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DEPARTMENT OF EDUCATION

Mathematics 8
Quarter 1 – Module 8
Systems of Linear Equations

ALTERNATIVE DELIVERY MODE
ADM

Personal Development
Alternative Delivery Mode
Quarter 1 – Module 8: Systems of Linear Equations
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Mathematics

Quarter 1 – Module 8

Systems of Linear Equations

DEPARTMENT OF EDUCATION

Introductory Message

For the facilitator:

Welcome to the **Mathematics 8** Alternative Delivery Mode (ADM) Module on **Systems of Linear Equations!**

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the **Mathematics 8** Alternative Delivery Mode (ADM) Module on **Systems of Linear Equations**!

The hand is one of the most symbolized parts of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish many things. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.

What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

SYSTEMS OF LINEAR EQUATIONS

As waves approach the beach, you can see them resembling parallel lines. There seems to be a system that the part of the wave that is farther offshore catches up with the part that reaches the shallow water first, and then it crashes on the shore more nearly parallel to the shore.

Incidentally, in Mathematics, we have systems of linear equations. The graphs of the equations of one system are parallel lines, the graphs of the equations of the second system are coincident lines, that is one graph falls exactly on the other graph, while the graphs of the equations of the third system intersect at a point.

This module includes**:** Illustrate Systems of Linear Equations,
Graphing System of Linear Equations and Categorize System of Linear Equations.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the Systems of Linear Equations. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is divided into three lessons, namely:

- Lesson 1 – Illustrate System of Linear Equations
- Lesson 2 – Graphing System of Linear Equations
- Lesson 3 – Categorize System of Linear Equations

After going through this module, you are expected to:

1. illustrate system of linear equations;
2. describe a system of linear equations and its solution;
3. state whether the given ordered pair is a solution of the given system;
4. graph the equations in each system, then identify the kind of system;
5. identify the kind of system using the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$;
6. develop analytical thinking while performing the assigned task.

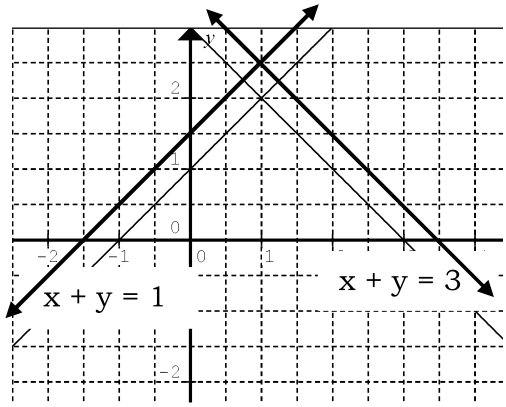
Lesson 1 Illustrate Systems of Linear Equations

In the existing situation the demands for a laptop computer rises. A laptop computer producer introduces a new model based totally on the student's need. The manufacturer tracks its costs, which is the amount it spends to produce the laptops, and its revenue, which is the amount it earns through sales of its laptops. How can the business enterprise determine if it is making an income with its new model? How many laptops should be produced and sold before a profit is possible? To answer these and comparable questions we will reflect inconsideration on system of linear equations with two variables particularly we focus on two equations only.

What I Know

Directions: Choose the letter of the best answer. Write your answers on a separate sheet of paper.

1. Which of the following is a system of linear equations in two variables?
   a. $x + y = 5$
   b. $\begin{cases} 2x + y = 4 \\ x - 2y = 6 \end{cases}$
   c. $\begin{cases} x + 3y = 2 \\ 2x - 3y < 1 \end{cases}$
   d. $3x - 2 = 5$
2. How many solutions does a consistent and dependent systems of linear equations have?
   a. 0 b. 1 c. 2 d. infinite
3. What is the solution set of the system $\begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$?
   a. $(1,2)$ b. $(2,1)$ c. $(4,1)$ d. $(1,4)$
4. How many solutions are there in the system $\begin{cases} x + y = 4 \\ 2x + 2y = 8 \end{cases}$?
   a. no solution b. one solution c. infinite solutions d. 2 solutions
5. What is the solution set of the system below?
   a. $\{(1,2)\}$ c. $\{(-1,2)\}$
   b. $\{(2,1)\}$ d. $\{(2,-1)\}$



What's In

Linear Equation in Two Variables

➢ Is an algebraic equation whose variable quantity or quantities are in the first power only and it is written in the form of Ax + By + C = 0, where a, b, and c are real numbers and the coefficients of x and y which are a and b respectively are not equal to zero.

Let's recall:

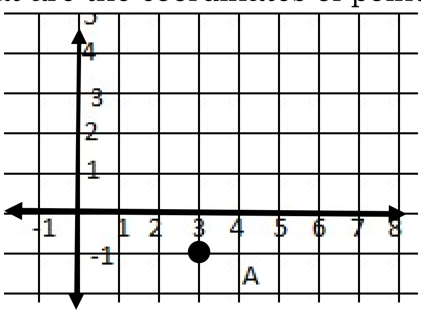
1. Which of the following is a linear equation in two variables?

   A. xy = 6 B. 2x + y – 4 = 0 C. $\frac{2}{x} = y$ D. $y = x^2 - 1$

   The answer is B because (1) it is a first-degree equation and (2) it is written in the form of Ax + By + C = 0 wherein x and y are the variables and a and b are the coefficient and c is the constant.

2. Simplify: 4(-1) + 4. Is your answer equal to zero? Good!

3. What are the coordinates of point A?



My answer is (3, -1).
Do we have the same answer?

4. Is (1,3) a solution of the linear equation x+y = 5? No it is not a solution because it doesn't satisfies the equation, that is 1 + 3 ≠ 5.

