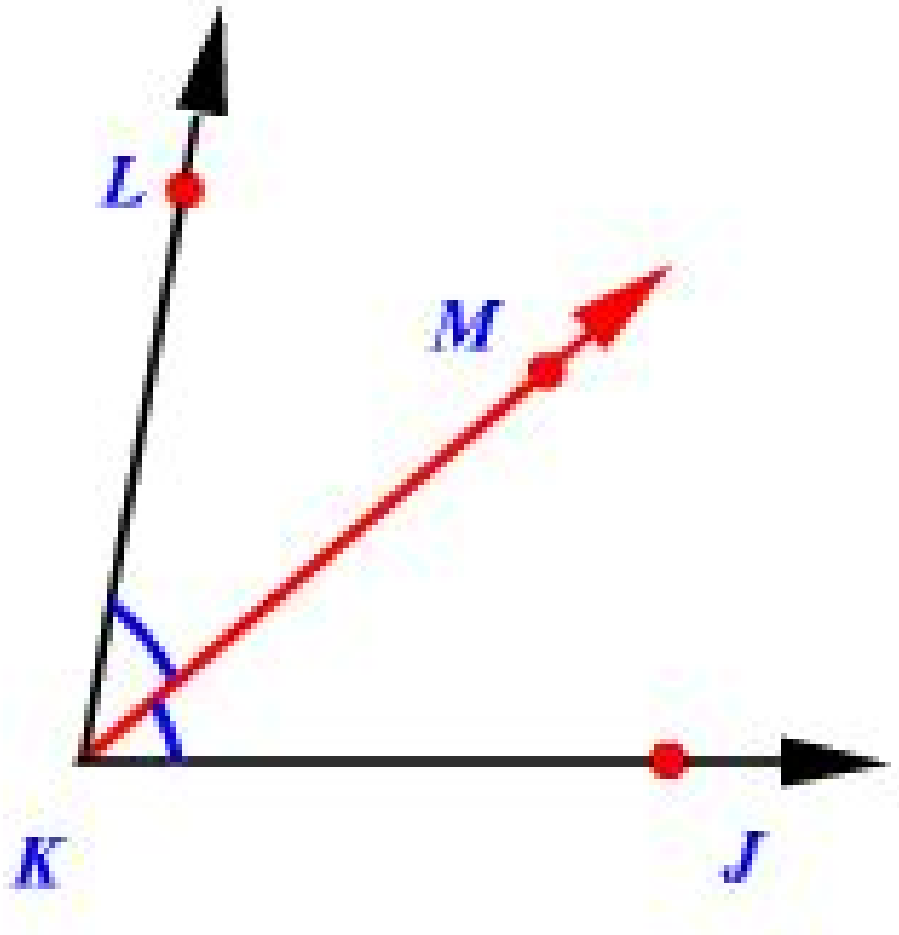
Applying Triangle Congruence in Constructing Angle Bisector

Jonathan R. Bacolod

Sauyo High School

What is an Angle Bisector?

An angle bisector or the bisector of an angle is a line that divides an angle into two equal parts.



What You Need

1. Compass

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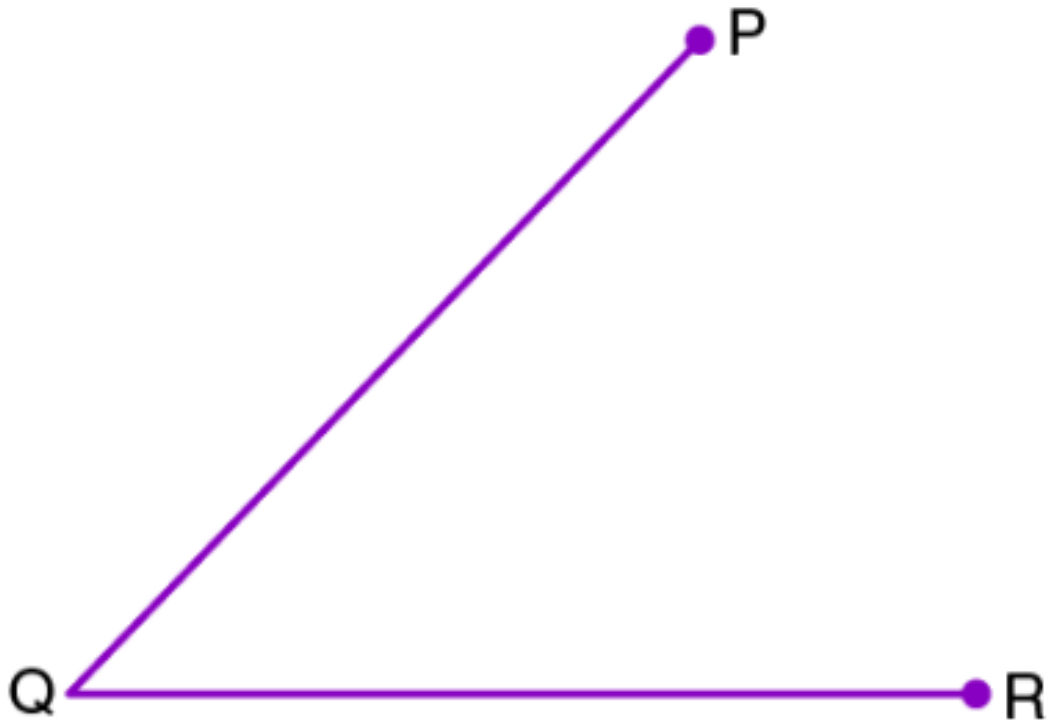
1. Compass
2. Ruler

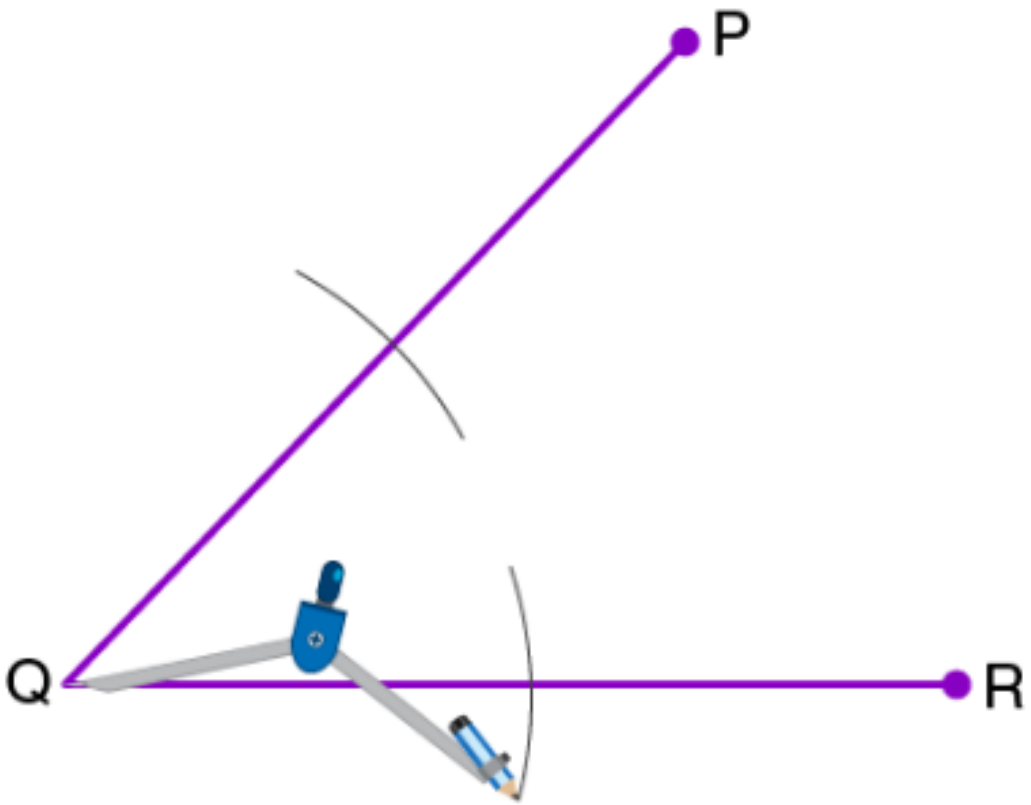
How to Construct an Angle Bisector?

