8

Mathematics

Quarter 1 – Module 3:
Rational Algebraic Expressions

ALTERNATIVE DELIVERY MODE

ADM

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Alternative Delivery Mode
Quarter 1 – Module 3: Rational Algebraic Expressions
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Rational Algebraic Expressions

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DEPARTMENT OF EDUCATION

Introductory Message

For the facilitator:

Welcome to the Grade 8 Mathematics Alternative Delivery Mode (ADM) Module on Rational Algebraic Expressions.

This module was collaboratively designed, developed and reviewed by educators from both public and private institutions to assist you, the teacher or facilitator in helping learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage students into guided and independent learning activities at their own pace. Furthermore, this also aims to help learners acquire the needed 21st Century Skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist learners as they do the tasks included in the module.

For the learner:

Welcome to the Mathematics 8 Alternative Delivery Mode (ADM) Module on Rational Algebraic Expressions.

The hand is one of the most symbolized parts of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish many things. Hence, the hand in this learning resource signifies that you as a learner are capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

 ***What I Need to Know***	This will give you an idea of the skills or competencies you are expected to learn in the module.
 ***What I Know***	This part includes an activity that aims to check what you already know about the lesson you are to take. If you get all the answers correctly (100%), you may skip this module.
 ***What's In***	This is a brief drill or review to help you link the current lesson with the previous one.
 ***What's New***	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
 ***What is It***	This section provides a brief discussion of the lesson to help you discover and understand new concepts and skills.
 ***What's More***	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
 ***What I Have Learned***	This part offers questions, or a fill-in-the-blank sentence/paragraph, to enable you to process what you have learned from the lesson.
 ***What I Can Do***	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.

 ***Assessment***	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
 ***Additional Activities***	In this portion, other activities will be given to you to enrich your knowledge or skill of the lesson learned. This also aids in the retention of learned concepts.
 ***Answer Key***	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer ***What I Know*** before moving on to the other activities included in the module.
3. Read the instructions carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

CHAPTER 2

RATIONAL ALGEBRAIC EXPRESSIONS

A landscape designer creates attractive, functional outdoor spaces for homeowners, businesses, schools and other organizations. Drawing plans, selecting plants and installing structures like walkways and water features are among his job duties. In small gardens where space is limited, landscape designers can create a series of mini views within the garden itself, offering interest in every direction while using plants to provide frameworks. Knowing the total lot area, landscape designers can divide a garden into small parts, even when small, to make it seem larger.

Fraction, on the other hand, is part of a whole. The numerator tells what part of the whole was taken while the denominator tells us into how many parts the whole was divided. Meanwhile, a rational expression is an expression that can be expressed in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0. If p and q are polynomials, then a rational algebraic expression comes in.

What I Need to Know

This module was designed and written with you in mind, and to help you master the nature and operations on Rational Algebraic Expressions. The scope of this module is helpful in answering many real-life problems. The language used recognizes the diverse vocabulary level among students, and the lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is divided into three lessons, namely:

- Lesson 1 – Illustrate Rational Algebraic Expressions
- Lesson 2 – Simplify Rational Algebraic Expressions

- Lesson 3 – Multiplication and Division of Rational Algebraic Expressions

After going through this module, you are expected to:

1. describe a rational algebraic expression;
2. give the domain of a rational algebraic expression;
3. discuss how to simplify rational algebraic expressions;
4. simplify rational algebraic expressions;
5. discuss the steps in multiplying and dividing rational algebraic expressions;
6. multiply and divide rational algebraic expressions; and
7. perform the assigned task accurately.

Lesson 1 Rational Algebraic Expressions

We seldom encounter problems involving distances, area and work problems. You might ask, how long will it take two people to finish a job if one works three times faster than the other? Or how fast does a car travel as it covers a certain distance for an hour? These problems contain expressions called rational algebraic expressions. These are the type of expressions we will study in this module.

What I Know

Direction: Choose the letter of the best answer and write it on a separate sheet of paper.

1. Which of the following is NOT a rational algebraic expression?

A. $\frac{3x}{2}$ B. $\frac{4}{y+7}$ C. $a^2 + 2b$ D. $5\sqrt{c}$

2. Which of the following is NOT a rational expression?

A. $3a + 5$ B. $\frac{2}{3}$ C. $\frac{x-3}{2x^2-2}$ D. $\frac{4x}{\sqrt{x}}$

3. For which value of x will $\frac{5x}{3x-2}$ be meaningless?

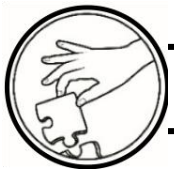
A. $\frac{-3}{2}$ B. $\frac{-2}{3}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

4. For what value of y will $\frac{7}{3y-6}$ be meaningless?

A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. -2 D. 2

5. State the value/s for which $\frac{4+a}{a^2+2a}$ is undefined.

A. 2 and 1 B. -2 and 0 C. 2 and 0 D. 1 and 0



What's In

Let us recall:

1. Is $0.\overline{33}$ a rational number?
2. What are the factors of $x^2 + 5x + 6$??
3. If $x + 2 = 0$, what is the value of x?
4. Is $5x + 7$ a polynomial?
5. Can we consider $5\sqrt{x}$ as monomial?

