Systems of Linear Equations in Two Variables

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What is a System of Linear Equations?

- It consists of two or more linear equations with the same variables considered together for which a common solution is desired.

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- It consists of two or more linear equations with the same variables considered together for which a common solution is desired.
- It is also called Simultaneous Equations.

Examples and Non-Examples of Systems of Linear Equations in Two Variables

Examples

$$\begin{cases} 2x + y = 10 \\ \quad x + y = 6 \end{cases}$$

$$\begin{cases} y = 3x - 1 \\ y = x - 1 \end{cases}$$

$$\begin{cases} 2x \quad = y + 1 \\ 3x + y = 2 \end{cases}$$

$$\begin{cases} x \quad = 3y \\ \quad y = 6x \end{cases}$$

Non-Examples

$$x + y = 6$$

$$\begin{cases} x \quad = 1 \\ x + y = 2 \end{cases}$$

$$\begin{cases} x + \quad 2y \leq 3 \\ x + 3y \neq 2 \end{cases}$$

$$\begin{cases} x + y \quad = 1 \\ x + y - z = 4 \end{cases}$$

Example

Write Yes if the given is a system of linear equations in two variables or No if it is not.

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$$\begin{cases} y > 3x - 1 \\ y = x - 1 \end{cases}$$

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No

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No

$$\begin{cases} 2x = y + 1 \\ 3x = 2y \end{cases}$$

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Write Yes if the given is a system of linear equations in two variables or No if it is not.

$$\begin{cases} y = x - 1 \\ x + y = 6 \end{cases}$$ Yes

$$\begin{cases} y > 3x - 1 \\ y = x - 1 \end{cases}$$ No

$$\begin{cases} 2x = y + 1 \\ 3x = 2y \end{cases}$$ Yes

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Write Yes if the given is a system of linear equations in two variables or No if it is not.

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$$\begin{cases} y > 3x - 1 \\ y = x - 1 \end{cases}$$

No

$$\begin{cases} 2x = y + 1 \\ 3x = 2y \end{cases}$$

Yes

$$\begin{cases} x \quad\;\; = 3y \\ \quad\;\; y = 2 \end{cases}$$

Example

Write Yes if the given is a system of linear equations in two variables or No if it is not.

$$\begin{cases} \quad\;\; y = x - 1 \\ x + y = 6 \end{cases}$$

Yes

$$\begin{cases} y > 3x - 1 \\ y = x - 1 \end{cases}$$

No

$$\begin{cases} 2x = y + 1 \\ 3x = 2y \end{cases}$$

Yes

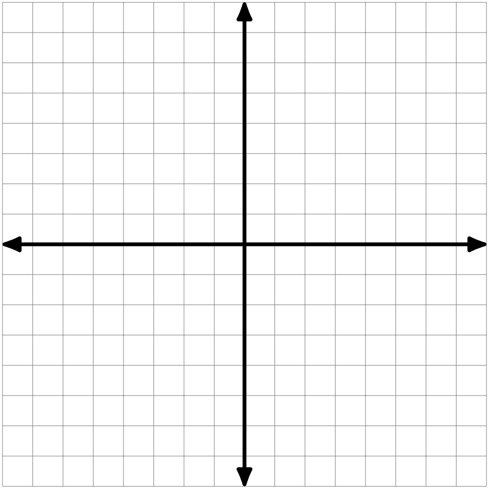
$$\begin{cases} x \qquad\; = 3y \\ \qquad y = 2 \end{cases}$$

No

What is a Solution Set of a System of Linear Equations?

A solution set is an ordered pair of real numbers that satisfies both equations of the system.

Example 1



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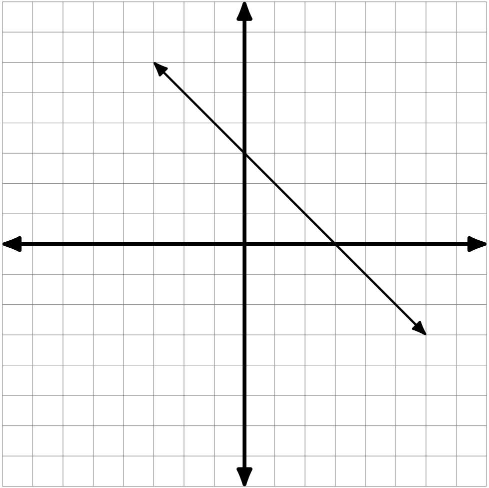