Construct an angle bisector for the following angle:



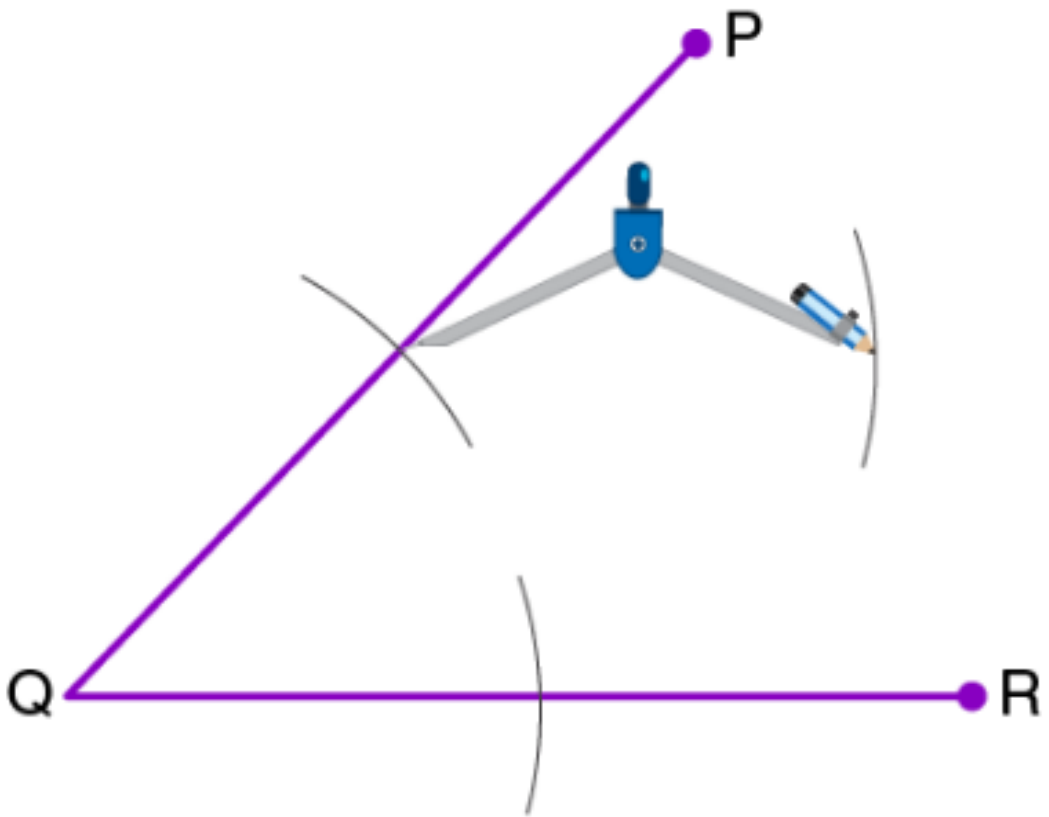
How to Construct an Angle Bisector?

1. Place the compass pointer at Q and make an arc that cuts the two arms of the angle at two different points.



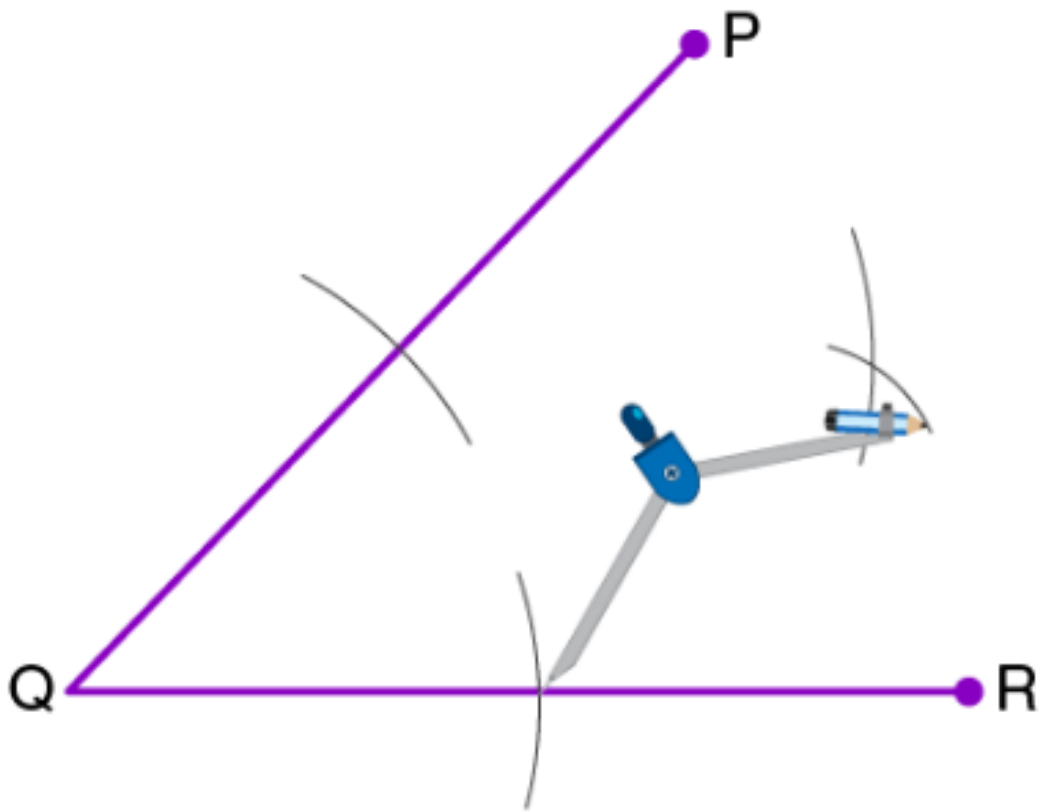
How to Construct an Angle Bisector?

2. From the point where the first arc cut the arm QP, make another arc towards the interior of the angle.



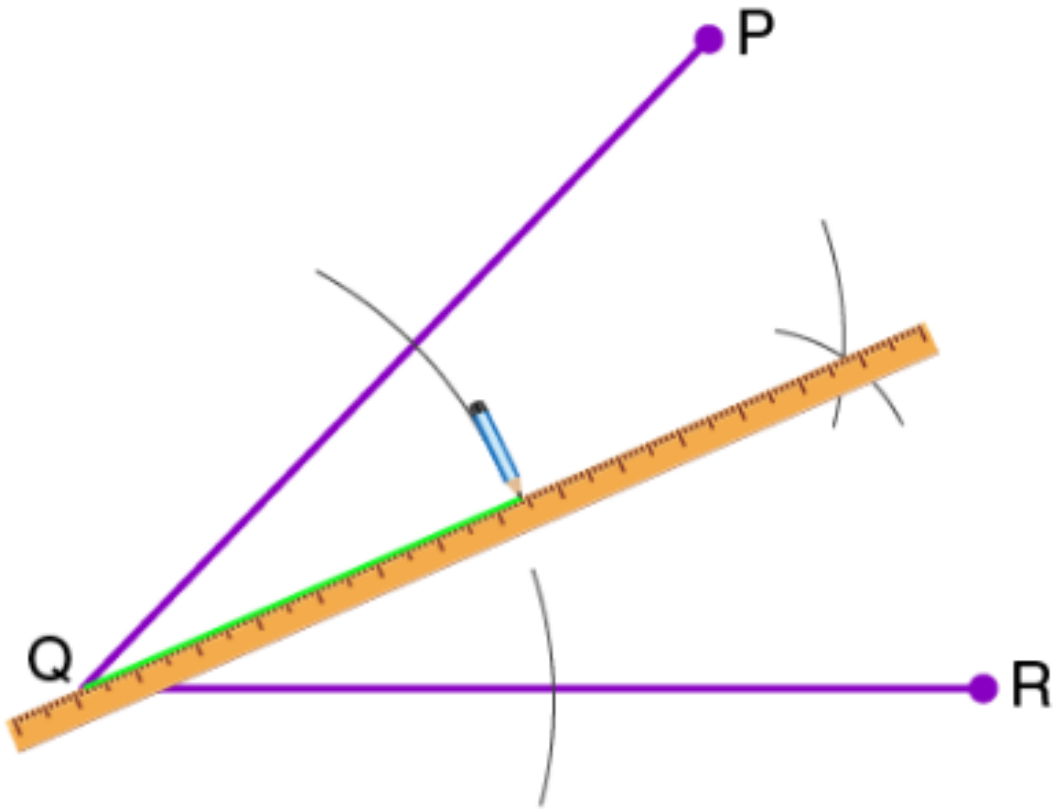
How to Construct an Angle Bisector?

