit\}$
 $E = \{A\clubsuit, A\spadesuit\}$
 $P(E) = \frac{n(E)}{n(S)}$

$$S = \{A\heartsuit, \dots, K\heartsuit, \qquad n(S) \\ A\diamondsuit, \dots, K\diamondsuit, \qquad n(E) \\ A\clubsuit, \dots, K\clubsuit, \\ A\spadesuit, \dots, K\spadesuit\}$$

$$E = \{A\clubsuit, A\spadesuit\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{52}$$

$$n(S) = 52$$

 $n(E) = 2$

$$S = \{A\heartsuit, \dots, K\heartsuit, \qquad n(S) \\ A\diamondsuit, \dots, K\diamondsuit, \qquad n(E) \\ A\clubsuit, \dots, K\clubsuit, \\ A\spadesuit, \dots, K\spadesuit\}$$

$$E = \{A\clubsuit, A\spadesuit\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{52}$$

$$P(E) = \frac{1}{24}$$

$$n(S) = 52$$

 $n(E) = 2$

What are the Properties of Probability?

1. The probability of an event, P(E), is a number from 0 to 1 which tells how likely the event is to happen.

$$0 \le P(E) \le 1$$

What are the Properties of Probability?

Impossible	Unlikely	Even Chance	Likely	Certain
•	•	•	•	•
0	1 4	1/2	3 4	1
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

The closer the probability of an event to 1, the more likely the event is to happen and the closer the probability of an event to zero, the less likely it is to happen.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{2\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{2\}$ $n(E) = 1$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{2\}$$

$$n(S) = 6$$
$$n(E) = 1$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$S = \{1, 2, 3, 4, 5, 6\} \\ E = \{2\}$$

$$n(S) = 6$$

 $n(E) = 1$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E)=\frac{1}{6}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{2\}$$

$$n(S) = 6$$
$$n(E) = 1$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{6} \text{ or } 0.167$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{1, 2, 3, 4, 5\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{1, 2, 3, 4, 5\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{1, 2, 3, 4, 5\}$ $n(E) = 5$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{1, 2, 3, 4, 5\}$

$$n(S) = 6$$
$$n(E) = 5$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{1, 2, 3, 4, 5\}$

$$n(S) = 6$$
$$n(E) = 5$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E)=\frac{5}{6}$$

A die is rolled. What is the probability of getting a number less than 6?

n(S) = 6n(E) = 5

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{1, 2, 3, 4, 5\}$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{5}{6} \text{ or } 0.83$$

What are the Properties of Probability?

2. The probability of an event that cannot happen is 0.

$$P(\varnothing) = 0$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{\}$ $n(E) = 0$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{\}$

$$n(S) = 6$$
$$n(E) = 0$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{\}$$

$$n(S) = 6$$

 $n(E) = 0$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E)=\frac{0}{6}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{\}$

$$n(S) = 6$$
$$n(E) = 0$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{0}{6} \text{ or } 0$$

What are the Properties of Probability?

3. If an event is sure to happen, then the probability is 1.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{1, 2, 3, 4, 5, 6\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{1, 2, 3, 4, 5, 6\}$ $n(E) = 6$

$$S = \{1, 2, 3, 4, 5, 6\}$$
 $n(S) = 6$
 $E = \{1, 2, 3, 4, 5, 6\}$ $n(E) = 6$

$$P(E) = \frac{n(E)}{n(S)}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

 $E = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

 $n(E) = 6$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E)=\frac{6}{6}$$

$$S = \{1, 2, 3, 4, 5, 6\} \\ E = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6$$

 $n(E) = 6$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{6}$$
 or 1

What are the Properties of Probability?

4. The sum of the probabilities of all the outcomes in the sample space is 1.

$$P(H)=\frac{1}{2}$$

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

$$P(S) = P(H) + P(T)$$

$$P(H) = \frac{1}{2}$$
 $P(T) = \frac{1}{2}$
 $P(S) = P(H) + P(T)$
 $P(S) = \frac{1}{2} + \frac{1}{2}$

