

Domain and Range of Functions

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What is Function Notation?

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- ▶ The $f(x)$ notation can also be used to define a function.
- ▶ If f is a function, the symbol $f(x)$ is used to denote the value of the function f at a given value of x .
- ▶ $f(x)$ denotes the y -value that the function f associates with x -value.

Example 1

Using the function notation, how do you write the rule of function f such that $y = -2x + 1$?

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$$f(x) = -2x + 1$$

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Using the function notation, how do you write the rule of function g such that $y = 5x - 2$?

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Using the function notation, how do you write the rule of function g such that $y = 5x - 2$?

$$g(x) = 5x - 2$$

What is the Domain of a Function?

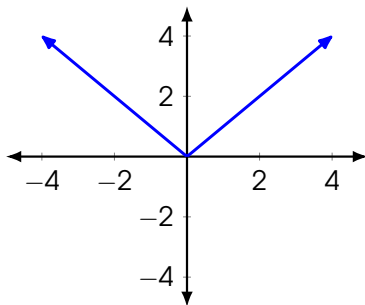
Domain: the set of all permissible values of x that give real values for y

What is the Range of a Function?

Range: the set of permissible values for y or $f(x)$ that give the values of x real numbers

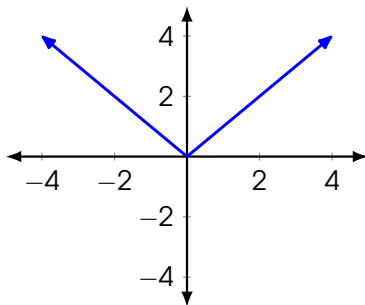
Example 1

Determine the domain and the range of the function described in this graph.



Example 1

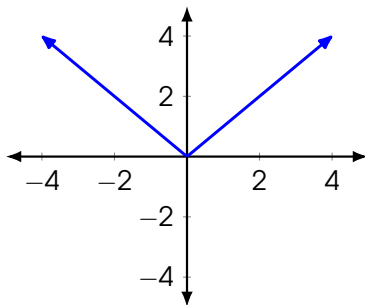
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\}$$

Example 1

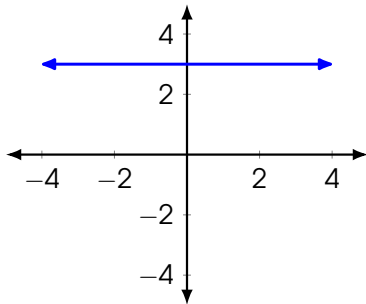
Determine the domain and the range of the function described in this graph.



$$D = \{x|x \in \mathbb{R}\} , R = \{y|y \in \mathbb{R}, y \geq 0\}$$

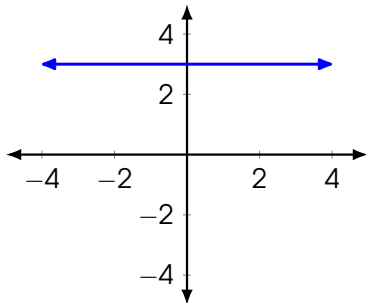
Example 2

Determine the domain and the range of the function described in this graph.



Example 2

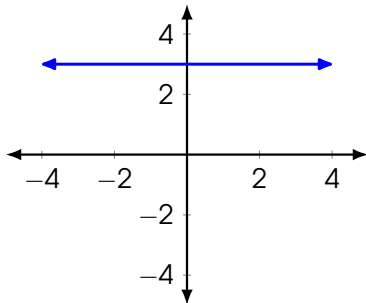
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Example 2

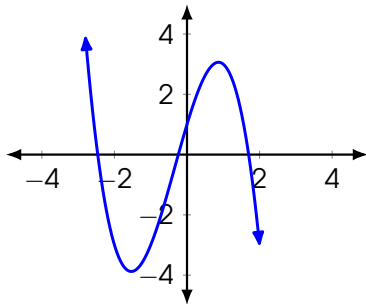
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\} , R = \{y | y = 3\}$$

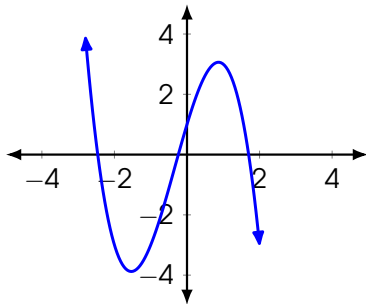
Example 3

Determine the domain and the range of the function described in this graph.



