Lesson 3.6.1: Proving the Congruence of Triangles

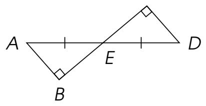
Triangle Congruence Theorems

SAA (Side-Angle-Angle) Theorem: If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-induded side of another triangle, then the triangles are congruent.
LL Congruence Theorem: If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.
LA (Leg-Acute angle) Congruence Theorem: If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.
HA (Hypotenuse-Acute angle) Congruence Theorem: If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and acute angle of another right triangle, then the triangles are congruent.
HL (Hypotenuse-Leg) Congruence Theorem: If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and leg of another right triangle, then the triangles are congruent.

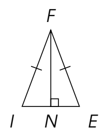
Practice Exercises 3.6.1

Complete the following proofs.

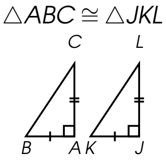
1. Given: $\overline{AE} \cong \overline{DE}$
   Prove: $\triangle ABE \cong \triangle DCE$



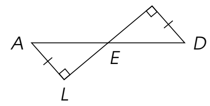
2. Given: $\overline{FN} \perp \overline{EI}$, $\overline{FI} \cong \overline{FE}$
   Prove: $\triangle FNI \cong \triangle FNE$



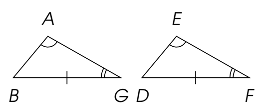
3. Given: $\overline{AB} \cong \overline{JK}$, $\overline{AC} \cong \overline{JL}$
   Prove: $\triangle ABC \cong \triangle JKL$



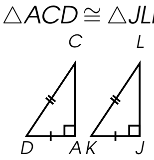
4. Given: $\overline{AL} \cong \overline{DC}$
   Prove: $\triangle ALE \cong \triangle DCE$



5. Given: $\angle A \cong \angle E$, $\angle G \cong \angle F$, $\overline{BG} \cong \overline{DF}$
   Prove: $\triangle ABG \cong \triangle DEF$



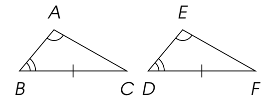
6. Given: $\overline{CD} \cong \overline{LK}$, $\overline{AD} \cong \overline{JK}$
   Prove: $\triangle ACD \cong \triangle JLK$



Activity 3.6.1

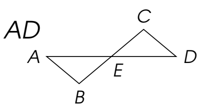
Complete the following proofs.

1. Given: $\angle A \cong \angle E$, $\angle B \cong \angle D$, $\overline{BC} \cong \overline{DF}$
   Prove: $\triangle ABC \cong \triangle EDF$
   Proof:



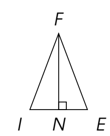
Statements	Reasons
1. $\angle A + \angle B + \angle C = 180°$ $\angle D + \angle E + \angle F = 180°$	1.
2. $\angle A + \angle B + \angle C =$ $\angle E + \angle D + \angle F$	2.
3. $\angle A = \angle E$, $\angle B = \angle D$	3.
4. $\angle C = \angle F$	4.
5. $\overline{BC} \cong \overline{DF}$	5.
6. $\triangle ABC \cong \triangle EDF$	6.

2. Given: E is the midpoint of segments AD and BC.
   Prove: $\triangle AEB \cong \triangle DEC$
   Proof:



Statements	Reasons
1. E is the midpoint of segments AD and BC.	1.
2. $\overline{AE} \cong \overline{DE}$	2.
3. $\angle AEB \cong \angle DEC$	3.
4. $\overline{BE} \cong \overline{CE}$	4.
5. $\triangle AEB \cong \triangle DEC$	5.

3. Given: $\overline{FN} \perp \overline{EI}$, $\overline{FN}$ bisects $\angle EFI$
   Prove: $\triangle FNI \cong \triangle FNE$
   Proof:



Statements	Reasons
1. $\overline{FN} \perp \overline{EI}$	1.
2. $\angle FNI = 90°$, $\angle FNE = 90°$	2.
3. $\angle FNI \cong \angle FNE$	3.
4. $\overline{FN} \cong \overline{FN}$	4.
5. $\overline{FN}$ bisects $\angle EFI$	5.
6. $\angle EFN \cong \angle IFN$	6.
7. $\triangle FNI \cong \triangle FNE$	7.