5. Is (3,1) a solution of 3x – y = 8? What is your answer? Yes, it is a solution because when you replace x with 3, y with 1 and performing the indicated operations, you'll get a true statement, that is 3(3) – 1 = 8.

What's New

Think about this!

1. What two numbers have a sum of 5?
2. What two numbers have a difference of 3?

Notes to the Teacher

Point out that for each of the problem given, there are many possible answers. Each problem can be presented by an equation.

Possible Answers:

1. 2 and 3, 1 and 4, -2 and 7, -1 and 6, etc.
2. 4 and 1, 5 and 2, 6 and 3, 8 and 5, 10 and 7, etc.

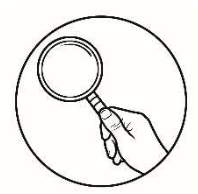
Rewrite the given problem into an equation:

Let x be the first number and y be the second number.

1. $x + y = 5$
2. $x - y = 3$

Go over your lists of answers and determine if there is a pair of numbers which appear in both lists, that is two numbers which have the sum of 5 and a difference of 3. (In my list I have 1 and 4, do we have the same answer?)

The equations written above if considered together and a common solution for them is desired is called **system of linear equation**.



What is It

Two or more linear equations with the same variables considered together for which a common solution is desired is a **system of linear equations**. The equations in a system are to be solved simultaneously for a common solution. A **solution** of a system of linear equations is an ordered pair of real numbers that satisfies each of the equations. This is why it is called a system. The particular system of equations included in our study are systems of linear equations in two variables and we're going to focus only on two equations. For example, let us consider

$$x + y = 3 \text{ and } \quad x - y = 1$$

This system may also be written in the form

$$\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$$

The **solution set** of this system of linear equations is the set of all ordered pairs, each of which is a solution of both equations. Let us represent the solution set of the first equation by A and the solution set of the second equation by B. The solution set of the system is the intersection of A and B or the ordered pair that is common to both sets.

A = {…,(0,3), (1,2), (2,1),(3,0), (4,-1),…}

B = {…(0,-1), (1,0), (2, 1), (3,2), …}

This means that A ⋂ B = {(2,1)}. To check,

x + y = 3, but x = 2 and y = 1

2 + 1 = 3

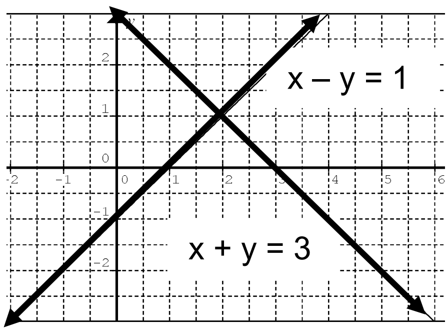
3 = 3 → true

x – y = 1, but x = 2 and y = 1

2 – 1 = 1

1 = 1 → true

The solution (2,1) means that x = 2 and y =1.



The graphs of x + y = 3 and x – y = 1 are given at the right. Notice that the point (2,1) is the intersection of the graphs. Since no other point lie on both lines, then, (2,1) is the only solution.

A system of linear equations is often called a system of **simultaneous equations**, because it imposes two conditions on the variables at the same time. After finding all the solutions of a system of linear equations, it means we have **solved the system**.

There are three kinds of systems of linear equation.

1. Let's observe this system: $\begin{cases} x + 2y = 4 \\ 2x + 4y = 6 \end{cases}$

   These equations make contradictory statements. If we rewrite the first equation by multiplying with 2, the new equation is 2x + 4y = 8. This would mean that our two equations have identical expressions on their left sides but have different answers. That is why they are contradicting. This is called an **inconsistent system**. Such a system has no solution, since there are no values of x and y that satisfy both equations.

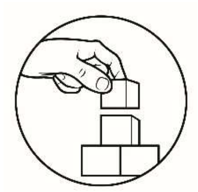
2. Now, examine this system: $\begin{cases} x - y = 3 \\ 3x - 3y = 9 \end{cases}$

   Let us rewrite the first equation by multiplying it with three, (3)(x – y = 3), which results to 3x – 3y = 9. The first equation is now identical to our second equation. If a system of linear equation can be written as the same equation, then it is called a **consistent-dependent system**. In such system there are an infinite number of solutions. The solutions of one equation are also solutions to the other since the equations are equivalent.

3. Refer to the equation given earlier: $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

   There are systems of equations that no matter how we rewrite them, the equations cannot be equivalent.

   From our example, x + y is already different from x – y, even if we multiply them by any integers, they stay different. This kind of system is called **consistent-independent system**, and they only have one solution.



What's More

Activity 1:

Identify what kind of systems are given and state how many solution/s each system has.

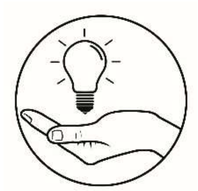
1. $\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$

2. $\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$

3. $\begin{cases} 2x + 3y = 4 \\ 6x + 9y = 8 \end{cases}$

4. $\begin{cases} \mathbf{x - 2y = 1} \\ \mathbf{2x + 4y = 3} \end{cases}$

5. $\begin{cases} x - 4y = 3 \\ 4x - 16y = 12 \end{cases}$



What I Have Learned

System of Linear Equations

- consists of two or more linear equations with the same variables considered together for which a common solution is desired.

Solution Set

- is an ordered pair of real numbers that satisfies both equations of the system.

To test whether an ordered pair is a solution of the given system:
a. Replace x and y with the given values in both equations.
b. Perform the indicated operations and find out if the ordered pair satisfies both equations, that is the left-hand side of each equation is equal to its right-hand side.

