Lesson 3.7.1: Proving Statements on Triangle Congruence

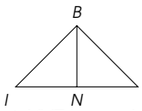
Isosceles Triangle: A triangle is isosceles if two of its sides are congruent. The congruent sides are its legs; the third side is the base; the angles opposite the congruent sides are the base angles; and the angle included by the legs is the vertex angle.
Equilateral Triangle: a triangle in which all three sides have the same length
Equiangular Triangle: a triangle which has all three interior angles congruent
Theorems on Isosceles Triangles
Isosceles Triangle Theorem: If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
Converse of Isosceles Triangle Theorem: If two angles of a triangle are congruent, then the sides opposite those angles are also congruent.
Corollaries

1. An equilateral triangle is also equiangular.
2. An equilateral triangle has three 60° angles.
3. An equiangular triangle is also equilateral.

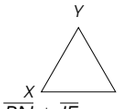
Practice Exercises 3.7.1

Complete the following proofs.

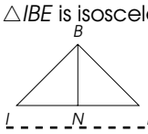
1. Given: $\overline{BI} \cong \overline{BE}$, N is the midpoint of $\overline{IE}$
   Prove: $\angle IBN \cong \angle EBN$



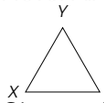
2. Given: $\triangle XYZ$ is equiangular
   Prove: $\triangle XYZ$ is equilateral



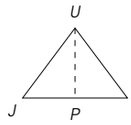
3. Given: $\overline{BN} \perp \overline{IE}$, N is the midpoint of $\overline{IE}$
   Prove: $\triangle IBE$ is isosceles



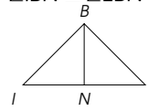
4. Given: $\triangle XYZ$ is equilateral
   Prove: All angles measure 60°



5. Given: $\angle J \cong \angle M$
   Prove: $\overline{UJ} \cong \overline{UM}$



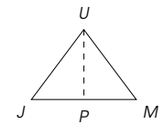
6. Given: $\overline{BN}$ bisects $\overline{IE}$, $\angle BNI \cong \angle BNE$
   Prove: $\angle IBN \cong \angle EBN$



Activity 3.7.1

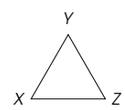
Complete the following proofs.

1. Given: $\triangle JUM$ with $\overline{UJ} \cong \overline{UM}$
   Prove: $\angle J \cong \angle M$
   Proof:



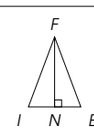
Statements	Reasons
1. $\overline{UJ} \cong \overline{UM}$	1.
2. Let P be the midpoint of $\overline{JM}$	2.
3. $\overline{JP} \cong \overline{MP}$	3.
4. Connect $\overline{UP}$	4.
5. $\overline{UP} \cong \overline{UP}$	5.
6. $\triangle JUP \cong \triangle MUP$	6.
7. $\angle J \cong \angle M$	7.

2. Given: $\triangle XYZ$ is equilateral
   Prove: $\triangle XYZ$ is equiangular
   Proof:



Statements	Reasons
1. $\triangle XYZ$ is equilateral	1.
2. $\overline{XY} \cong \overline{YZ}$	2.
3. $\angle X \cong \angle Z$	3.
4. $\overline{XY} \cong \overline{XZ}$	4.
5. $\angle Y \cong \angle Z$	5.
6. $\angle X \cong \angle Y \cong \angle Z$	6.
7. $\triangle XYZ$ is equiangular	7.

3. Given: $\overline{FN} \perp \overline{EI}$, $\angle I \cong \angle E$
   Prove: $\overline{FI} \cong \overline{FE}$
   Proof:



Statements	Reasons
1. $\overline{FN} \perp \overline{EI}$	1.
2. $\angle FNI = 90°$, $\angle FNE = 90°$	2.
3. $\triangle FNI$ and $\triangle FNE$ are right triangles	3.
4. $\overline{FN} \cong \overline{FN}$	4.
5. $\angle I \cong \angle E$	5.
6. $\triangle FNI \cong \triangle FNE$	6.
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Practice Exercises 3.7.1

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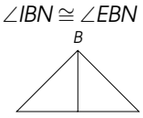

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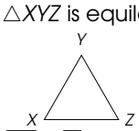
   Prove: $\triangle XYZ$ is equilateral



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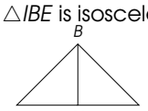

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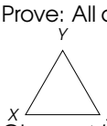

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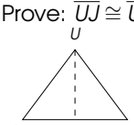

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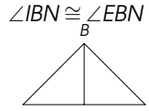

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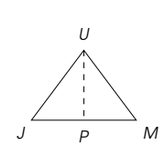

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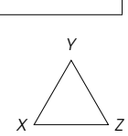

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