8

Mathematics

Quarter 1 – Module 1:
Factoring
(Part 1)

ALTERNATIVE DELIVERY MODE

ADM

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Alternative Delivery Mode
Quarter 1 – Module 1: Factoring (Part 1)
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Secretary: Leonor Magtolis Briones
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Development Team of the Module

Writers: Rosalia A. Vicente, Neylinda M. Moldogo, Melvic Borja

Editors: Rex Arcadio R. San Diego II (H.C.)

Reviewers: Joel Feliciano, Ma. Nimfa R. Gabertan

Illustrator: Leilanie S. Yutiampo

Layout Artist: Brian Spencer B. Reyes, Heidee F. Ferrer

Management Team: JENILYN ROSE B. CORPUZ, CESO VI, SDS
FREDIE V. AVENDANO, ASDS
JUAN C. OBIERNA, Chief, CID
JOEL FELICIANO, EPS, Mathematics
HEIDEE F. FERRER, EPS, LRMS

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Department of Education **– National Capital Region**
Office Address: Nueva Ecija Street, Bago Bantay, Quezon City
Telefax: 352-68-07
E-mail Address:sdoqcactioncenter@gmail.com/sdo.quezoncity@deped.gov.ph

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(Part 1)

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Introductory Message

For the facilitator:

Welcome to the **Grade 8 Mathematics** Alternative Delivery Mode (ADM) **Module on Rational Algebraic Expressions.**

This module was collaboratively designed, developed and reviewed by educators from both public and private institutions to assist you, the teacher or facilitator in helping learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage students into guided and independent learning activities at their own pace. Furthermore, this also aims to help learners acquire the needed 21st Century Skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist learners as they do the tasks included in the module.

For the learner:

Welcome to the **Mathematics 8** Alternative Delivery Mode (ADM) **Module on Factoring.**

The hand is one of the most symbolized parts of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish many things. Hence, the hand in this learning resource signifies that you as a learner are capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know

This will give you an idea of the skills or competencies you are expected to learn in the module.

What I Know

This part includes an activity that aims to check what you already know about the lesson you are to take. If you get all the answers correctly (100%), you may skip this module.

What's In

This is a brief drill or review to help you link the current lesson with the previous one.

What's New

In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.

What is It

This section provides a brief discussion of the lesson to help you discover and understand new concepts and skills.

What's More

This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.

What I Have Learned

This part offers questions, or a fill-in-the-blank sentence/paragraph, to enable you to process what you have learned from the lesson.

What I Can Do

This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.

Assessment

	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
 ***Additional Activities***	In this portion, other activities will be given to you to enrich your knowledge or skill of the lesson learned. This also aids in the retention of learned concepts.
 ***Answer Key***	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer ***What I Know*** before moving on to the other activities included in the module.
3. Read the instructions carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

CHAPTER 1

FACTORING

Each tourist destination in our country has unique characteristics to a particular place. The tarsier and Chocolate Hills of Bohol, the windmills of Ilocos, the Underground River in Palawan, Mayon Volcano of Bicol, and Taal Volcano of Batangas are some of the most scenic spots our country has to offer.

Factoring has distinct arrangements that should be understood and recognized for easy calculation. Factoring also deals with looking for and recognizing patterns and sometimes making estimates.

A. Factoring Polynomials with Common Monomial Factor and by Grouping
B. Factoring the Difference of Two Squares
C. Factoring the Sum or Difference of Two Cubes

The second part includes:

D. Factoring Quadratic Trinomial
E. Factoring Perfect Square Trinomials
F. Solving Problems Involving Factors of Polynomials

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the concept of Factoring. The scope of this module is helpful in answering many real-life problems. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module is the first part of the chapter and is divided into three lessons, namely:

- Lesson 1 – Factoring Polynomials with Common Monomial Factors and by Grouping.
- Lesson 2 – Factoring the Difference of Two Squares; and
- Lesson 3 – Factoring the Sum or Difference of Two Cubes

After going through this module, you are expected to:

1. Identify factors and prime factors of a given integer and of a monomial;
2. Factor the polynomials with common monomial factors;
3. Factor the polynomials by grouping in terms;
4. Identify if the given polynomial is the difference of two squares, and sum or difference of two cubes;
5. Factor the difference of two squares;
6. Factor the sum or difference of two cubes;
7. Exhibit patience; and
8. Develop analytical skills in doing tasks.