3. Without changing the radius on the compass, repeat step 2 from the point where the first arc cut QR.



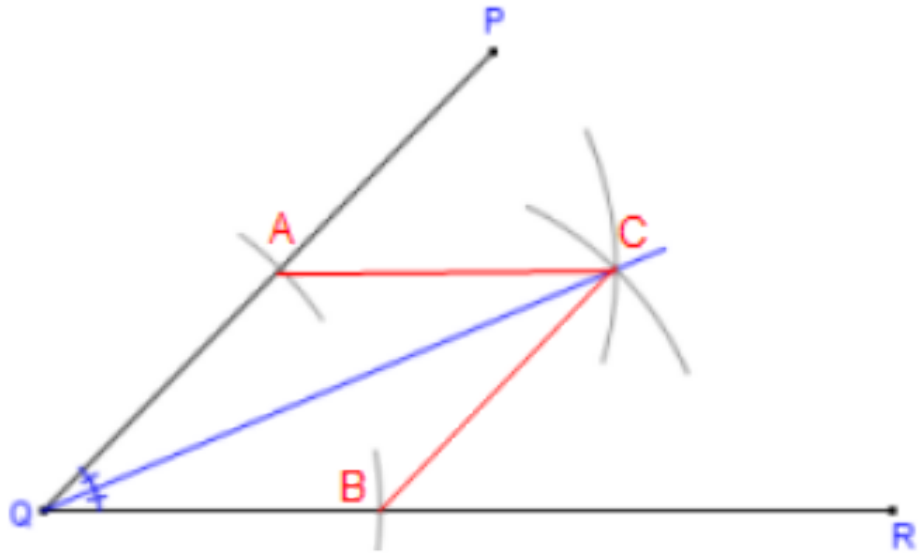
How to Construct an Angle Bisector?

4. Using a ruler, draw a line from Q to the point where the arcs intersect.



Proof

Prove: $\angle AQC \cong \angle BQC$



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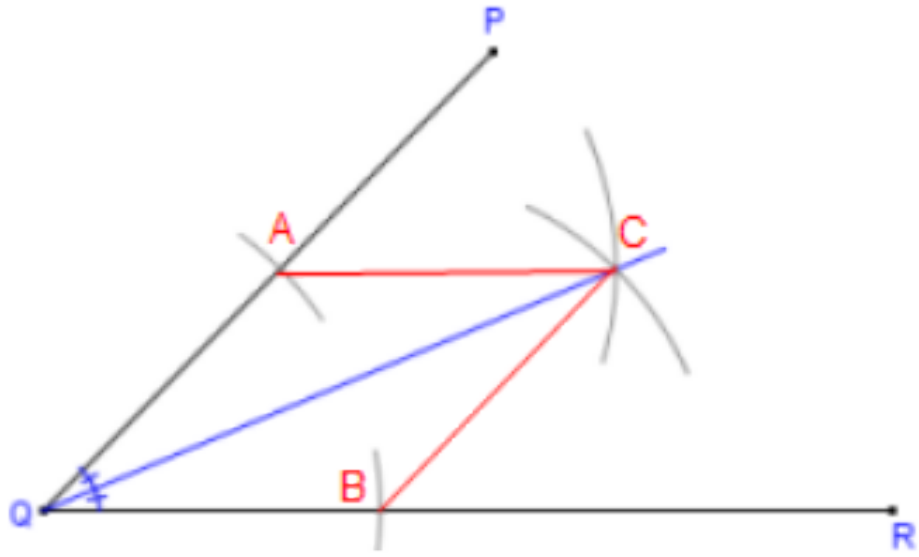

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Statements	Reasons

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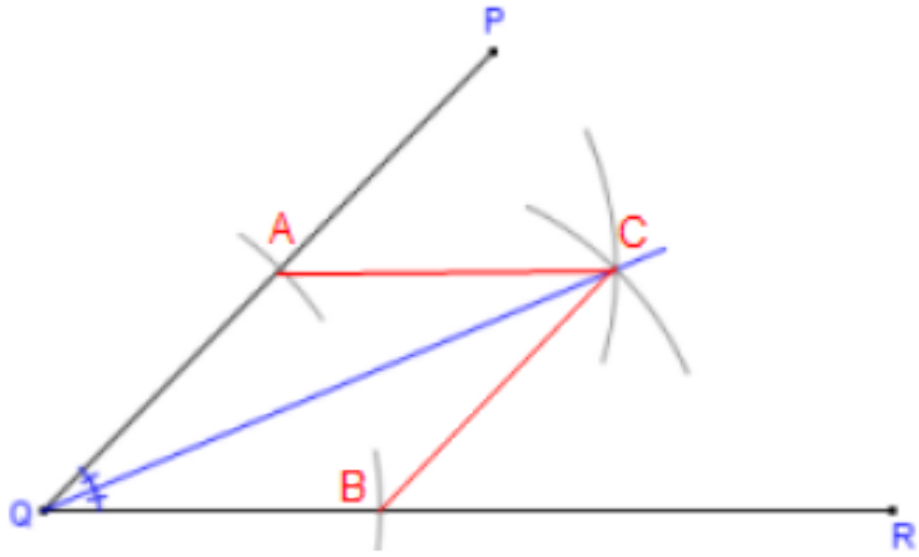

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Statements	Reasons
1. $\overline{AQ} \cong \overline{AC}, \overline{BQ} \cong \overline{BC}$	1. They were drawn with the same compass width.

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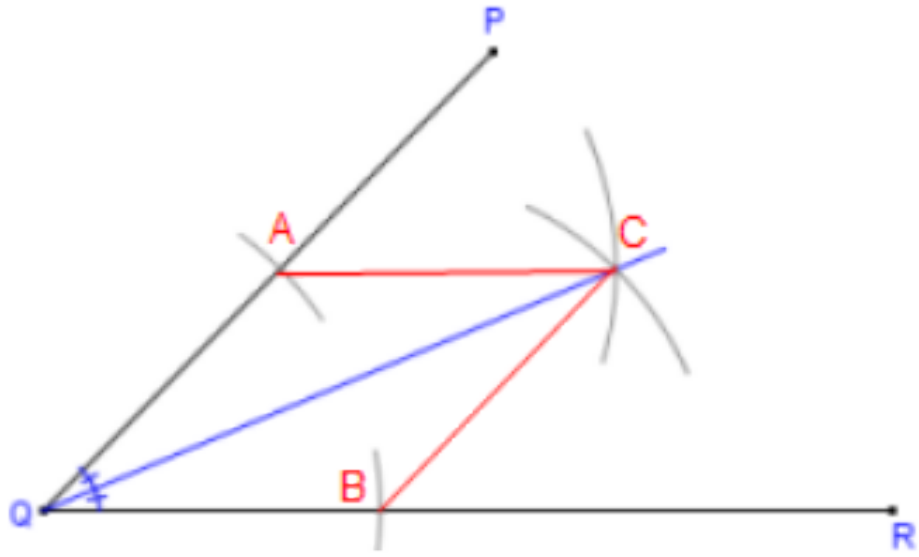

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1. $\overline{AQ} \cong \overline{AC}, \overline{BQ} \cong \overline{BC}$	1. They were drawn with the same compass width.
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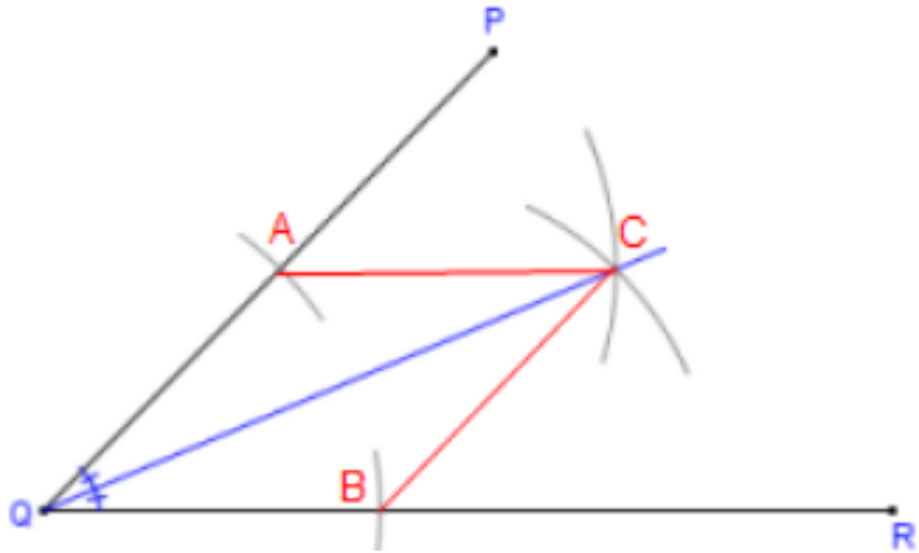

