Systems of Linear Equations in Two Variables

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What is a System of Linear Equations?

- It consists of two or more linear equations with the same variables considered together for which a common solution is desired.

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- It consists of two or more linear equations with the same variables considered together for which a common solution is desired.
- It is also called Simultaneous Equations.

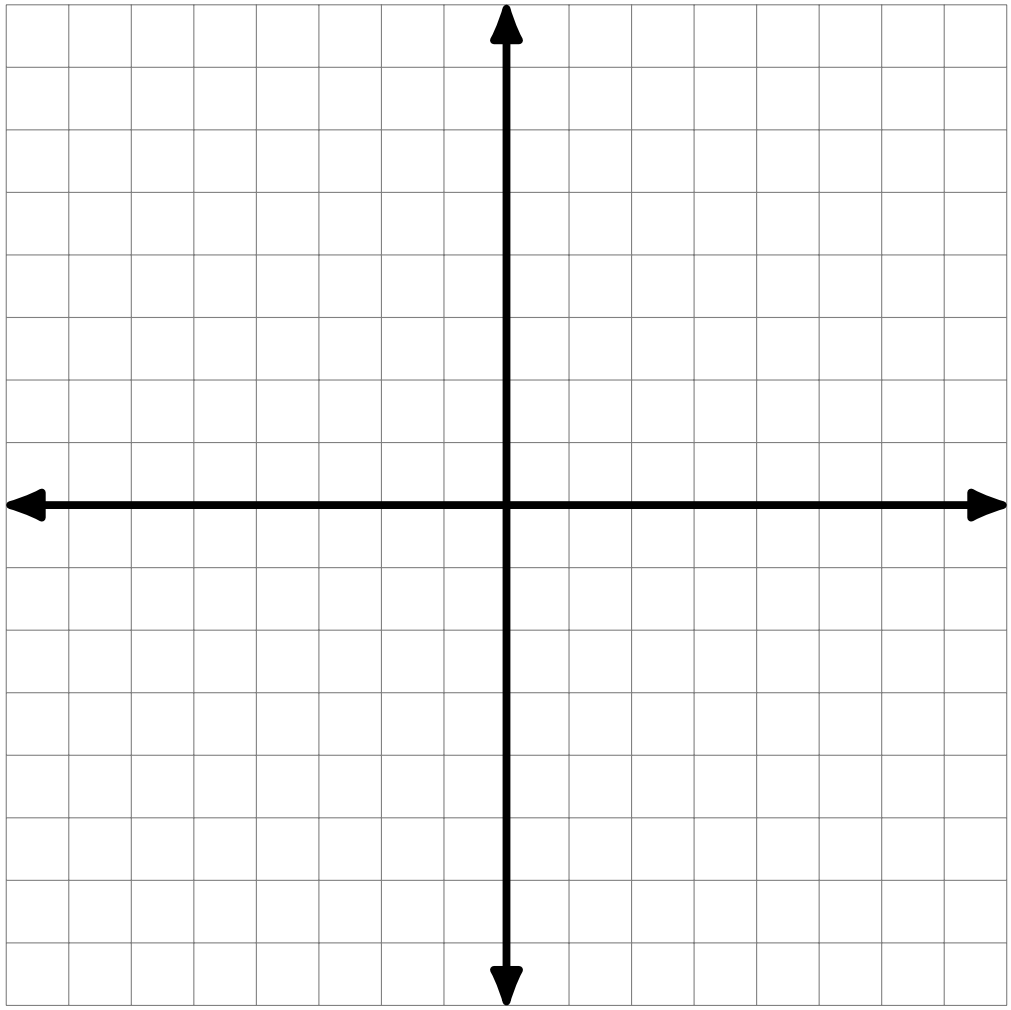
Example

$$\begin{cases} 2x + y = 10 \\ x + y = 6 \\ x + 3y = 13 \end{cases}$$

What is a Solution Set of a System of Linear Equations?

A solution set is an ordered pair of real numbers that satisfies both equations of the system.

Example 1



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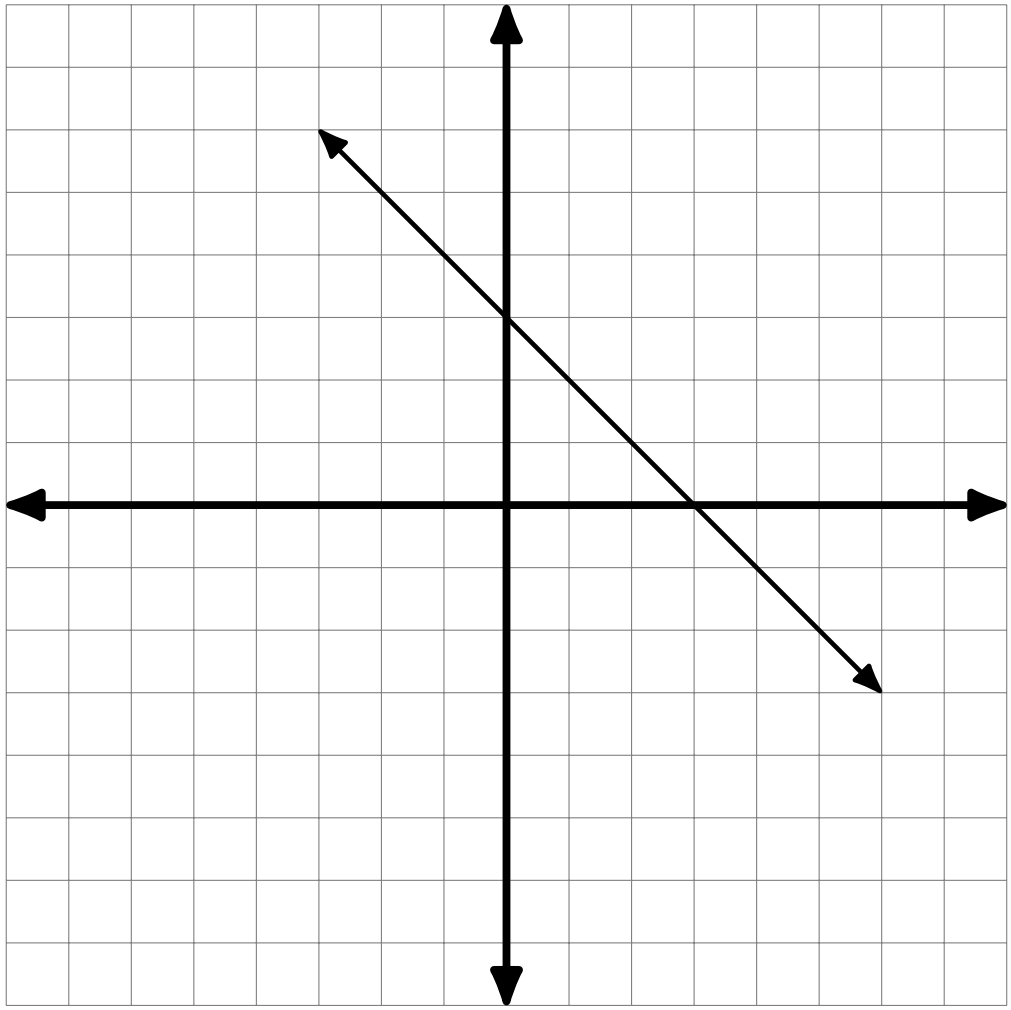

