

Systems of Linear Equations in Two Variables

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What is a System of Linear Equations?

- ▶ It consists of two or more linear equations with the same variables considered together for which a common solution is desired.

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- ▶ It consists of two or more linear equations with the same variables considered together for which a common solution is desired.
- ▶ It is also called Simultaneous Equations.

Examples and Non-Examples of Systems of Linear Equations in Two Variables

Examples

$$\begin{cases} 2x + y = 10 \\ x + y = 6 \end{cases}$$

$$\begin{cases} y = 3x - 1 \\ y = x - 1 \end{cases}$$

$$\begin{cases} 2x = y + 1 \\ 3x + y = 2 \end{cases}$$

$$\begin{cases} x = 3y \\ y = 6x \end{cases}$$

Non-Examples

$$x + y = 6$$

$$\begin{cases} x = 1 \\ x + y = 2 \end{cases}$$

$$\begin{cases} x + 2y \leq 3 \\ x + 3y \neq 2 \end{cases}$$

$$\begin{cases} x + y = 1 \\ x + y - z = 4 \end{cases}$$

Example

Write Yes if the given is a system of linear equations in two variables or No if it is not.

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$$\begin{cases} y > 3x - 1 \\ y = x - 1 \end{cases}$$

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$$\begin{cases} x = 3y \\ y = 2 \end{cases}$$

Example

Write Yes if the given is a system of linear equations in two variables or No if it is not.

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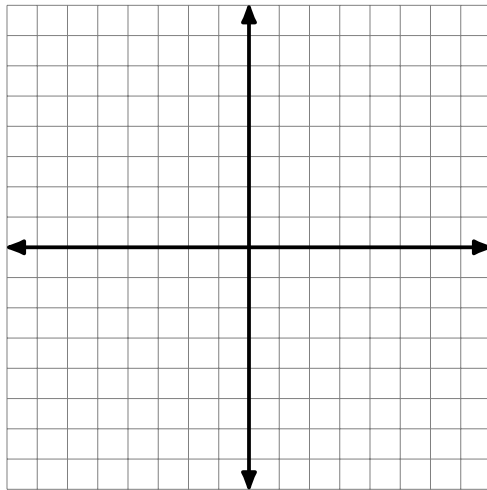
$$\begin{cases} 2x = y + 1 \\ 3x = 2y \end{cases} \quad \text{Yes}$$

$$\begin{cases} x = 3y \\ y = 2 \end{cases} \quad \text{No}$$

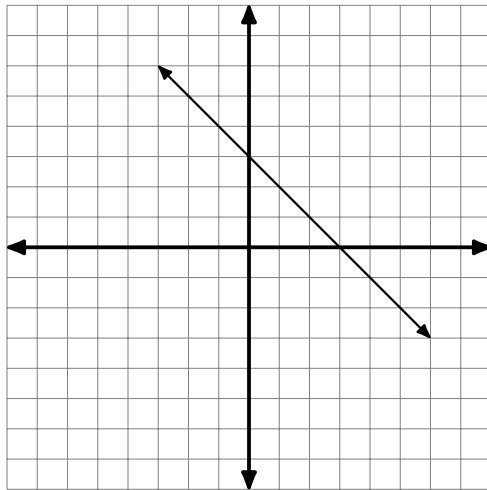
What is a Solution Set of a System of Linear Equations?

A solution set is an ordered pair of real numbers that satisfies both equations of the system.