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3. $\triangle AQC \cong \triangle BQC$	3. SSS Triangle Congruence Postulate

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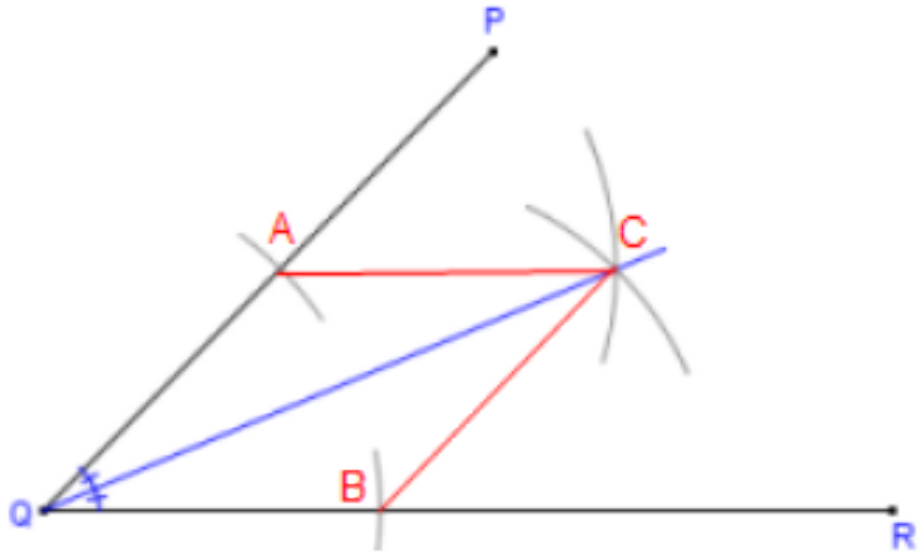

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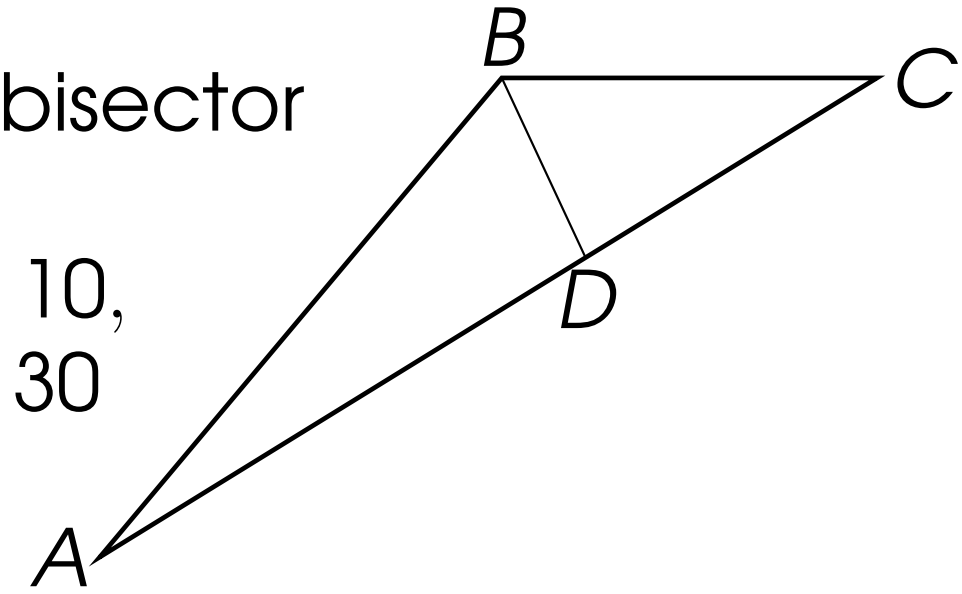
3. $\triangle AQC \cong \triangle BQC$	3. SSS Triangle Congruence Postulate
4. $\angle AQC \cong \angle BQC$	4. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)

Example 1

Given: $\overline{BD}$ is an angle bisector of $\angle ABC$
$m\angle CBD = 3x + 10$,
$m\angle ABD = 2x + 30$

Find: $m\angle CBD$



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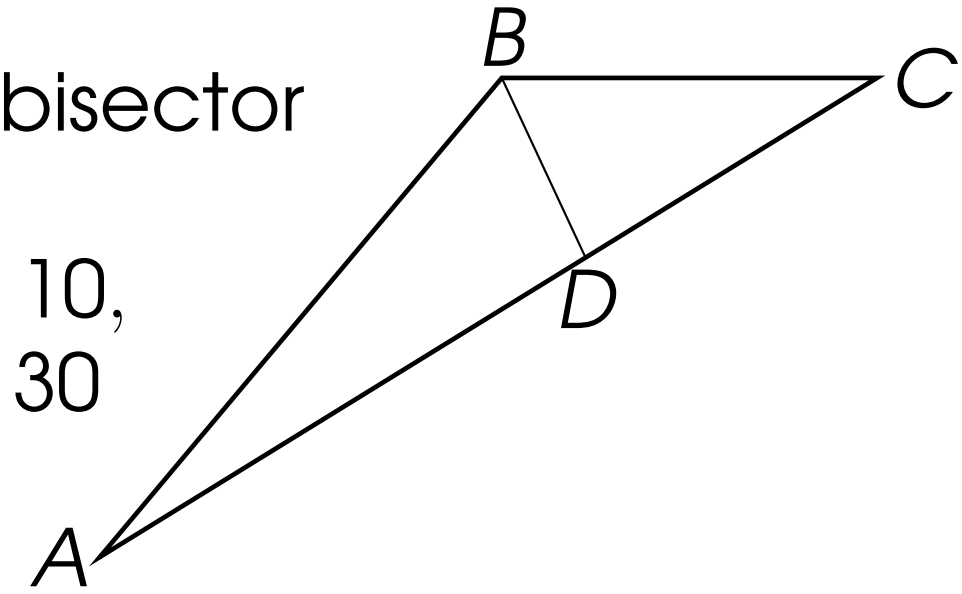

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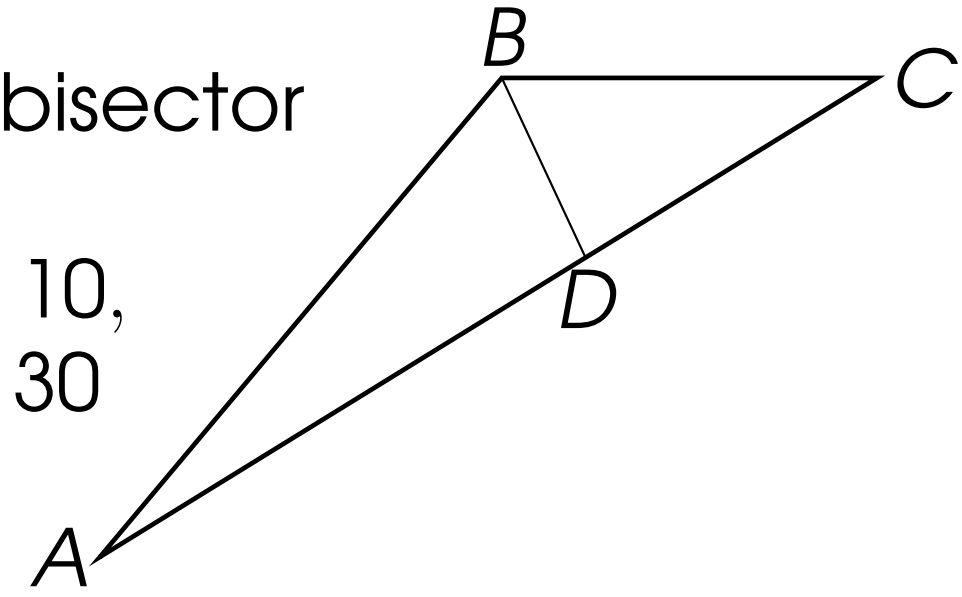