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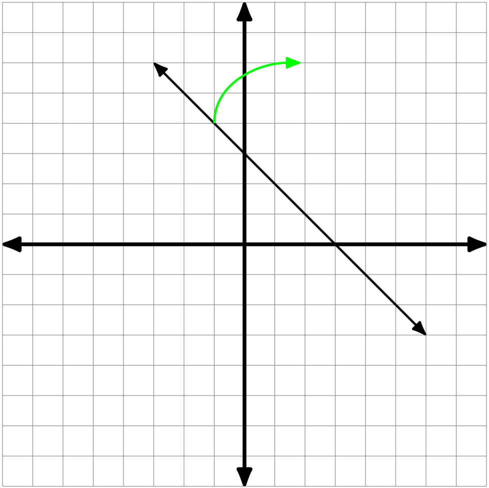

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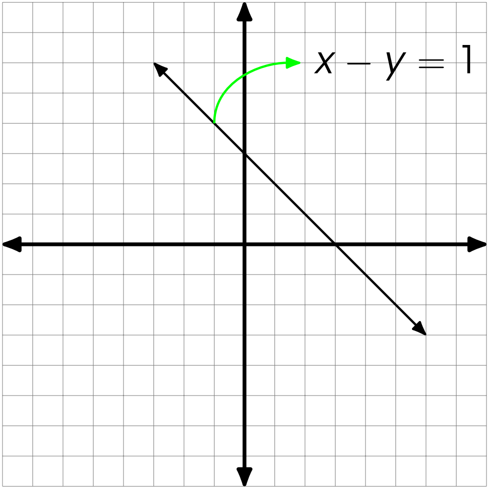

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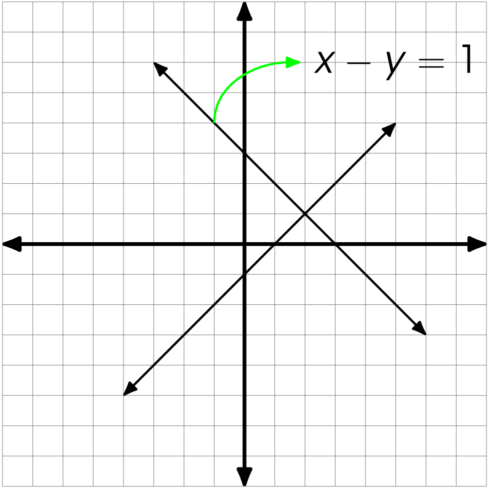

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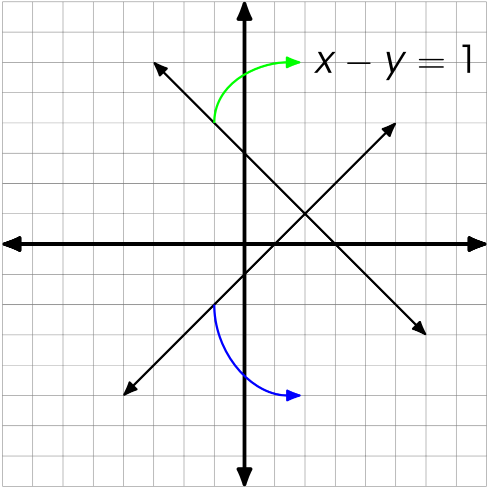

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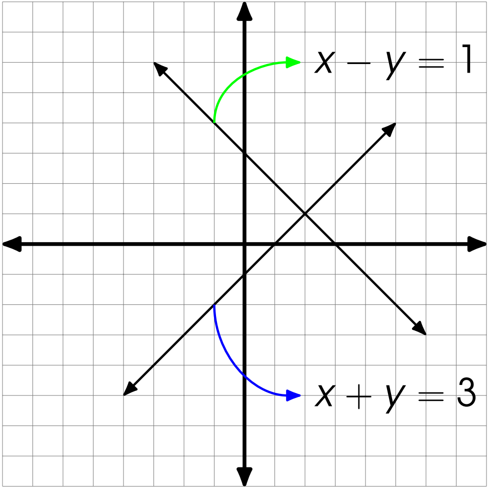

