Proving Statements on Triangle Congruence
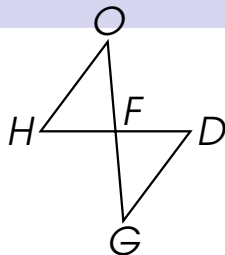
Jonathan R. Bacolod

Sauyo High School

AAS (Angle-Angle-Side) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-included side of another triangle, then the triangles are congruent.

Example 1

Given: $\overline{OH} \cong \overline{GD}$
$\angle H \cong \angle D$
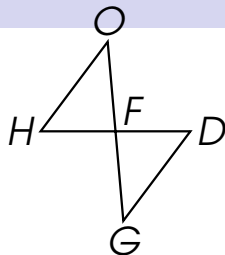
Prove: $\triangle OFH \cong \triangle GFD$

Example 1

Given: $\overline{OH} \cong \overline{GD}$
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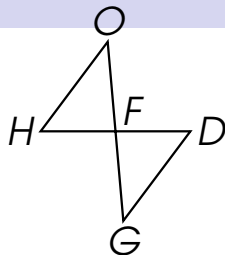
Prove: $\triangle OFH \cong \triangle GFD$

Proof:

Statements	Reasons

Example 1

Given: $\overline{OH} \cong \overline{GD}$
$\angle H \cong \angle D$

Prove: $\triangle OFH \cong \triangle GFD$

Proof:

Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given

Example 1

Given: $\overline{OH} \cong \overline{GD}$
$\angle H \cong \angle D$

Prove: $\triangle OFH \cong \triangle GFD$

Proof:

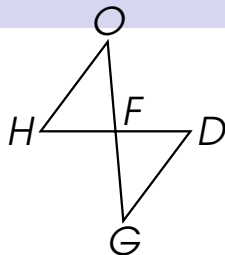| Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.

Example 1

Given: $\overline{OH} \cong \overline{GD}$
$\angle H \cong \angle D$
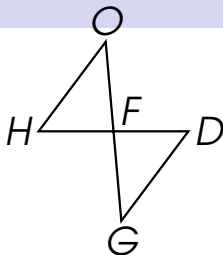
Prove: $\triangle OFH \cong \triangle GFD$

Proof:

Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180°, m\angle GFD + m\angle G + m\angle D = 180°$	3. Angle Sum Thm.

Example 1

Given: $\overline{OH} \cong \overline{GD}$
$\angle H \cong \angle D$
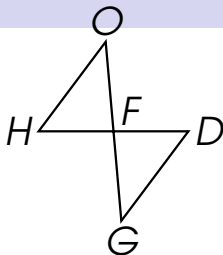
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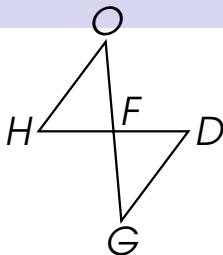| Statements	Reasons
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2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180°, m\angle GFD + m\angle G + m\angle D = 180°$	3. Angle Sum Thm.
4. $m\angle OFH + m\angle O + m\angle H = m\angle GFD + m\angle G + m\angle D$	4. Transitive Prop.

Example 1

Given: $\overline{OH} \cong \overline{GD}$
   $\angle H \cong \angle D$
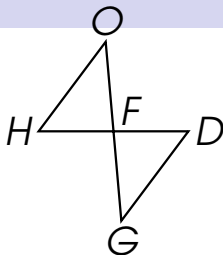
Prove: $\triangle OFH \cong \triangle GFD$

Proof:

Statements	Reasons
1. $\overline{OH} \cong \overline{GD}, \angle H \cong \angle D$	1. Given
2. $\angle OFH \cong \angle GFD$	2. Vert. Angle Thm.
3. $m\angle OFH + m\angle O + m\angle H = 180°, m\angle GFD + m\angle G + m\angle D = 180°$	3. Angle Sum Thm.
4. $m\angle OFH + m\angle O + m\angle H = m\angle GFD + m\angle G + m\angle D$	4. Transitive Prop.
5. $m\angle O = m\angle G$	5. Subtraction Prop.