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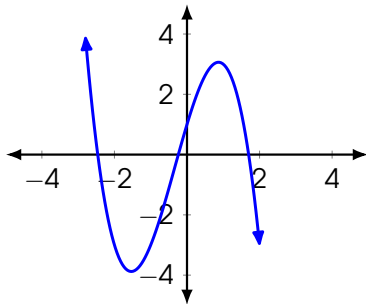
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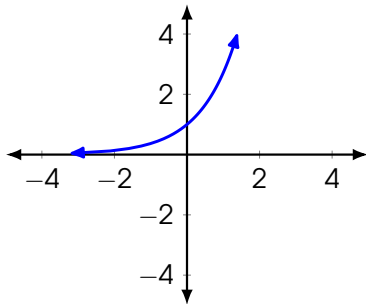
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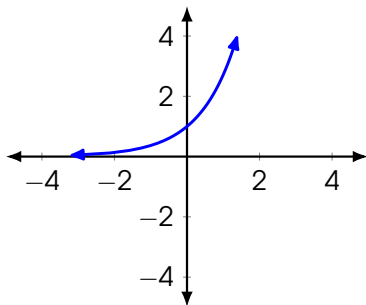
Example 4

Determine the domain and the range of the function described in this graph.



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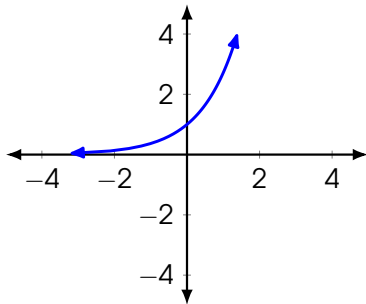
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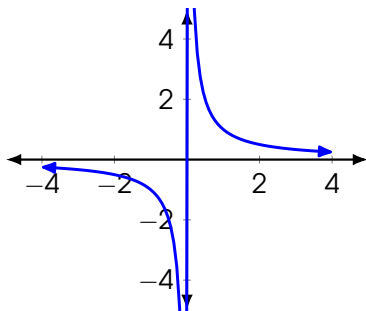
$$D = \{x | x \in \mathbb{R}\} , R = \{y | y \in \mathbb{R}, y > 0\}$$

What is an Asymptote?

Asymptote: a line that the graph of a function approaches, but never intersects

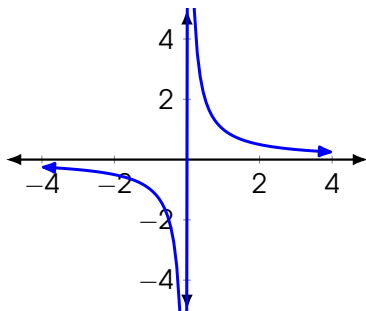
Example 5

Determine the domain and the range of the function described in this graph.



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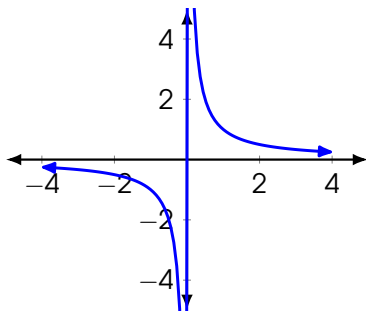
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \neq 0\}$$

Example 5

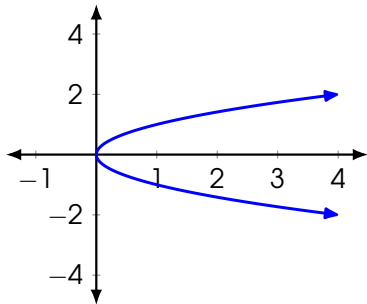
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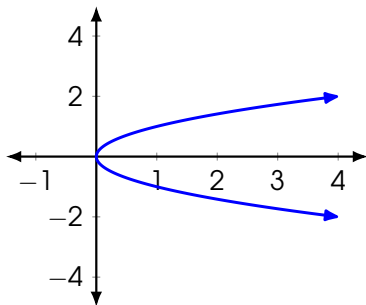
Example 6

Determine the domain and the range of the function described in this graph.