What I Know

Direction: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is a product of 4 and (x + 7) ?

A. 4x + 7 B. x + 28 C. 4 (x + 7) D. 4x + 28

2. Find the greatest common factor (GCF) for $12p^2q^3$ and $36p^3q^2$.

A. $12p^2q^2$ B. $6p^2q^2$ C. $4p^2q^2$ D. $2p^2q^2$

3. What should be multiplied by 5x to have $10x^3 + 15x^2 - 20x$?

A. $2x + 3x - 4$ B. $x^2 + 3x - 4$ C. $2x^2 + 3x - 4$ D. $2x^2 - 3x + 4$

4. Factor completely: ab + 7a + 4b + 28

A. (a + b)(4 + 7) B. (a + 4)(b + 7) C. (a – 4)(b – 7) D. (a + 7)(b + 4)

5.. Which of the following is the factor of $4x^2 - 25y^2$?

A. (x + 5y)(x – 5y) C.(2x + y)(2x – y)

B. (2x + 5y)(2x – 5y) D.(4x + 5y)(4x – 5y)

6. What polynomial should be paired with $(a^2 - 5a + 25)$ to get $a^3 + 125$?

A. $(a^2 + 25)$ B. (a + 5) C. (a - 5) D. (a + 5a)

7. What expression should be placed in the blank to make

$27x^3 - 64y^3 = (\_\_ - 4y)(9x^2 + 12xy + 16y^2)$?

A. 3x B. 4x C.9x D. 16x

8. If the height of a box is (h + 9) cm and its volume is $(h^3 + 729)$ cubic cm, what is the area of the base?

A. $(h^2 - 81)$ sq cm C. $(h^2 + 81)$ sq cm

B. $(h^2 - 9h + 81)$ sq cm D. $(h^2 + 9h + 81)$ sq cm

Lesson 1

Factoring Polynomials with Common Monomial Factor and by Grouping

What is considered in getting the common factor of a given polynomial? When do we say that a factor is a greatest common factor of all the terms in a polynomial? What happens when you have a negative common factor?

These are the things that you will learn in lesson 1 of this module.

What I Know

Activity 1

A. Supply the missing term.

1. $3a + 3b =$____ $(a + b)$
2. $bx + by + bz =$ _____ $(x + y + z)$
3. $a^2 b – ab^2 = ab$ (_____)
4. $4x + 6y =$____ $(2x + 3y)$
5. $m^3 – m =$ ____ $(m^2 – 1)$

B. Factor the following polynomials.

6. $16a^2 + 12a$
7. $12am + 6a^2m$
8. $72x^2 + 36xy - 27x$
9. $5a^3 + a^2b - 5a - b$
10. $30a - 5ay + 36z - 6yz$

What's In

Activity 2

Multiply the following.

1. (13) (4)
2. -7 (5xy)
3. -6ab (-4ab)
4. 9 (2m + 5)
5. -10abc $(3b^2c – 5a)$
6. (a - b) (c + d)
7. (3m + n) (p – 2q)

What's New

Activity 3: Let us explore.

1. What is the greatest common factor of 6, 9 and 15?
2. What is the greatest common factor of 12, 7 and 5?
3. What is $x^5 \div x^2$?
4. What is $2x\ (x - 5)$?
5. How about the product of $3xy^2\ (2x - 1 + 5y)$? Is it $6x^2y^2 - 3xy^2 + 15xy^3$?

What is It

The reverse process of getting the product of any number or algebraic expression is called **Factoring**. To factor a polynomial means to describe the algebraic expression as the product of two or more expressions. When a factor is contained in every algebraic expression term, it is called the common factor. To factor algebraic expressions, the greatest common monomial of all terms of the given expression has to be factored out, then each term of the expression is divided by the greatest common factor. The resulting expression is the other factor.

What's More

Recall that distributive property of multiplication is applied when a polynomial is multiplied by a monomial. That is, $a\ (b + c + d) = ab + ac + ad$. Similarly, the polynomial $ab + ac + ad = a\ (b + c + d)$, where a is said to be the greatest common monomial factor and the $(b + c + d)$ is the other polynomial factor, which is the same kind as the given polynomial to be factored.

