Zeros of a Polynomial Function

If $(x-c)$ is a factor of a polynomial $P(x)$, then c is called a **zero of the polynomial function**.

Multiple Zeros of a Polynomial: If a polynomial $P(x)$ has $x-c$ occurring as a factor exactly k times, then c is a **zero of multiplicity** k of the polynomial function $y=P(x)$.

Fundamental Theorem of Algebra: A polynomial function $P(x)$ of degree n has exactly n complex zeros.

Integral Zero Theorem: If an integer is a zero of a given integral polynomial function, then it is a divisor of the constant term of the polynomial.

Practice Exercises

A. State the degree of the polynomial function, then find the zeros and their multiplicity.

1. $f(x)=x^5(x-3)^2(x+1)^3$
2. $f(x)=(2x+1)^2(x-2)^3$
3. $f(x)=(x-3)^2(x-5)(x+5)$
4. $f(x)=(5x-3)^3(x-1)(3x+4)^2$
5. $f(x)=x(x-7)^2(6x+5)(4x-3)^4$

B. Find all the zeros of each polynomial function.

1. $f(x)=x^3-3x^2-25x+75$
2. $f(x)=x^3-7x^2-5x+75$
3. $f(x)=-3x^3+12x^2+15x$
4. $f(x)=x^3-x^2-x+1$
5. $f(x)=3x^3+5x^2-10x-16$

C. Find a polynomial function with the following sets of zeros.

1. $\left\{-4,2,\frac{2}{3}\right\}$
2. $\left\{1,-1,-\frac{2}{5},\frac{1}{3}\right\}$
3. $\{4,2+i,2-i\}$
4. $\{7,3+i,3-i\}$

Problem Set

A. Find the roots of each polynomial function. Indicate the multiplicity of each root.

1. $f(x)=(x+3)^3(x-1)^5$
2. $h(x)=x^2(x-5)^3(x+6)^4$
3. $P(x)=x^4(x-5)$
4. $F(x)=(x+3)^4(x-7)$
5. $P(x)=(x+3)^4(x-3)^4$

B. Find the zeros of each function.

1. $P(x)=x^3-3x-2$
2. $P(x)=x^4-13x^2+36$
3. $P(x)=x^4-3x^3-53x^2-9x$
4. $P(x)=x^3+3x^2-4x-12$
5. $P(x)=x^3+7x^2+2x-40$

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