

# Proving Statements on Triangle Congruence

Jonathan R. Bacolod

Sauyo High School

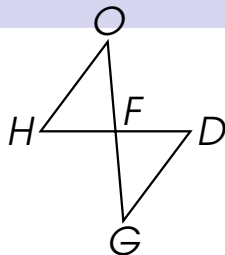
# AAS (Angle-Angle-Side) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-included side of another triangle, then the triangles are congruent.

# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

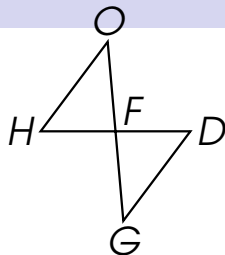


# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



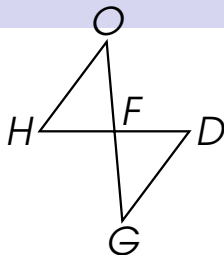
Statements	Reasons
------------	---------

# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



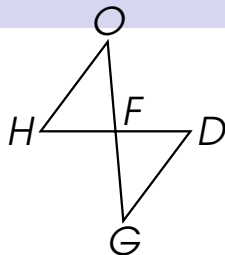
Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given

# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



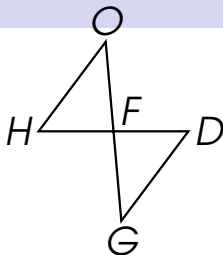
Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.

# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



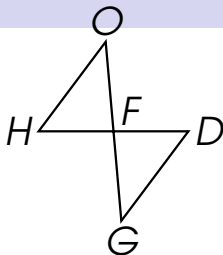
Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180^\circ, m\angle GFD + m\angle G + m\angle D = 180^\circ$	3. Angle Sum Thm.

# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180^\circ, m\angle GFD + m\angle G + m\angle D = 180^\circ$	3. Angle Sum Thm.
4. $m\angle OFH + m\angle O + m\angle H = m\angle GFD + m\angle G + m\angle D$	4. Transitive Prop.

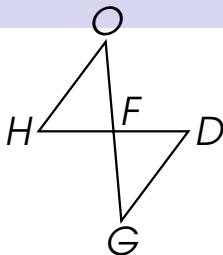


# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



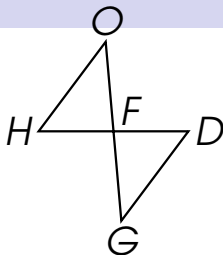
Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180^\circ, m\angle GFD + m\angle G + m\angle D = 180^\circ$	3. Angle Sum Thm.
4. $m\angle OFH + m\angle O + m\angle H = m\angle GFD + m\angle G + m\angle D$	4. Transitive Prop.
5. $m\angle O = m\angle G$	5. Subtraction Prop.

# Example 1

Given:  $\overline{OH} \cong \overline{GD}$   
 $\angle H \cong \angle D$

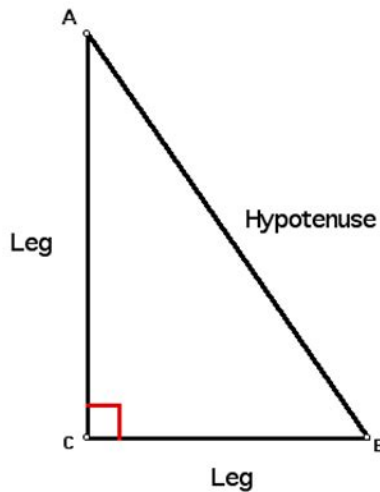
Prove:  $\triangle OFH \cong \triangle GFD$

Proof:



Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180^\circ, m\angle GFD + m\angle G + m\angle D = 180^\circ$	3. Angle Sum Thm.
4. $m\angle OFH + m\angle O + m\angle H = m\angle GFD + m\angle G + m\angle D$	4. Transitive Prop.
5. $m\angle O = m\angle G$	5. Subtraction Prop.
6. $\triangle OFH \cong \triangle GFD$	6. ASA Triangle Congruence Post.