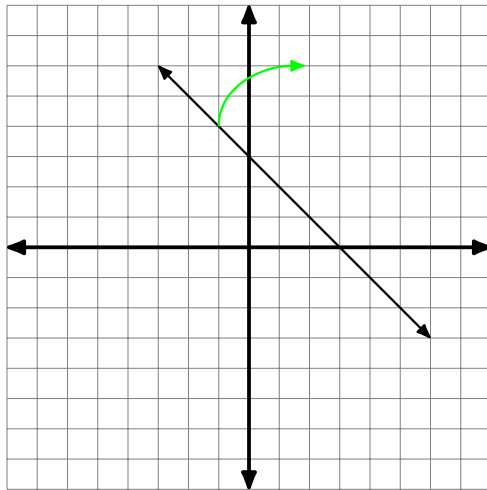
Example 1



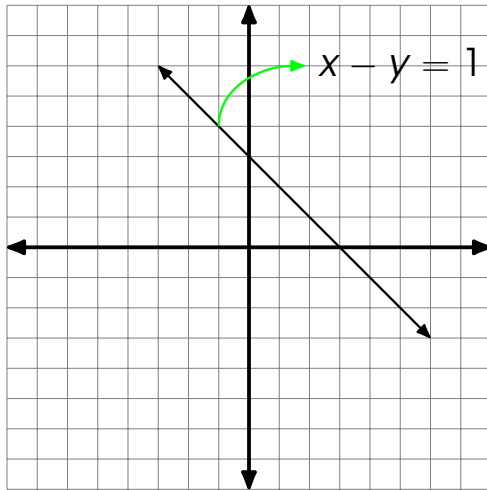
Example 1



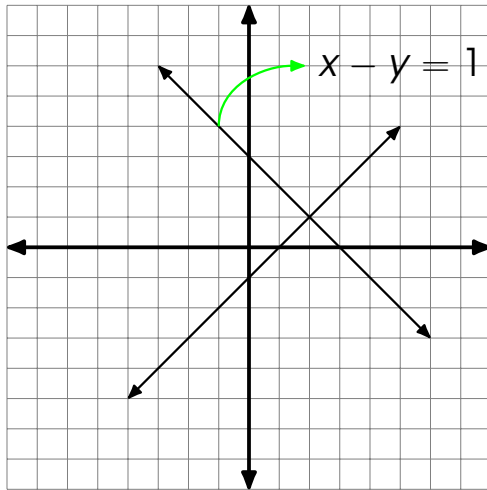
Example 1



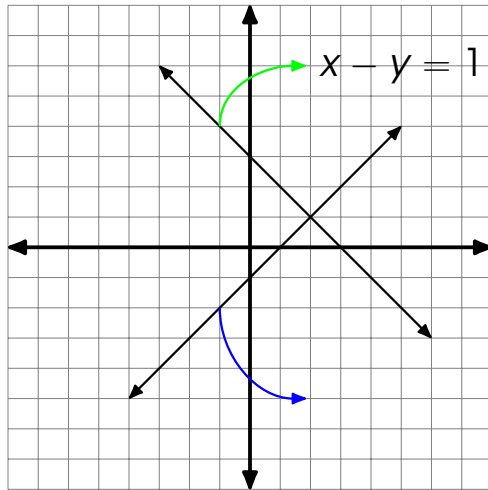
Example 1



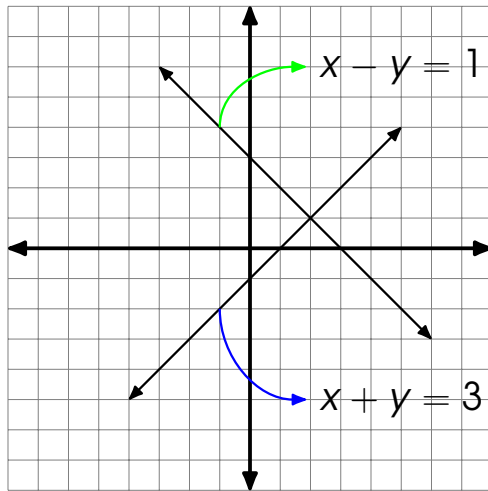
Example 1



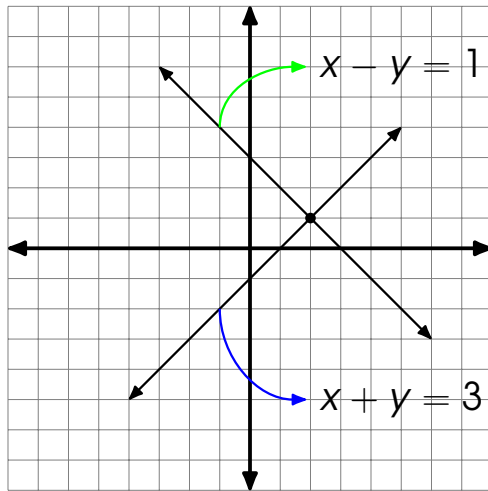
Example 1



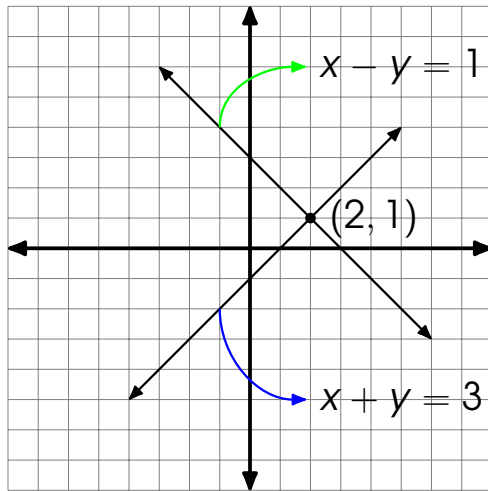
Example 1



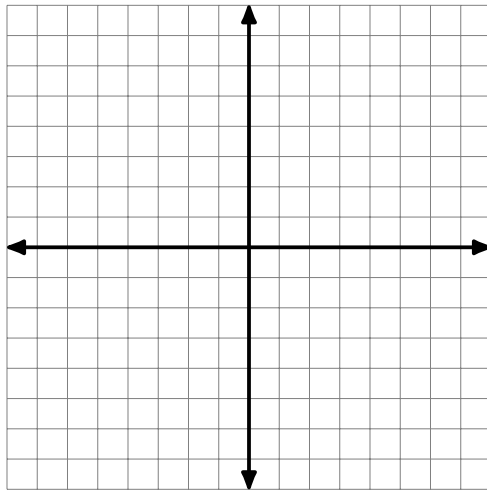
Example 1