Example 1. What is the greatest common monomial factor of $12ab^2 - 4a^2b^2$?

Solution:

STEP 1: *Find the GCF of the numerical coefficients. 12 and -4 of the binomial and that is 4.*

STEP 2: *Write the common variable of the two terms of the binomial is* ab^2.

Thus, the greatest common monomial factor of $12ab^2 - 4a^2b^2$ is $4ab^2$

Example 2. What should be multiplied by $3y^3$ to get $6y^4 + 12y^3 – 15y^5$?

Solution: *Simply divide each term of the trinomial* $6y^4 + 12y^3 – 15y^5$ *by* $3y^3$*. And it will yield (*$2y + 4 – 5y^2$*) and it is referred as the other factor of* $6y^4 + 12y^3 – 15y^5$.

Example 3. What are the factors of $4ax + 2x + 6bx$?

Solution:

STEP 1: *Determine the GCMF of the given polynomial, which is 2x.*

STEP 2: *Solve for the other factor by dividing 4ax + 2x + 6bx by 2x. And will result to 2a + 1 + 3b.*

Hence, the factors of the given trinomial are 2x and 2a + 1 + 3b.

Example 4. Factor $12x^2y^2 – 9x^3y^2 + 6xy^3$.

Solution: *Find the GCMF and the other factor. Therefore, the factored form of* $12x^2y^2 – 9x^3y^2 + 6xy^3$ *is* $3xy^2 (4x – 3x^2 + 2y)$*. Did you get it?*

Example 5. How high is a triangle with a base measuring $4x^2$, if its area is $(8x^3 – 4x^2 + 12x^5 + 20x^4)$ square cm?

Solution: *Notice that is the GCMF of* $8x^3 – 4x^2 + 12x^5 + 20x^4$ *is* $4x^2$*, which implies that* $8x^3– 4x^2 +12x^5 +20x^4 = 4x^2 (2x – 1 + 3x^3 + 5x^2)$.

Hence, the height of the given triangle is $(2x – 1 + 3x^3 + 5x^2)$ *cm.*

There are cases when a polynomial with four or more terms has no common monomial factor. In this case, grouping methods may be applied. Get the greatest common monomial factor of each group, then take the common binomial factor and the other as factors of the polynomials.

Example 6. Factor by grouping $xy + xp + y^2 + yp$

Solution:

STEP 1: *Group the terms into two binomials, in terms of x and y as their common variable,* $xy + xp + y^2 + yp$.

STEP 2: *Get the GCMF of each binomial term,* $x (y + p) + y (y + p)$

STEP 3: *Take out the common binomial factor and get the other factor. Therefore,* $xy + xp + y^2 + yp = x (y + p) + y (y + p) = (y + p) (x + y)$

Example 7. Is $(x + 3y)$ one of the factors of $15ay – 2bx – 6by + 5ax$?

Solution:

STEP 1: *Group the terms into two binomials, in terms of x and y or a and b as their common variable, suppose we group* $(15ay + 5ax) – (2bx + 6by)$

STEP 2: *Get the GCMF of each binomial term,* $5a (3y + x) – 2b (x + 3y)$

STEP 3: *Take out the common binomial factor and get the other factor, that is* $(15ay – 5ax) – (2bx + 6by) = 5a(3y + x) – 2b (x + 3y) = (3y + x)(5a – 2b)$.

Therefore, the factors are $(x + 3y) (5a – 2b)$*, and since* $(3x + y)$ *is one of the factors the answer is YES.*

What I Have Learned

Factoring is the reverse process of getting the product. To factor polynomials is to describe the algebraic expression as the product of two or more expressions. When a factor ends up contained in every term of an algebraic expression, it is identified as the **common factor**.

The following are the steps to factor any algebraic expressions with the greatest common monomial factor:

- factor out the greatest common monomial of all terms of the given expression, then
- divide each term of the expression by the greatest common factor, so that the resulting expression is the other factor.

There are cases when a polynomial with four or more terms have no common monomial factor. In such cases, grouping methods may be applied by getting the greatest common monomial factor of each group, then taking the common binomial factor and the other as factors of the polynomials.