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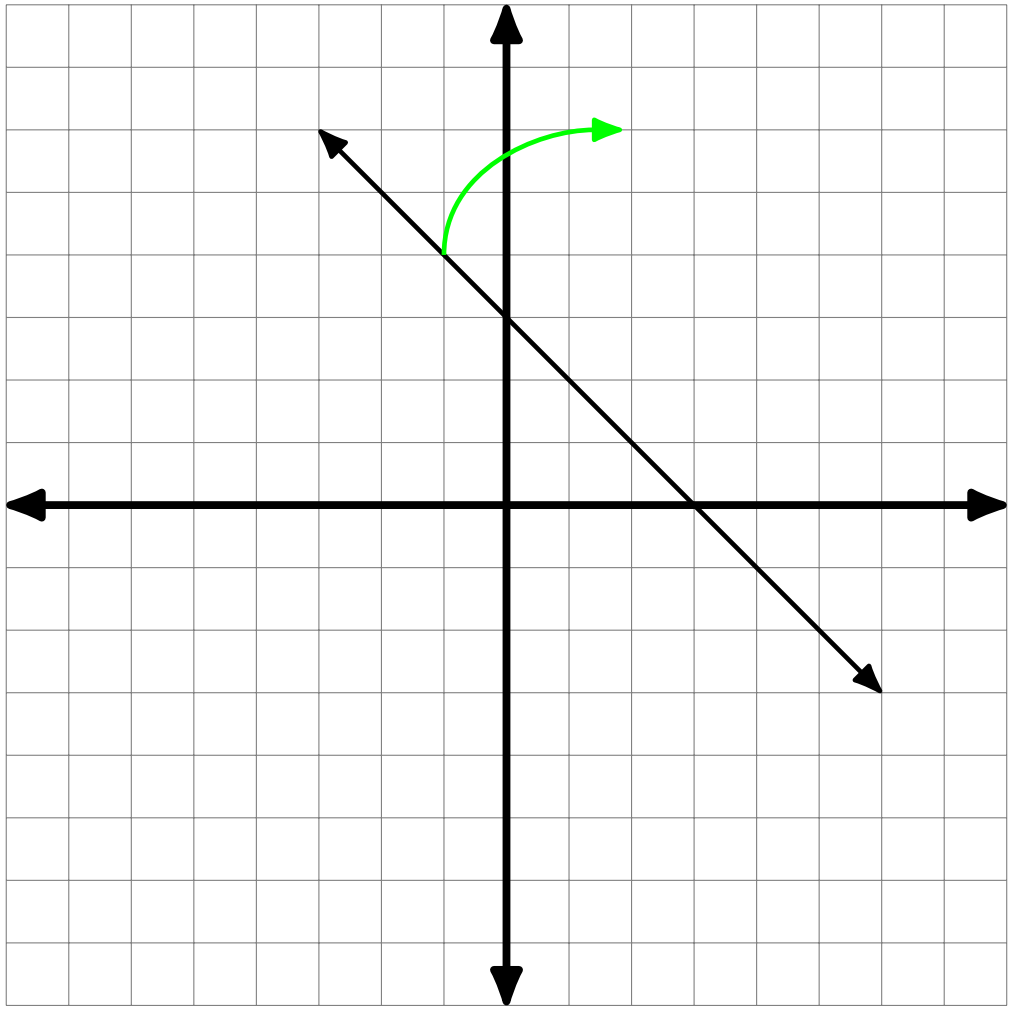

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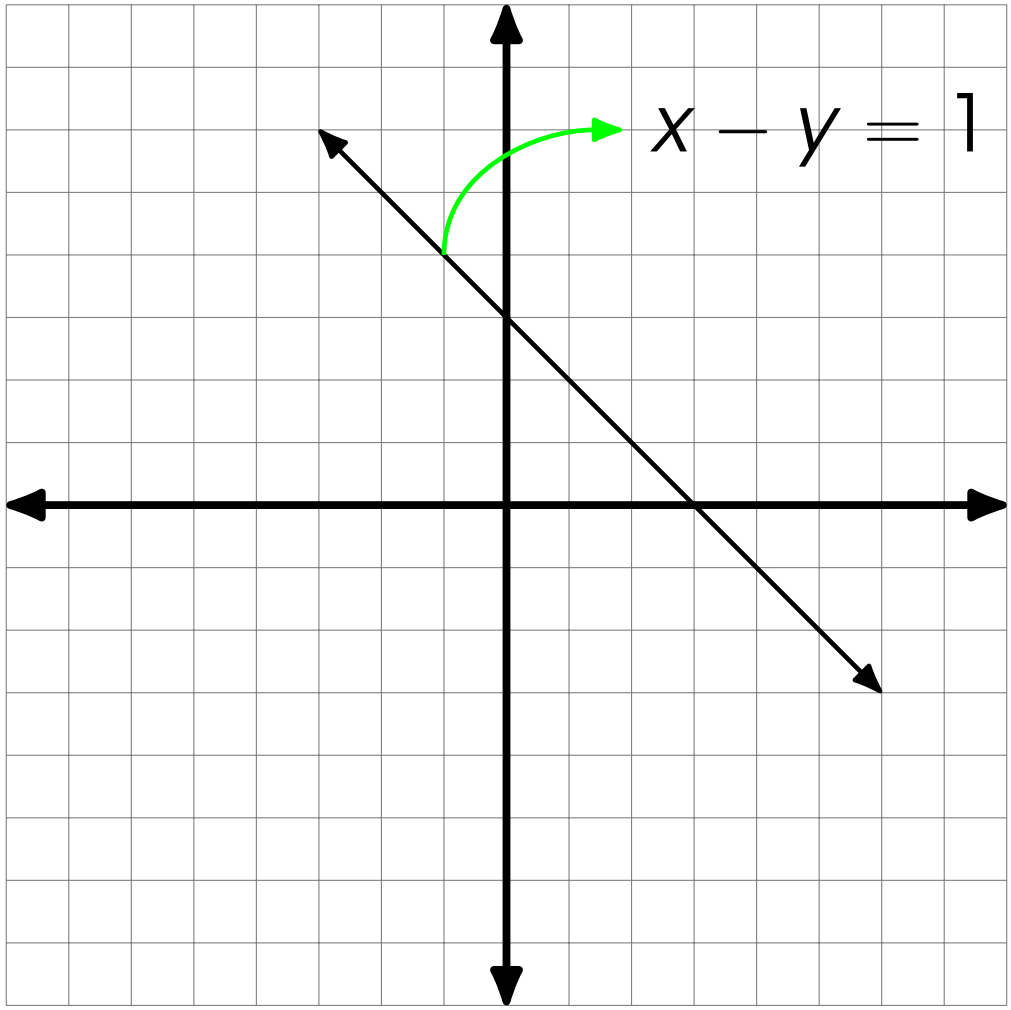

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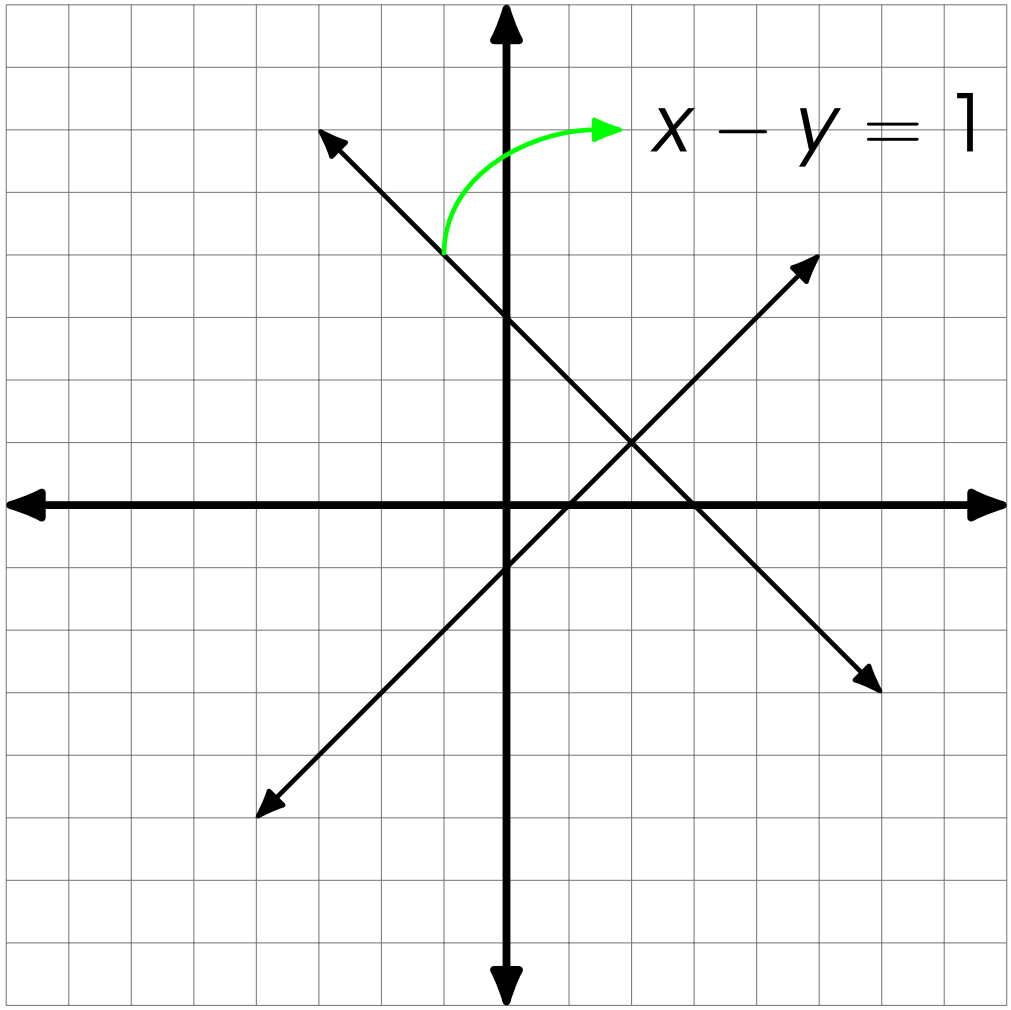