Example 1

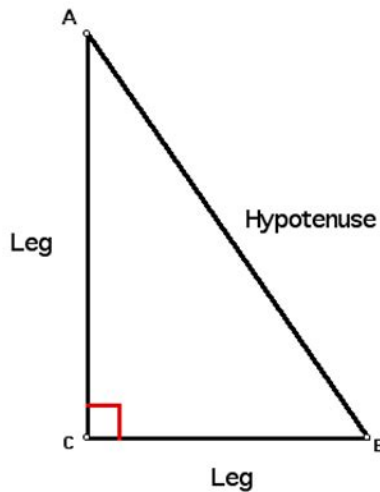Given: $\overline{OH} \cong \overline{GD}$
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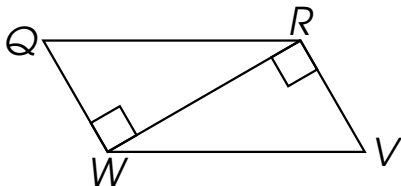| Statements	Reasons
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3. $m\angle OFH + m\angle O + m\angle H = 180°, m\angle GFD + m\angle G + m\angle D = 180°$	3. Angle Sum Thm.
4. $m\angle OFH + m\angle O + m\angle H = m\angle GFD + m\angle G + m\angle D$	4. Transitive Prop.
5. $m\angle O = m\angle G$	5. Subtraction Prop.
6. $\triangle OFH \cong \triangle GFD$	6. ASA Triangle Congruence Post.

Right Triangle

LL (Leg-Leg) Congruence Theorem

If the legs of one right triangle are congruent respectively to the legs of another right triangle, then the triangles are congruent.

Example 2

Given: $\overline{QW} \cong \overline{VR}$

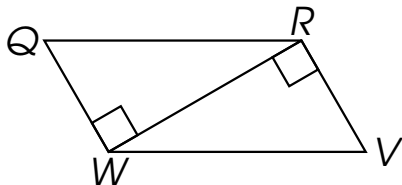Prove: $\triangle QRW \cong \triangle VWR$

Example 2

Given: $\overline{QW} \cong \overline{VR}$
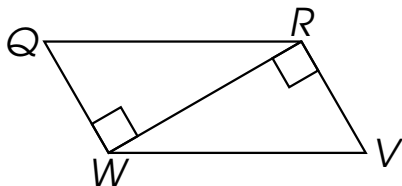
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Statements	Reasons

Example 2

Given: $\overline{QW} \cong \overline{VR}$

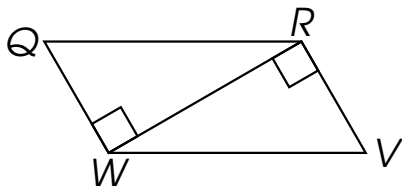Prove: $\triangle QRW \cong \triangle VWR$

Proof:

Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given

Example 2

Given: $\overline{QW} \cong \overline{VR}$

Prove: $\triangle QRW \cong \triangle VWR$

Proof:

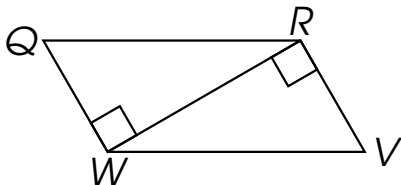| Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property

Example 2

Given: $\overline{QW} \cong \overline{VR}$
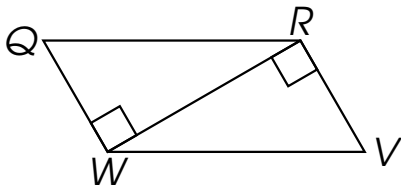
Prove: $\triangle QRW \cong \triangle VWR$

Proof:

Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Definition of Right Angles

Example 2

Given: $\overline{QW} \cong \overline{VR}$
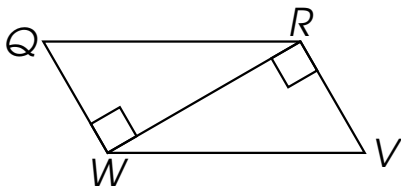
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Proof:

Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Definition of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Definition of Congruent Angles

Example 2

Given: $\overline{QW} \cong \overline{VR}$

Prove: $\triangle QRW \cong \triangle VWR$

Proof:

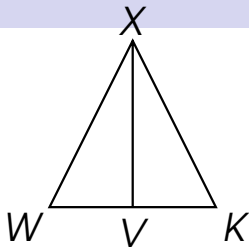| Statements	Reasons
1. $\overline{QW} \cong \overline{VR}$	1. Given
2. $\overline{WR} \cong \overline{RW}$	2. Reflexive Property
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Definition of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Definition of Congruent Angles
5. $\triangle QRW \cong \triangle VWR$	5. SAS Triangle Congruence Postulate

LA (Leg-Acute Angle) Congruence Theorem

If a leg and an acute angle of one triangle are congruent respectively to a leg and an acute angle of another right triangle, then the triangles are congruent.

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$
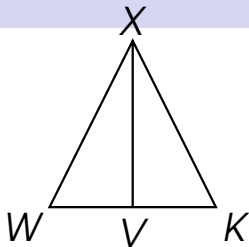
Prove: $\triangle WXV \cong \triangle KXV$

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$

Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$

Prove: $\triangle WXV \cong \triangle KXV$
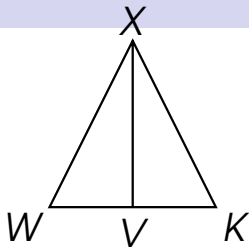
Proof:

Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK, \overline{XV} \perp \overline{WK}$	1. Given

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$
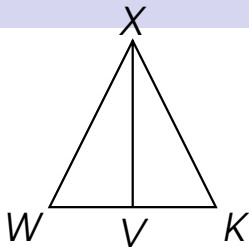
Prove: $\triangle WXV \cong \triangle KXV$

Proof:

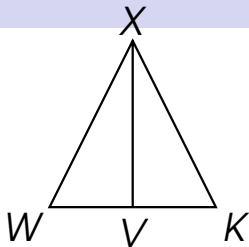| Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK$, $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
         $\overline{XV} \perp \overline{WK}$

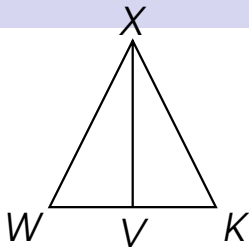Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK, \overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW, \angle XVK$ are right angles	3. Def. of Perp. Line Segments

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$

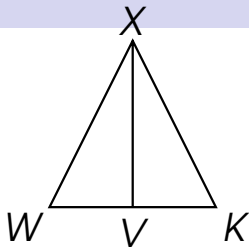Prove: $\triangle WXV \cong \triangle KXV$

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Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK$, $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW$, $\angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle RWQ = 90°$, $m\angle WRV = 90°$	4. Def. of Right Angles

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$

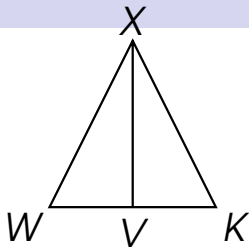Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK$, $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW, \angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle RWQ = 90°, m\angle WRV = 90°$	4. Def. of Right Angles
5. $\angle RWQ \cong \angle WRV$	5. Def. of Cong. $\angle s$

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
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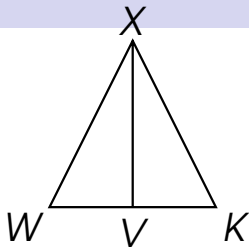
Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK, \overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW, \angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle RWQ = 90°, m\angle WRV = 90°$	4. Def. of Right Angles
5. $\angle RWQ \cong \angle WRV$	5. Def. of Cong. $\angle s$
6. $\overline{XV} \cong \overline{XV}$	6. Reflexive Property