What I Can Do

A. Answer the following items as indicated.

1. What is the greatest common monomial factor of $24x^2y^5 – 20x^4y^3$?
2. What should be multiplied by $5a^2$ to get $10a^3 – 20a^2 + 15a^4$?
3. What are the factors of $9xy + 6xz + 3x$?
4. Factor $2ab^4 – 5a^2b^3 + 3a^3b^2$.
5. How high is a triangle with a base measuring $7y^3$, if its area is $(14y^2 – 21y^4 + 7y^5)$ square cm?
6. Factor completely:
   a. $4ay + 4az + 3by + 3bz$
   b. $9a^2 + 12 – 8a – 6a^3$

B. ENRICHMENT ACTIVITY

Question: What is the leading product of Tagaytay City?

Match the products in Column A with the factors in Column B to decode the answer.

Column A	Column B
1. $6x^2 + 3xy$	E: $3x (2x + y)$
2. $24abx + 20ab$	A: $5x (3x + 4)$
3. $15x^2y – 6xy$	P: $3xy (5x – 2)$
4. $36ab + 60b$	L: $4ab (6x + 5)$
5. $18a^3 – 24a^2$	E: $6a^2 (3a – 4)$
6. $12x^2y + 12xy^2 + 12xyz$	N: $12b (3a + 5)$
7. $3x^4 – 6x^3y$	P: $4mn (4x + 7)$
8. $9x^2 + 9xy$	I: $12xy (x + y + z)$
9. $16mnx + 28mn$	M: $9x (x + y)$
10. $15x^2 + 20x$	P: $3x^3 (x – 2y)$

3	6	4	1	10	7	9	2	5

Assessment

A. Direction: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is a product of 5 and (2x - 3) ?
   A. 10x + 3 B. 10x - 15 C. 10x + 15 D. 10 – 5x
2. What should be multiplied to 3x to get $15x^2 + 6xy$?
   A. 5x + 2y B. 5x - 2y C. 2x + 5y D. 2x - 5y
3. Find the greatest common factor (GCF) of $4a^3$, $8a^2$, 12a.
   A. 2a B. 4a C. 8a D. 12a
4. Which of the following are factors of 35x – 21y + 14z ?
   A. 7xyz (5 – 3 +2) B. 7(x –y + z) C. 7(5x - 3y + 2z) D. 5(7x – 3y + 2z)
5. If $(5ax^2 + 3x^2 – 5a – 3)$ is divided by (5a + 3), what is the other factor?
   A. x + 1 B. x - 1 C. $x^2 + 1$ D. $x^2 - 1$

B. Factor the following.

6. 5x + 5y + 5z
7. 4ax + 6ay + 10az
8. $4x^3 + 8x^2 – 12x$
9. $6xy +18y^2 – 9x – 27y$
10. $15a^3 – 20a^2 + 3a – 4$

Lesson 2 Factoring a Difference of Two Squares

When will you know if a given set of numerals represent perfect square numbers? If algebraic expressions are in the form of the difference of two squares, how will you know? What steps should you take when factoring polynomials in the Difference of Two Squares?

What I Know

Activity 1

A. Fill in the blanks.

1. _____ $(x - 8) = x^2 - 64$
2. $(30 + 12)$ _____ $= 30^2 - 12^2$
3. _____ $(3m - 11n) = 9m^2 - 121n^2$
4. $(6x^2y + 3ab)(6x^2y - 3ab) =$ _____ $- 9a^2b^2$
5. $(11 + 10x)(11 - 10x) = 121 -$ _____

B. Factor completely

6. $25 - 100m^2$
7. $36d^2 - 49k^2$
8. $-81a^2 + (4/9)b^2$
9. $x^6 - 400$
10. $9h^4 - 256f^6$

What's In

Activity 2: Answer as indicated.

1. Factor $c^2 - 49$.
2. What is the factored form of $4b^2 - 25$?
3. How wide is a rectangular lot (6b +7) cm long with an area of $(36b^2 - 49)$ cm^2?

4. What are the factors of $9y^4 - 100$?

5. What should be paired with $(5n - 9)$ to get $(25n^2 - 81)$?

What's New

Activity 3: Read the following and answer.

1. Is $4b^2 - 81$ a difference of 2 squares? Explain your answer.
2. Is $y - 36$ a difference of 2 squares? Why, why not?
3. What is $\sqrt{81}$?
4. What is $(a + 4)(a - 4)$?
5. How about the product of $(k - 5)(k + 5)$? Is it $k^2 - 25$?