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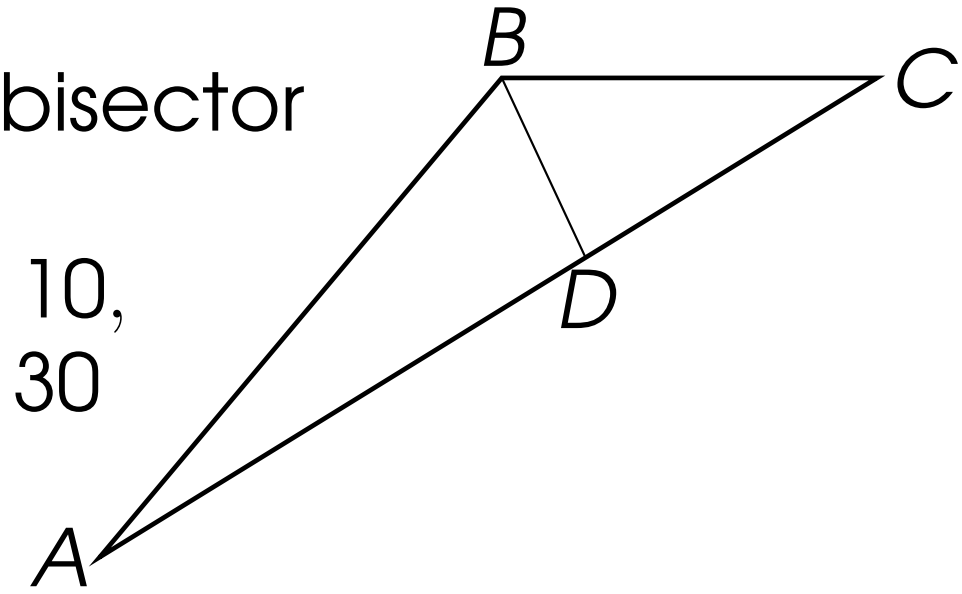

$\angle CBD \cong \angle ABD$

$m\angle CBD = m\angle ABD$

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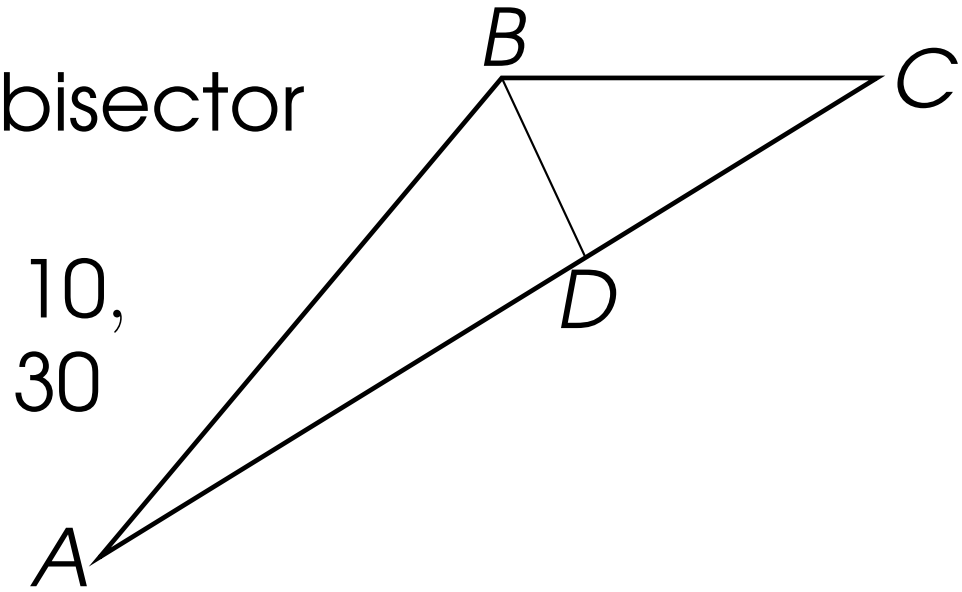

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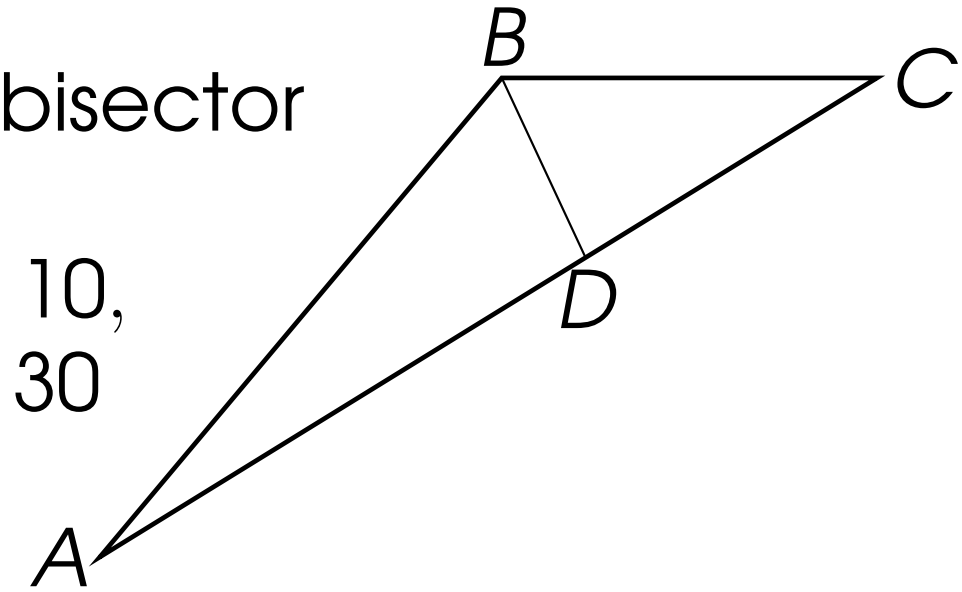

Find: $m\angle CBD$

$\angle CBD \cong \angle ABD$

$m\angle CBD = m\angle ABD$

$3x + 10 = 2x + 30$

$3x - 2x + 10 - 10 = 2x - 2x + 30 - 10$

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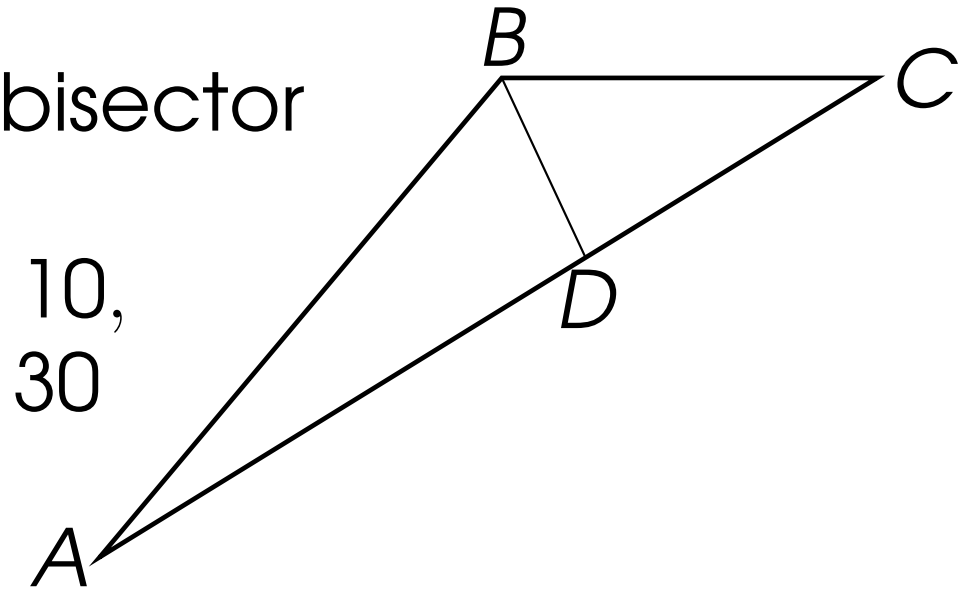