Kinds of Systems of Linear Equations

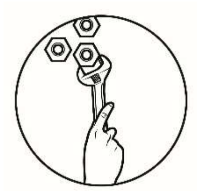
1. **Consistent-dependent**
   - a system of equations that could be rewritten as identical equations and have an **infinite solution**.
2. **Consistent-independent**
   - a system of equations that could NOT be rewritten as contradicting equations or identical equations, they stay different and has **one solution**.
3. **Inconsistent**
   - a system of equations that could be rewritten as contradicting equations and has **no solution**

ACTIVITY 2:

Complete the following diagram to summarize the three kinds of systems.

Kinds of Systems of Linear Equations

- **1.** ____________
  - **4.** ____________
- **Consistent**
  - **2.** ____________
    - **5.** ____________
  - **3.** ____________
    - **Infinite Solutions**



What I Can Do

ACTIVITY 3

A. State whether the given ordered pair is a solution of the system

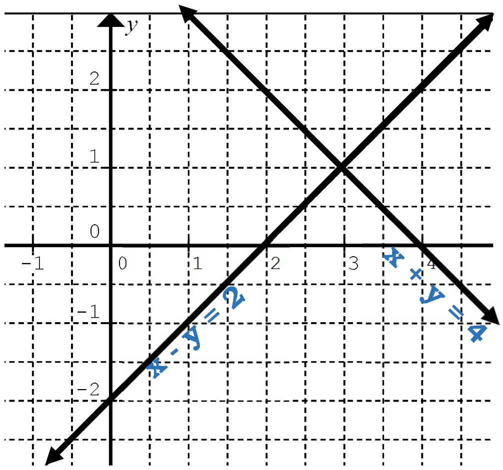
1. (2,1)
$$\begin{cases} 2x - y = 3 \\ x + y = 2 \end{cases}$$

2. (-1,2)
$$\begin{cases} 4x + y = -2 \\ 3x + 2y = 1 \end{cases}$$

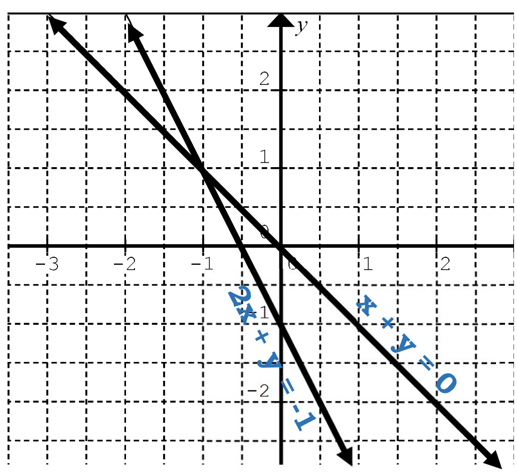
3. (3,-1)
$$\begin{cases} 2x = 3 - 3y \\ y = 2x - 7 \end{cases}$$

B. State the solution of each system.

4.



5.



Assessment

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

Determine whether the given systems are:

A. Consistent-dependent
B. Consistent-independent
C. Inconsistent

1. $\begin{cases} x - y = 3 \\ x + y = 2 \end{cases}$

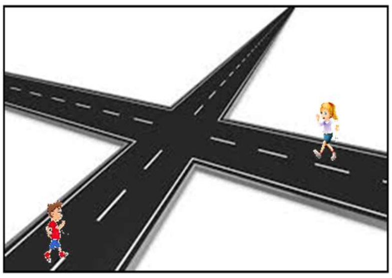
2. $\begin{cases} 3x + 3y = 6 \\ x + y = 2 \end{cases}$

3. $\begin{cases} x - 3y = 4 \\ 3x - 9y = 8 \end{cases}$

4. $\begin{cases} 2x - y = 1 \\ x + 4y = 3 \end{cases}$

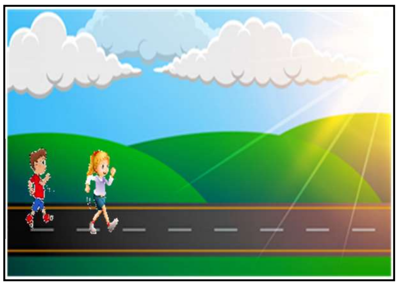
5. $\begin{cases} 5x - 4y = 3 \\ 10x - 8y = 6 \end{cases}$

Lesson 2 Graphing Systems of Linear Equations in Two Variables



Crossing Road

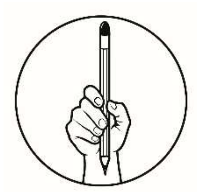
Aligned Road



Single Road

The given figures show cases where two friends walk on different roads and a case where they walk on the same road. The first case shows them walk on crossroads. Is it possible for them to meet? If so, how many times will they meet? The second case shows them walk on parallel roads. Is there a chance for them to meet? The last case shows a lone road. The two friends walk on the same road. The road taken by them is one and the same.

In algebra, one way of solving system of linear equations is by graphing them. By graphing the equations of a system, we can find out if the lines produced are crossing, aligned or jus t a single line.

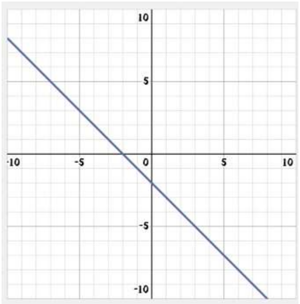


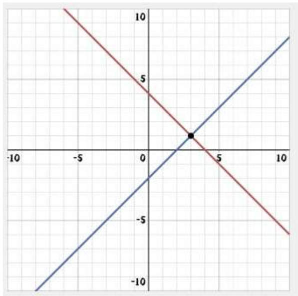
What I Know

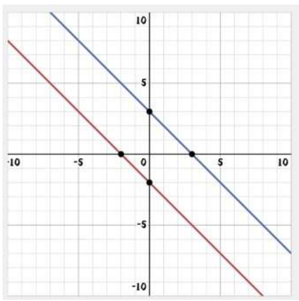
Choose the letter of the best answer. Write your answers on a separate sheet of paper.