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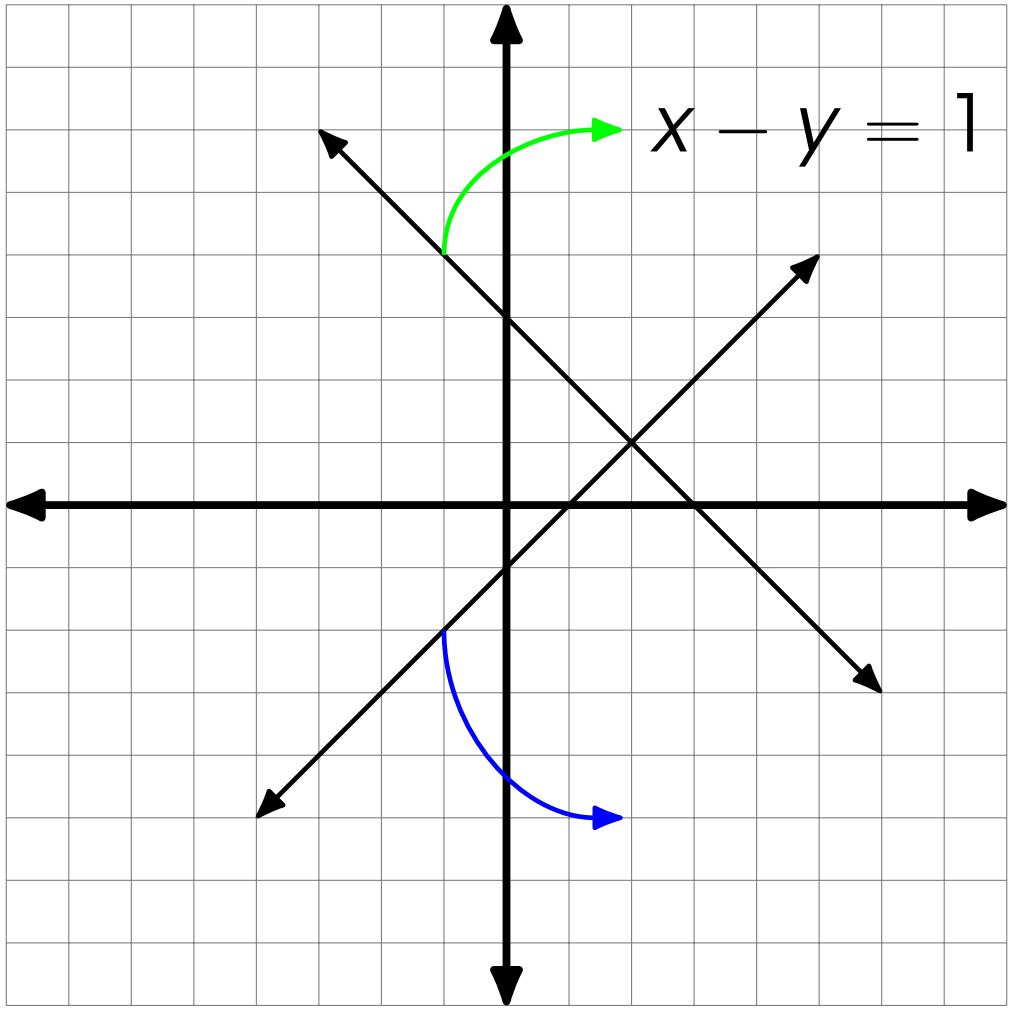

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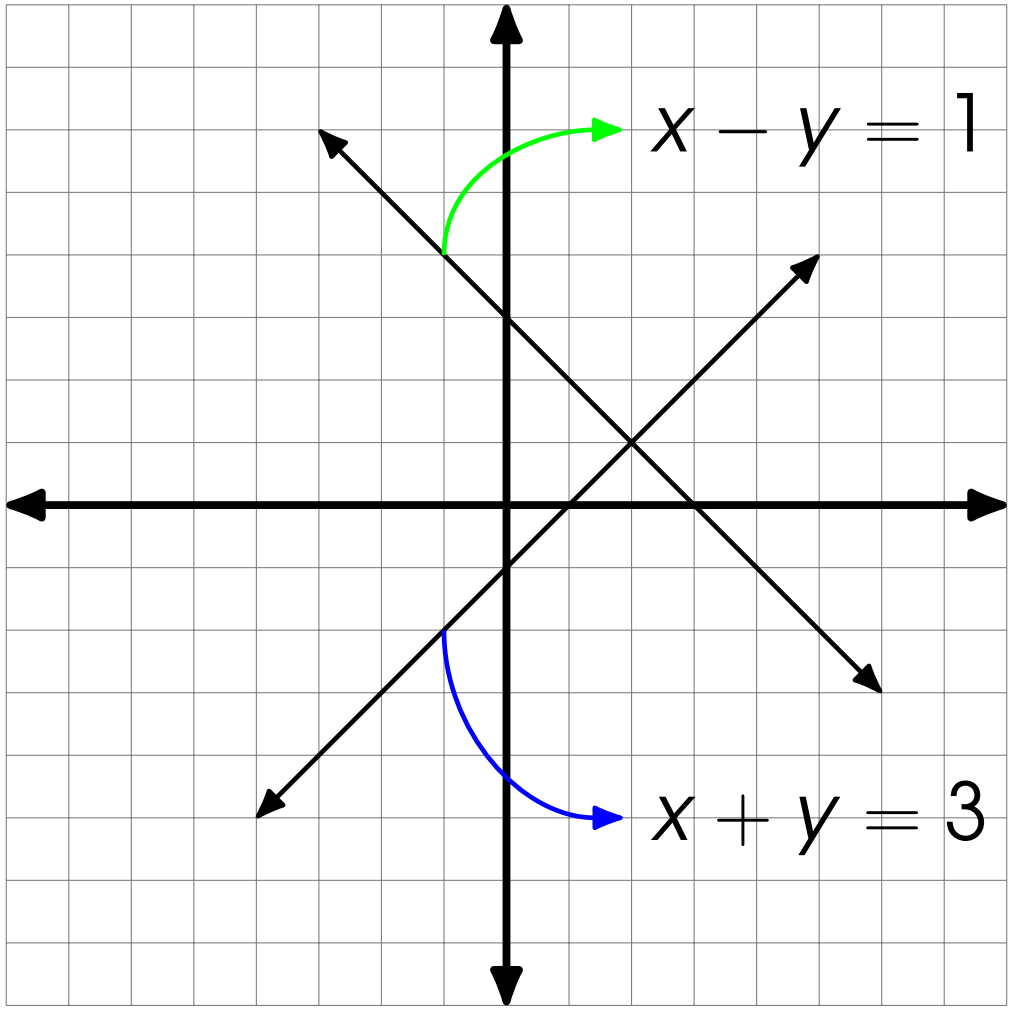