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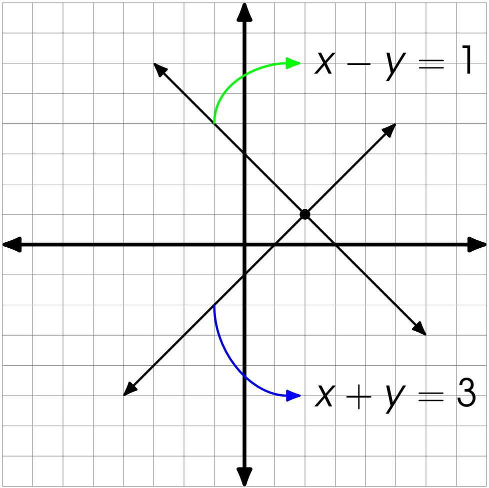

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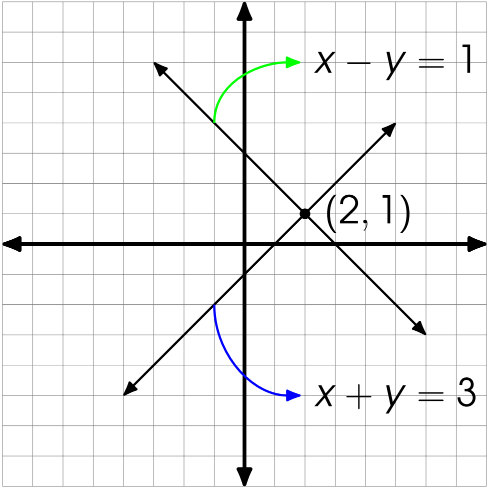

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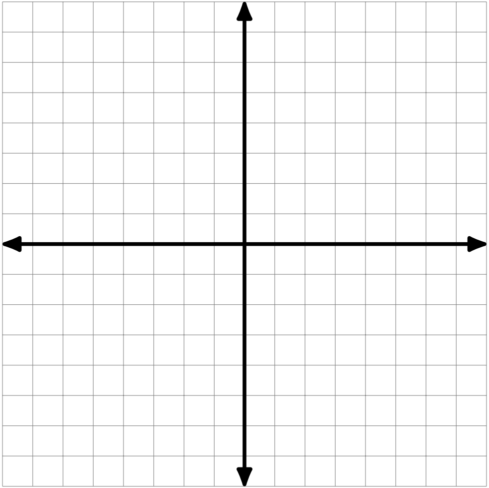

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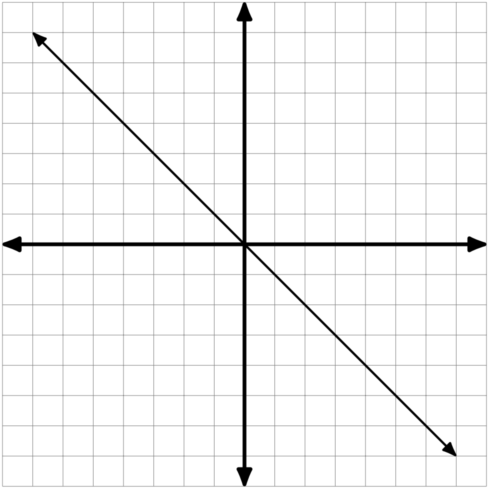

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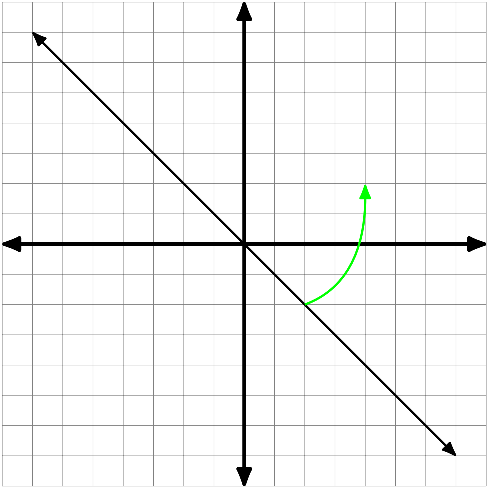