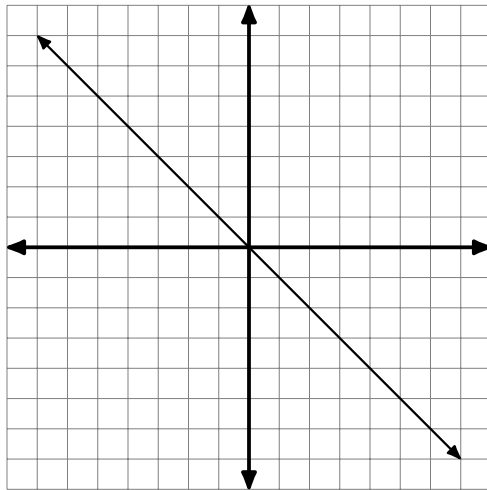
Example 1



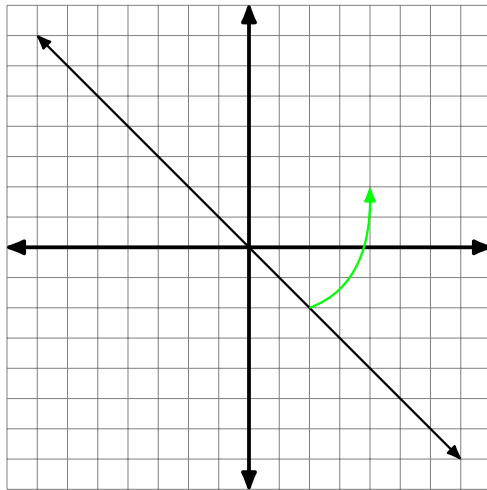
Example 2



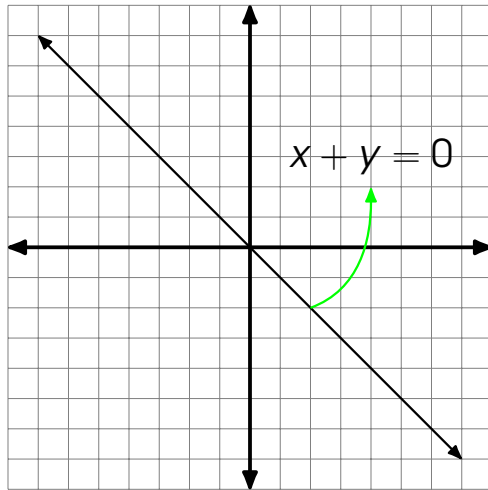
Example 2



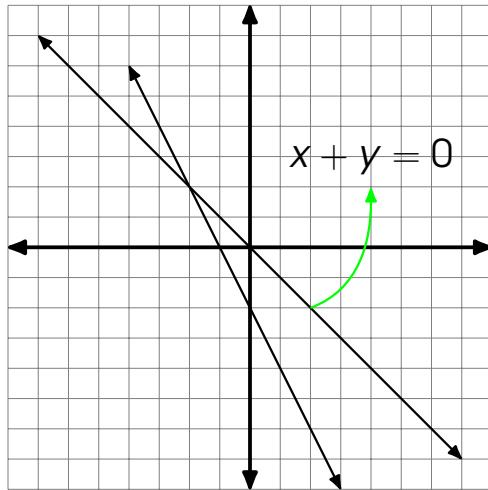
Example 2



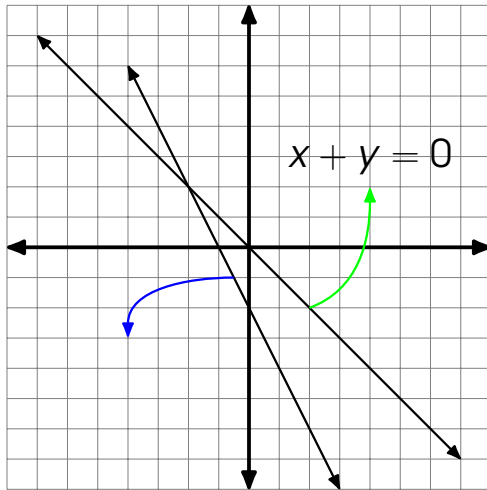
Example 2



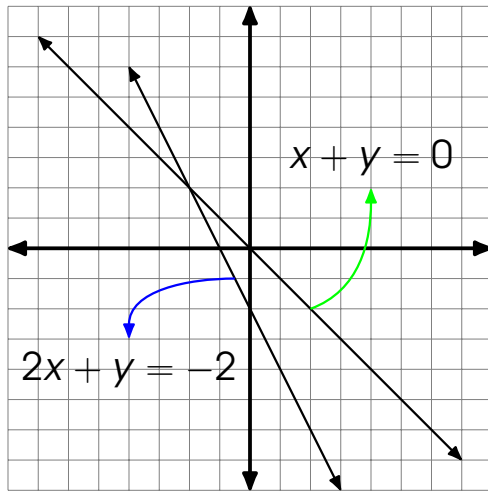
Example 2



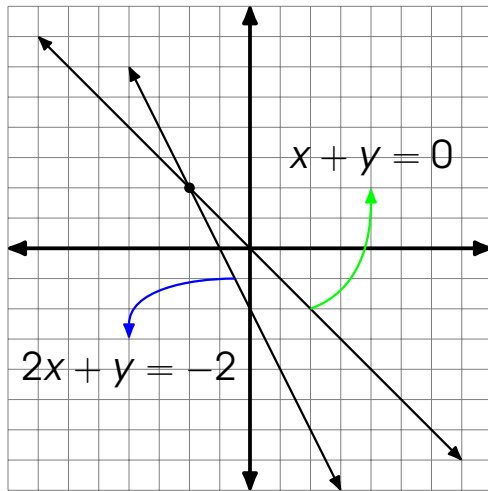
Example 2



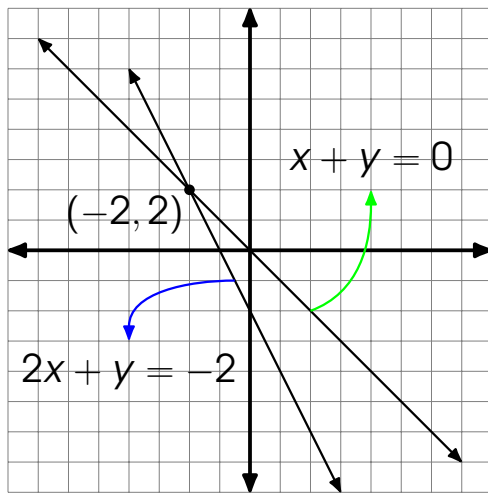
Example 2



Example 2



Example 2



How to Check Whether an Ordered Pair is a Solution to a Linear System?

1. Replace x and y with the given values in both equations.

How to Check Whether an Ordered Pair is a Solution to a Linear System?

1. Replace x and y with the given values in both equations.
2. Simplify. Check if the ordered pair satisfies both equations.

Example 1

Is the ordered pair $(2, 1)$ a solution to the system $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$?