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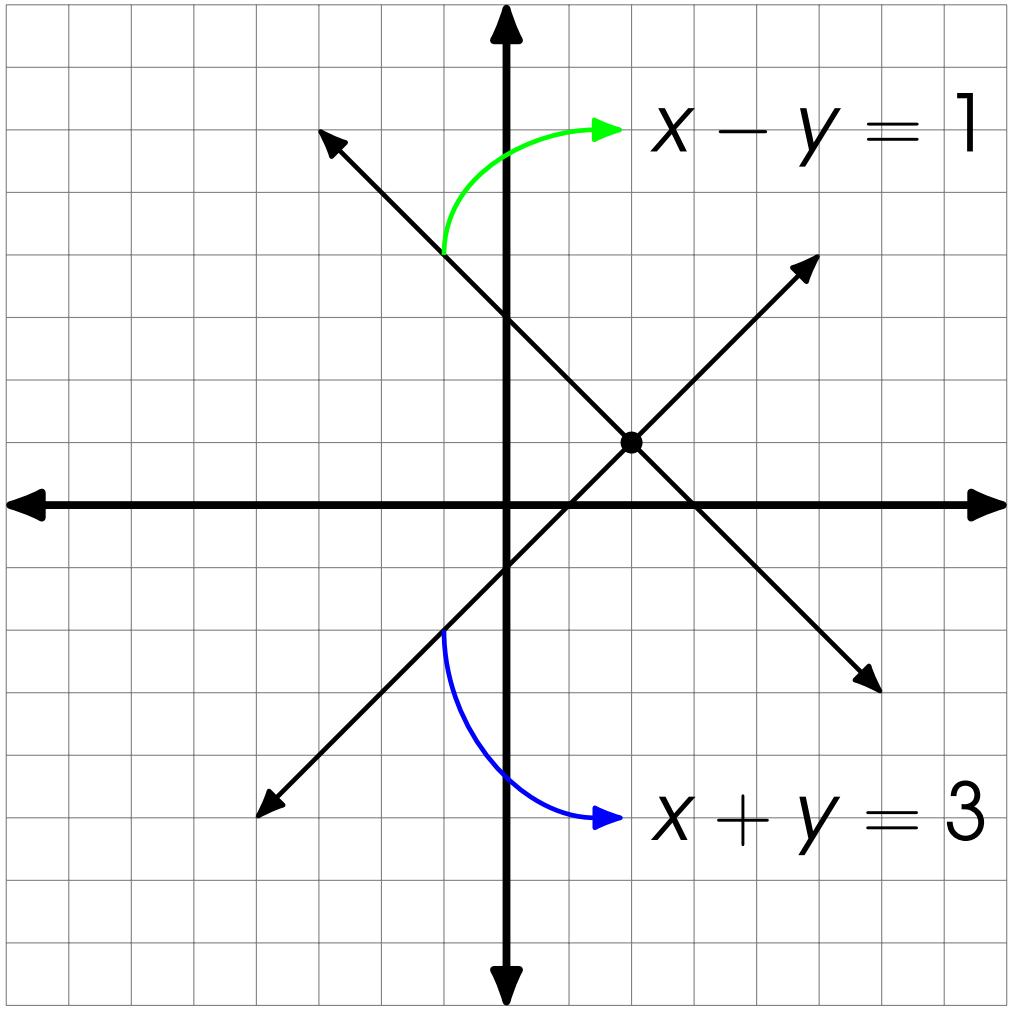

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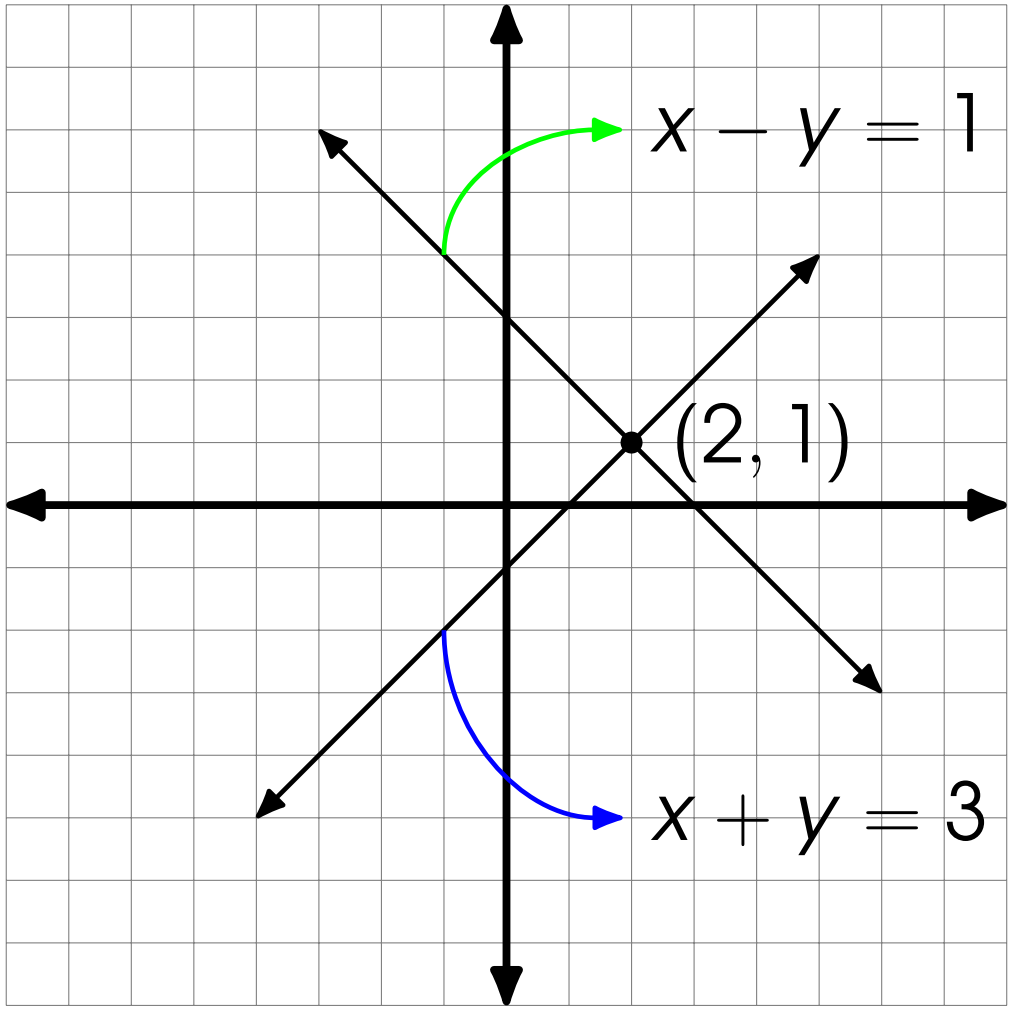