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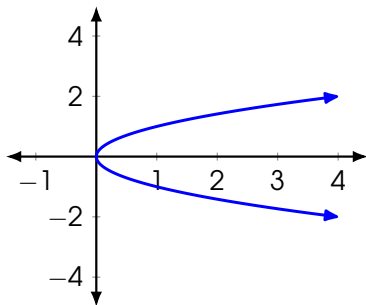
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \geq 0\}$$

Example 6

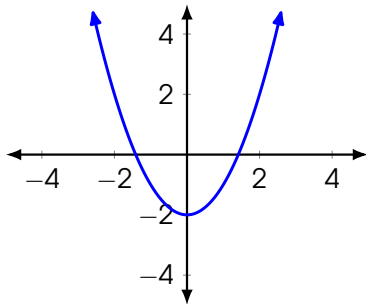
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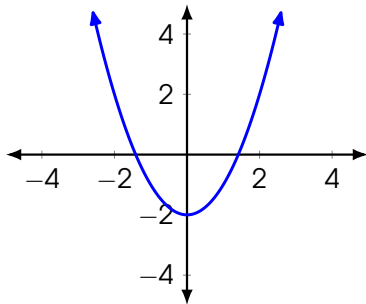
Example 7

Determine the domain and the range of the function described in this graph.



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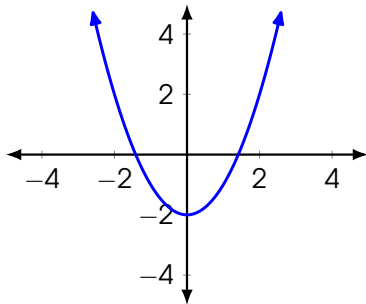
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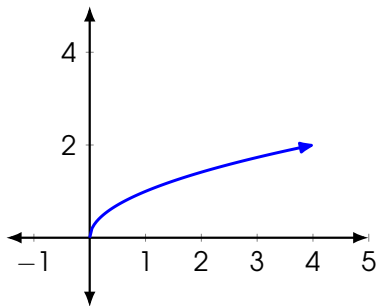
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}\} , R = \{y | y \in \mathbb{R}, y \geq -2\}$$

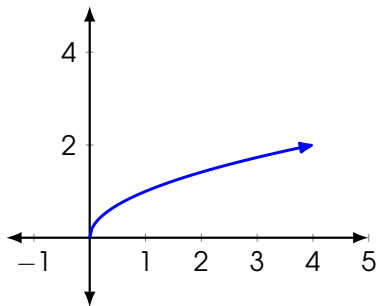
Example 8

Determine the domain and the range of the function described in this graph.



Example 8

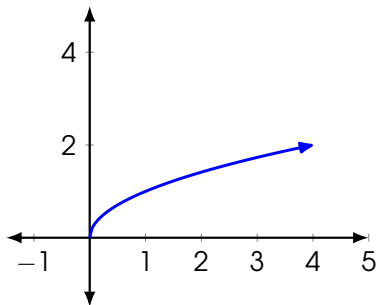
Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \geq 0\}$$

Example 8

Determine the domain and the range of the function described in this graph.



$$D = \{x | x \in \mathbb{R}, x \geq 0\} , R = \{y | y \in \mathbb{R}, y \geq 0\}$$

Example 9

Determine the domain and the range of the following function.

$$f(x) = 3x$$

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$$D = \{x|x \in \mathbb{R}\} , R = \{y|y \in \mathbb{R}\}$$

Example 10

Determine the domain and the range of the following function.

$$f(x) = x^2$$

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$$f(x) = x^2$$

$$D = \{x|x \in \mathbb{R}\} , R = \{y|y \in \mathbb{R}, y \geq 0\}$$

Take Note!

The value of the function will not be a real number if it is an imaginary number ($\sqrt{-1}$) or undefined (division by 0).

How to Determine the Domain of a Function with Square Root?

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- ▶ Write the radicand as an expression greater than or equal to zero.

How to Determine the Domain of a Function with Square Root?

- ▶ Write the radicand as an expression greater than or equal to zero.
- ▶ Solve for x .

How to Determine the Domain of a Rational Function?

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- ▶ Write the denominator as an expression not equal to zero.

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- ▶ Write the denominator as an expression not equal to zero.
- ▶ Solve for x .

Example 11

Determine the domain of the following function.

$$f(x) = \sqrt{x - 2}$$

Example 11

Step 1: Write the radicand as an expression greater than or equal to zero.

$$f(x) = \sqrt{x - 2}$$

Example 11

Step 2: Solve for x .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Example 11

Step 2: Solve for x .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

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$$x - 2 \geq 0$$

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$$x - 2 + 2 \geq 0 + 2$$

Example 11

Step 2: Solve for x .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

$$x - 2 + 2 \geq 0 + 2$$

Simplify

Example 11

Step 2: Solve for x .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

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Simplify

$$x \geq 2$$

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Step 2: Solve for x .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

$$x - 2 + 2 \geq 0 + 2$$

Simplify

$$x \geq 2$$

Domain:

Example 11

Step 2: Solve for x .

$$f(x) = \sqrt{x - 2}$$

Radicand

$$x - 2 \geq 0$$

Use Addition Prop.

$$x - 2 + 2 \geq 0 + 2$$

Simplify

$$x \geq 2$$

Domain:

$$D = \{x | x \in \mathbb{R}, x \geq 2\}$$

Example 12

Determine the domain of the following function.

$$f(x) = \frac{x+1}{x-1}$$

Example 12

Step 1: Write the denominator as an expression not equal to zero.

$$f(x) = \frac{x+1}{x-1}$$

Example 12

Step 2: Solve for x .

$$f(x) = \frac{x+1}{x-1}$$

Denominator

$$x - 1 \neq 0$$

Example 12

Step 2: Solve for x .