What's New

Activity 1: Let us explore.

1.Given a set of algebraic expression cards

$y+1$	x^2+x-12	$x^{2/3}+1$	9
$y-3$	$2y^2+13y$+15	$x^3+\sqrt{x}-3$	

2. Form a fraction whose numerator and denominator are both polynomials. Put it under the column Rational Algebraic Expression.

3. Form a fraction whose numerator is not a polynomial but the denominator is a polynomial and vice-versa. Put it under the column Not a Rational Algebraic Expression.

Rational Algebraic Expression	Not a Rational Algebraic Expression

4. Define in your own words rational algebraic expression.

What is It

The following are examples of rational algebraic expressions:

$$\frac{2y^2+13y+15}{y+1}, \frac{y+1}{y-3}, \frac{9}{x^2+x-12}$$

On the other hand, the following cannot be considered as rational algebraic expressions: $\frac{x^{2/3}+1}{9}$, $\frac{x^2+\sqrt{x}-3}{y+1}$, $\frac{y-3}{x^2+\sqrt{x}-3}$. Remember that dividing by zero is not allowable, or else the expression becomes meaningless. In $\frac{k}{k+3}$, what value of k will make the denominator zero? Is it -3?

Definition

A **rational algebraic expression** can be expressed as a quotient of two polynomials, where the denominator is not equal to zero. In symbols; $\frac{P}{Q}$, where $P \text{ and } Q$ are polynomials and $Q \neq 0$.

The set of numbers which can be used as replacements for the variable in a rational expression is known as its **domain.** What is the domain of $\frac{a}{2}$? Is there variable in the denominator? None? So, there is no possibility that the denominator will become zero, its domain is the set of all real numbers.

Examples*:*

A. Which of the following is **NOT** a rational algebraic expression?

a. $\frac{3x}{2}$ b. $\frac{4}{y+7}$ c. a^2+2b d. $5\sqrt{c}$

$5\sqrt{c}$ is not a monomial because there is a variable in the radicand, so it is not a rational algebraic expression. How about the other expressions?

Yes, they are all rational algebraic expressions.

B. Give the domain of $\frac{2+4c}{c+5}$.

What number or numbers can you use to replace c? The meaningful replacements are all real numbers except (-5), that is, its domain is the set of all real numbers except (-5). How did we get (-5)? If $c + 5 = 0$, then $c = -5$.

C. For what value of y will $\frac{7}{3y-6}$ be meaningless?

The expression is meaningless or undefined if the denominator is zero.

If $3y - 6 = 0$,	then,
$3y = 0 + 6$	Addition property
$y = \frac{6}{3}$	Multiplication property
$y = 2$	Hence, $\frac{7}{3y-6}$ is meaningless if $y = 2$.

D. Find the domain of $\frac{3xy}{x^2+5x+6}$.

Can you readily give its domain? No? Let us get the factors of the denominator. Do you agree that $x^2 + 5x + 6 = (x + 2)(x + 3)$?
Can you now give its domain? Not yet? Let us set the denominator equal to zero, then by Zero Product property

If $(x + 2)(x + 3) = 0$, then $(x + 2) = 0$ or $(x + 3) = 0$. This implies that $x = -2$ or $x = -3$ and these two numbers cannot replace x. What is now the domain of the given expression? Is your domain the set of all real numbers except -2 and -3?

Congratulations! You are correct.

What's More

A. Determine if the following are rational algebraic expressions. Explain your answer.

1. $\frac{3x-1}{9}$ 2. $\frac{x^2-4x+4}{\sqrt{x}-5}$ 3. $2x^3 + 3x^2 + x + 1$

B. What value(s) of x will make the expression undefined?

1. $\frac{3x^2-2x+1}{5x}$ 2. $\frac{x^3+1}{x^2-5x+6}$ 3. $\frac{x^2+x+1}{5x+10}$

What I Have Learned

A **rational algebraic expression is** a representation which can be written as the quotient of two polynomials, where the denominator is not zero.

Domain is the set of real numbers which can be used to replace a variable in a rational algebraic expression.

To get the domain of a rational algebraic expression

- set the denominator to zero
- find the solution set of the resulting equation

What I Can Do

Answer the following.