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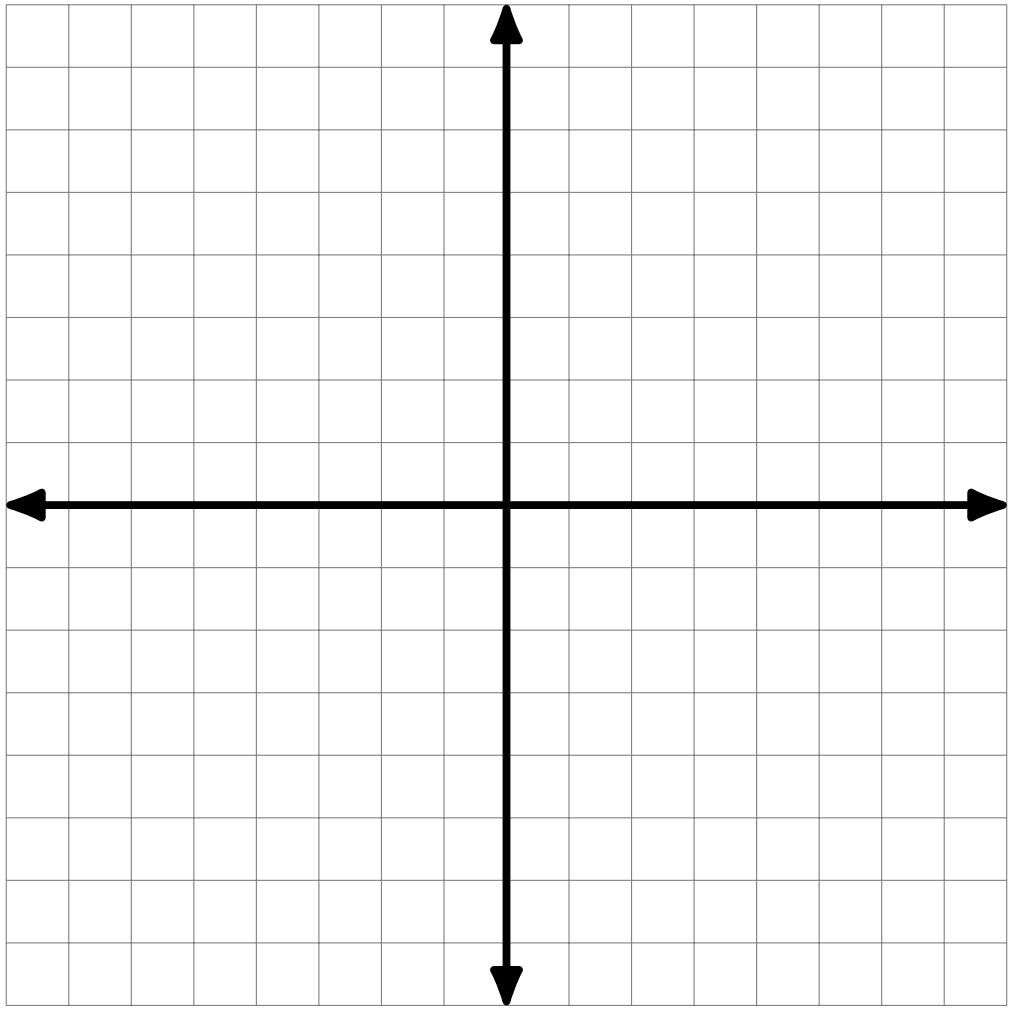

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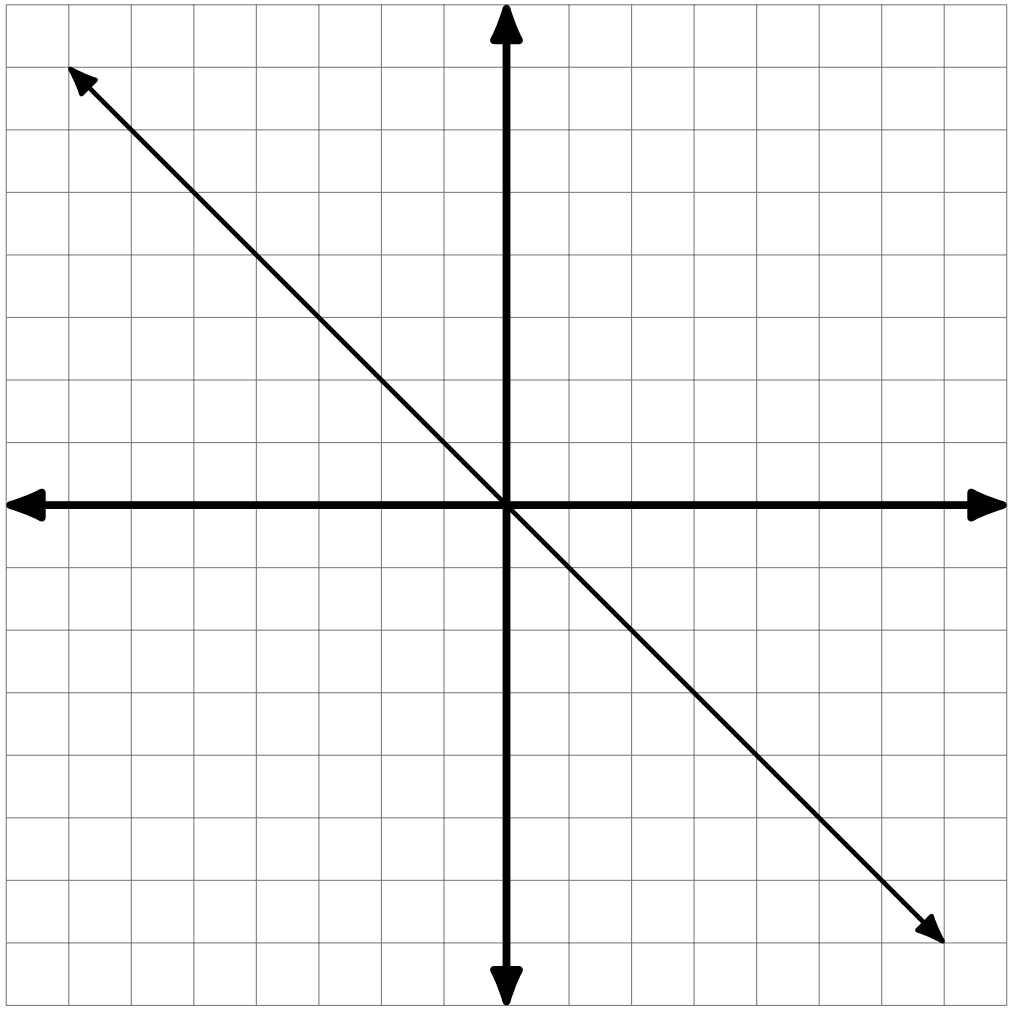

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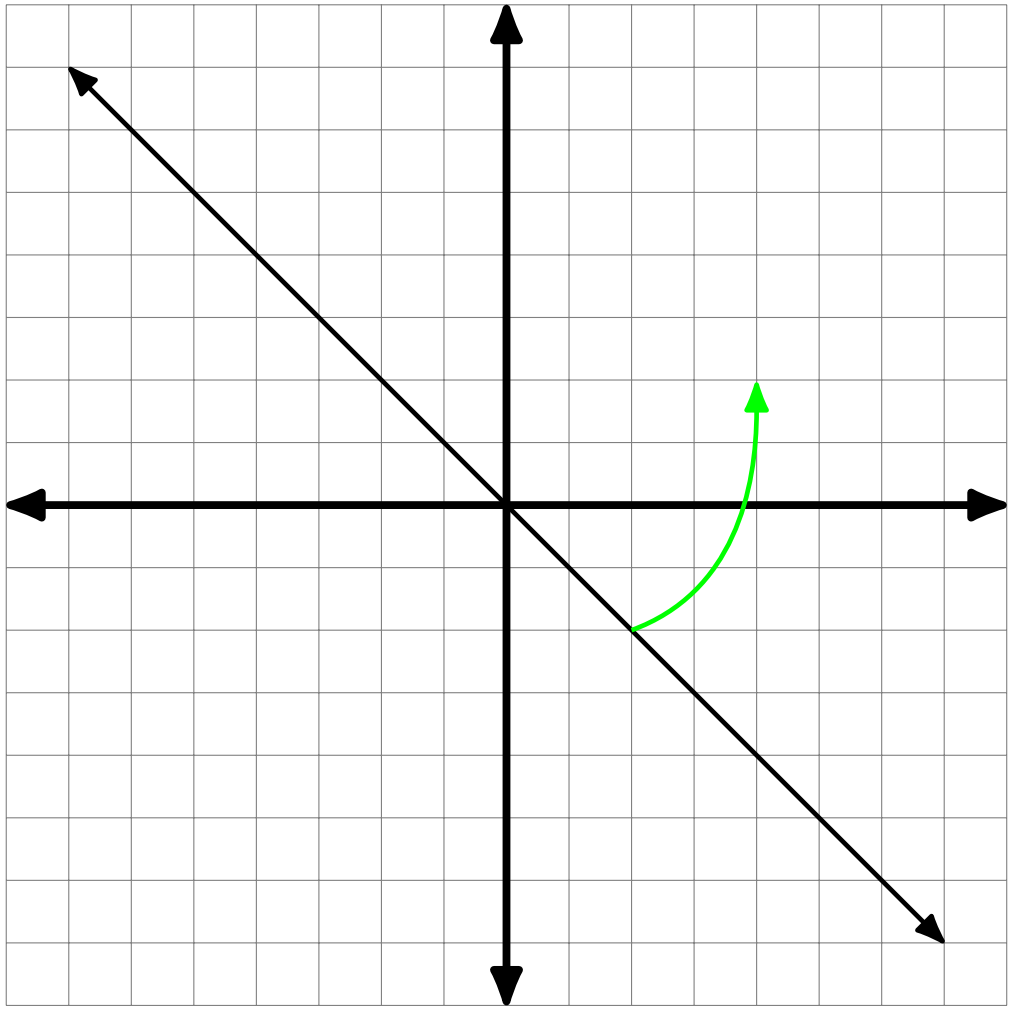