Find: $m\angle CBD$

$\angle CBD \cong \angle ABD$

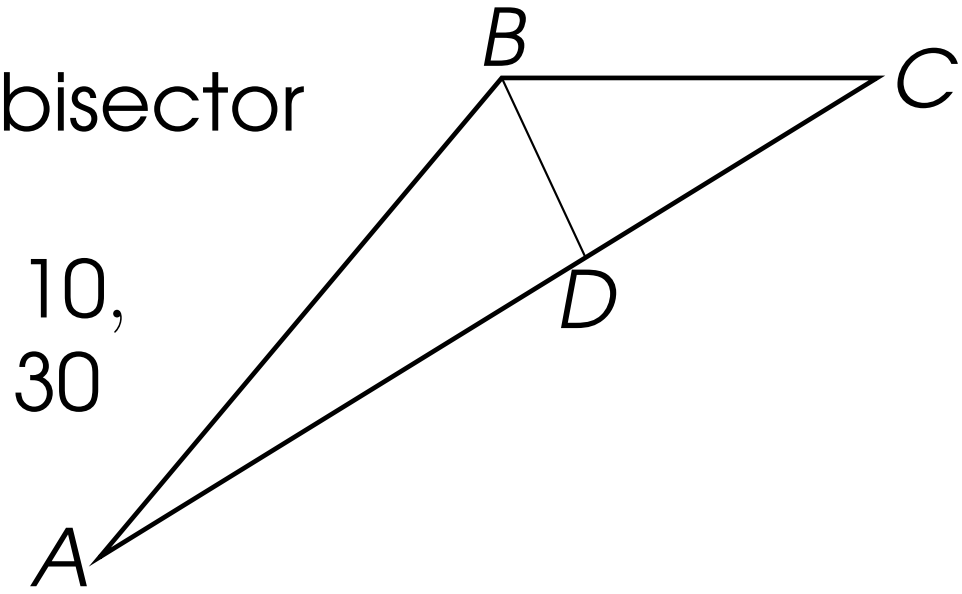
$m\angle CBD = m\angle ABD$

$3x + 10 = 2x + 30$

$3x - 2x + 10 - 10 = 2x - 2x + 30 - 10$

$x = 20$

Example 1



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$m\angle CBD = m\angle ABD$

$3x + 10 = 2x + 30$

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$x = 20$

$m\angle CBD = 3x + 10$

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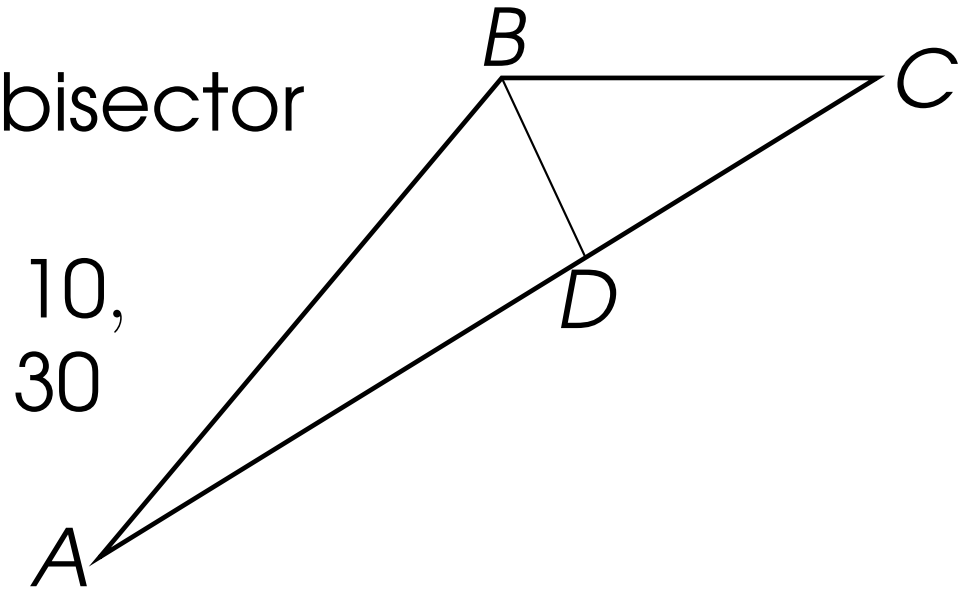

Find: $m\angle CBD$

$\angle CBD \cong \angle ABD$

$m\angle CBD = m\angle ABD$

$3x + 10 = 2x + 30$

$3x - 2x + 10 - 10 = 2x - 2x + 30 - 10$

$x = 20$

$m\angle CBD = 3x + 10$

$m\angle CBD = 3(20) + 10$

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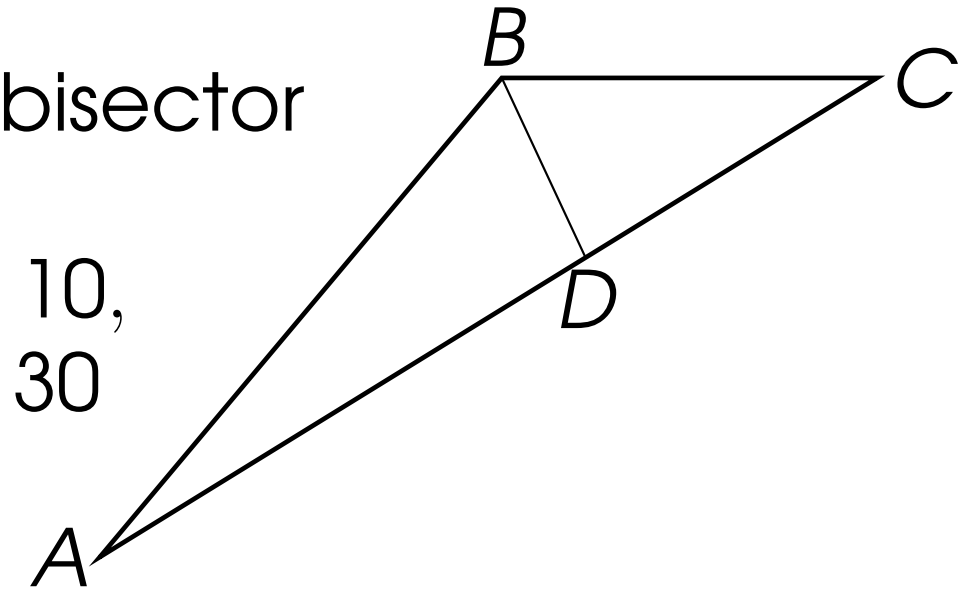


Find: $m\angle CBD$

$\angle CBD \cong \angle ABD$

$m\angle CBD = m\angle ABD$

$3x + 10 = 2x + 30$

$3x - 2x + 10 - 10 = 2x - 2x + 30 - 10$

$x = 20$

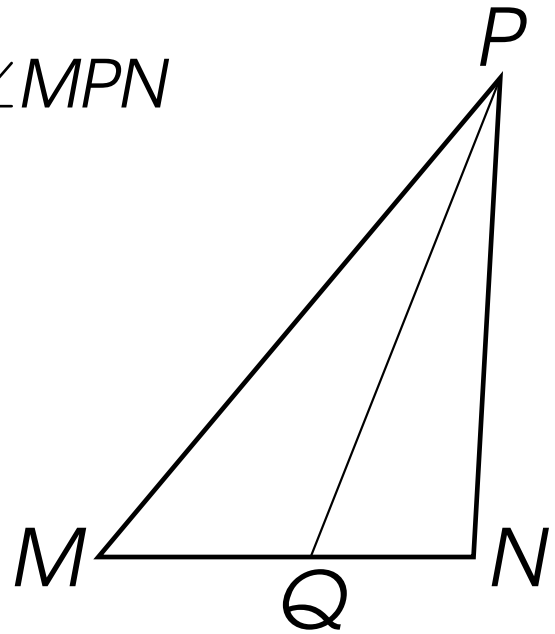
$m\angle CBD = 3x + 10$

$m\angle CBD = 3(20) + 10$

$m\angle CBD = 70^\circ$

Example 2

Given: $\overline{PQ}$ is an angle bisector of $\angle MPN$
$m\angle MPQ = 3x + 9$,
$m\angle NPQ = 5x - 5$
Find: $m\angle MPN$