# Right Triangle



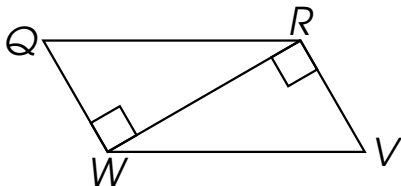
# LL (Leg-Leg) Congruence Theorem

If the legs of one right triangle are congruent respectively to the legs of another right triangle, then the triangles are congruent.

# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

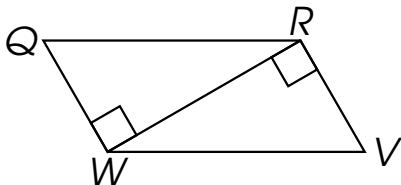


# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

Proof:



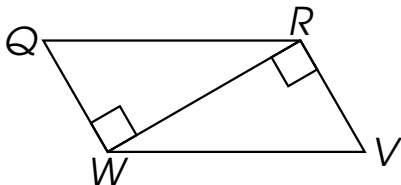
Statements	Reasons
------------	---------

# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

Proof:



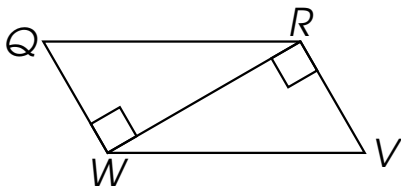
Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given

# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

Proof:



Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property

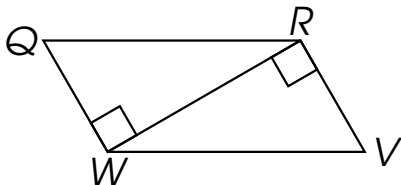


# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

Proof:



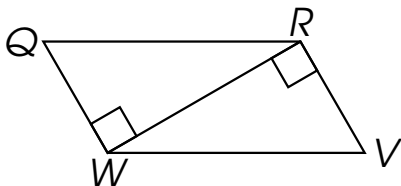
Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property
3. $m\angle RWQ = 90^\circ, m\angle WRV = 90^\circ$	3. Definition of Right Angles

# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

Proof:



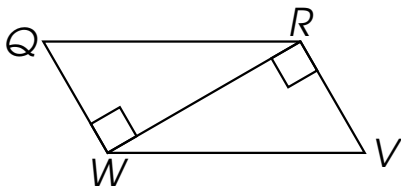
Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property
3. $m\angle RWQ = 90^\circ, m\angle WRV = 90^\circ$	3. Definition of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Definition of Congruent Angles

# Example 2

Given:  $\overline{QW} \cong \overline{VR}$

Prove:  $\triangle QRW \cong \triangle VWR$

Proof:



Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property
3. $m\angle RWQ = 90^\circ, m\angle WRV = 90^\circ$	3. Definition of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Definition of Congruent Angles
5. $\triangle QRW \cong \triangle VWR$	5. SAS Triangle Congruence Postulate

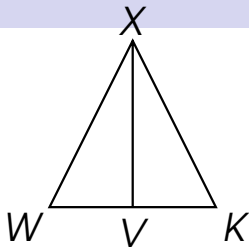
# LA (Leg-Acute Angle) Congruence Theorem

If a leg and an acute angle of one triangle are congruent respectively to a leg and an acute angle of another right triangle, then the triangles are congruent.