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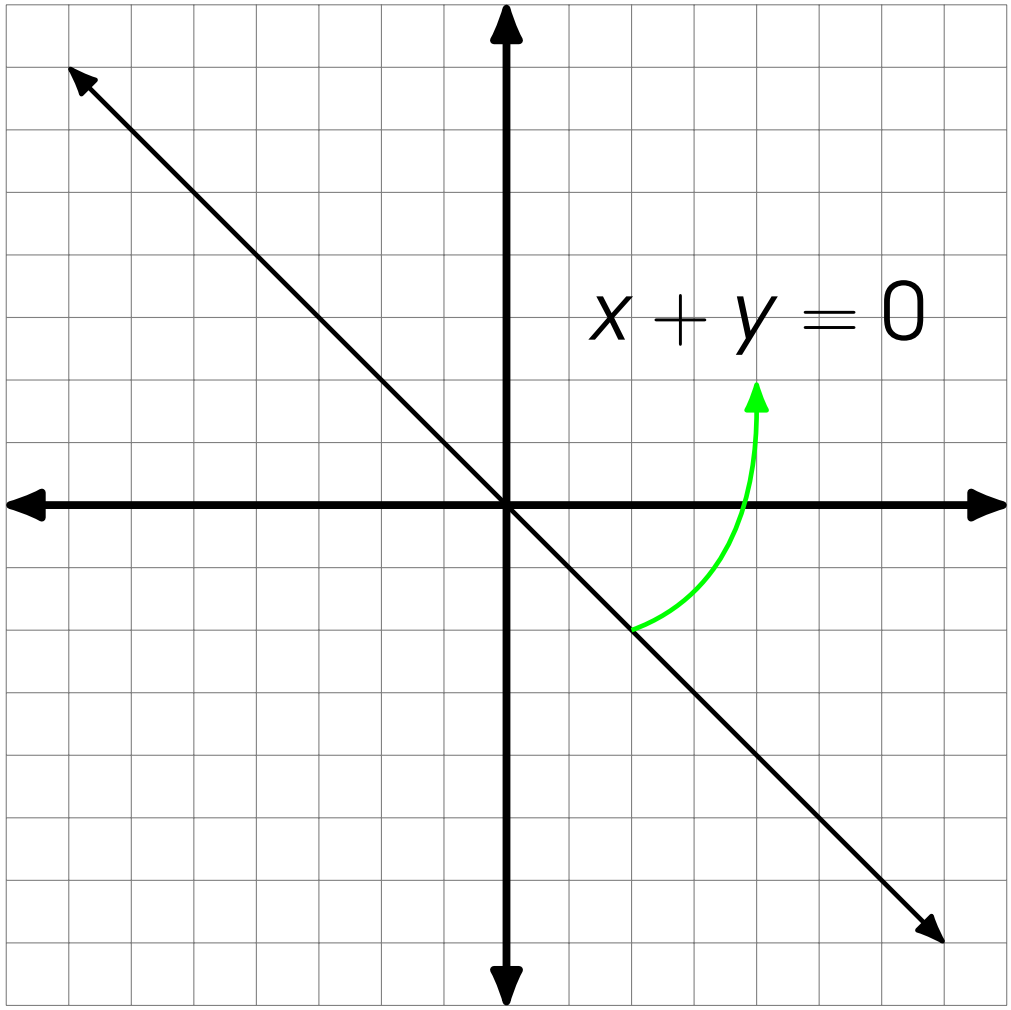

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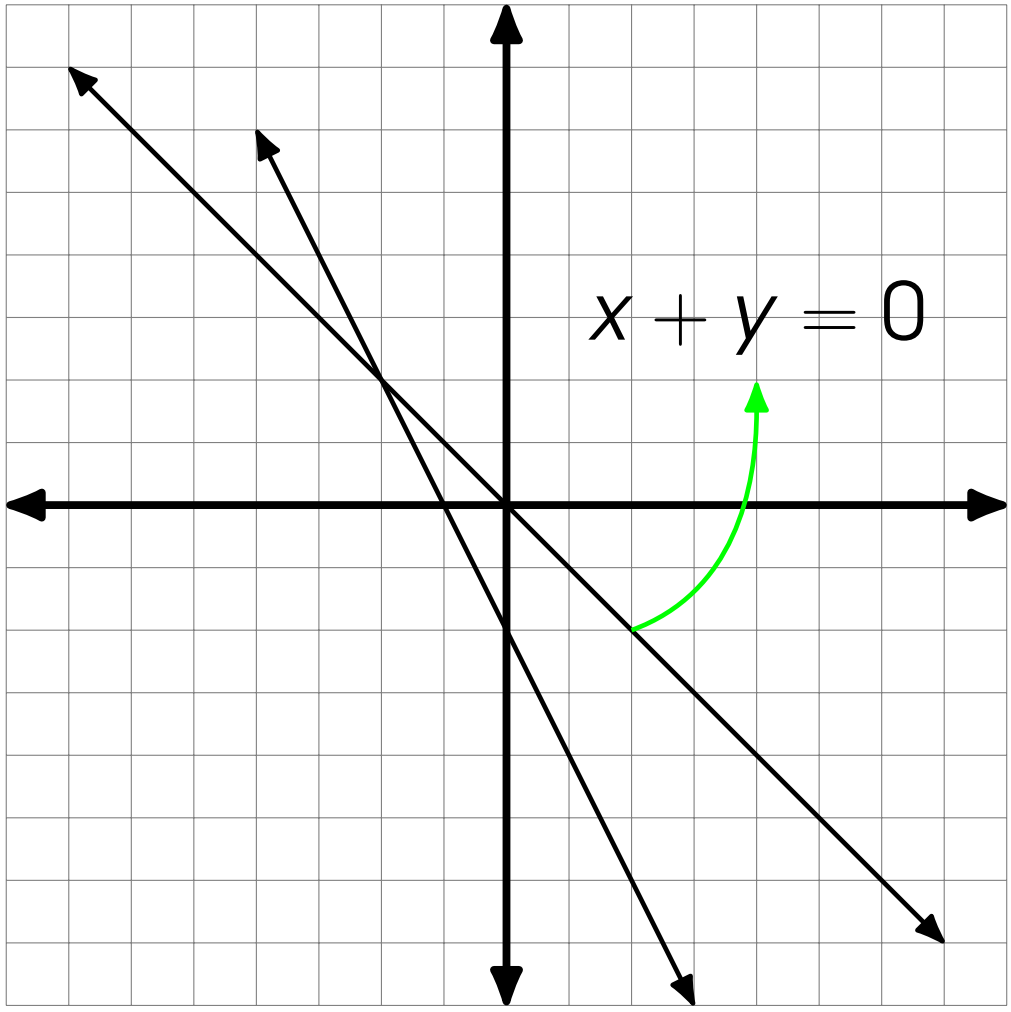