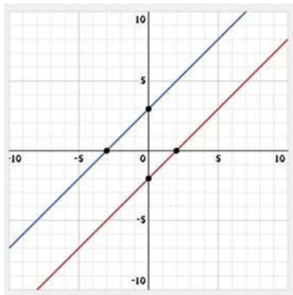
1. What kind of system of linear equations has intersecting lines as graphs of the equations?
   a. consistent-dependent   c. inconsistent
   b. consistent-independent   d. none of the above
2. Which of the following systems of linear equations has parallel lines as graphs of the equations?
   a. $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ c. $\begin{cases} x + y = -2 \\ x + y = 3 \end{cases}$
   b. $\begin{cases} x + y = -2 \\ 2x + 2y = -4 \end{cases}$ d. none of the above

3. Which of the following is the graph of a consistent-dependent system?

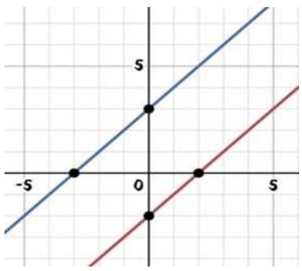
   a.


   c.


   b.


   d.


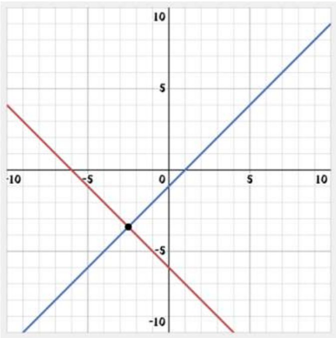
4. Which of the following systems of linear equations is being illustrated by the graph?

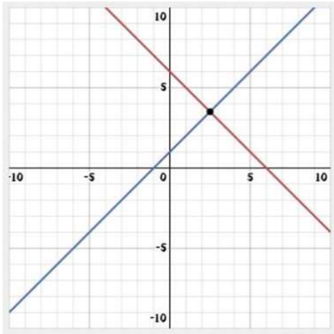


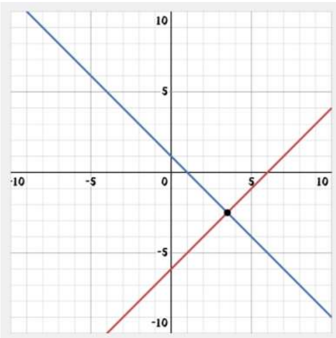
   a. $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ c. $\begin{cases} x + y = -2 \\ x + y = 3 \end{cases}$

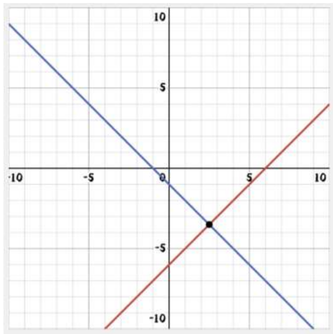
   b. $\begin{cases} x + y = -2 \\ 2x + 2y = -4 \end{cases}$ d. $\begin{cases} y - x = -2 \\ y - x = 3 \end{cases}$

5. Which of the following is the graph of $\begin{cases} x + y = 6 \\ x - y = -1 \end{cases}$?

a.


c.


b.


d.


What's In

RECALL

1. What is the x – intercept of x + y = 5?
   - ❖ Setting y to zero, you should get **5**.
2. What is the y – intercept of x – y = 3?
   - ❖ Replacing x with 0, your y –intercept should be **– 3**.
3. Rewrite x + y = 3 to y = mx + b, give its slope and y - intercept.
   - ❖ Is your slope – intercept form the same as **y = – x + 3**? Is your **m = – 1**? Is your **b = 3**? Very Good!
4. If the x – intercept of x + 2y = 5 is 5, at what point does its graph intersect the x – axis?
   - ❖ The graph intersects the x – axis at **(5,0)**.
5. If the y – intercept of x - 2y = 6 is -3, at what point does its graph intersect the y– axis?
   - ❖ The graph intersects the y – axis at **(0,-3)**. Is this also your answer? Good job!

Good, you are now ready for the next topic.

What's New

The easiest way to graph the system of linear equations is by using intercepts. For us to graph it we need to solve first the x-intercept and y-intercept of each equation in the system. Let's try it.

1. $\begin{cases} x + 2y = 4 \\ 2x + 4y = 6 \end{cases}$

Equation 1: $x + 2y = 4$

x – intercept, (Let y = 0)
$x + 2(0) = 4$
$x = 4$
$(4,0)$

y – intercept, (Let x = 0)
$0 + 2y = 4$
$2y = 4$
$\frac{2y}{2} = \frac{4}{2}$
$y = 2$
$(0,2)$

Equation 2: $2x + 4y = 6$

x – intercept, (Let y = 0)
$2x + 4(0) = 6$
$2x = 6$
$\frac{2x}{2} = \frac{6}{2}$
$(3,0)$

y – intercept, (Let x = 0)
$0 + 4y = 6$
$4y = 6$
$\frac{4y}{4} = \frac{6}{4}$
$y = \frac{3}{2}$ or 1.5
$(0,1.5)$

2. $\begin{cases} x - y = 3 \\ 3x - 3y = 9 \end{cases}$

Equation 1: $x - y = 3$

x – intercept, (Let y = 0)
$x - (0) = 3$
$x = 3$
$(3,0)$

y – intercept, (Let x = 0)
$0 - y = 3$
$-y = 3$
$\frac{-y}{-1} = \frac{3}{-1}$
$y = -3$
$(0,-3)$