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

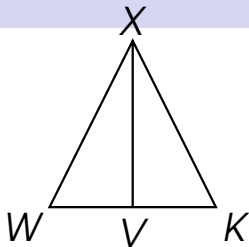


# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



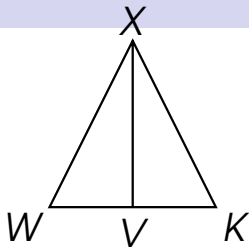
Statements	Reasons
------------	---------

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



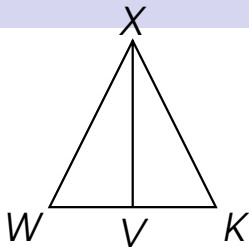
Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector

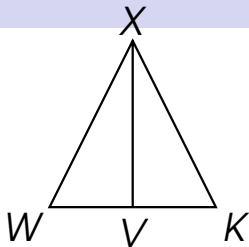


# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



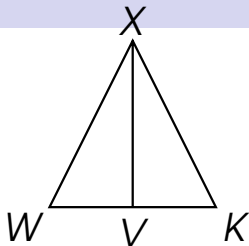
Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW, \angle XVK$ are right angles	3. Def. of Perp. Line Segments

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



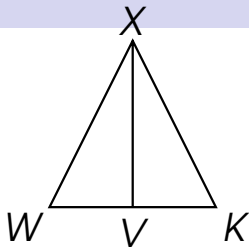
Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW$ , $\angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle XVW = 90^\circ$ , $m\angle XVK = 90^\circ$	4. Def. of Right Angles

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



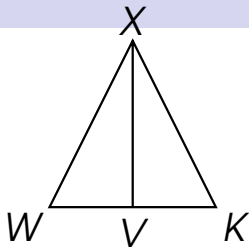
Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW$ , $\angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle XVW = 90^\circ$ , $m\angle XVK = 90^\circ$	4. Def. of Right Angles
5. $\angle XVW \cong \angle XVK$	5. Def. of Cong. $\angle$ s

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



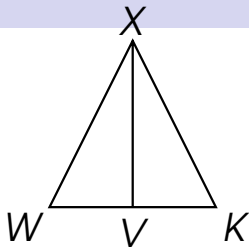
Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW$ , $\angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle XVW = 90^\circ$ , $m\angle XVK = 90^\circ$	4. Def. of Right Angles
5. $\angle XVW \cong \angle XVK$	5. Def. of Cong. $\angle$ s
6. $\overline{XV} \cong \overline{XV}$	6. Reflexive Property

# Example 3

Given:  $\overline{XV}$  bisects  $\angle W XK$   
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



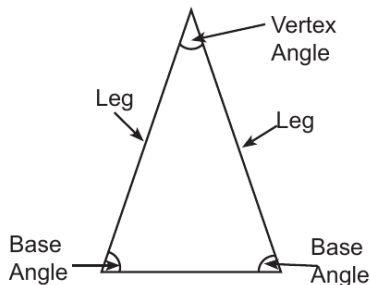
Statements	Reasons
1. $\overline{XV}$ bisects $\angle W XK$ , $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW$ , $\angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle XVW = 90^\circ$ , $m\angle XVK = 90^\circ$	4. Def. of Right Angles
5. $\angle XVW \cong \angle XVK$	5. Def. of Cong. $\angle$ s
6. $\overline{XV} \cong \overline{XV}$	6. Reflexive Property
7. $\triangle WXV \cong \triangle KXV$	7. ASA Post.

# HyL (Hypotenuse-Leg) Congruence Theorem

If the hypotenuse and a leg of one right triangle are congruent respectively to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

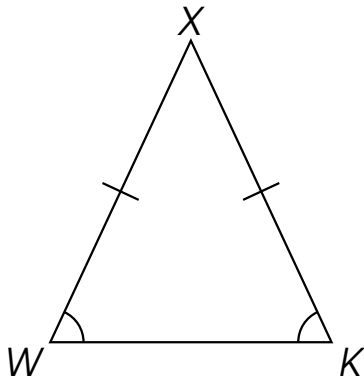
# Isosceles Triangle

A triangle is isosceles if two of its sides are congruent. The congruent sides are its legs; the third side is the base; the angles opposite the congruent sides are the base angles; and the angle included by the legs is the vertex angle.



# Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

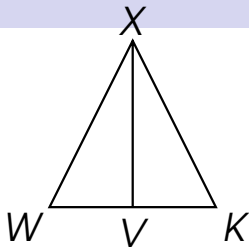




# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

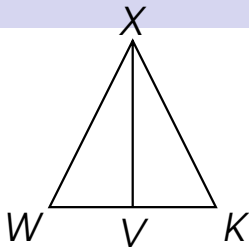


# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



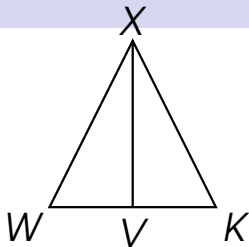
Statements	Reasons
------------	---------

# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



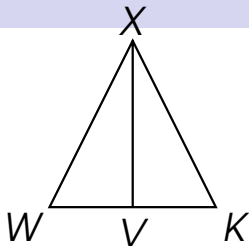
Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given

# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



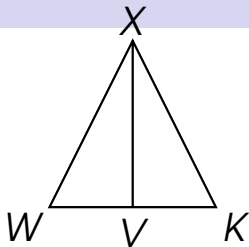
Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments

# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



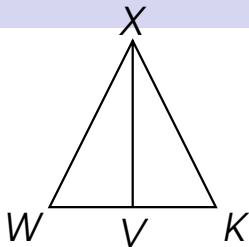
Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle XVW = 90^\circ, m\angle XVK = 90^\circ$	3. Def. of Right Angles

# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



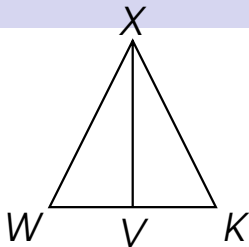
Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle XVW = 90^\circ, m\angle XVK = 90^\circ$	3. Def. of Right Angles
4. $\angle XVW \cong \angle XVK$	4. Def. of Cong. $\angle$ s

# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



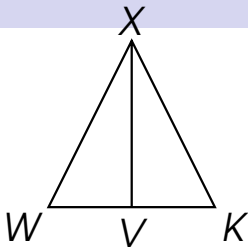
Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle XVW = 90^\circ, m\angle XVK = 90^\circ$	3. Def. of Right Angles
4. $\angle XVW \cong \angle XVK$	4. Def. of Cong. $\angle$ s
5. $\overline{XW} \cong \overline{XK}$	5. Def. of Isosceles Triangle

# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle XVW = 90^\circ, m\angle XVK = 90^\circ$	3. Def. of Right Angles
4. $\angle XVW \cong \angle XVK$	4. Def. of Cong. $\angle$ s
5. $\overline{XW} \cong \overline{XK}$	5. Def. of Isosceles Triangle
6. $\angle W \cong \angle K$	6. Isosceles Triangle Theorem

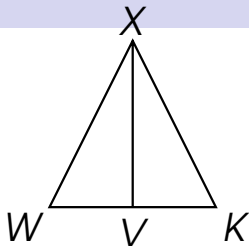


# Example 4

Given:  $\triangle XWK$  is an isosceles triangle  
 $\overline{XV} \perp \overline{WK}$

Prove:  $\triangle WXV \cong \triangle KXV$

Proof:



Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle XVW = 90^\circ, m\angle XVK = 90^\circ$	3. Def. of Right Angles
4. $\angle XVW \cong \angle XVK$	4. Def. of Cong. $\angle$ s
5. $\overline{XW} \cong \overline{XK}$	5. Def. of Isosceles Triangle
6. $\angle W \cong \angle K$	6. Isosceles Triangle Theorem
7. $\triangle WXV \cong \triangle KXV$	7. AAS Cong. Thm.