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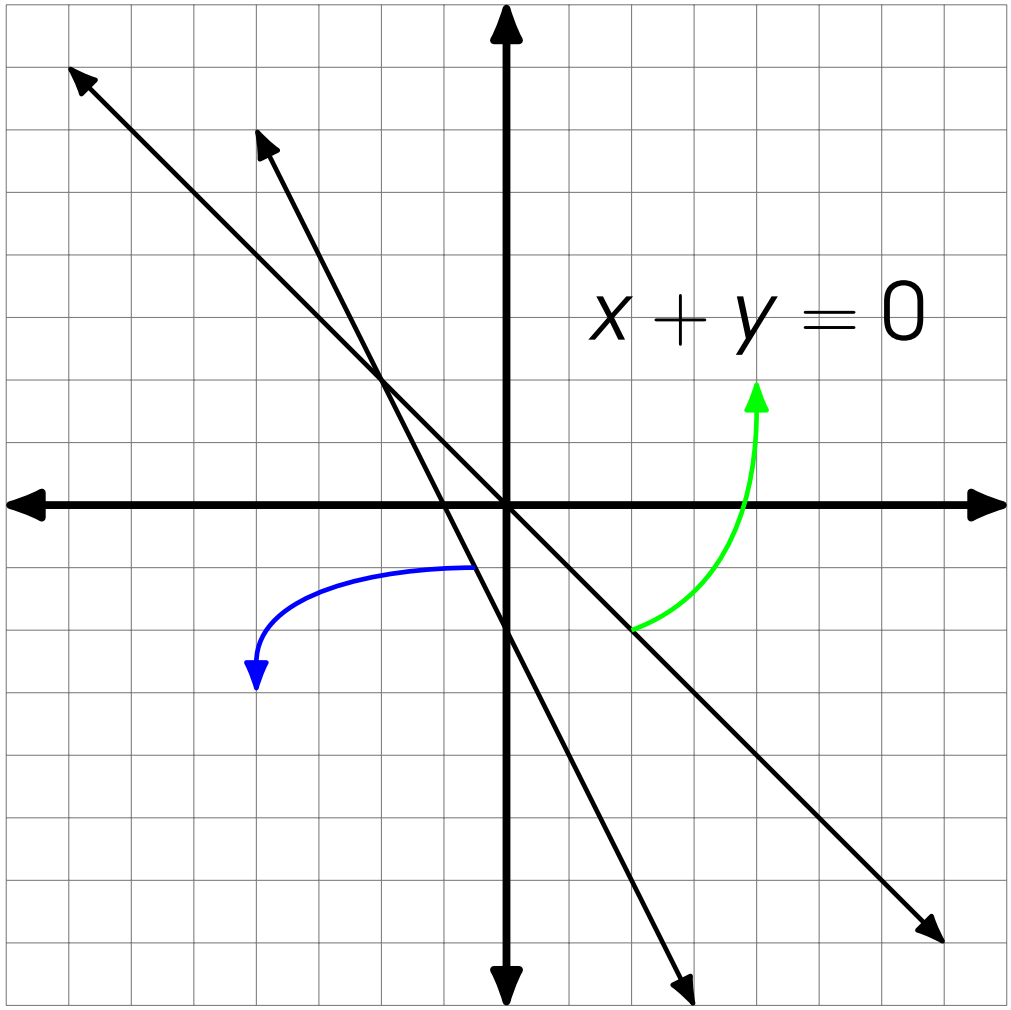

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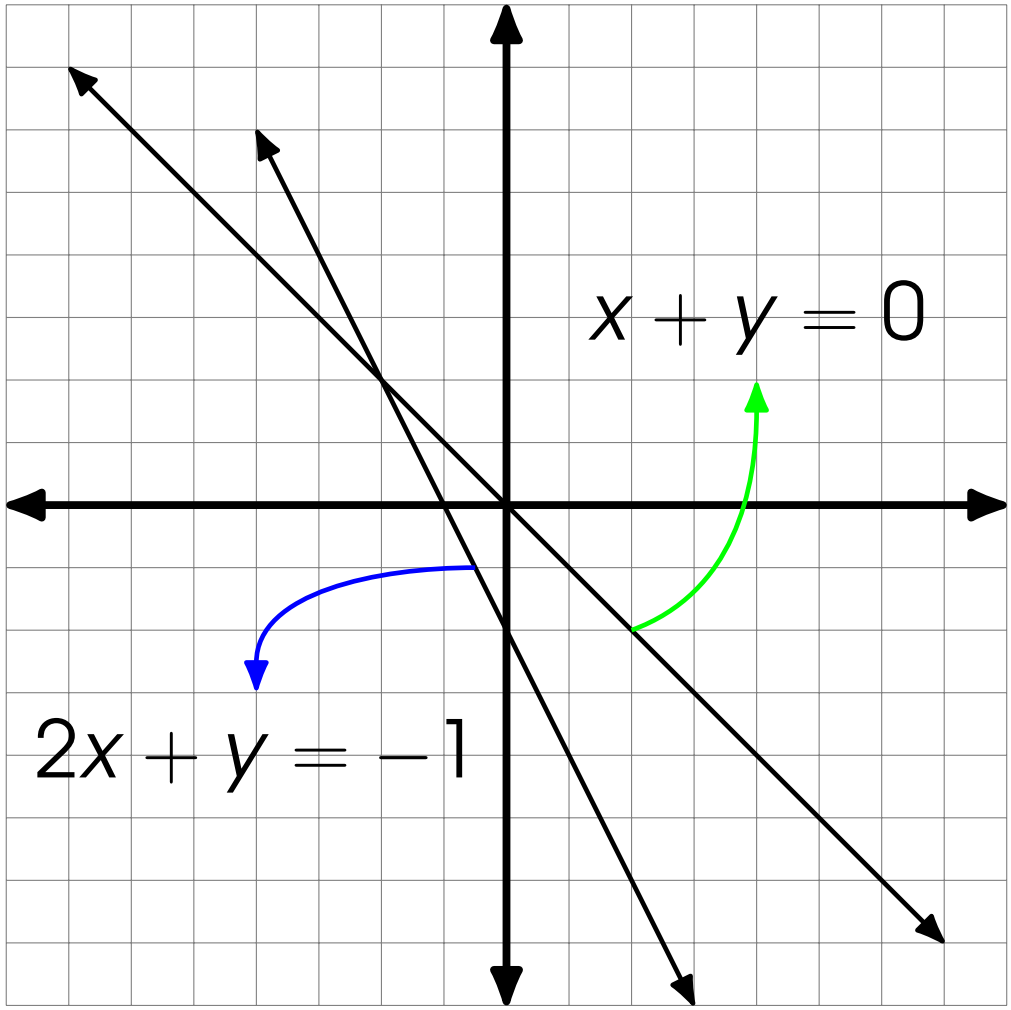