What is It

When we multiply a sum and difference of the same two terms, the product is called the difference of two squares. That is, if $(a + b)(a - b) = a^2 - b^2$, then, $a^2 - b^2$ is the difference of two squares.

Generally, to get the factors of the difference of two squares, such as

$$a^2 - b^2 = (a + b)(a - b)$$

Notice that the factors of the difference between two squares are the sum and difference of their square roots.

What's More

You see, after identifying if a polynomial is a difference of two squares, it is easy to find the factors.

Example 1. Factor $c^2 - 49$.

Solution: If you are to factor $c^2 - 49$, c^2 is $c \bullet c$ and 49 is $7 \bullet 7$, the polynomial is a difference of two squares. Hence, $c^2 - 49 = (c + 7)(c - 7)$.

Example 2. What is the factored form of $4b^2 - 25$?

Solution: Is it a difference of two squares? The factored form of $4b^2$ is $(2b)^2$ and 25 is $(5)^2$. That means $4b^2$ - 25 is a difference of two squares and its factored form is (2b – 5)(2b + 5).

Example 3. How wide is a rectangular lot (6b + 7) cm long with an area of $(36b^2 – 49)$ square cm?

Solution: Notice that $36b^2 = (6b)^2$ and $49 = 7^2$, so $36b^2 – 49$ is a difference of two squares. Since $36b^2 – 49 = (6b + 7)(6b – 7)$ and the length is (6b + 7) cm, this implies that the width is (6b – 7) cm.

Example 4. What are the factors of $9y^4 –100$? Is it a difference of two squares?

Solution: Yes! $9y^4$ is $(3y^2)^2$ and 100 is 10^2. The factors are $(3y^2 – 10)$ $(3y^2 +10)$. Did you get it?

Example 5. What should be paired with (5n – 9) to get $25n^2 – 81$?

Solution: In this case we should look for the other factor of $25n^2 – 81$. Although (5n – 9) offers the clue for the other factor, you may have observed that $25n^2 = (5n)^2$ and $81 = 9^2$. Obviously, the answer is (5n + 9). Do you agree?

What I Have Learned

Difference of Two Squares is the product when you multiply a sum and difference of the same two terms.

In general, a polynomial that is a difference of two squares can be factored out as the sum and difference of the square roots of the squared numbers. In symbols: $a^2 – b^2 = (a + b)(a – b)$.

What I Can Do

A. Answer as indicated

1. Factor $b^2 – 25$.
2. What is the factored form of $9k^2 – 16$?

3. How wide is a rectangular lot (2c + 3) cm long with an area of ($4c^2 - 9$) cm^2 ?

4. What are the factors of $h^2 - 121$?

5. What should be paired with (3m – 7) to get $9m^2 - 49$?

B. **ENRICHMENT ACTIVITY**

Motivation: **SECRET MESSAGE.** Find the square roots and solve the secret message by putting the letter of the corresponding square root of the given number.

4 = _____ 16 = _____ 121 = _____ 4 = _____ 49 = _____ 64 = _____

9 = _____ 81 = _____ 49 = _____ 81 = _____ 100 = ____ 25 = _____

36 = _____ 169 = _____ 64 = _____ 144 = _____ 196 = _____ 225 = _____

169 = _____ 256 = _____ 169 = _____ 25 = _____ 49 = _____ 144 = _____

289 = _____ 196 = _____ 225 = _____ 400 = _____

A	B	C	D	E	F	G	H	I	J	K	L	M
2	6	18	3	13	21	20	17	14	1	24	4	27

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
15	9	21	19	5	8	12	10	14	11	25	7	23

Assessment

A. Fill in the frames to make the sides of each equation equivalent.

1. _____ $(x - 9) = x^2 - 81$

2. $(20 + 4)$ _____ $= 20^2 - 4^2$

3. _____ $(7m - 11n) = 49m^2 - 121n^2$

4. $(6x^2y + 3ab)(6x^2y - 3ab) =$ _____ $- 9a^2b^2$

5. $(13 + x)(13 - x) =$ _____ $- x^2$

B. Factor completely

1. $144a^2 - 169b^2$

2. $1 - 0.09a^2$

3. $16x^2 - 121$

4. $-64a^2 + (9/25)\,b^2$

5. $x^4 - 256$

Lesson 3 Factoring a Sum or Difference of Two Cubes

When do you know if the given numbers are perfect cube numbers? How will you know if an algebraic expression is in the form of the sum or difference of two cubes? What are the steps when factoring polynomials in the sum or difference of two cubes?