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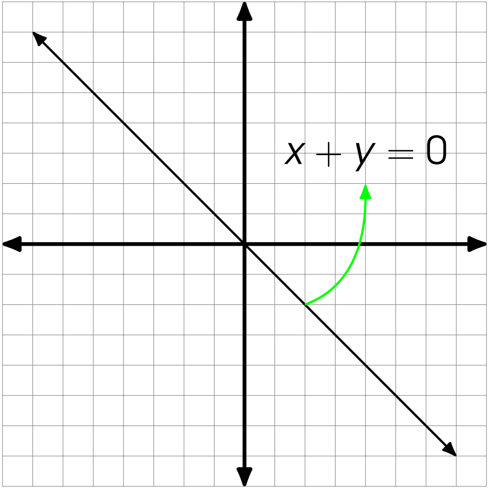

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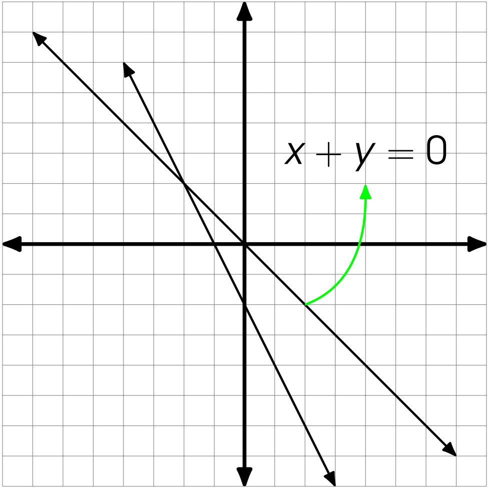

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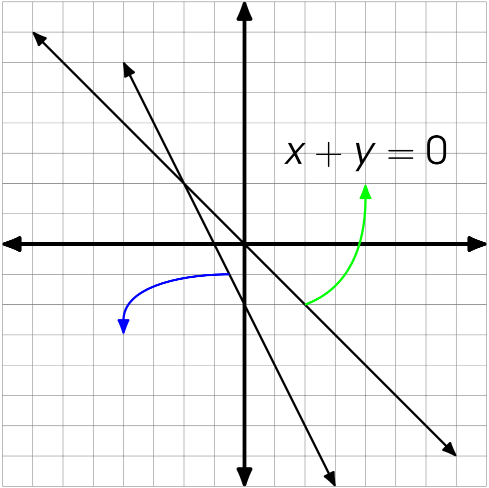

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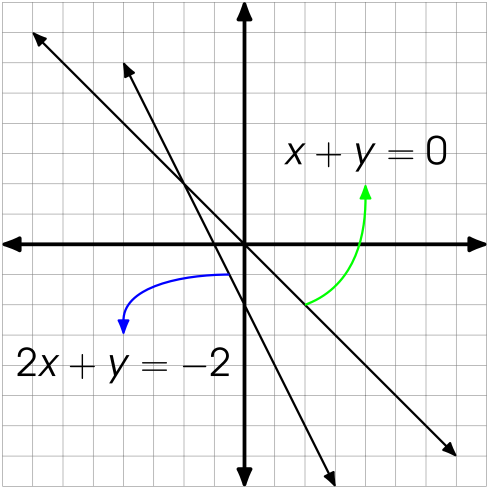

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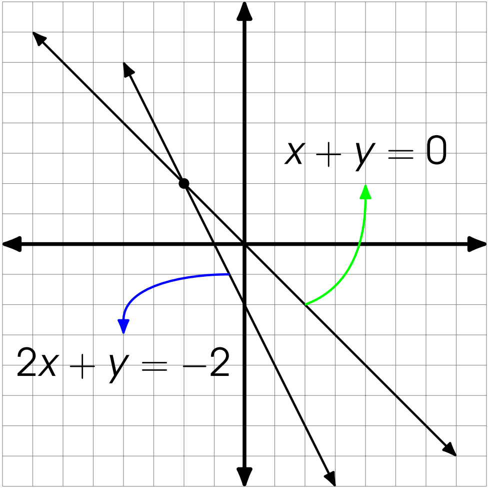