Equation 2: $3x - 3y = 9$

x – intercept, (Let y = 0)
$3x - 3(0) = 9$
$3x = 9$
$\frac{3x}{3} = \frac{9}{3}$
$(3,0)$

y – intercept, (Let x = 0)
$3(0) - 3y = 9$
$-3y = 9$
$\frac{-3y}{-3} = \frac{9}{-3}$
$y = -3$
$(0,-3)$

3. $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

Equation 1: $x + y = 3$

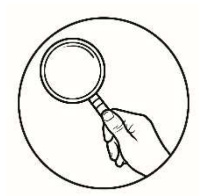
x – intercept, (Let y = 0)
$x + (0) = 3$
$x = 3$
$(3,0)$

y – intercept, (Let x = 0)
$0 + y = 3$
$y = 3$
$(0,3)$

Equation 2: $x - y = 1$

x − intercept, (Let y = 0)
$x - (0) = 1$
$x = 1$
$(1,0)$

y − intercept, (Let x = 0)
$0 - y = 1$
$-y = 1$
$(0, -1)$

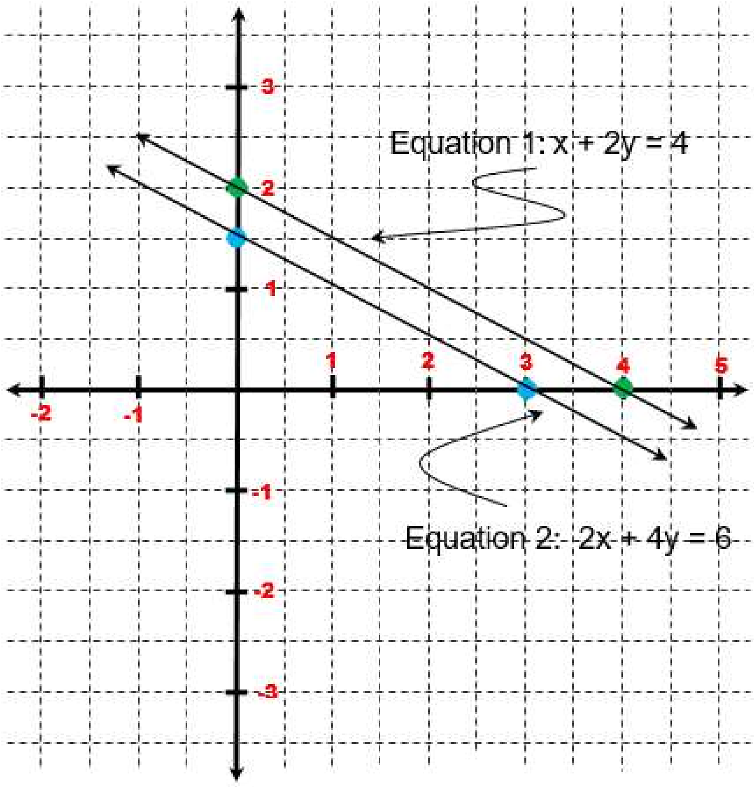


What is It

Let's draw the graph of each system of linear equations.

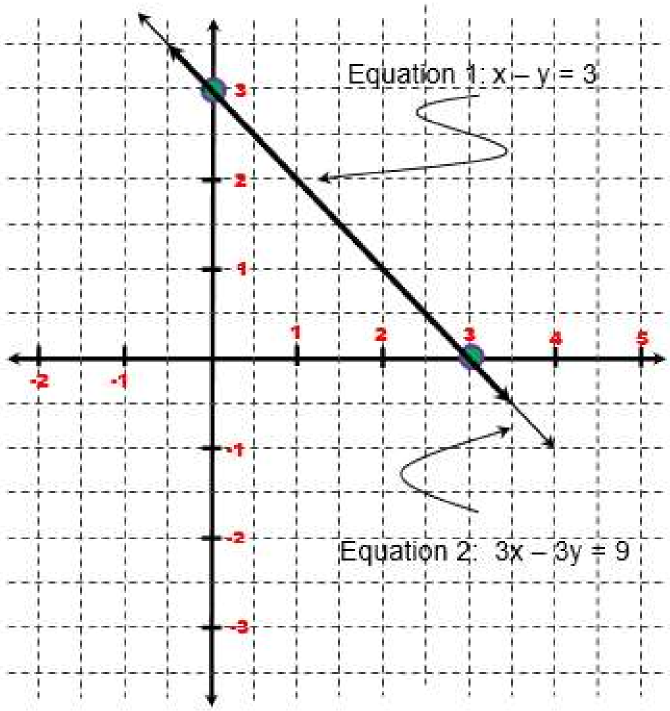
Steps in Graphing System of Linear Equations using the Intercepts
1. Identify the x-intercept and y-intercept of each equation in the system. 2. Plot the intercepts of both equations on the same Cartesian plane. 3. Connect the x-intercept and y-intercept.