Example 3

Given: $\overline{XV}$ bisects $\angle WXK$
$\overline{XV} \perp \overline{WK}$
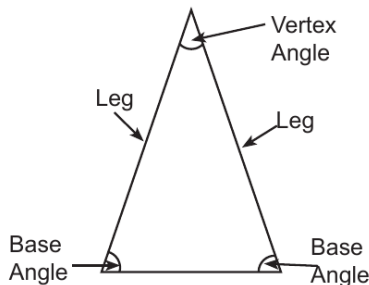
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Proof:

Statements	Reasons
1. $\overline{XV}$ bisects $\angle WXK, \overline{XV} \perp \overline{WK}$	1. Given
2. $\angle WXV \cong \angle KXV$	2. Def. of Angle Bisector
3. $\angle XVW, \angle XVK$ are right angles	3. Def. of Perp. Line Segments
4. $m\angle RWQ = 90°, m\angle WRV = 90°$	4. Def. of Right Angles
5. $\angle RWQ \cong \angle WRV$	5. Def. of Cong. $\angle s$
6. $\overline{XV} \cong \overline{XV}$	6. Reflexive Property
7. $\triangle WXV \cong \triangle KXV$	7. ASA Post.

HyL (Hypotenuse-Leg) Congruence Theorem

If the hypotenuse and a leg of one right triangle are congruent respectively to the hypotenuse and a leg of another right triangle, then the triangles are congruent.
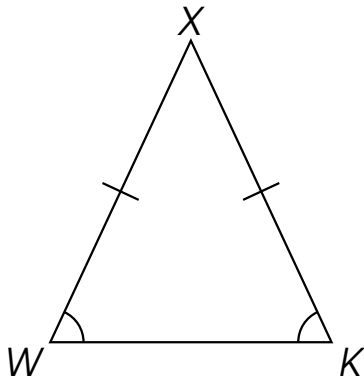
Isosceles Triangle

A triangle is isosceles if two of its sides are congruent. The congruent sides are its legs; the third side is the base; the angles opposite the congruent sides are the base angles; and the angle included by the legs is the vertex angle.

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
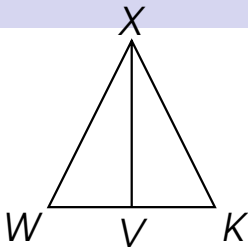
Example 4

Given: △XWK is an isosceles triangle
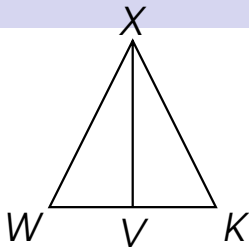$\overline{XV} \perp \overline{WK}$

Prove: △WXV ≅ △KXV

Example 4

Given: $\triangle XWK$ is an isosceles triangle
$\overline{XV} \perp \overline{WK}$
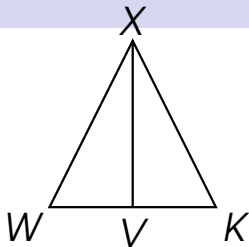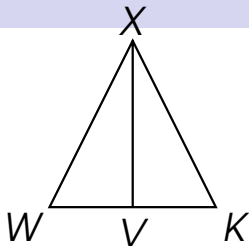
Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons

Example 4

Given: △XWK is an isosceles triangle
$\overline{XV} \perp \overline{WK}$

Prove: △WXV ≅ △KXV

Proof:

Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given

Example 4

Given: $\triangle XWK$ is an isosceles triangle
$\overline{XV} \perp \overline{WK}$

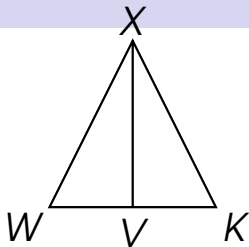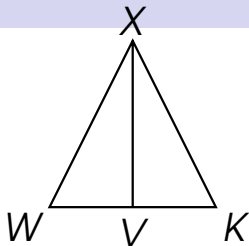Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW$, $\angle XVK$ are right angles	2. Def. of Perp. Line Segments

Example 4

Given: △XWK is an isosceles triangle
$\overline{XV} \perp \overline{WK}$

Prove: △WXV ≅ △KXV

Proof:

Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. ∠XVW, ∠XVK are right angles	2. Def. of Perp. Line Segments
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Def. of Right Angles

Example 4

Given: $\triangle XWK$ is an isosceles triangle
$\overline{XV} \perp \overline{WK}$

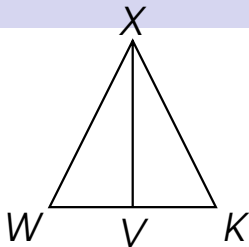Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Def. of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Def. of Cong. $\angle s$

Example 4

Given: $\triangle XWK$ is an isosceles triangle
$\overline{XV} \perp \overline{WK}$
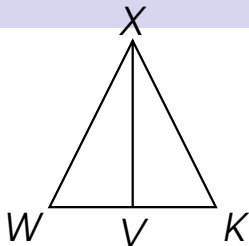
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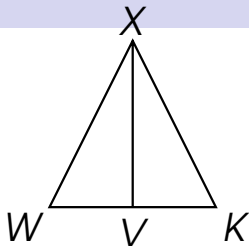| Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Def. of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Def. of Cong. $\angle s$
5. $\overline{XW} \cong \overline{XK}$	5. Def. of Isosceles Triangle

Example 4

Given: $\triangle XWK$ is an isosceles triangle
$\overline{XV} \perp \overline{WK}$

Prove: $\triangle WXV \cong \triangle KXV$

Proof:

Statements	Reasons
1. $\overline{XV} \perp \overline{WK}$	1. Given
2. $\angle XVW, \angle XVK$ are right angles	2. Def. of Perp. Line Segments
3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Def. of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Def. of Cong. $\angle s$
5. $\overline{XW} \cong \overline{XK}$	5. Def. of Isosceles Triangle
6. $\angle W \cong \angle K$	6. Isosceles Triangle Theorem

Example 4

Given: $\triangle XWK$ is an isosceles triangle
$\overline{XV} \perp \overline{WK}$

Prove: $\triangle WXV \cong \triangle KXV$

Proof:

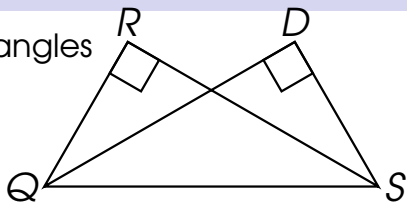| Statements	Reasons
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3. $m\angle RWQ = 90°, m\angle WRV = 90°$	3. Def. of Right Angles
4. $\angle RWQ \cong \angle WRV$	4. Def. of Cong. $\angle s$
5. $\overline{XW} \cong \overline{XK}$	5. Def. of Isosceles Triangle
6. $\angle W \cong \angle K$	6. Isosceles Triangle Theorem
7. $\triangle WXV \cong \triangle KXV$	7. AAS Cong. Thm.

HyA (Hypotenuse-Angle) Congruence Theorem

If the hypotenuse and an acute angle of one right triangle are congruent respectively to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

Example 5

Given: ∠R and ∠D are right angles
∠DQS ≅ ∠RSQ
Prove: △DQS ≅ △RSQ

Example 5

Given: ∠R and ∠D are right angles
       ∠DQS ≅ ∠RSQ
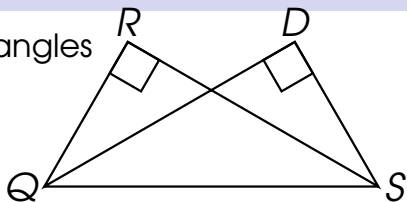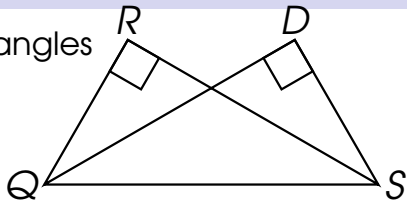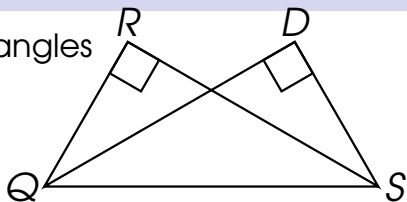Prove: △DQS ≅ △RSQ
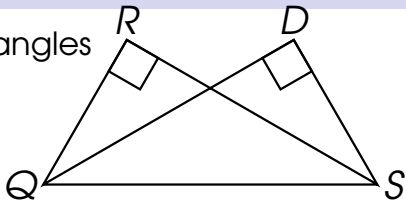
Proof:

