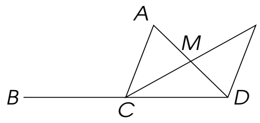
Lesson 4.3.1: Proving Inequalities in a Triangle

Angle-Side Relationship theorem: In a triangle, the side opposite the larger angle is the longer side.
Base Angles theorem: If two sides of a triangle are congruent, then the angles opposite them are congruent.
"The whole is greater than its parts" theorem

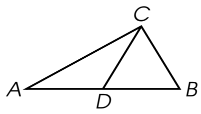
Practice Exercises 4.3.1

Complete the following proofs.

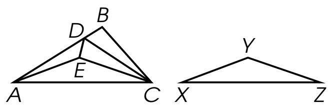
1. Given: M is the midpoint of $\overline{AD}$ and $\overline{CE}$
Prove: $m\angle ECB > m\angle E$



2. Given: D is the midpoint of $\overline{AB}$
$\overline{CD} \cong \overline{BC}$
Prove: $AD + CD > AC$



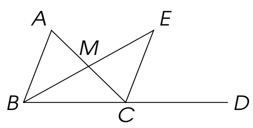
3. Given: $\overline{CD}$ bisects $\angle BCE$
$\overline{BC} \cong \overline{YZ}, \overline{AC} \cong \overline{XZ}$
$\triangle AEC \cong \triangle XYZ$
Prove: $AB > XY$



Activity 4.3.1

Complete the following proofs.

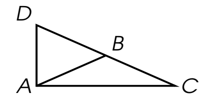
1. Given: M is the midpoint of $\overline{AC}$ and $\overline{BE}$
Prove: $m\angle ACD > m\angle A$



Proof:

Statements	Reasons
1. M is the midpoint of $\overline{AC}$ and $\overline{BE}$	1.
2. $\overline{AM} \cong \overline{CM}, \overline{BM} \cong \overline{EM}$	2.
3. $\angle AMB \cong \angle CME$	3.
4. $\triangle AMB \cong \triangle CME$	4.
5. $\angle A \cong \angle ECM$	5.
6. $m\angle A = m\angle ECM$	6.
7. $m\angle ACD = m\angle ECD + m\angle ECM$	7.
8. $m\angle ACD = m\angle ECD + m\angle A$	8.
9. $m\angle ACD > m\angle A$	9.

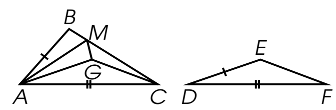
2. Given: B is the midpoint of $\overline{CD}$
$\overline{AB} \cong \overline{BC}$
Prove: $AB + BC > AC$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{BC}$, B is the midpoint of $\overline{CD}$	1.
2. $\overline{BC} \cong \overline{DB}$	2.
3. $\overline{AB} \cong \overline{DB}$	3.
4. $\angle D \cong \angle DAB$	4.
5. $m\angle DAC = m\angle DAB + m\angle BAC$	5.
6. $m\angle DAC = m\angle D + m\angle BAC$	6.
7. $m\angle DAC > m\angle D$	7.
8. $DC > AC$	8.
9. $DC = DB + BC$	9.
10. $DC = AB + BC$	10.
11. $AB + BC > AC$	11.

3. Given: $\overline{AM}$ bisects $\angle BAG$
$\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}$
$\triangle AGC \cong \triangle DEF$



Prove: $BC > EF$

Proof:

Statements	Reasons
1. $\triangle AGC \cong \triangle DEF$	1.
2. $\overline{AG} \cong \overline{DE}, \overline{GC} \cong \overline{EF}$	2.
3. $\angle BAM \cong \angle GAM$	3.
4. $\overline{AM} \cong \overline{AM}$	4.
5. $\triangle BAM \cong \triangle GAM$	5.
6. $\overline{BM} \cong \overline{GM}$	6.
7. $CM + GM > GC$	7.
8. $CM + BM > GC$	8.
9. $BC = BM + CM$	9.
10. $BC > GC$	10.
11. $BC > EF$	11.

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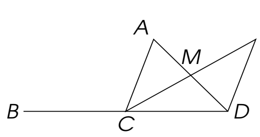

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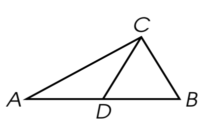

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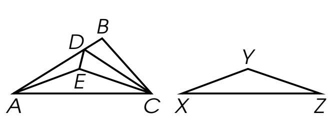

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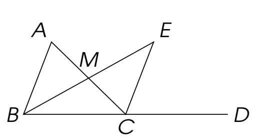

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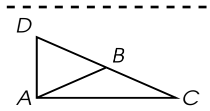

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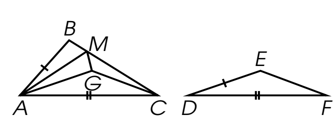

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