1. $\begin{cases} x + 2y = 4 \\ 2x + 4y = 6 \end{cases}$



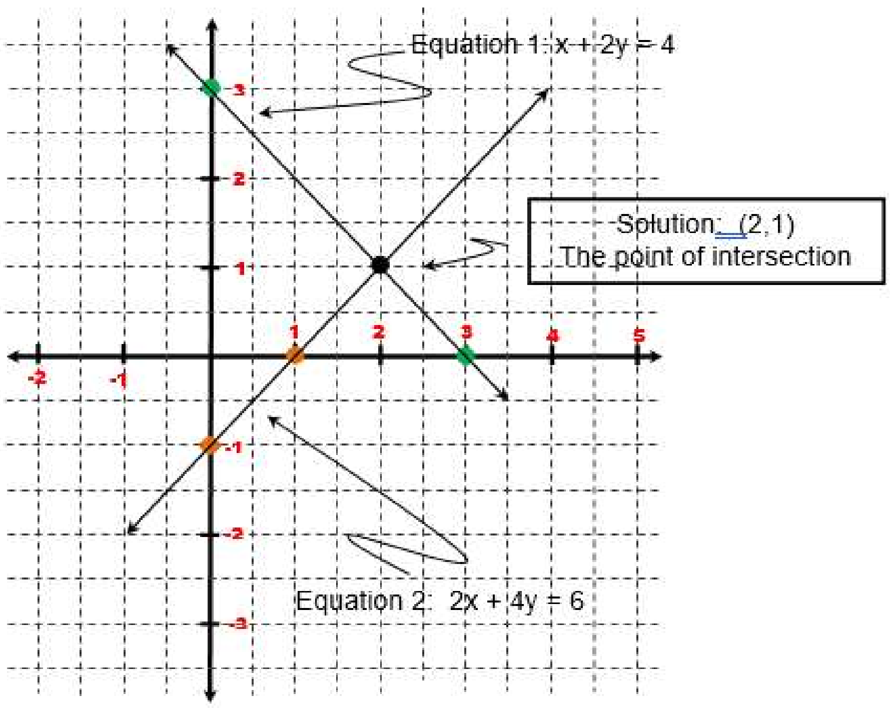
❖ The given system is **inconsistent** and the lines are **parallel**. If you have a doubt you can check whether both equations have the same slope.

2. $\begin{cases} x - y = 3 \\ 3x - 3y = 9 \end{cases}$

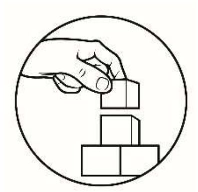


❖ The given system is **consistent-dependent** and the lines are **coinciding.**

3. $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$



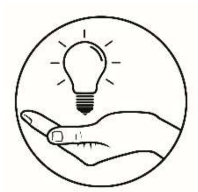
❖ The given system is **consistent-independent** and the lines are **intersecting.**



What's More

ACTIVITY 1: Graph, identify the kind of system and describe the graph of the following systems of linear equations.

1. $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ 2. $\begin{cases} x + y = -2 \\ x + y = 3 \end{cases}$ 3. $\begin{cases} x + y = -2 \\ 2x + 2y = -4 \end{cases}$



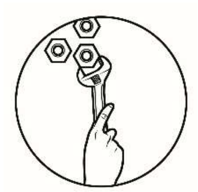
What I Have Learned

To solve a system of linear equations in two variables by graphing:

1. Find the intercepts of each linear equation.
2. Draw the graph of each linear equation in the same coordinate plane.
   a. If the lines **intersect** at a point, the coordinates of the intersection point give the solution of the system. This system has a unique solution; it is consistent and the equations are independent. The system is called **consistent-independent.**

   b. If the lines are **parallel**, there is no solution or the solution set is { }. This system is **inconsistent**.

   c. If the lines **coincide**, then the system has infinitely many solutions. The system is consistent and the equations are dependent also known as **consistent-dependent**.



What I Can Do

ACTIVITY 2: Complete the table.

Kind of System	Number of Solution	Graphs
inconsistent	1.______________________	2.______________________
consistent-independent	3.______________________	intersecting
4.______________________	infinite solutions	5.______________________

Assessment

Choose the letter of the corresponding graph of each system of linear equations.

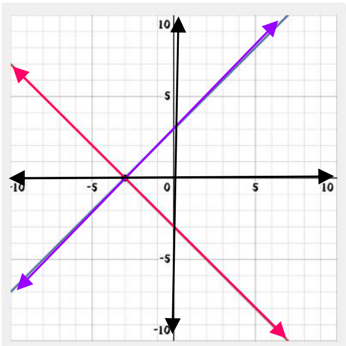
1. $\begin{cases} x + 2y = 2 \\ x + 2y = 4 \end{cases}$

2. $\begin{cases} x + y = 2 \\ 2x - y = 4 \end{cases}$

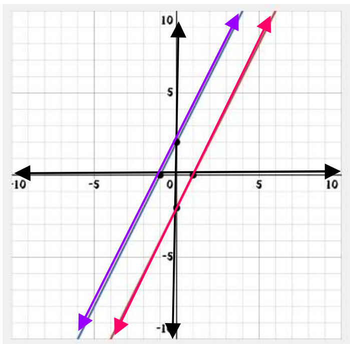
3. $\begin{cases} 2x - y = 2 \\ 2x - y = -2 \end{cases}$

4. $\begin{cases} x - y = -3 \\ 2x - 2y = -6 \end{cases}$

5. $\begin{cases} x + y = -3 \\ 2x - 2y = -6 \end{cases}$



A



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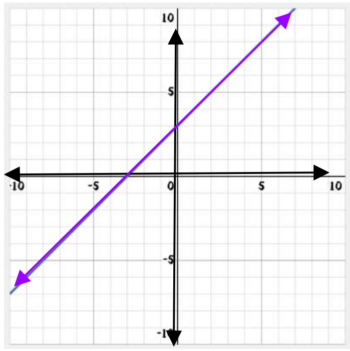

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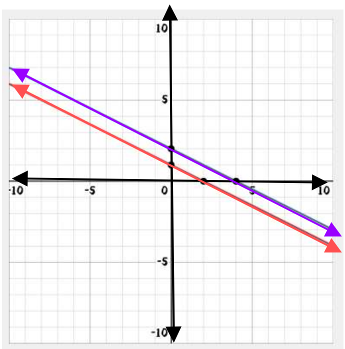

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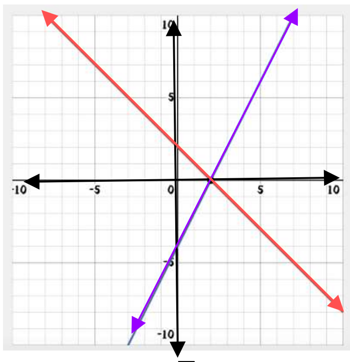
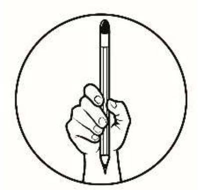


E

Lesson 3	Categorize Systems of Linear Equations

Categorizing, organizing and classifying are some essential things to be done to unify ideas. Hence, even in Mathematics you will encounter these words. In the previous discussion, we classify system of linear equations according to the number of solutions and by its graph. In this lesson, we will be using ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ to categorize the kinds of system being illustrated.