Experiment: Tossing a coin

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

$$P(S) = P(H) + P(T)$$

$$P(S) = \frac{1}{2} + \frac{1}{2}$$

$$P(S) = \frac{2}{2}$$

Experiment: Tossing a coin

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

$$P(S) = P(H) + P(T)$$

$$P(S) = \frac{1}{2} + \frac{1}{2}$$

$$P(S) = \frac{2}{2} = 1$$

$$P(1)=\frac{1}{6}$$

$$P(1) = \frac{1}{6}$$

$$P(2) = \frac{1}{6}$$

$$P(1) = \frac{1}{6}$$
 $P(3) = \frac{1}{6}$ $P(2) = \frac{1}{6}$

$$P(1) = \frac{1}{6}$$
 $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$

$$P(1) = \frac{1}{6}$$
 $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$

$$P(5)=\frac{1}{6}$$

$$P(1) = \frac{1}{6}$$

$$P(2) = \frac{1}{6}$$

$$P(3) = \frac{1}{6}$$

 $P(4) = \frac{1}{6}$

$$P(4) = \frac{1}{6}$$

$$P(5) = \frac{1}{6}$$

 $P(6) = \frac{1}{6}$

$$P(6) = \frac{1}{6}$$

$$P(1) = \frac{1}{6}$$
 $P(3) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$

$$P(S) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$P(1) = \frac{1}{6} \qquad P(3) = \frac{1}{6} \qquad P(5) = \frac{1}{6}$$

$$P(2) = \frac{1}{6} \qquad P(4) = \frac{1}{6} \qquad P(6) = \frac{1}{6}$$

$$P(S) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$P(S) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$P(1) = \frac{1}{6} \qquad P(3) = \frac{1}{6} \qquad P(5) = \frac{1}{6}$$

$$P(2) = \frac{1}{6} \qquad P(4) = \frac{1}{6} \qquad P(6) = \frac{1}{6}$$

$$P(S) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$P(S) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$P(S) = \frac{6}{6}$$

$$P(1) = \frac{1}{6} \qquad P(3) = \frac{1}{6} \qquad P(5) = \frac{1}{6}$$

$$P(2) = \frac{1}{6} \qquad P(4) = \frac{1}{6} \qquad P(6) = \frac{1}{6}$$

$$P(S) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$P(S) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$P(S) = \frac{6}{6} = 1$$

$$n(S) = 52$$

$$n(S) = 52$$
$$n(\heartsuit) = 13$$

$$n(S) = 52$$

 $n(\heartsuit) = 13$
 $P(\heartsuit) = \frac{n(\heartsuit)}{n(S)}$

$$n(S) = 52$$

$$n(\heartsuit) = 13$$

$$P(\heartsuit) = \frac{n(\heartsuit)}{n(S)}$$

$$P(\heartsuit) = \frac{13}{52}$$

$$n(S) = 52$$

$$n(\heartsuit) = 13$$

$$P(\heartsuit) = \frac{n(\heartsuit)}{n(S)}$$

$$P(\heartsuit) = \frac{13}{52}$$

$$P(\heartsuit) = \frac{1}{13}$$

$$n(S) = 20$$

$$n(S) = 20$$

 $n(blue marble) = 5$

$$n(S) = 20$$

 $n(\text{blue marble}) = 5$
 $P(\text{blue marble}) = \frac{n(\text{blue marble})}{n(S)}$

$$n(S) = 20$$

 $n(\text{blue marble}) = 5$
 $P(\text{blue marble}) = \frac{n(\text{blue marble})}{n(S)}$
 $P(\text{blue marble}) = \frac{5}{20}$

$$n(S) = 20$$

 $n(\text{blue marble}) = 5$
 $P(\text{blue marble}) = \frac{n(\text{blue marble})}{n(S)}$
 $P(\text{blue marble}) = \frac{5}{20}$
 $P(\text{blue marble}) = \frac{1}{4}$

$$n(S) = 12$$

$$n(S) = 12$$

 $n(E) = 4$

$$n(S) = 12$$

$$n(E) = 4$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$n(S) = 12$$

$$n(E) = 4$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{12}$$

$$n(S) = 12$$

$$n(E) = 4$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{12}$$

$$P(E) = \frac{1}{3}$$

$$n(S) = 45$$

$$n(S) = 45$$
$$n(girl) = 20$$

$$n(S) = 45$$

 $n(girl) = 20$
 $P(girl) = \frac{n(girl)}{n(S)}$

$$n(S) = 45$$
 $n(girl) = 20$
 $P(girl) = \frac{n(girl)}{n(S)}$
 $P(girl) = \frac{20}{45}$

$$n(S) = 45$$

$$n(girl) = 20$$

$$P(girl) = \frac{n(girl)}{n(S)}$$

$$P(girl) = \frac{20}{45}$$

$$P(girl) = \frac{4}{9}$$

$$n(S) = 4$$

$$n(S) = 4$$
$$n(TH) = 1$$

$$n(S) = 4$$

 $n(TH) = 1$
 $P(TH) = \frac{n(TH)}{n(S)}$

$$n(S) = 4$$
 $n(TH) = 1$
 $P(TH) = \frac{n(TH)}{n(S)}$
 $P(TH) = \frac{1}{\Delta}$

Thank you for attending the virtual class.