The Equation of a Circle

Standard Form

The equation of a circle centered at the origin (0,0) and radius \mathbf{r} is given by

$$x^2 + y^2 = r^2.$$

The equation of a circle centered at (h, k) having a radius of length \mathbf{r}

$$(x-h)^2 + (y-k)^2 = r^2$$
.

General Form

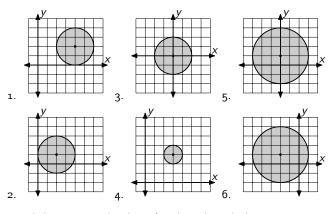
The general equation of a circle is

$$x^2 + y^2 + Dx + Ey + F = 0,$$

where D, E, and F are real numbers, D = -2h, E = -2k, $F = h^2 + k^2 - r^2$.

Practice Exercises

A. Give the radius and coordinates of the center of the circle. Then write the equation in standard form.



B. Find the center and radius of each circle with the given equation.

1.
$$2x^2 + 2y^2 = 32$$

3.
$$(x-5)^2 + y^2 = 169$$

2.
$$x^2 + (y+5)^2 = 100$$

4.
$$(x+2)^2 + (y-4)^2 = 36$$

C. Write an equation of circle ${\it C}$ based on the given information.

- 1. Center at (5, 4) and touching the x-axis
- 2. Center at (10, 4) and passing through (2, 2)
- 3. Center at (3, 8) and passing through the origin
- 4. Center at (2, 5) and tangent to the y-axis
- 5. A circle with area 49 cm² and center at (2, 5)

The Equation of a Circle

Standard Form

The equation of a circle centered at the origin (o,o) and radius \boldsymbol{r} is

$$x^2 + y^2 = r^2.$$

The equation of a circle centered at (h, k) having a radius of length \mathbf{r} is

$$(x-h)^2 + (y-k)^2 = r^2$$
.

General Form

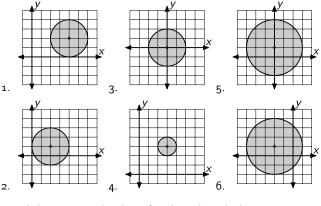
The general equation of a circle is

$$x^2 + y^2 + Dx + Ey + F = 0,$$

where D, E, and F are real numbers, D = -2h, E = -2k, $F = h^2 + k^2 - r^2$

Practice Exercises

A. Give the radius and coordinates of the center of the circle. Then write the equation in standard form.



B. Find the center and radius of each circle with the given equation.

1.
$$2x^2 + 2y^2 = 32$$

3.
$$(x-5)^2 + y^2 = 169$$

2.
$$x^2 + (y+5)^2 = 100$$

4.
$$(x+2)^2 + (y-4)^2 = 36$$

C. Write an equation of circle C based on the given information.

- 1. Center at (5, 4) and touching the x-axis
- 2. Center at (10, 4) and passing through (2, 2)
- 3. Center at (3, 8) and passing through the origin
- 4. Center at (2, 5) and tangent to the y-axis 5. A circle with area 49 cm² and center at (2, 5)

D. Write each equation of a circle in general form

1.
$$(x-2)^2 + (y-4)^2 = 36$$
 3. $(x-6)^2 + (y-1)^2 = 81$

2.
$$(x+4)^2 + (y-9)^2 = 144$$
 4. $x^2 + (y-5)^2 = 36$

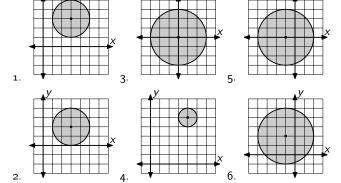
E. Transform each equation to standard form, then give the coordinates of the center and the radius.

