Conditional Probability

For any two dependent events A and B with $P(B) > 0$, the conditional probability of A given that B has occurred is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

For any two independent events A and B with $P(B) > 0$, the conditional probability of A given that B has occurred is defined by:

$$P(A|B) = P(A)$$

Practice Exercises

Solve each problem completely.

1. In a school, the probability that a student takes environmental science and geography is 0.25. The probability that a student takes environmental science is 0.72. What is probability that a student takes geography given that the student is taking environmental science?

2. In certain population, the probability that a men lives to at least seventy-five years long is 0.75. The probability that he will live to at least eighty-five years is 0.65. If a man is seventy-five years old, what is the probability that he will survive to eighty-five years?

3. In a bag are economics and math books. Two books are chosen at random without replacement. The probability of selecting an economics book and then a math book is 0.42. The probability of selecting an economics book on the first draw is 0.62. What is probability of selecting a math book on the second draw given that the first book was economics?

4. In a box are a carton of dairy milk and Boonville chocolates. Two items are chosen at random from the box, without replacement. The probability of selecting dairy milk and then a Boonville chocolate is 0.47. The probability of selecting dairy milk on the first draw is 0.71. What is probability of selecting a Boonville chocolate on the second draw given that the first chocolate drawn was dairy milk?

5. You roll two dice. The first die shows a TWO and the other die rolls under the table and you cannot see it. Now, what is the probability that both die show TWO?

Problem Set

Solve each problem completely.

1. Andrea is a very good student. The probability that she studies and passes her mathematics test is $\frac{17}{20}$. If the probability that Andrea studies is $\frac{15}{16}$, find the probability that Andrea passes her mathematics test, given that she has studied.

2. The probability that Janice smokes is $\frac{3}{10}$. The probability that she smokes and develops lung cancer is $\frac{4}{15}$. Find the probability that Janice develops lung cancer, given that she smokes.

3. The probability that Sue will go to Mexico in the winter and to France in the summer is 0.40. The probability that she will go to Mexico in the winter is 0.60. Find the probability that she will go to France this summer, given that she just returned from her winter vacation in Mexico.

4. A peso coin and a five-peso coin are tossed. Find the probability that the five-peso coin shows heads, given that the peso coin shows heads.

5. A box contains three blue marbles, five red marbles, and four white marbles. If two marbles are drawn at random, find:
   a. P(not white | blue)
   b. P(not red | white)