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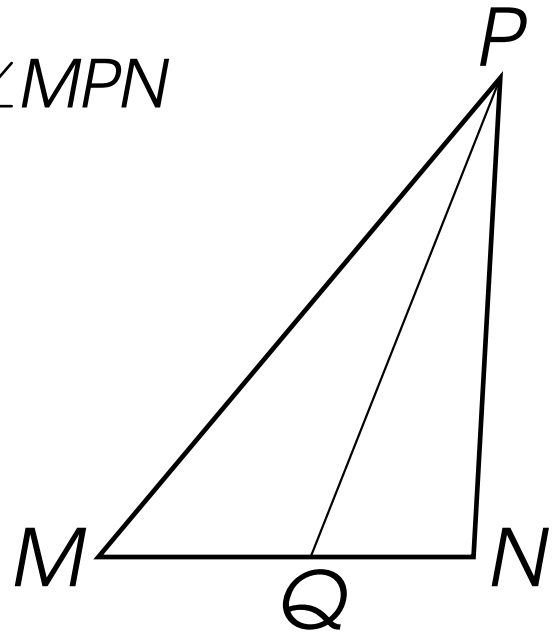

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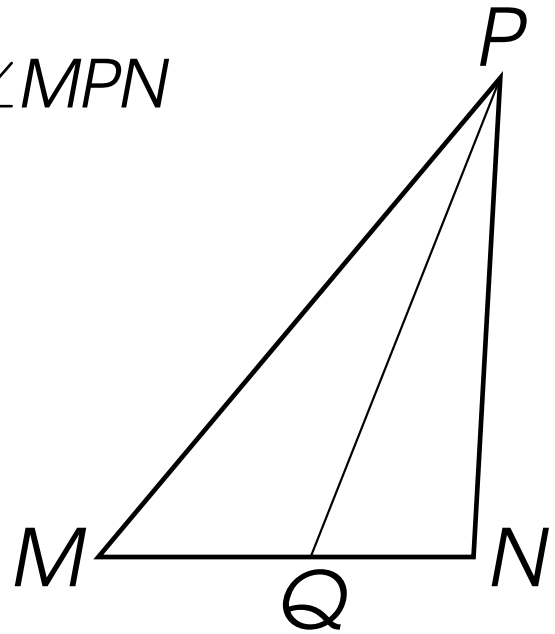

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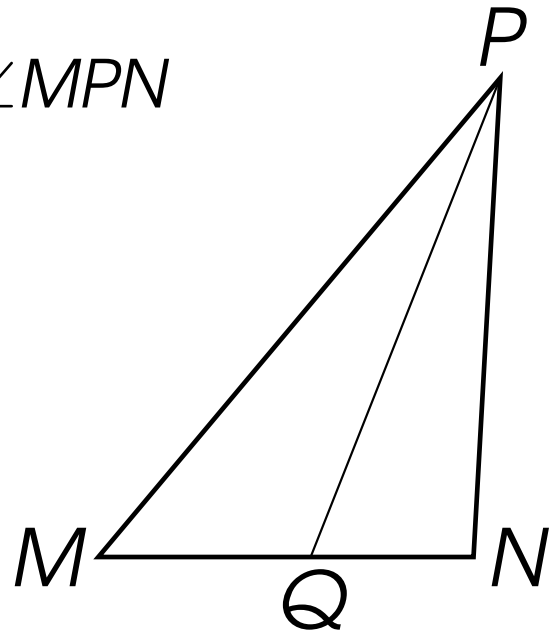

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$\angle MPQ \cong \angle NPQ$
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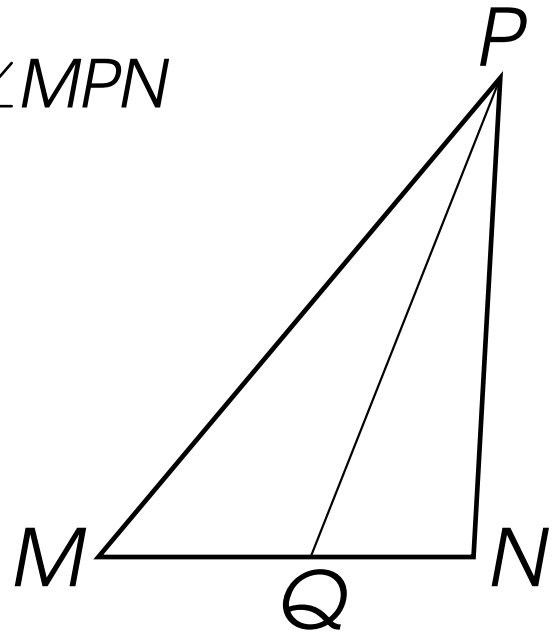
$3x + 9 = 5x - 5$



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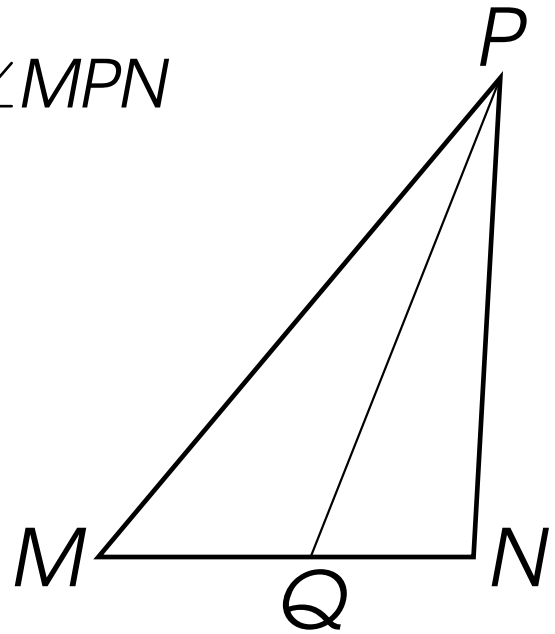

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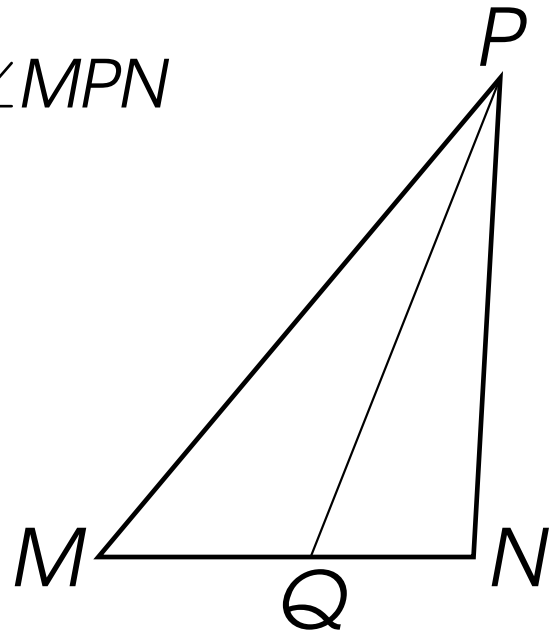

$\angle MPQ \cong \angle NPQ$

$m\angle MPQ = m\angle NPQ$

$3x + 9 = 5x - 5$

$3x - 5x + 9 - 9 = 5x - 5x - 5 - 9$

$-2x = -14$

Example 2

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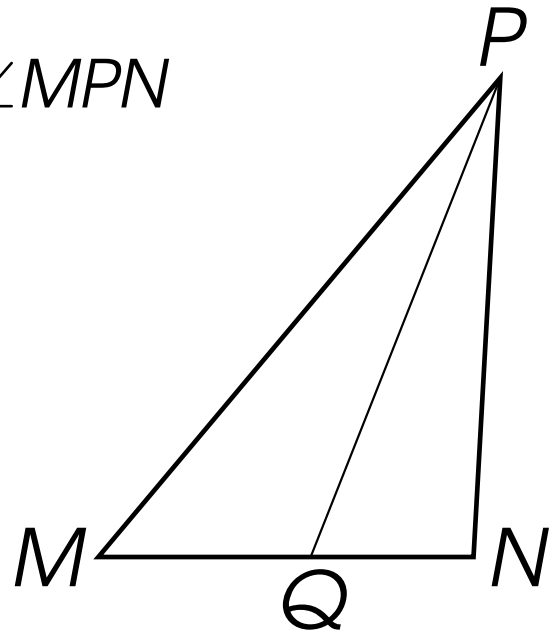

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$\frac{-2x}{-2} = \frac{-14}{-2}$