Example 1

Step 1: Replace x and y with the given values in both equations.

Given: $x = 2$,

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Given: $x = 2$, $y = 1$

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Given: $x = 2, y = 1$

$$x - y = 1$$

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Given: $x = 2, y = 1$

$$x - y = 1$$

$$2 - 1 = 1$$

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, y = 1$

$$x - y = 1$$

$$2 - 1 = 1$$

Substitution Property

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = 2, \quad y = 1$$

$$x - y = 1$$

$$2 - 1 = 1$$

Substitution Property

$$1 = 1$$

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, y = 1$

$$x - y = 1$$

$$2 - 1 = 1$$

Substitution Property

$$1 = 1$$

Simplification

\therefore the ordered pair $(2, 1)$ satisfies the equation $x - y = 1$.

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Given: $x = 2$, $y = 1$

$$x + y = 3$$

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$$x + y = 3$$

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Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2$, $y = 1$

$$x + y = 3$$

$$2 + 1 = 3$$

Substitution Property

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2$, $y = 1$

$$x + y = 3$$

$$2 + 1 = 3$$

Substitution Property

$$3 = 3$$

Example 1

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2, y = 1$

$$x + y = 3$$

$$2 + 1 = 3$$

Substitution Property

$$3 = 3$$

Simplification

\therefore the ordered pair $(2, 1)$ satisfies the equation $x + y = 3$.

Example 1

\therefore since the ordered pair $(2, 1)$ satisfies both the equations $x - y = 1$ and $x + y = 3$, it is a solution to the system
$$\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}.$$

Example 2

Is the ordered pair $(-1, 1)$ a solution to the system $\begin{cases} x + y = 0 \\ 2x + y = 1 \end{cases}$?

Example 2

Step 1: Replace x and y with the given values in both equations.

Given: $x = -1$,

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Given: $x = -1$, $y = 1$

$$x + y = 0$$

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$$-1 + 1 = 0$$

Example 2

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Given: $x = -1$, $y = 1$

$$x + y = 0$$

$$-1 + 1 = 0$$

Substitution Property

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = -1, \quad y = 1$$

$$x + y = 0$$

$$-1 + 1 = 0$$

Substitution Property

$$0 = 0$$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = -1, \quad y = 1$$

$$x + y = 0$$

$$-1 + 1 = 0$$

Substitution Property

$$0 = 0$$

Simplification

\therefore the ordered pair $(-1, 1)$ satisfies the equation $x + y = 0$.

Example 2

Step 1: Replace x and y with the given values in both equations.

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$$2x + y = 1$$

Example 2

Step 1: Replace x and y with the given values in both equations.

Given: $x = -1$, $y = 1$

$$2x + y = 1$$

$$2(-1) + 1 = 1$$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1$, $y = 1$

$$2x + y = 1$$

$$2(-1) + 1 = 1 \quad \text{Substitution Property}$$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = -1$, $y = 1$

$$2x + y = 1$$

$$2(-1) + 1 = 1 \quad \text{Substitution Property}$$

$$-1 \neq 1$$

Example 2

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = -1, \quad y = 1$$

$$2x + y = 1$$

$$2(-1) + 1 = 1 \quad \text{Substitution Property}$$

$$-1 \neq 1 \quad \text{Simplification}$$

\therefore the ordered pair $(-1, 1)$ does not satisfy the equation $2x + y = 1$.

Example 2

\therefore since the ordered pair $(-1, 1)$ does not satisfy the equation $2x + y = 1$, it is not a solution to the system $\begin{cases} x + y = 0 \\ 2x + y = 1 \end{cases}$.

Example 3

Is the ordered pair $(2, -1)$ a solution to the system $\begin{cases} x - 2y = 4 \\ x + 2y = 0 \end{cases}$?

Example 3

Step 1: Replace x and y with the given values in both equations.

Given: $x = 2$,

Example 3

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Given: $x = 2$, $y = -1$

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Given: $x = 2$, $y = -1$

$$x - 2y = 4$$

Example 3

Step 1: Replace x and y with the given values in both equations.