Statements	Reasons

Example 5

Given: ∠R and ∠D are right angles
$\angle DQS \cong \angle RSQ$
Prove: $\triangle DQS \cong \triangle RSQ$

Proof:

Statements	Reasons
1. $\angle DQS \cong \angle RSQ$	1. Given

Example 5

Given: ∠R and ∠D are right angles
          ∠DQS ≅ ∠RSQ
Prove: △DQS ≅ △RSQ

Proof:

Statements	Reasons
1. ∠DQS ≅ ∠RSQ	1. Given
2. ∠R and ∠D are right angles	2. Given

Example 5

Given: ∠R and ∠D are right angles
       ∠DQS ≅ ∠RSQ
Prove: △DQS ≅ △RSQ

Proof:

Statements	Reasons
1. ∠DQS ≅ ∠RSQ	1. Given
2. ∠R and ∠D are right angles	2. Given
3. $m∠RWQ = 90°, m∠WRV = 90°$	3. Def. of Right Angles

Example 5

Given: ∠R and ∠D are right angles
       ∠DQS ≅ ∠RSQ
Prove: △DQS ≅ △RSQ
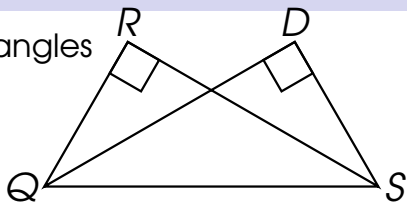
Proof:

Statements	Reasons
1. ∠DQS ≅ ∠RSQ	1. Given
2. ∠R and ∠D are right angles	2. Given
3. m∠RWQ = 90°, m∠WRV = 90°	3. Def. of Right Angles
4. ∠RWQ ≅ ∠WRV	4. Def. of Cong. ∠s

Example 5

Given: ∠R and ∠D are right angles
        ∠DQS ≅ ∠RSQ
Prove: △DQS ≅ △RSQ
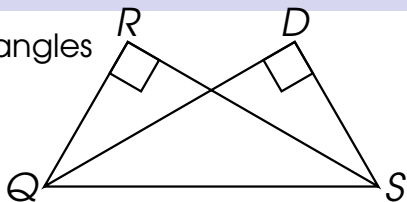
Proof:

Statements	Reasons
1. ∠DQS ≅ ∠RSQ	1. Given
2. ∠R and ∠D are right angles	2. Given
3. $m∠RWQ = 90°, m∠WRV = 90°$	3. Def. of Right Angles
4. ∠RWQ ≅ ∠WRV	4. Def. of Cong. ∠s
5. $\overline{QS} ≅ \overline{SQ}$	5. Reflexive Property

Example 5

Given: ∠R and ∠D are right angles
       ∠DQS ≅ ∠RSQ
Prove: △DQS ≅ △RSQ
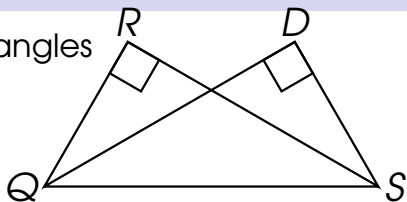
Proof:

Statements	Reasons
1. ∠DQS ≅ ∠RSQ	1. Given
2. ∠R and ∠D are right angles	2. Given
3. $m\angle RWQ = 90°$, $m\angle WRV = 90°$	3. Def. of Right Angles
4. ∠RWQ ≅ ∠WRV	4. Def. of Cong. ∠s
5. $\overline{QS} \cong \overline{SQ}$	5. Reflexive Property
6. △DQS ≅ △RSQ	6. AAS Triangle Congruence Theorem

Thank you for watching.