What I Know

Choose the letter of the best answer. Write your answers on a separate sheet of paper.

1. What kind of system of linear equations is $\begin{cases} x - 2y = 4 \\ 3x - 6y = 8 \end{cases}$?

   a. consistent-dependent　　c. inconsistent
   b. consistent-independent　　d. none of the above

2. What is the ratio of a_1 to a_2 of the system $\begin{cases} x - y = 8 \\ 3x - 2y = 4 \end{cases}$?

   a. $\frac{1}{3}$　　b. $\frac{1}{2}$　　c. 2　　d. 3

3. What kind of system of linear equations has the ratio $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$?

   c. consistent-dependent　　c. inconsistent
   b. consistent-independent　　d. none of the above

4. What is the ratio of the systems of linear equations with infinite solutions?

   a. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$　　b. $\frac{a_1}{a_2} = \frac{b_1}{b_2}$　　c. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$　　d. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

5. Which of the following systems of linear equations has the ratio $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$?

   a. consistent-dependent　　c. inconsistent
   b. consistent-independent　　d. none of the above

What's In

RECALL

1. Change 2x + y – 5 = 0 to standard form.
   - ❖ The answer is 2x + y = 5.
2. What is the standard form of 2x = - y + 2?
   - ❖ Is it 2x + y = 2? Correct!
3. What is the simplest form of $\frac{3}{6}$?
   - ❖ Is your answer equal to $\frac{1}{2}$? Very Good!
4. In the equation ax + by = c, a is the coefficient of x. What is the value of a in 2x + 3y = 6?
   - ❖ Is it 2? Good Job!
5. In the equation ax + by = c, b is the coefficient of y while c is the constant term. What is the value of b in 2x + 3y = 6? What is the value of c?
   - ❖ Is it 3? Is it 6? Well done!

I think you are now ready for the next topic.

What's New

How do you find the previous lesson about graphing system of linear equations? Is it easy or hard? If you find it easy it's good to know, but if you find it hard let me help you. Let's sum up the important keys of Lesson 2.

We have learned earlier how to: (1) graph the equations by intercepts (we need to have some paper and writing materials for this), (2) look at the graphs of the equations and describe them, (3) categorize the kinds of system of linear equations.

But what if we could describe and categorize system of linear equations without graphing them?

Confusing? Let's explore this idea.

Remember that the standard form of the linear equation is $ax + by = c$, where "a" is the coefficient of x, "b" is the coefficient of y and "c" is the constant. And since we have two equations, it means we have two sets of coefficients and constant.

$$a_1x + b_1y = c_1 \qquad Equation\ 1$$
$$a_2x + b_2y = c_2 \qquad Equation\ 2$$

Now, make a ratio from the coefficient of x, it should look like this $\frac{a_1}{a_2}$. A ratio from the coefficient of b, looks like $\frac{b_1}{b_2}$. And finally, a ratio of the constants $\frac{c_1}{c_2}$. Now, let's have some drill.

We can now categorize the system of linear equations. How? By comparing the ratios, we just made, see if they are equal or not.

Note: In order to categorize correctly the ratios, all equations should be in standard form, $ax + by = c$.

Let's try it.

Categorize the given system of linear equations. $\begin{cases} x + 2y = 6 \\ x = 2y + 2 \end{cases}$

$x + 2y = 6$	Equation 1 is in standard form
$x = 2y + 2$	Equation 2 is in NOT standard form, rewrite
$x - 2y = 2$	Equation 2 now is in standard form

Now that all our equations are in standard form, let's get the ratios

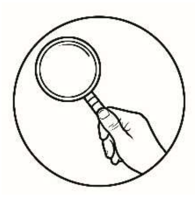
$\frac{a_1}{a_2} = \frac{1}{1} or\ 1$, $\frac{b_1}{b_2} = \frac{2}{-2} or -1$, and $\frac{c_1}{c_2} = \frac{6}{2}\ or\ 3$

Now, let's compare, $1 \neq -1 \neq 3$. All ratios are not equal to each other. Then what?

To categorize the system let us be guided by this guideline.

Ratios	**Kinds of System of Linear Equations**
$\boldsymbol{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$	consistent-independent
$\boldsymbol{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$	inconsistent
$\boldsymbol{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$	consistent-dependent

Therefore, the given system is **consistent-independent**, since all of the ratios are not equal to each other.



What is It

Let us verify by answering the system of equations given from the previous lesson.

1. $\begin{cases} x + 2y = 4 \\ 2x + 4y = 6 \end{cases}$ is an inconsistent system;

2. $\begin{cases} x - y = 3 \\ 3x - 3y = 9 \end{cases}$ is a consistent-dependent system; and

3. $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ is a consistent-independent system.

Let us now compare the ratios $\frac{a_1}{a_2}$**,** $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ in each system.