# HyA (Hypotenuse-Angle) Congruence Theorem

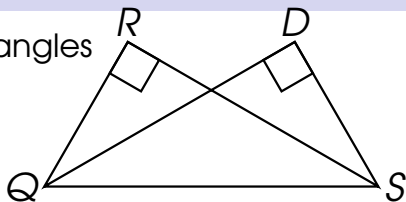
If the hypotenuse and an acute angle of one right triangle are congruent respectively to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$



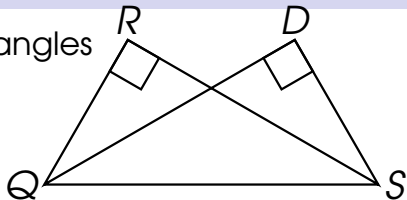
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
------------	---------

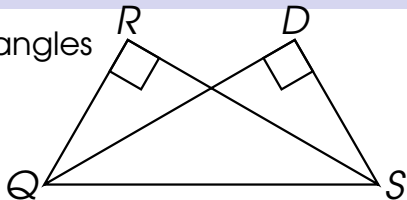
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given

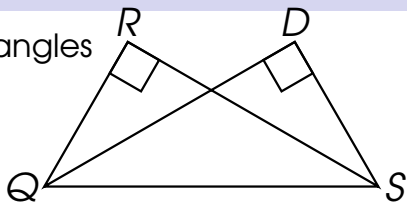
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given
2. $\angle R$ and $\angle D$ are right angles	2. Given

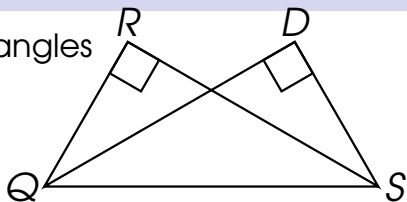
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given
2. $\angle R$ and $\angle D$ are right angles	2. Given
3. $m\angle R = 90^\circ, m\angle D = 90^\circ$	3. Def. of Right Angles

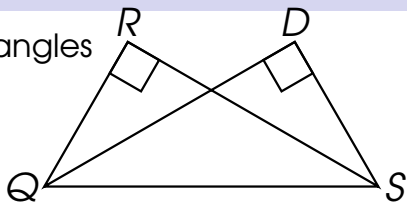
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given
2. $\angle R$ and $\angle D$ are right angles	2. Given
3. $m\angle R = 90^\circ, m\angle D = 90^\circ$	3. Def. of Right Angles
4. $\angle R \cong \angle D$	4. Def. of Cong. $\angle$ s



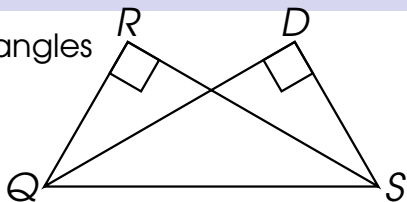
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given
2. $\angle R$ and $\angle D$ are right angles	2. Given
3. $m\angle R = 90^\circ, m\angle D = 90^\circ$	3. Def. of Right Angles
4. $\angle R \cong \angle D$	4. Def. of Cong. $\angle$ s
5. $\overline{QS} \cong \overline{SQ}$	5. Reflexive Property

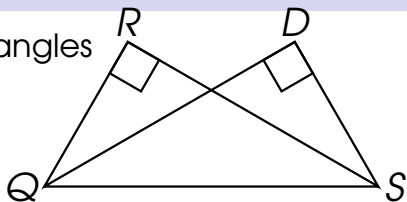
# Example 5

Given:  $\angle R$  and  $\angle D$  are right angles

$$\angle DQS \cong \angle RSQ$$

Prove:  $\triangle DQS \cong \triangle RSQ$

Proof:



Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given
2. $\angle R$ and $\angle D$ are right angles	2. Given
3. $m\angle R = 90^\circ, m\angle D = 90^\circ$	3. Def. of Right Angles
4. $\angle R \cong \angle D$	4. Def. of Cong. $\angle$ s
5. $\overline{QS} \cong \overline{SQ}$	5. Reflexive Property
6. $\triangle DQS \cong \triangle RSQ$	6. AAS Triangle Congruence Theorem

**Thank you for watching.**