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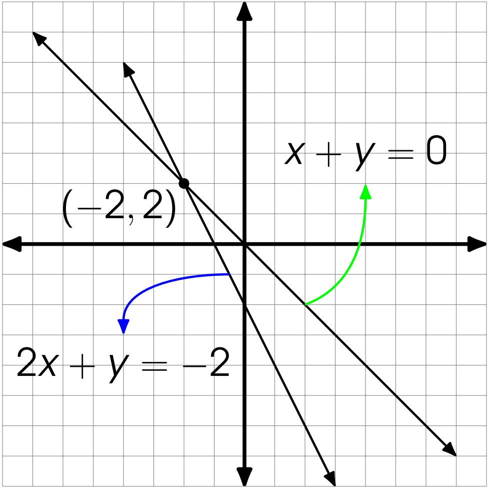

How to Check Whether an Ordered Pair is a Solution to a Linear System?

1. Replace x and y with the given values in both equations.

How to Check Whether an Ordered Pair is a Solution to a Linear System?

1. Replace x and y with the given values in both equations.
2. Simplify. Check if the ordered pair satisfies both equations.

Example 1

Is the ordered pair $(2, 1)$ a solution to the system $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$?

Example 1

Step 1: Replace x and y with the given values in both equations.

Given: $x = 2$,

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$x - y = 1$

Example 1

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Given: $x = 2, \quad y = 1$

$x - y = 1$

$2 - 1 = 1$

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = 1$

$x - y = 1$

$2 - 1 = 1$ Substitution Property

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = 1$

$x - y = 1$

$2 - 1 = 1$ Substitution Property

$1 = 1$

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = 1$

$x - y = 1$

$2 - 1 = 1$ Substitution Property

$1 = 1$ Simplification

$\therefore$ the ordered pair $(2, 1)$ satisfies the equation $x - y = 1$.

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$x + y = 3$

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = 1$

$x + y = 3$

$2 + 1 = 3$ Substitution Property

$3 = 1$

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = 1$

$x + y = 3$

$2 + 1 = 3$ Substitution Property

$3 = 1$ Simplification

$\therefore$ the ordered pair (2, 1) satisfies the equation $x + y = 3$.

Example 1

$\therefore$ since the ordered pair $(2, 1)$ satisfies both the equations $x - y = 1$ and $x + y = 3$, it is a solution to the system $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$.

Example 2

Is the ordered pair $(-1, 1)$ a solution to the system $\begin{cases} x + y = 0 \\ 2x + y = 1 \end{cases}$?

Example 2

Step 1: Replace x and y with the given values in both equations.

Given: $x = -1$,

Example 2

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$x + y = 0$

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Given: $x = -1, \quad y = 1$

$x + y = 0$

$-1 + 1 = 0$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1, \quad y = 1$

$x + y = 0$

$-1 + 1 = 0$ Substitution Property

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Given: $x = -1, \quad y = 1$

$x + y = 0$

$-1 + 1 = 0$ Substitution Property

$0 = 0$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1, \quad y = 1$

$x + y = 0$

$-1 + 1 = 0$ Substitution Property

$0 = 0$ Simplification

$\therefore$ the ordered pair $(-1, 1)$ satisfies the equation $x + y = 0$.

Example 2

Step 1: Replace x and y with the given values in both equations.

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Given: $x = -1, \quad y = 1$

$2x + y = 1$

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Given: $x = -1, \quad y = 1$

$2x + y = 1$

$2(-1) + 1 = 1$

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1, \quad y = 1$

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$2(-1) + 1 = 1$ Substitution Property

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1, \quad y = 1$

$2x + y = 1$

$2(-1) + 1 = 1$ Substitution Property

$-1 \neq 1$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1, \quad y = 1$

$2x + y = 1$	
$2(-1) + 1 = 1$	Substitution Property
$-1 \neq 1$	Simplification

$\therefore$ the ordered pair $(-1, 1)$ does not satisfy the equation $2x + y = 1$.

Example 2

$\therefore$ since the ordered pair $(-1, 1)$ does not satisfy the equation $2x + y = 1$, it is not a solution to the system $\begin{cases} x + y = 0 \\ 2x + y = 1 \end{cases}$.

Example 3

Is the ordered pair $(2, -1)$ a solution to the system $\begin{cases} x - 2y = 4 \\ x + 2y = 0 \end{cases}$?

Example 3

Step 1: Replace x and y with the given values in both equations.