2. $\begin{cases} x + 2y = 4 & \rightarrow a = 1, b = 2, c = 4 \\ 2x + 4y = 6 \rightarrow a = 2, b = 4, c = 6 \end{cases}$

The ratio of $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{4}$ or $\frac{1}{2}$,

and $\frac{c_1}{c_2} = \frac{4}{6}$ or $\frac{2}{3}$

Notice that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and we know that the given system is an inconsistent.

Hence if $\boldsymbol{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$, the system is **inconsistent** and its graph are **parallel**.

1. $\begin{cases} x - y = 3 & \rightarrow a = 1, b = -1, c = 3 \\ 3x - 3y = 9 \rightarrow a = 3, b = -3, c = 9 \end{cases}$

The ratio of $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-1}{-3}$ or $\frac{1}{3}$,

and $\frac{c_1}{c_2} = \frac{3}{9}$ or $\frac{1}{3}$

Notice that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and we know that the given system is a consistent-dependent.

Hence if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the system is **consistent-dependent** and its graph are **coinciding**.

3. $\begin{cases} x + y = 3 & \rightarrow a = 1, b = 1, c = 3 \\ x - y = 1 \rightarrow a = 1, b = -1, c = 1 \end{cases}$

The ratio of $\frac{a_1}{a_2} = \frac{1}{1}$ or 1, $\frac{b_1}{b_2} = \frac{1}{-1}$ or -1, and $\frac{c_1}{c_2} = \frac{3}{1}$ or 3

Notice that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and we know that the given system is a consistent-independent.

Note that in this kind of system the ratio $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$ only matters. Hence if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the system is **consistent-independent** and its graph are **intersecting**.

What's More

ACTIVITY 1: Identify the kind of system and describe the graph of each of the following systems of linear equations using ratios.

1. $\begin{cases} 2x + y = 3 \\ 2x - 2y = 3 \end{cases}$

2. $\begin{cases} 2x - y = 2 \\ 2x = y + 3 \end{cases}$

3. $\begin{cases} 4x + 4y = -4 \\ 3x + 3y = -3 \end{cases}$

4. $\begin{cases} 2x - y - 2 = 0 \\ y = 2x + 1 \end{cases}$

5. $\begin{cases} x + 2y = 4 \\ x + 3y = 2 \end{cases}$

What I Have Learned

To identify the kind of system of linear equations in 2 variables like

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

Get the ratios of $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. If

a. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system is inconsistent;

b. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the system is consistent-dependent;

c. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the system is consistent-independent.

What I Can Do

ACTIVITY 2: Complete the table.

Kind of System	Number of Solution	Graphs	Ratios
1.______________	no solution	parallel	2. ____________
consistent-independent	one solution	3._________________	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
consistent-dependent	infinite solutions	4.________________	5.______________

Assessment

Directions: Choose the letter of the best answer. Write your answers on a separate sheet of paper.

1. What kind of system of linear equations is $\begin{cases} x + 2y = 2 \\ 3x + 6y = 6 \end{cases}$?
   a. consistent-dependent
   b. consistent-independent
   c. inconsistent
   d. none of the above

2. What is the ratio of b_1 to b_2 of the system $\begin{cases} 2x - y = 8 \\ 3x - 2y = 4 \end{cases}$?
   a. $\frac{1}{3}$
   b. $\frac{1}{2}$
   c. 2
   d. 3

3. What kind of system of linear equations has the ratio $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$?
   a. consistent-dependent
   b. consistent-independent
   c. inconsistent
   d. none of the above

4. What kind of system of linear equations is represented by graphs that coincide?
   a. consistent-dependent
   b. consistent-independent
   c. inconsistent
   d. none of the above

5. Which of the following systems of linear equation has intersecting lines as graphs of the equations?
   a. $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$
   b. $\begin{cases} x + y = -2 \\ 2x + 2y = -4 \end{cases}$
   c. $\begin{cases} x + y = -2 \\ x + y = 3 \end{cases}$
   d. none of the above

Answer Key

Lesson 1:

What I Know

1. b
2. d
3. c
4. c
5. a

What's More

1. consistent-independent / one solution
2. consistent-dependent / finite solutions
3. inconsistent / no solution
4. consistent-independent / one solution
5. consistent-dependent / finite solutions

What I Have Learned

1. Inconsistent
2. Independent
3. Dependent
4. No Solution
5. One Solution

What I Can Do

1. a solution
2. a solution
3. a solution
4. (3,1)
5. (-1,1)

Assessment

1. b
2. a
3. c
4. b
5. a

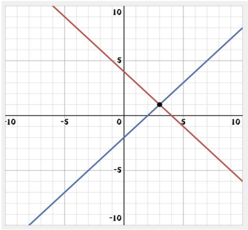
Lesson 2:

What I Know

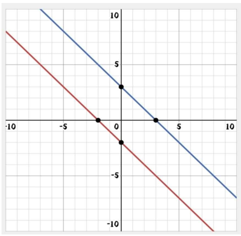
1. b
2. c
3. a
4. d
5. c

What's More

1.



2.



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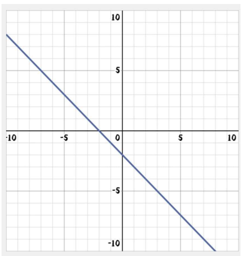

What I Can Do

1. no solution
2. parallel
3. one solution
4. consistent-dependent
5. coinciding

Assessment

1. D
2. E
3. B
4. C
5. A

Lesson 3:

What I Know

1. c
2. a
3. b
4. d
5. c

What's More

1. consistent-independent / intersecting
2. inconsistent / parallel
3. consistent-dependent / coinciding
4. inconsistent / parallel
5. consistent-independent / intersecting

What I Can Do

1. inconsistent
2. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
3. intersecting
4. coinciding
5. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Assessment

1. A
2. B
3. C
4. A
5. A

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