Almost the same questions asked in the previous lesson. Let's see the difference between the two.

What I Know

Activity1

A. Complete the factoring.

1. $t^3 – w^3 = (t – w)$ (_____)
2. $z^3 + v^3 = (z + v)$ (_____)
3. $8x^3 + 27 =$ (____) $(4x^2 – 6x + 9)$
4. $y^3 – 64 =$ (_____) $(y^2 + 4y + 16)$
5. $125a^3b^3 + 343c^6 = (5ab + 7c^2)$ (_____)

B. Factor the following:

6. $x^3 - 8$
7. $27x^3 + 1$
8. $x^3 y^6 - 64$
9. $m^3 + 125$
10. $x^3 + 343$

What's In

Activity 2: Read the following items carefully, then write your answers in your notebook.

1. Factor $c^3 – d^3$.
2. What is the factored form of $8b^3 + 216$?

3. What expression should be placed in the blank to make

$27x^3 + y^3 = (\_\_ + y)\ (9x^2 – 3xy + y^2)$?

4. What polynomial should be paired with $(a^2 + 5a + 25)$ to get $a^3 – 125$?

5. If the area of the base of a box is $(h + 9)$ square cm and its volume is $(h^3 + 729)$ cubic cm, how high is the box?

What's New

Activity 3: Answer the following questions

1. Is $b^3 – d^3$ a difference of two cubes?
2. Is $y^2 + 27$ a sum of two cubes?
3. What is $\sqrt[3]{27}$?
4. What is $(a + 4)(a^2 – 4a + 16)$?
5. Give the product of $(k – 5)(k^2 + 5k + 25)$? Is it $k^3 + 125$?

What is It

The sum and difference of two cubes have these factors:

$$\mathbf{a^3 + b^3 = (a + b)\ (a^2 – ab + b^2)}$$

$$\mathbf{a^3 – b^3 = (a – b)\ (a^2 + ab + b^2)}$$

Notice that the factors are one binomial and one trinomial. The sum of the cubes of the two terms is equal to the sum of the cube roots of the two terms multiplied by the sum of the squares of these terms minus the product of these two terms. The difference of the cubes of the two terms is equal to the difference of their cube roots multiplied by the sum of the squares of these two terms plus the product of these two terms.

What's More

To factor a sum or difference of two cubes, simply reverse the procedure of multiplying polynomials. Let's take a look at the following example;

Example 1: Let us take $c^3 – d^3$ as an example. Is it a difference of two cubes?

Solution: Yes because both c^3 and d^3 are perfect cubes. Using our pattern, $c^3 – d^3 = (c – d)(c^2 + cd + d^2)$.

Example 2: What is the factored form of $8b^3 - 216$?

Solution: Take note that $8b^3 = (2b)^3$ and $216 = 6^3$, we are talking here of a difference of two cubes. So, $8b^3 - 216 = (2b - 6)[(2b)^2 + 2b(6) + (6)^2]$. Thus, $8b^3 - 216 = (2b - 6)(4b^2 + 12b + 36)$. Were you able to follow the steps?

Example 3: What expression should be placed in the blank to make $27x^3 + y^3$ equal to $(\_\_ + y)(9x^2 - 3xy + y^2)$?

Solution: Here we have a sum of two cubes because $27x^3 = (3x)^3$ and y^3 is a perfect cube. $27x^3 + y^3 = (3x + y)[(3x)^2 - (3x)(y) + y^2]$, which also equal to $27x^3 + y^3 = (3x + y)(9x^2 - 3xy + y^2)$. I hope the steps are now getting clearer. Do you think so?

Example 4: What polynomial should be paired with $(a^2 + 5a + 25)$ to get $a^3 - 125$?

Solution: Aside from a^3 being a perfect cube 125 is also a perfect cube because $125 = (5)^3$, we have here a difference of two cubes. Can you see that? $a^3 - 125 = (a - 5)[a^2 + a(5) + (5)^2]$, therefore $a^3 - 125 = (a - 5)(a^2 + 5a + 25)$, meaning the missing factor is $(a - 5)$. Is it right?

