Direct and Indirect Proofs

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What is a Proof?

Proof: a form of logical reasoning in which each statement is organized and backed up by given information, definitions, axioms, postulates, or theorems

What is a Direct Proof?

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- can be done in three ways: paragraph form, flowchart form, and two column form

How to Write a Direct Proof?

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1. Take the original conditional statement.

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2. Assume that the hypothesis is true, and show that the conclusion is true.

How to Write a Two-Column Proof?

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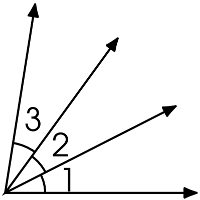
How to Write a Two-Column Proof?

1. Write all the series of statements in the first column of the table in a logical order starting with the given statements and ends it with the statement that needs to be proven.
2. In a step-by-step manner, write all the reasons for each statement.

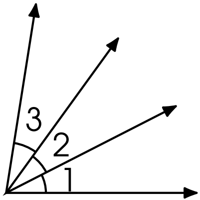
Example 1



Given: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$
Prove: $m\angle 1 = m\angle 3$
Proof:

Statements	**Reasons**

Example 1



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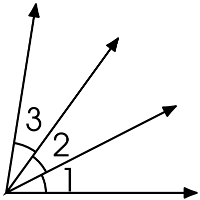

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Proof:

Statements	**Reasons**
1. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	1. Given
2. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 2 - m\angle 2 + m\angle 3$	2. Subtraction Property

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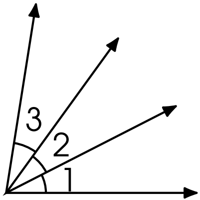

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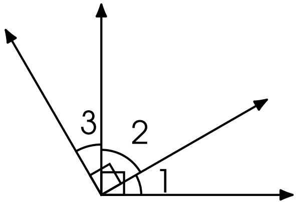
2. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 2 - m\angle 2 + m\angle 3$	2. Subtraction Property
3. $m\angle 1 = m\angle 3$	3. Simplification

Example 2



Given: $m\angle 1 + m\angle 2 = 90^\circ$

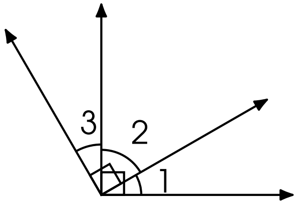
$m\angle 3 + m\angle 2 = 90^\circ$

Prove: $m\angle 1 = m\angle 3$

Proof:

Statements	**Reasons**

Example 2



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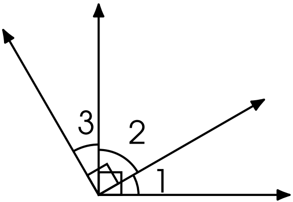

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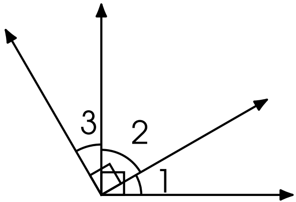

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Proof:

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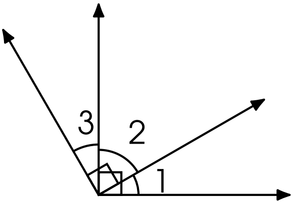
1. $m\angle 1 + m\angle 2 = 90^\circ$	1. Given
2. $m\angle 3 + m\angle 2 = 90^\circ$	2. Given
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property

Example 2



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Proof:

Statements	**Reasons**
1. $m\angle 1 + m\angle 2 = 90^\circ$	1. Given
2. $m\angle 3 + m\angle 2 = 90^\circ$	2. Given
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property
4. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 3 + m\angle 2 - m\angle 2$	4. Subtraction Property

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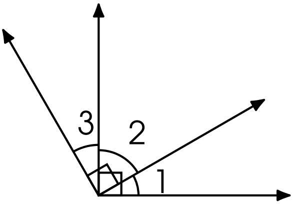

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Prove: $m\angle 1 = m\angle 3$

Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 90°$	1. Given
2. $m\angle 3 + m\angle 2 = 90°$	2. Given
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property
4. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 3 + m\angle 2 - m\angle 2$	4. Subtraction Property
5. $m\angle 1 = m\angle 3$	5. Simplification

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- a type of proof where the opposite of the statement to be proven is assumed true until the assumption leads to contradiction

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- a type of proof where the opposite of the statement to be proven is assumed true until the assumption leads to contradiction
- is a method of reasoning usually written in paragraph form

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1. Write the opposite of the statement to be proven.
2. Proceed as if this assumption is true to find the contradiction.
3. Once there is contradiction, the original statement is true.

Example 3

Prove: If $x = 2$, then $3x - 5 \neq 10$.

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$\frac{3x}{3} = \frac{15}{3}$

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Prove: If $x = 2$, then $3x - 5 \neq 10$.

If $x = 2$, then $3x - 5 = 10$.

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$\frac{3x}{3} = \frac{15}{3}$

$x = 5 \rightarrow$ Contradiction

Example 3

Prove: If $x = 2$, then $3x - 5 \neq 10$.

If $x = 2$, then $3x - 5 = 10$.

$3x - 5 + 5 = 10 + 5$

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5 \rightarrow$ Contradiction

Therefore, the original statement is true.

Example 4

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$$\frac{4x}{4} = \frac{16}{4}$$

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Example 4

Prove: If $x = 3$, then $4x - 4 \neq 12$.

If $x = 3$, then $4x - 4 = 12$.

$4x - 4 + 4 = 12 + 4$

$4x = 16$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4 \rightarrow$ Contradiction

Example 4

Prove: If $x = 3$, then $4x - 4 \neq 12$.

If $x = 3$, then $4x - 4 = 12$.

$4x - 4 + 4 = 12 + 4$

$4x = 16$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4 \rightarrow$ Contradiction

Therefore, the original statement is true.

Thank you for watching.