Lesson 3.6.1: Proving the Congruence of Triangles

Triangle Congruence Theorems

SAA (Side-Angle-Angle) Theorem: If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-induded side of another triangle, then the triangles are congruent.

LL Congruence Theorem: If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

LA (Leg-Acute angle) Congruence Theorem: If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.

HA (Hypotenuse-Acute angle) Congruence Theorem: If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and acute angle of another right triangle, then the triangles are congruent.

HL (Hypotenuse-Leg) Congruence Theorem: If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and leg of another right triangle, then the triangles are congruent.
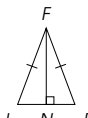
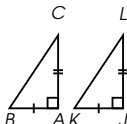### Practice Exercises 3.6.1
Complete the following proofs.

1. Given: $\overline{AE} \cong \overline{DE}$
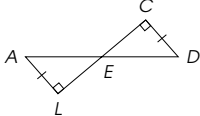   Prove: $\triangle ABE \cong \triangle DCE$
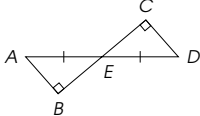
2. Given: $\overline{FN} \perp \overline{EI}$, $\overline{FI} \cong \overline{FE}$
   Prove: $\triangle FNI \cong \triangle FNE$

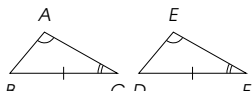3. Given: $\overline{AB} \cong \overline{JK}$, $\overline{AC} \cong \overline{JL}$
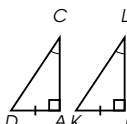   Prove: $\triangle ABC \cong \triangle JKL$

4. Given: $\overline{AL} \cong \overline{DC}$
   Prove: $\triangle ALE \cong \triangle DCE$

5. Given: $\angle A \cong \angle E$, $\angle G \cong \angle F$, $\overline{BG} \cong \overline{DF}$
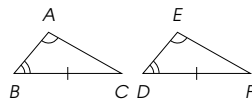   Prove: $\triangle ABG \cong \triangle DEF$

6. Given: $\overline{CD} \cong \overline{LK}$, $\overline{AD} \cong \overline{JK}$
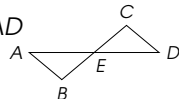   Prove: $\triangle ACD \cong \triangle JLK$

Activity 3.6.1

Complete the following proofs.

1. Given: $\angle A \cong \angle E$, $\angle B \cong \angle D$, $\overline{BC} \cong \overline{DF}$
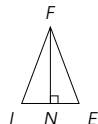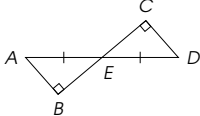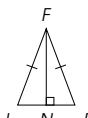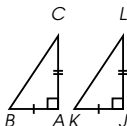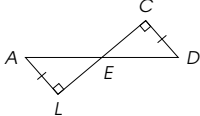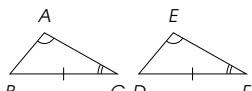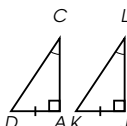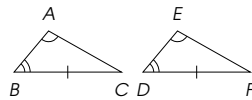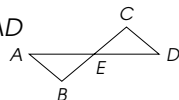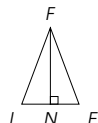   Prove: $\triangle ABC \cong \triangle EDF$
   Proof:

Statements	Reasons
1. $\angle A + \angle B + \angle C = 180°$ $\angle D + \angle E + \angle F = 180°$	1.
2. $\angle A + \angle B + \angle C = \angle E + \angle D + \angle F$	2.
3. $\angle A = \angle E$, $\angle B = \angle D$	3.
4. $\angle C = \angle F$	4.
5. $\overline{BC} \cong \overline{DF}$	5.
6. $\triangle ABC \cong \triangle EDF$	6.

2. Given: E is the midpoint of segments AD and BC.
   Prove: $\triangle AEB \cong \triangle DEC$
   Proof:

Statements	Reasons
1. E is the midpoint of segments AD and BC.	1.
2. $\overline{AE} \cong \overline{DE}$	2.
3. $\angle AEB \cong \angle DEC$	3.
4. $\overline{BE} \cong \overline{CE}$	4.
5. $\triangle AEB \cong \triangle DEC$	5.

3. Given: $\overline{FN} \perp \overline{EI}$, $\overline{FN}$ bisects $\angle EFI$
   Prove: $\triangle FNI \cong \triangle FNE$
   Proof:

Statements	Reasons
1. $\overline{FN} \perp \overline{EI}$	1.
2. $\angle FNI = 90°$, $\angle FNE = 90°$	2.
3. $\angle FNI \cong \angle FNE$	3.
4. $\overline{FN} \cong \overline{FN}$	4.
5. $\overline{FN}$ bisects $\angle EFI$	5.
6. $\angle EFN \cong \angle IFN$	6.
7. $\triangle FNI \cong \triangle FNE$	7.