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Denominator

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Use Addition Prop.

$$x - 1 + 1 \neq 0 + 1$$

Simplify

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Step 2: Solve for x .

$$f(x) = \frac{x+1}{x-1}$$

Denominator

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Use Addition Prop.

$$x - 1 + 1 \neq 0 + 1$$

Simplify

$$x \neq 1$$

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Denominator

$$x - 1 \neq 0$$

Use Addition Prop.

$$x - 1 + 1 \neq 0 + 1$$

Simplify

$$x \neq 1$$

Domain:

Example 12

Step 2: Solve for x .

$$f(x) = \frac{x+1}{x-1}$$

Denominator

$$x - 1 \neq 0$$

Use Addition Prop.

$$x - 1 + 1 \neq 0 + 1$$

Simplify

$$x \neq 1$$

Domain:

$$D = \{x | x \in \mathbb{R}, x \neq 1\}$$

Example 13

Determine the domain of the following function.

$$f(x) = \sqrt{2x + 3}$$

Example 13

Step 1: Write the radicand as an expression greater than or equal to zero.

$$f(x) = \sqrt{2x + 3}$$

Example 13

Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Example 13

Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

Example 13

Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

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Step 2: Solve for x .

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Simplify

$$2x \geq -3$$

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Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

Example 13

Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

Example 13

Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

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Use Subtraction Prop.

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Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

Simplify

$$x \geq -\frac{3}{2}$$

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Step 2: Solve for x .

$$f(x) = \sqrt{2x + 3}$$

Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

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Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

Simplify

$$x \geq -\frac{3}{2}$$

Domain:

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Step 2: Solve for x .

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Radicand

$$2x + 3 \geq 0$$

Use Subtraction Prop.

$$2x + 3 - 3 \geq 0 - 3$$

Simplify

$$2x \geq -3$$

Use Division Prop.

$$\frac{2x}{2} \geq \frac{-3}{2}$$

Simplify

$$x \geq -\frac{3}{2}$$

Domain:

$$D = \left\{ x \mid x \in \mathbb{R}, x \geq -\frac{3}{2} \right\}$$

Example 14

Determine the domain of the following function.

$$f(x) = \frac{x - 3}{3x + 2}$$

Example 14

Step 1: Write the denominator as an expression not equal to zero.

$$f(x) = \frac{x - 3}{3x + 2}$$

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

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Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator $3x + 2 \neq 0$

Use Subtraction Prop. $3x + 2 - 2 \neq 0 - 2$

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator $3x + 2 \neq 0$

Use Subtraction Prop. $3x + 2 - 2 \neq 0 - 2$

Simplify

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator	$3x + 2 \neq 0$
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Use Subtraction Prop.	$3x + 2 - 2 \neq 0 - 2$
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Simplify	$3x \neq -2$
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Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator $3x + 2 \neq 0$

Use Subtraction Prop. $3x + 2 - 2 \neq 0 - 2$

Simplify $3x \neq -2$

Use Division Prop.

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator $3x + 2 \neq 0$

Use Subtraction Prop. $3x + 2 - 2 \neq 0 - 2$

Simplify $3x \neq -2$

Use Division Prop. $\frac{3x}{3} \neq \frac{-2}{3}$

Simplify

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Simplify

$$x \neq -\frac{2}{3}$$

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Simplify

$$x \neq -\frac{2}{3}$$

Domain:

Example 14

Step 2: Solve for x .

$$f(x) = \frac{x - 3}{3x + 2}$$

Denominator

$$3x + 2 \neq 0$$

Use Subtraction Prop.

$$3x + 2 - 2 \neq 0 - 2$$

Simplify

$$3x \neq -2$$

Use Division Prop.

$$\frac{3x}{3} \neq \frac{-2}{3}$$

Simplify

$$x \neq -\frac{2}{3}$$

Domain:

$$D = \left\{ x \mid x \in \mathbb{R}, x \neq -\frac{2}{3} \right\}$$

Thank you for watching.