

Practice Exercises

A. Write *Yes* if the sequence is harmonic. Otherwise, write *Not*.

1. $\frac{1}{5}, \frac{1}{8}, \frac{1}{11}$

2. $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}$

3. $\frac{1}{2}, \frac{3}{8}, \frac{3}{10}$

4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$

5. $-\frac{1}{2}, \frac{1}{2}, \frac{1}{6}$

B. Find the specified term of each harmonic sequence

$$1. \quad \frac{4}{3}, 2, 4, \dots a_7$$

$$2. \quad \frac{1}{3}, \frac{3}{10}, \frac{3}{11}, \dots a_9$$

$$3. \quad a_1 = \frac{1}{6}, a_2 = \frac{1}{7}, a_n = \frac{1}{25}, n = ?$$

$$4. \quad a_1 = \frac{1}{15}, a_{10} = \frac{1}{27}, a_7 = ?$$

$$5. \quad a_8 = 4, a_{14} = \frac{4}{19}, a_{13} = ?$$

C. Find the harmonic mean between the two given numbers.

1. 40 and 60

2. 80 and 120

3. -30 and 60

4. $-\frac{3}{7}$ and $\frac{5}{6}$

Problem Set

A. Write *Yes* if the sequence is harmonic. Otherwise, write *Not*.

1. $\frac{1}{3}, \frac{2}{3}, 1$

2. $\frac{1}{4}, \frac{1}{7}, \frac{1}{9}$

3. $\frac{1}{8}, \frac{3}{8}, \frac{5}{11}$

4. $\frac{4}{7}, \frac{1}{2}, \frac{3}{2}$

5. $\frac{1}{5}, \frac{6}{5}, \frac{11}{5}$

B. Find the specified term of each harmonic sequence

$$1. \quad \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots, a_7$$

$$2. \quad \frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \dots, a_{10}$$

$$3. \quad \frac{1}{4}, \frac{1}{11}, \frac{1}{18}, \dots, a_9$$

$$4. \quad -\frac{1}{10}, -\frac{1}{3}, \frac{1}{4}, \dots, a_{14}$$

$$5. \quad 1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \dots, a_{10}$$

C. Find the harmonic mean between the two given numbers.

$$1. \quad 20 \text{ and } 4$$

$$2. \quad 10 \text{ and } 5$$

3. 15 and 45

4. 9 and 25

Problem Set

A.

1. $3, \frac{3}{2}, 1$

$$d = \frac{3}{2} - 3 = -\frac{3}{2}$$

$$d = 1 - \frac{3}{2} = -\frac{1}{2}$$

No

2. $4, 7, 9$

$$d = 7 - 4 = 3$$

$$d = 9 - 7 = 2$$

No

3. $8, \frac{8}{3}, \frac{11}{5}$

$$d = \frac{8}{3} - 8 = -\frac{16}{3}$$

$$d = \frac{11}{5} - \frac{8}{3} = -\frac{7}{15}$$

No

4. $\frac{7}{4}, 2, \frac{2}{3}$

$$d = 2 - \frac{7}{4} = \frac{1}{4}$$

$$d = \frac{2}{3} - 2 = -\frac{4}{3}$$

No

5. $5, \frac{5}{6}, \frac{5}{11}$

$$d = \frac{5}{6} - 5 = -\frac{25}{6}$$

$$d = \frac{5}{11} - \frac{5}{6} = -\frac{25}{66}$$

No

B.

1. $2, 4, 6, \dots a_7$

$$d = 4 - 2 = 2$$

$$a_n = a_1 + (n - 1)d$$

$$a_7 = 2 + (7 - 1)(2)$$

$$a_7 = 2 + (6)(2)$$

$$a_7 = 14$$

$$a_7 = \frac{1}{14}$$

2. $5, 10, 15, \dots a_{10}$

$$d = 10 - 5 = 5$$

$$a_n = a_1 + (n - 1)d$$

$$a_{10} = 5 + (10 - 1)(5)$$

$$a_{10} = 5 + (9)(5)$$

$$a_{10} = 50$$

$$a_{10} = \frac{1}{50}$$

$$a_{10} = \frac{50}{1}$$

3. $4, 11, 18, \dots a_9$

$$d = 11 - 4 = 7$$

$$a_n = a_1 + (n - 1)d$$

$$a_9 = 4 + (9 - 1)(7)$$

$$a_9 = 4 + (8)(7)$$

$$a_9 = 60$$

$$a_9 = \frac{1}{60}$$

$$a_9 = \frac{60}{1}$$

4. $-10, -3, 4, \dots a_{14}$

$$d = -3 - (-10) = 7$$

$$a_n = a_1 + (n - 1)d$$

$$a_{14} = -10 + (14 - 1)(7)$$

$$a_{14} = -10 + (13)(7)$$

$$a_{14} = \frac{81}{1}$$

$$a_{14} = \frac{1}{81}$$

$$5. 1, \frac{3}{2}, 2, \frac{5}{2}, \dots, a_{10}$$

$$d = \frac{3}{2} - 1 = \frac{1}{2}$$

$$a_n = a_1 + (n - 1)d$$

$$a_{10} = 1 + (10 - 1) \left(\frac{1}{2} \right)$$

$$a_{10} = 1 + (9) \left(\frac{1}{2} \right)$$

$$a_{10} = \frac{11}{2}$$

$$a_{10} = \frac{2}{11}$$

C.

$$1. \frac{1}{20} \text{ and } \frac{1}{4}$$

$$A.M. = \frac{\frac{1}{20} + \frac{1}{4}}{2}$$

$$A.M. = \frac{\frac{1+5}{20}}{2}$$

$$A.M. = \frac{\frac{6}{20}}{2}$$

$$A.M. = \left(\frac{\frac{6}{20}}{2} \right) \left(\frac{1}{2} \right)$$

$$A.M. = \frac{3}{20}$$

$$H.M. = \frac{20}{3}$$

$$A.M. = \left(\frac{3}{10}\right) \left(\frac{1}{2}\right)$$

$$A.M. = \frac{3}{20}$$

$$H.M. = \frac{20}{3}$$

2. $\frac{1}{10}$ and $\frac{1}{5}$

$$A.M. = \frac{\frac{1}{10} + \frac{1}{5}}{2}$$

$$A.M. = \frac{10}{3}$$

$$A.M. = \frac{10}{2}$$

3. $\frac{1}{15}$ and $\frac{1}{45}$

$$A.M. = \frac{\frac{1}{15} + \frac{1}{45}}{2}$$

$$A.M. = \frac{45}{2}$$

$$A.M. = \frac{\frac{4}{45}}{2} \left(\frac{1}{2} \right)$$

$$A.M. = \frac{2}{45}$$

$$H.M. = \frac{45}{2}$$

4. $\frac{1}{9}$ and $\frac{1}{25}$

$$A.M. = \frac{\frac{1}{9} + \frac{1}{25}}{2}$$

$$A.M. = \frac{\frac{25+9}{225}}{2}$$

$$A.M. = \frac{2}{225}$$

$$A.M. = \left(\frac{\frac{34}{225}}{2} \right) \left(\frac{1}{2} \right)$$

$$A.M. = \frac{17}{225}$$

$$H.M. = \frac{225}{17}$$