Lesson 3.6.1: Proving the Congruence of Triangles

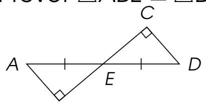
Triangle Congruence Theorems

SAA (Side-Angle-Angle) Theorem: If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-induded side of another triangle, then the triangles are congruent.
LL Congruence Theorem: If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.
LA (Leg-Acute angle) Congruence Theorem: If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.
HA (Hypotenuse-Acute angle) Congruence Theorem: If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and acute angle of another right triangle, then the triangles are congruent.
HL (Hypotenuse-Leg) Congruence Theorem: If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and leg of another right triangle, then the triangles are congruent.

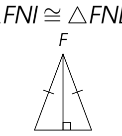
Practice Exercises 3.6.1

Complete the following proofs.

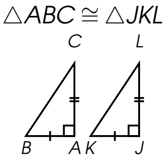
1. Given: $\overline{AE} \cong \overline{DE}$
   Prove: $\triangle ABE \cong \triangle DCE$



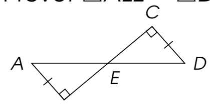
2. Given: $\overline{FN} \perp \overline{EI}$, $\overline{FI} \cong \overline{FE}$
   Prove: $\triangle FNI \cong \triangle FNE$



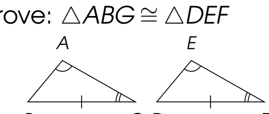
3. Given: $\overline{AB} \cong \overline{JK}$, $\overline{AC} \cong \overline{JL}$
   Prove: $\triangle ABC \cong \triangle JKL$



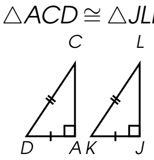
4. Given: $\overline{AL} \cong \overline{DC}$
   Prove: $\triangle ALE \cong \triangle DCE$



5. Given: $\angle A \cong \angle E$, $\angle G \cong \angle F$, $\overline{BG} \cong \overline{DF}$
   Prove: $\triangle ABG \cong \triangle DEF$



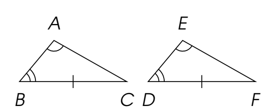
6. Given: $\overline{CD} \cong \overline{LK}$, $\overline{AD} \cong \overline{JK}$
   Prove: $\triangle ACD \cong \triangle JLK$



Activity 3.6.1

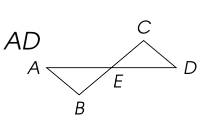
Complete the following proofs.

1. Given: $\angle A \cong \angle E$, $\angle B \cong \angle D$, $\overline{BC} \cong \overline{DF}$
   Prove: $\triangle ABC \cong \triangle EDF$
   Proof:



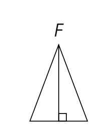
Statements	Reasons
1. $\angle A + \angle B + \angle C = 180°$ $\angle D + \angle E + \angle F = 180°$	1.
2. $\angle A + \angle B + \angle C =$ $\angle E + \angle D + \angle F$	2.
3. $\angle A = \angle E$, $\angle B = \angle D$	3.
4. $\angle C = \angle F$	4.
5. $\overline{BC} \cong \overline{DF}$	5.
6. $\triangle ABC \cong \triangle EDF$	6.

2. Given: E is the midpoint of segments AD and BC.
   Prove: $\triangle AEB \cong \triangle DEC$
   Proof:



Statements	Reasons
1. E is the midpoint of segments AD and BC.	1.
2. $\overline{AE} \cong \overline{DE}$	2.
3. $\angle AEB \cong \angle DEC$	3.
4. $\overline{BE} \cong \overline{CE}$	4.
5. $\triangle AEB \cong \triangle DEC$	5.

3. Given: $\overline{FN} \perp \overline{EI}$, $\overline{FN}$ bisects $\angle EFI$
   Prove: $\triangle FNI \cong \triangle FNE$
   Proof:



Statements	Reasons
1. $\overline{FN} \perp \overline{EI}$	1.
2. $\angle FNI = 90°$, $\angle FNE = 90°$	2.
3. $\angle FNI \cong \angle FNE$	3.
4. $\overline{FN} \cong \overline{FN}$	4.
5. $\overline{FN}$ bisects $\angle EFI$	5.
6. $\angle EFN \cong \angle IFN$	6.
7. $\triangle FNI \cong \triangle FNE$	7.