Example 5: If the height of a box is $(h + 9)$ cm and its volume is $(h^3 + 729)$ cubic cm, what is the area of the base?

Solution: The volume of the box is equal to the area of the base times its height. Factoring the sum of two cubes given $h^3 + 729$, yields to $h^3 + 729 = (h + 9)[h^2 - h(9) + 9^2]$, simplified to, $h^3 + 729 = (h + 9)(h^2 - 9h + 81)$. If the height is $(h + 9)$ then the area of the base is $(h^2 - 9h + 81)$. Therefore the answer is $(h^2 - 9h + 81)$ square cm.

What I Have Learned

The difference of two cubes, like $a^3 - b^3$, is factored as

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

Although we cannot factor the sum of two squares, the sum of cubes such as $a^3 + b^3$ can be factored as:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2).$$

What I Can Do

A. Solve for the following. Write down your answers in your notebook.

1. What are the factors of $m^3 - n^3$?

2. Give the factors of $27a^3 - 1$.

3. What expression should be placed in the blank to make

$$8x^3 + y^3 = (\_\_ + y)(4x^2 - 2xy + y^2)?$$

4. What is the other factor of $r^3 + 64$ if one factor is $(r + 4)$?

5. If the area of the base of a box is $(d^2 - 10d + 100)$ square cm and its volume is $(d^3 + 1000)$ cm^3, how high is the box?

B. **ENRICHMENT ACTIVITY**

Directions. Find the cube roots. Then, match each solution to the numbers. Write the corresponding letter in each blank to the question.
In the survey, the best place for family picnics in Tagaytay City?

27	512	343	216	1728	729	1	729	1331	1000	2197	0	64	125
C	**R**	**G**	**O**	**P**	**I**	**A**	**T**	**I**	**C**	**V**	**D**	**E**	**N**

12	**11**	**3**	**5**	**9**	**10**	**7**	**8**	**6**	**13**	**4**

Assessment

A. Supply the missing expression.

1. m^3 - 27 = (m – 3) _____
2. 64 + $27n^3$ = _____ (16 – 12n + $9n^2$)
3. _____ = (2p + 5q) ($4p^2$ – 10pq + $25q^2$)
4. x^6 + 1000 = _____ (x^4 – $10x^2$ + 100)
5. _____ = (6x – 7y) ($36x^2$ + 42xy + $49y^2$)

B. Factor each completely.

6. $8x^3 + 125$
7. $27a^3 + 64b^3$
8. $64m^3 - 1$
9. $343u^3 - 512t^6$
10. $1000 - 729q^6p^9$

What I Know

1. D
2. A
3. C
4. B
5. B
6. B
7. A
8. B

LESSON 1:

What I Know

1. 3
2. b
3. (a –b)
4. 2
5. m
6. 4a(4a+3)
7. 6am (2 + a)
8. 9x (8x +4y -3)
9. (5a +6) (a^2 – 1)
10. (6-y) (5a +6z)

What's In

1. 52
2. -35xy
3. $24a^2b^2$
4. 18m + 45
5. $-15ab^3c^2 + 50a^2bc$
6. ac + ad – bc - bd
7. 3mp - 6mq + np – 2nq

What's More

1. 3
2. 1
3. x^3
4. $2x^2$-10x
5. YES

What I Can Do

A. 1. $4x^2y^3(6y^2 – 5x^2)$

1. $2a - 4 + 3a^2$
2. 3x (3y+2z+1)
3. $ab^2(2b^2 - 5ab + 3a^2)$
4. $2 - 3y + y^2$
5. $a.(y + z)(4a + 3b)$

b. (3 - 2a)$(3a^2 + 4)$

B. ENRICHMENT

1. E	3x(2x+y)	6. I	12xy(x+y+z)
2. L	4ab(6x+5)	7. P	$3x^2$(x-2y)
3. P	3xy(5x-2)	8. M	9x(x+y)
4. N	12b(3a+5)	9. P	4mn(4x+7)
5. E	$6a^2$(3a-4)	10. A	5x(3x+4)
answer:	PINEAPPLE		