Given: $x = 2$,

Example 3

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Given: $x = 2, \quad y = -1$

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$x - 2y = 4$

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Step 1: Replace x and y with the given values in both equations.

Given: $x = 2, \quad y = -1$

$x - 2y = 4$

$2 - 2(-1) = 4$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = -1$

$x - 2y = 4$

$2 - 2(-1) = 4$ Substitution Property

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = -1$

$x - 2y = 4$

$2 - 2(-1) = 4$ Substitution Property

$4 = 4$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = -1$

$x - 2y = 4$

$2 - 2(-1) = 4$ Substitution Property

$4 = 4$ Simplification

$\therefore$ the ordered pair $(2, -1)$ satisfies the equation $x - 2y = 4$.

Example 3

Step 1: Replace x and y with the given values in both equations.

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Example 3

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$x + 2y = 0$

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Given: $x = 2, \quad y = -1$

$x + 2y = 0$

$2 + 2(-1) = 0$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = -1$

$x + 2y = 0$

$2 + 2(-1) = 0$ Substitution Property

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = -1$

$x + 2y = 0$

$2 + 2(-1) = 0$ Substitution Property

$0 = 0$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = -1$

$x + 2y = 0$

$2 + 2(-1) = 0$ Substitution Property

$0 = 0$ Simplification

$\therefore$ the ordered pair $(2, -1)$ satisfies the equation $x + 2y = 0$.

Example 3

$\therefore$ since the ordered pair $(2, -1)$ satisfies both the equations $x - 2y = 4$ and $x + 2y = 0$, it is a solution to the system $\begin{cases} x - 2y = 4 \\ x + 2y = 0 \end{cases}$.

What are the Kinds of Systems of Linear Equations?

1. Consistent-dependent: a system of equations that can be rewritten as identical equations and have an infinite solution

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2. Consistent-independent: a system of equations that can not be rewritten as contradicting equations nor identical equations; they stay different and have one solution

What are the Kinds of Systems of Linear Equations?

1. Consistent-dependent: a system of equations that can be rewritten as identical equations and have an infinite solution
2. Consistent-independent: a system of equations that can not be rewritten as contradicting equations nor identical equations; they stay different and have one solution
3. Inconsistent: a system of equations that can be rewritten as contradicting equations and has no solution

Example 1

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

$$\begin{cases} 2(2x + y) = 2(4) \\ \quad 4x + 2y = 8 \end{cases}$$

Multiplication Property

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

$$\begin{cases} 2(2x + y) = 2(4) \\ \quad 4x + 2y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 4x + 2y = 8 \\ 4x + 2y = 8 \end{cases}$$

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

$$\begin{cases} 2(2x + y) = 2(4) \\ 4x + 2y = 8 \end{cases}$$ Multiplication Property

$$\begin{cases} 4x + 2y = 8 \\ 4x + 2y = 8 \end{cases}$$ Distributive Property

$\therefore$ since the equations are identical, it is a **Consistent-dependent** system and has infinitely many solutions.

Example 2

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

$$\begin{cases} 3(2x + 3y) = 3(4) \\ \quad 6x + \ 9y = 8 \end{cases}$$

Multiplication Property

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

$$\begin{cases} 3(2x + 3y) = 3(4) \\ \quad 6x + \ 9y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 6x + 9y = 12 \\ 6x + 9y = 8 \end{cases}$$

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

$$\begin{cases} 3(2x + 3y) = 3(4) \\ \quad 6x + \ 9y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 6x + 9y = \ 12 \\ 6x + 9y = 8 \end{cases}$$

Distributive Property

$\therefore$ since the equations are contradicting, it is an **Inconsistent** system and has no solution.

Example 3

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

$$\begin{cases} 2(x - 2y) = 2(1) \\ 2x + 4y = 3 \end{cases}$$

Multiplication Property

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

$$\begin{cases} 2(x - 2y) = 2(1) \\ 2x + 4y = 3 \end{cases}$$ Multiplication Property

$$\begin{cases} 2x - 4y = 2 \\ 2x + 4y = 3 \end{cases}$$ Distributive Property

$\therefore$ since the equations are not contradicting and not identical, it is a **Consistent-independent** system and has one solution.

Thank you for watching.