Lesson 2.8.2: Direct and Indirect Proofs

Proof: a form of logical reasoning in which each statement is organized and backed up by given information, definitions, axioms, postulates, or theorems

Direct Proof:

- a sequence of statements which are either givens or deductions from previous statements, and whose last statement is the conclusion to be proved
- can be done in three ways: paragraph form, flowchart form, and two column form

How to Write a Direct Proof?

1. Take the original conditional statement.
2. Assume that the hypothesis is true, and show that the conclusion is true.

How to Write a Two-Column Proof?

1. Write all the series of statements in the first column of the table in a logical order starting with the given statements and ends it with the statement that needs to be proven.
2. In a step-by-step manner, write all the reasons for each statement.

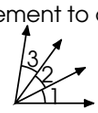
Indirect Proof:

- a type of proof where the opposite of the statement to be proven is assumed true until the assumption leads to contradiction
- a method of reasoning usually written in paragraph form

Practice Exercises 2.8.2

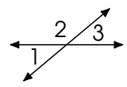
A. Provide the reason for each statement to complete the proof.

1. Given: $\angle 1 \cong \angle 3$
Prove: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$



2. Given: $LV = EO$
Prove: $LO = EV$

3. Given: $\angle 1$ and $\angle 2$ form a linear pair
$\angle 3$ and $\angle 2$ form a linear pair
Prove: $\angle 1 \cong \angle 3$



B. Use indirect proof to prove the following.

1. If $x = 4$, then $2x - 5 \neq 2$.
2. If $x = -2$, then $3x + 2 \neq -3$.
3. If $x = -1$, then $2x + 3 \neq 2$.

Activity 2.8.2

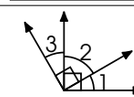
A. Provide the reason for each statement to complete the proof.

1. Given: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$
Prove: $m\angle 1 = m\angle 3$

Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	1. _____
2. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 2 - m\angle 2 + m\angle 3$	2. _____
3. $m\angle 1 = m\angle 3$	3. _____

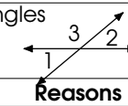
2. Given: $m\angle 1 + m\angle 2 = 90°$
$m\angle 3 + m\angle 2 = 90°$
Prove: $m\angle 1 = m\angle 3$



Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 90°$	1. _____
2. $m\angle 3 + m\angle 2 = 90°$	2. _____
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. _____
4. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 3 + m\angle 2 - m\angle 2$	4. _____
5. $m\angle 1 = m\angle 3$	5. _____

3. Given: $\angle 1$ and $\angle 2$ are vertical angles
Prove: $\angle 1 \cong \angle 2$



Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are vertical angles	1. _____
2. $\angle 1$ and $\angle 3$ form a linear pair	2. _____
3. $\angle 1$ and $\angle 3$ are supplementary	3. _____
4. $m\angle 1 + m\angle 3 = 180°$	4. _____
5. $\angle 2$ and $\angle 3$ form a linear pair	5. _____
6. $\angle 2$ and $\angle 3$ are supplementary	6. _____
7. $m\angle 2 + m\angle 3 = 180°$	7. _____
8. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	8. _____
9. $m\angle 1 = m\angle 2$	9. _____
10. $\angle 1 \cong \angle 2$	10. _____

B. Use indirect proof to prove the following.

1. If $x = 2$, then $3x - 5 \neq 10$.
2. If $x = 3$, then $4x - 4 \neq 12$.

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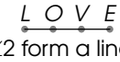

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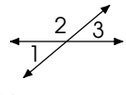

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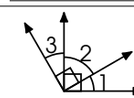

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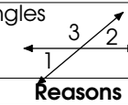

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