1.
$$x^2 + y^2 - 2x - 8y - 47 = 0$$
 3. $x^2 + y^2 + 10x + 4y - 3 = 0$

2.
$$x^2 + y^2 + 4x - 4y - 28 = 0$$
 4. $x^2 + y^2 + 8y - 84 = 0$

Problem Set

A. Give the radius and coordinates of the center of the circle. Then write the equation in standard form.



B. Find the center and radius of each circle with the given equation.

1.
$$3x^2 + 3y^2 = 12$$

3.
$$(x-3)^2+y^2=144$$

2.
$$x^2 + (y+4)^2 = 81$$

4.
$$(x+4)^2 + (y-3)^2 = 64$$

C. Write an equation of circle C based on the given information.

- 1. Center at (3, 2) and touching the x-axis
- 2. Center at (7, 5) and passing through (3, 4)
- 3. Center at (4, 7) and passing through the origin
- 4. Center at (3, 7) and tangent to the y-axis
- 5. A circle with area 64 cm^2 and center at (3, 6)

D. Write each equation of a circle in general form

1.
$$(x-7)^2 + y^2 = 64$$

3.
$$(x+2)^2 + y^2 = 100$$

2.
$$x^2 + (y+2)^2 = 49$$

4.
$$(x-5)^2+(y-5)^2=27$$

E. Transform each equation to standard form, then give the coordinates of the center and the radius

1.
$$x^2 + y^2 - 8x + 2y - 32 = 0$$
 3. $x^2 + y^2 - 6x - 2y - 15 = 0$

2.
$$x^2 + y^2 - 6x - 10y + 18 = 0$$
 4. $x^2 + y^2 - 4x + 6y + 4 = 0$

D. Write each equation of a circle in general form.

1.
$$(x-2)^2 + (y-4)^2 = 36$$
 3. $(x-6)^2 + (y-1)^2 = 81$

3.
$$(x-6)^2 + (y-1)^2 = 8$$

2.
$$(x+4)^2 + (y-9)^2 = 144$$
 4. $x^2 + (y-5)^2 = 36$

4.
$$x^2 + (y-5)^2 = 36$$

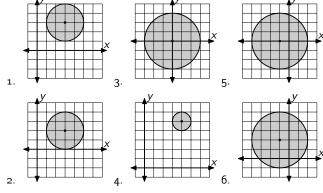
E. Transform each equation to standard form, then give the coordinates of the center and the radius.

1.
$$x^2 + y^2 - 2x - 8y - 47 = 0$$
 3. $x^2 + y^2 + 10x + 4y - 3 = 0$

2.
$$x^2 + y^2 + 4x - 4y - 28 = 0$$
 4. $x^2 + y^2 + 8y - 84 = 0$

Problem Set

A. Give the radius and coordinates of the center of the circle. Then write the equation in standard form.



B. Find the center and radius of each circle with the given equation.

1.
$$3x^2 + 3y^2 = 12$$

3.
$$(x-3)^2 + y^2 = 144$$

2.
$$x^2 + (y+4)^2 = 81$$

4.
$$(x+4)^2 + (y-3)^2 = 64$$

C. Write an equation of circle ${\it C}$ based on the given information.

- 1. Center at (3, 2) and touching the x-axis
- 2. Center at (7, 5) and passing through (3, 4)
- 3. Center at (4, 7) and passing through the origin
- 4. Center at (3, 7) and tangent to the y-axis 5. A circle with area 64 cm² and center at (3, 6)
- D. Write each equation of a circle in general form.

1.
$$(x-7)^2 + y^2 = 64$$

3.
$$(x+2)^2 + y^2 = 100$$

2.
$$x^2 + (y+2)^2 = 40$$

4.
$$(x-5)^2 + (y-5)^2 = 27$$

E. Transform each equation to standard form, then give the coordinates of the center and the radius.

1.
$$x^2 + y^2 - 8x + 2y - 32 = 0$$
 3. $x^2 + y^2 - 6x - 2y - 15 = 0$

2.
$$x^2 + y^2 - 6x - 10y + 18 = 0$$
 4. $x^2 + y^2 - 4x + 6y + 4 = 0$