Assessment

1. B
2. A
3. B
4. C
5. D
6. 5 (x + y + z)
7. 2a (2x+3y+5z)
8. 4x (x^2 + 2x + 3)
9. (2y – 3)(3x + 9y)
10. (3a – 4)($5a^2$ – 1)

LESSON 2

What I Know

1. (x + 8)
2. (30 – 12)
3. (3m +11n)
4. $36x^4y^2$
5. $100x^2$
6. (5 + 10m) (5 – 10m)
7. (6d + 7k) (6d – 7k)
8. (2/3b + 9c) (2/3b – 9c)
9. (x^2 + 20) (x^2 – 20)
10. $(3h^2 + 16t^3)(3h^2 - 16t^3)$

What's in:

1. (c+7)(c-7)
2. (2b + 5)(2b - 5)
3. (6b - 7)
4. $(3y^2+10)(3y^2-10)$
5. (5n + 9)

What's New:

1. YES
2. NO
3. 9
4. a^2-16
5. YES

What I can do:

A. 1. (b + 5)(b - 5)
2. (3k + 4)(3k - 4)
3. (2c - 3)
4. (h + 11)(h - 11)
5. (3m +7)

B. ENRICHMENT
Always do your best in everything

Assessment:

A. 1. (x+9)
2. 20-4)
3. 7m+11n
4. $36x^4y^2$
5. 169

B. 1. (12a+13b)(12a-13b)
2. (1+0.3a)(1-0.3a)
3. (4x+11)(4x-11)
4. $-1(8a+\frac{3}{5}b)(8a+\frac{3}{5}b)$
5. $(x^2+16)(x^2-16)$

LESSON 3

What I Know

1. $(t^2 + tw + w^2)$
2. $(z^2 – zv + v^2)$
3. $(2x + 3)$
4. $(y - 4)$
5. $(25a^2b^2 – 35abc^2 + 49c^4)$
6. $(x - 2)\ (x^2 + 2x + 4)$
7. $(3x + 1)\ (9x^2 – 3x + 1)$
8. $(xy^2 – 4)\ x^2y^4 + 4xy^2 + 16)$
9. $(m + 5)\ (m^2 – 5m + 25$
10. $(x – 7)\ (x^2 – 7x + 49)$

What's in:

1. $(c-d)(c^2+cd+d^2)$
2. $(2b+6)(4b^2-12b+36)$
3. 3
4. $(a-5)$
5. $(h^2-ah+81)$

What's New:

1. YES
2. NO
3. 3
4. a^3+64
5. No, it should be k^3-125

What I can do

A. 1. $(m-n)(m^2+mn+n^2)$
2. $(3a-1)(9a^2+3a+1)$
3. 2x
4. $r^2+4r+16$
5. $(d+10)$

B. ENRICHMENT
PICNIC GROVE

Assessment:

A. 1. (m^2+3m+9)
2. $(4+3n)$
3. $8p^3+125q^3$
4. x^2+10
5. $216x^3-343y^3$

B. 6. $(2a+5)(4a^2-10a+25)$
7. $(3a+4b)(9a^2-12ab+16b^2)$
8. $(4m-1)(16m^2+4m+1)$
9. $(7u-8t^2)(49u^2+56ut^2+64t^4)$
10. $(10 - 9q^2p^3)\ (100 + 90q^2p^3 + 81p^4q^6)$

References:

Jose M. Marasigan, et.al. 2002. *Intermediate Algebra*. Makati City. The Bookmark, Inc.

Government of the Philippines, Department of Education 2013. *Mathematics 8 Learner's Module*. Manila.

Ricardo M. Crisostomo, et. al 2013. *Our World Math*. Quezon City. Vibal Publishing House, Inc.

Maxima J. Acelajado. 2002. *Algebra: A Problem Solving Approach*. Makati City. Belgosa Media Systems, Inc.

Gladys C. Nivera. 2014.*Grade 8 Mathematics: Patterns and Practicalities*. Makati City. Salesiana Books by Don Bosco Press, Inc.

Module in Math 8

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For inquiries or feedback, please write or call:

Schools Division Office-Quezon City

Nueva Ecija St. Bago Bantay, Quezon City

Telephone no. 8352-6806/6809; Telefax: 34560343

Email Address: sdoqcactioncenter@gmail.com