Graph of a Polynomial Function

End Behavior: the behavior of the graph of a function at the far left or the far right

Turning Point: a point where the graph of a function changes direction from increasing to decreasing or vice versa

The graph of a polynomial function of degree n has, at most, $n-1$ turning points.

Leading Coefficient Test: as x increases or decreases without bound, the graph of the polynomial function

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$$

eventually rises or falls.

Case	Leading Coefficient	Degree	End Behavior	
			Left-hand	Right-hand
1	Positive	Odd	Falling	Rising
2	Negative	Odd	Rising	Falling
3	Positive	Even	Rising	Rising
4	Negative	Even	Falling	Falling

Steps in Graphing Polynomial Functions

1. Write the function in factored form.
2. Find the end behavior of the graph using the Leading Coefficient test.
3. Find the zeros of the polynomial function and their multiplicity.

Multiplicity of Zero (c)	Graph	Sign of $P(x)$
Even	touches the x-axis at c	does not change from one side to the other side of c
Odd	crosses the x-axis at c	changes from one side to the other side of c

4. Make a table of values of x and y.
5. Plot the points and connect them with a smooth continuous curve.
6. Make sure the graph follows the end behavior.

Practice Exercises

Sketch the graph of each polynomial function.

1. $f(x) = (x-2)(x+1)$
2. $f(x) = (x-2)(x+1)(x+3)$
3. $f(x) = (x-2)^2(x+2)^2$
4. $f(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$
5. $f(x) = -x^3 - 9x^2 - 27x - 27$

Problem Set

Sketch the graph of each polynomial function.

1. $f(x) = (x-2)(x-1)(x-3)$
2. $f(x) = x(x+1)^2$
3. $f(x) = x(x-2)(x+1)(x+3)$
4. $f(x) = (x+2)(x-1)(x-3)^2$
5. $f(x) = -x^5 - 6x^4 - 4x^3 - 8x^2$

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