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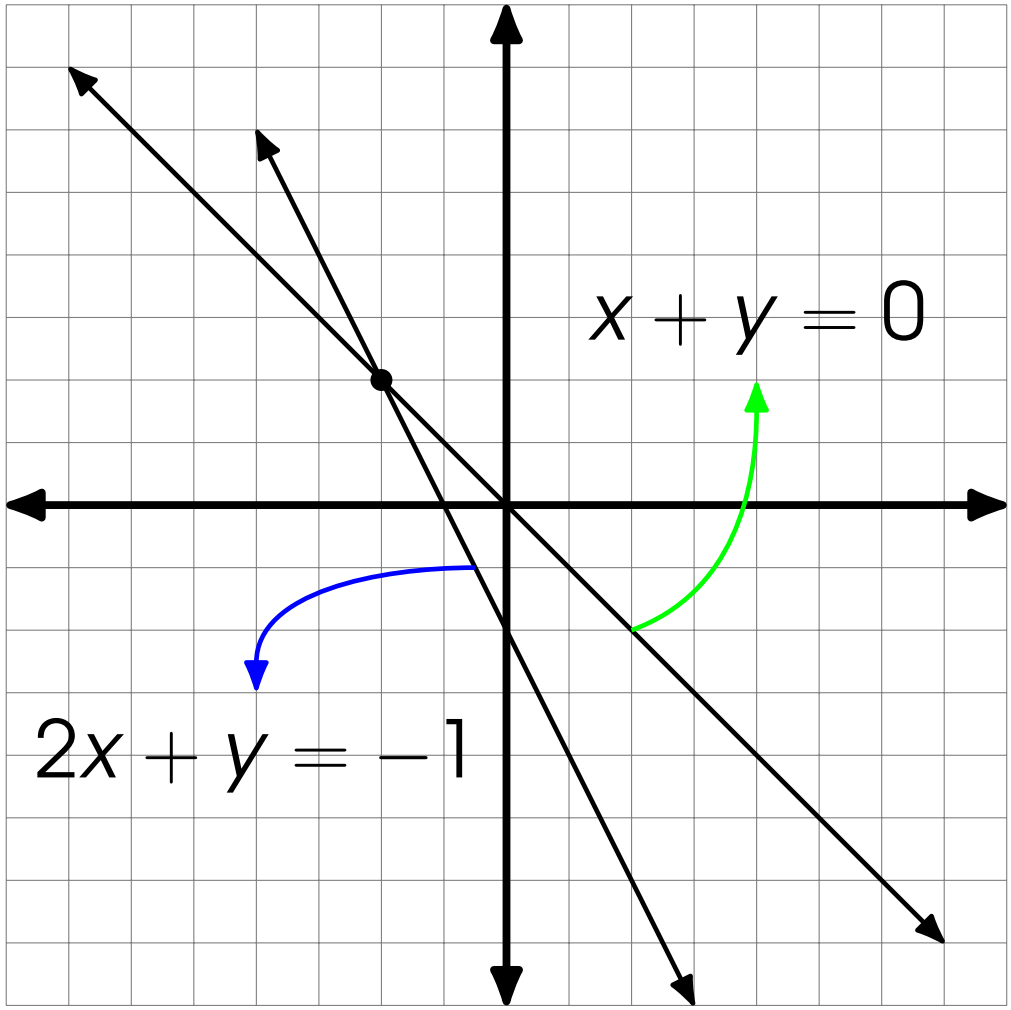

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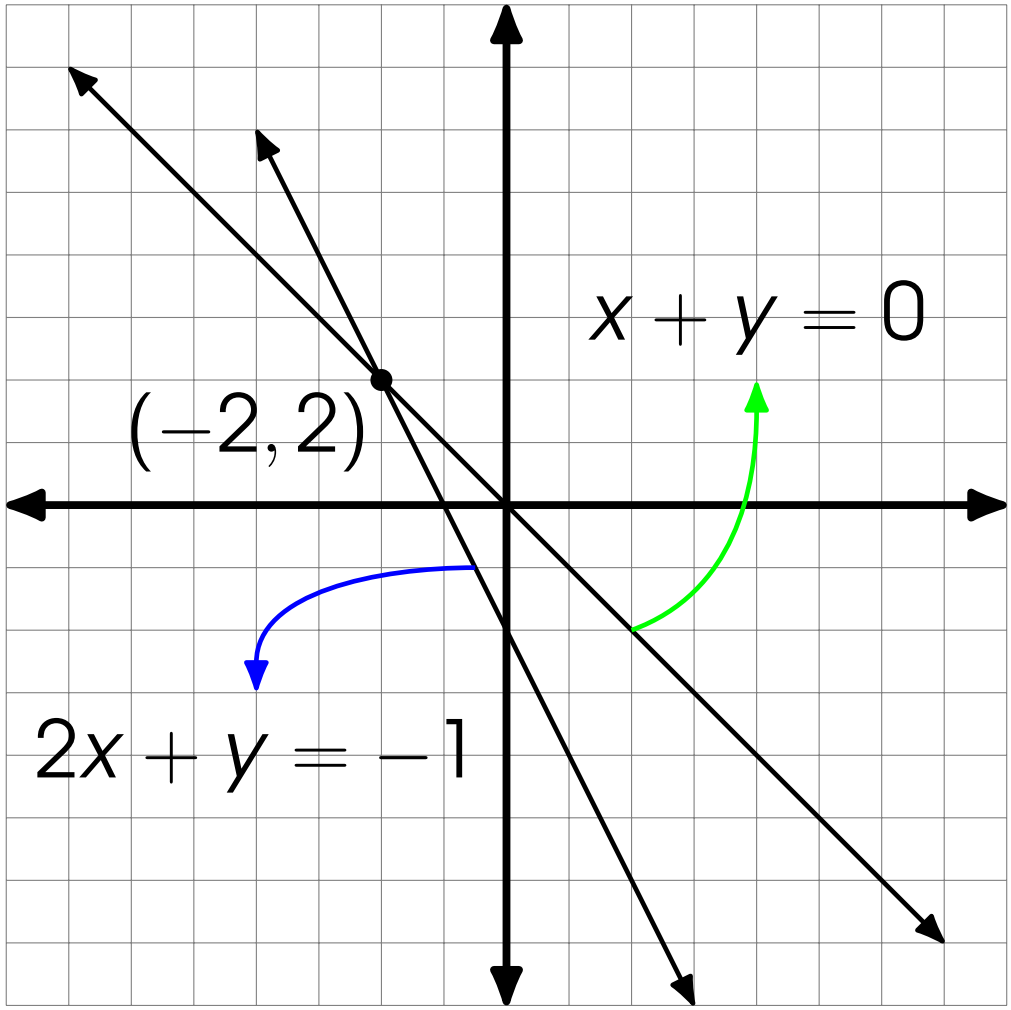

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2. Simplify. Check if the ordered pair satisfies both equations.

Example 2

Is the ordered pair $(2, 1)$ a solution to the system $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$?

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$$x - y = 1$$

$$2 - 1 = 1$$

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, \quad y = 1$

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$2 - 1 = 1$ Substitution Property

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$2 - 1 = 1$ Substitution Property

$1 = 1$ Simplification

$\therefore$ the ordered pair $(2, 1)$ satisfies the equation $x - y = 1$.

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$x + y = 3$

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$3 = 1$ Simplification

$\therefore$ the ordered pair $(2, 1)$ satisfies the equation $x + y = 3$.

Example 2

$\therefore$ since the ordered pair $(2, 1)$ satisfies both the equations $x - y = 1$ and $x + y = 3$, it is a solution to the system $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$.

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1. Consistent-dependent: a system of equations that can be rewritten as identical equations and have an infinite solution
2. Consistent-independent: a system of equations that can not be rewritten as contradicting equations nor identical equations; they stay different and have one solution
3. Inconsistent: a system of equations that can be rewritten as contradicting equations and has no solution

Example 1

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} 2x + \quad y = 4 \\ 4x + 2y = 8 \end{cases}$$

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$$\begin{cases} 2(2x + \ y) = 2(4) \\ \ \ 4x + 2y = 8 \end{cases}$$

Multiplication Property

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$$\begin{cases} 4x + 2y = 8 \\ 4x + 2y = 8 \end{cases}$$ Distributive Property

$\therefore$ since the equations are identical, it is a **Consistent-dependent** system and has infinitely many solutions.

Example 2

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

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Multiplication Property

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$$\begin{cases} 6x + 9y = 12 \\ 6x + 9y = 8 \end{cases}$$

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$$\begin{cases} 3(2x + 3y) = 3(4) \\ \quad 6x + \ \ 9y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 6x + 9y = 12 \\ 6x + 9y = 8 \end{cases}$$

Distributive Property

$\therefore$ since the equations are contradicting, it is an **Inconsistent** system and has no solution.

Example 3

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

$$\begin{cases} 2(x - 2y) = 2(1) \\ 2x + 4y = 3 \end{cases}$$

Multiplication Property

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

$$\begin{cases} 2(x - 2y) = 2(1) \\ 2x + 4y = 3 \end{cases}$$ Multiplication Property

$$\begin{cases} 2x - 4y = 2 \\ 2x + 4y = 3 \end{cases}$$ Distributive Property

$\therefore$ since the equations are not contradicting and not identical, it is a **Consistent-independent** system and has one solution.

Thank you for watching.