Given: $x = 2$, $y = -1$

$$x - 2y = 4$$

$$2 - 2(-1) = 4$$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2$, $y = -1$

$$x - 2y = 4$$

$$2 - 2(-1) = 4 \quad \text{Substitution Property}$$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = 2, \quad y = -1$$

$$x - 2y = 4$$

$$2 - 2(-1) = 4 \quad \text{Substitution Property}$$

$$4 = 4$$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = 2, \quad y = -1$$

$$x - 2y = 4$$

$$2 - 2(-1) = 4 \quad \text{Substitution Property}$$

$$4 = 4 \quad \text{Simplification}$$

\therefore the ordered pair $(2, -1)$ satisfies the equation $x - 2y = 4$.

Example 3

Step 1: Replace x and y with the given values in both equations.

Given: $x = 2$,

Example 3

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$$x + 2y = 0$$

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Given: $x = 2$, $y = -1$

$$x + 2y = 0$$

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Step 2: Simplify. Check if the ordered pair satisfies both equations.

Given: $x = 2$, $y = -1$

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$$2 + 2(-1) = 0 \quad \text{Substitution Property}$$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = 2, \quad y = -1$$

$$x + 2y = 0$$

$$2 + 2(-1) = 0 \quad \text{Substitution Property}$$

$$0 = 0$$

Example 3

Step 2: Simplify. Check if the ordered pair satisfies both equations.

$$\text{Given: } x = 2, \quad y = -1$$

$$x + 2y = 0$$

$$2 + 2(-1) = 0 \quad \text{Substitution Property}$$

$$0 = 0 \quad \text{Simplification}$$

\therefore the ordered pair $(2, -1)$ satisfies the equation $x + 2y = 0$.

Example 3

\therefore since the ordered pair $(2, -1)$ satisfies both the equations $x - 2y = 4$ and $x + 2y = 0$, it is a solution to the system $\begin{cases} x - 2y = 4 \\ x + 2y = 0 \end{cases}$.

What are the Kinds of Systems of Linear Equations?

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What are the Kinds of Systems of Linear Equations?

1. Consistent-dependent: a system of equations that can be rewritten as identical equations and have an infinite solution
2. Consistent-independent: a system of equations that can not be rewritten as contradicting equations nor identical equations; they stay different and have one solution
3. Inconsistent: a system of equations that can be rewritten as contradicting equations and has no solution

Example 1

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

$$\begin{cases} 2(2x + y) = 2(4) \\ 4x + 2y = 8 \end{cases}$$

Multiplication Property

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

$$\begin{cases} 2(2x + y) = 2(4) \\ 4x + 2y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 4x + 2y = 8 \\ 4x + 2y = 8 \end{cases}$$

Example 1

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

$$\begin{cases} 2(2x + y) = 2(4) \\ 4x + 2y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 4x + 2y = 8 \\ 4x + 2y = 8 \end{cases}$$

Distributive Property

\therefore since the equations are identical, it is a **Consistent-dependent** system and has infinitely many solutions.

Example 2

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

$$\begin{cases} 3(2x + 3y) = 3(4) \\ 6x + 9y = 8 \end{cases}$$

Multiplication Property

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

$$\begin{cases} 3(2x + 3y) = 3(4) \\ 6x + 9y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 6x + 9y = 12 \\ 6x + 9y = 8 \end{cases}$$

Example 2

$$\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$$

$$\begin{cases} 3(2x + 3y) = 3(4) \\ 6x + 9y = 8 \end{cases}$$

Multiplication Property

$$\begin{cases} 6x + 9y = 12 \\ 6x + 9y = 8 \end{cases}$$

Distributive Property

\therefore since the equations are contradicting, it is an **Inconsistent** system and has no solution.

Example 3

Determine whether the following system of linear equations is consistent-dependent, consistent-independent, or inconsistent. Then state the number of solution/s it has.

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

$$\begin{cases} 2(x - 2y) = 2(1) \\ 2x + 4y = 3 \end{cases}$$

Multiplication Property

Example 3

$$\begin{cases} x - 2y = 1 \\ 2x + 4y = 3 \end{cases}$$

$$\begin{cases} 2(x - 2y) = 2(1) \\ 2x + 4y = 3 \end{cases}$$

Multiplication Property

$$\begin{cases} 2x - 4y = 2 \\ 2x + 4y = 3 \end{cases}$$

Distributive Property

\therefore since the equations are not contradicting and not identical, it is a

Consistent-independent system and has one solution.

Thank you for watching.