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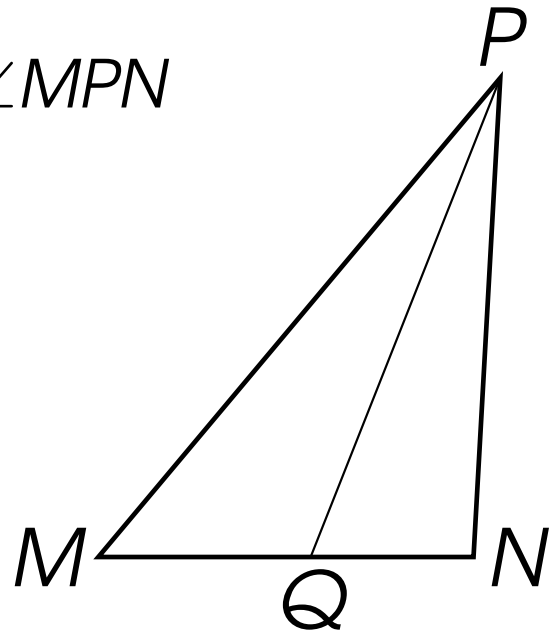

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$x = 7$

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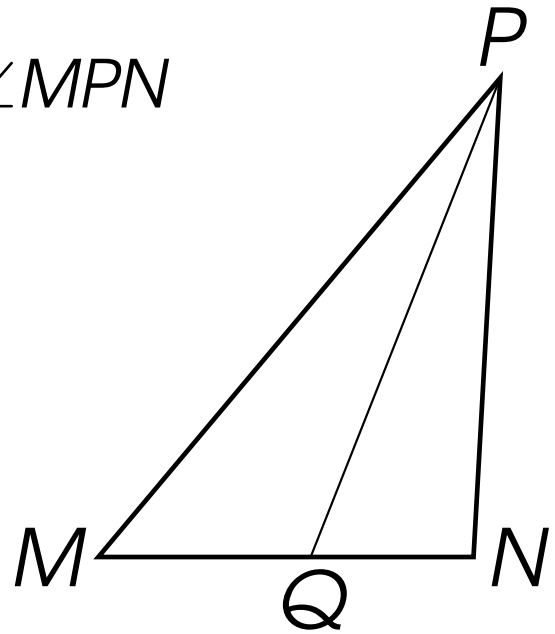

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$m\angle MPN = 2(m\angle MPQ)$

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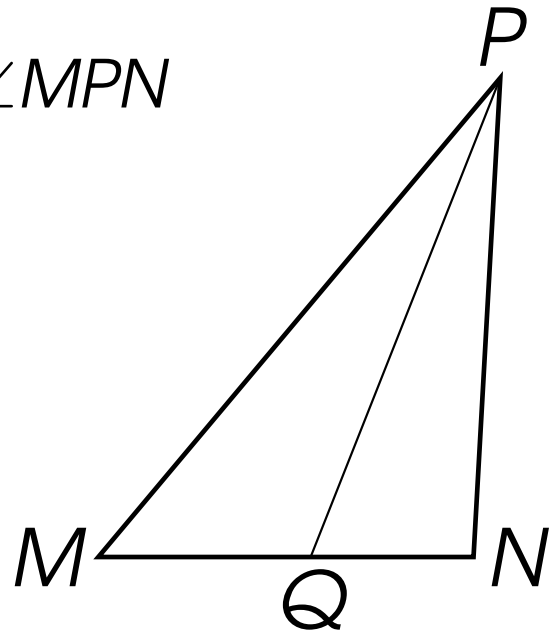

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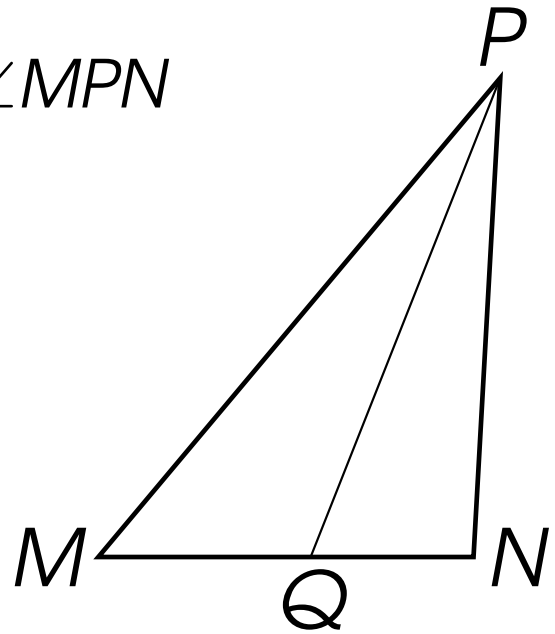

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$\frac{-2x}{-2} = \frac{-14}{-2}$

$x = 7$

$m\angle MPN = 2(m\angle MPQ)$

$m\angle MPN = 2(3x + 9)$

$m\angle MPN = 2[3(7) + 9]$

Example 2

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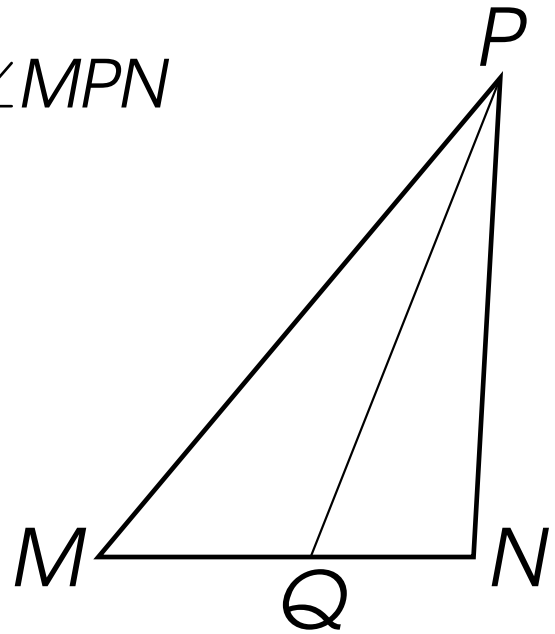
$m\angle NPQ = 5x - 5$

Find: $m\angle MPN$



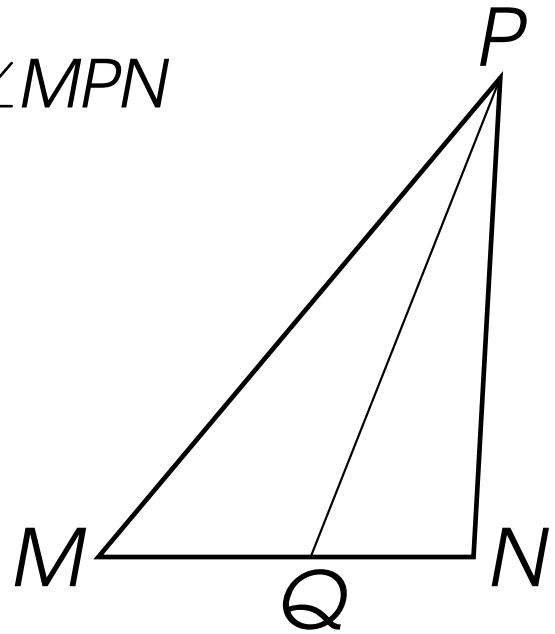
$\angle MPQ \cong \angle NPQ$
$m\angle MPQ = m\angle NPQ$
$3x + 9 = 5x - 5$
$3x - 5x + 9 - 9 = 5x - 5x - 5 - 9$
$-2x = -14$
$\frac{-2x}{-2} = \frac{-14}{-2}$
$x = 7$

$m\angle MPN = 2(m\angle MPQ)$
$m\angle MPN = 2(3x + 9)$
$m\angle MPN = 2[3(7) + 9]$
$m\angle MPN = 2(30)^\circ$

Example 2

Given: $\overline{PQ}$ is an angle bisector of $\angle MPN$
$m\angle MPQ = 3x + 9$,
$m\angle NPQ = 5x - 5$

Find: $m\angle MPN$



$\angle MPQ \cong \angle NPQ$

$m\angle MPQ = m\angle NPQ$

$3x + 9 = 5x - 5$

$3x - 5x + 9 - 9 = 5x - 5x - 5 - 9$

$-2x = -14$

$\frac{-2x}{-2} = \frac{-14}{-2}$

$x = 7$

$m\angle MPN = 2(m\angle MPQ)$

$m\angle MPN = 2(3x + 9)$

$m\angle MPN = 2[3(7) + 9]$

$m\angle MPN = 2(30)^\circ = 60^\circ$

Thank you for watching.