1. Which of the following are rational algebraic expressions?

A. $\frac{4x}{x+10}$ B. $\frac{3\sqrt{x}+1}{9}$ C. $\frac{x^2-4}{x^{2/3}}$ D. $\frac{x-1}{\sqrt{5x}+4}$

2. Determine the value of x that will make the expression undefined.

A. $\frac{x+5}{2x}$ B. $\frac{x^2-1}{x^2+x-12}$

Assessment

Answer the following.

1. Which of the following is NOT a rational algebraic expression?

A. $\frac{3}{\sqrt{2b}}$ B. $\frac{a-4}{5}$ C. c^2+2bc D. 5

2. Which of the following is a rational expression?

A. $\frac{2y}{\sqrt{x+1}}$ B. $2x^3-1$ C. $y^{\frac{1}{2}}+1$ D. $\sqrt{5x}+1$

3. Give the domain of $\frac{k-1}{k+1}$.

A. set of all real numbers except 1
B. set of all real numbers except -1
C. set of all real numbers except 0
D. set of all real numbers except ½.

4. For what value of a will $\frac{a^2}{2a+8}$ be meaningless?

A. $\frac{1}{-4}$ B. -4 C. $\frac{1}{4}$ D. 4

5. Give the value/s of v for which $\frac{10+v}{w^2+4w}$ is undefined.

A. 0 and 4 B. 0 and -4 C. 0 D. 4

Lesson 2 Simplifying Rational Algebraic Expressions

How do you simplify fractions? How will you know that a fraction is in its simplest form? Remember that a fraction, $\frac{a}{b}$, where $b \neq 0$, is said to be in simplest form or lowest term when a and b have no common factor except 1.

What I Know

Direction: Choose the letter of the best answer and write it on a separate sheet of paper.

1. Which of the following expressions CANNOT be simplified?

A. $\frac{m-1}{m^2-1}$ B. $\frac{x^2-y^2}{x+y}$ C. $\frac{4n+12}{n+3}$ D. $\frac{x+y}{x-y}$

2. Which fraction is written in the lowest terms?

A. $\frac{18}{27}$ B. $\frac{56}{49}$ C. $\frac{13}{52}$ D. $\frac{25}{39}$

3. Which expression is in simplest form?

A. $\frac{2y-1}{2y+1}$ B. $\frac{3y+6}{3}$ C. $\frac{y-1}{1-y}$ D. $\frac{y^2+y}{2y}$

4. Which of the following expressions is equal to -1?

A. $\frac{3m-1}{1-3m}$ B. $\frac{5-3x}{-3x+5}$ C. $\frac{x+y}{y-x}$ D. $\frac{x-1}{x}$

5. Write the rational expression $\frac{36x^7y^3}{12x^2y}$ in lowest term.

A. $\frac{3x^5}{y^2}$ B. $3x^5y^2$ C. $3x^4y^2$ D. $3x^5y$

What's In

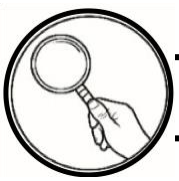
Recall:

1. What are the factors of $x^2 - y^2$?
2. What is the factored form of $3c - 12$?
3. What is the simplest form of $\frac{4}{12}$?
4. What is the common factor of $(m - 1)$ and $(m^2 - 1)$?
5. Is $(3 - c)$ the same as $(c - 3)$?

What's

Activity 1: Match column A to its equivalent fraction in column B.

Column A	Column B
$\frac{36}{24}$	$\frac{5}{7}$
$\frac{15}{25}$	$\frac{3}{2}$
$\frac{16}{56}$	$\frac{2}{3}$
$\frac{6}{9}$	$\frac{3}{5}$
$\frac{10}{14}$	$\frac{2}{7}$



What is It

A rational expression is in the simplest form when the numerator and the denominator have no common factor other than 1. We can simplify rational expressions in the same way as we simplify fractions. Simplifying rational expressions

requires good factoring skills. To simplify rational expressions, just factor the numerator and denominator, then cancel out factors equivalent to 1.

Examples:

1. Which of the following expressions CANNOT be simplified?

a. $\frac{m-1}{m^2-1}$ b. $\frac{x^2-y^2}{x+y}$ c. $\frac{4n+12}{n+3}$ d. $\frac{x+y}{x-y}$

Solutions:

a. $\frac{m-1}{m^2-1} = \frac{m-1}{(m-1)(m+1)}$

Since m^2-1 is a difference of two squares and $\frac{m-1}{m-1} = 1$

Hence, $\frac{m-1}{m^2-1} = \frac{\cancel{m-1}}{\cancel{(m-1)}(m+1)} = \frac{1}{m+1}$

b. $\frac{x^2-y^2}{x+y} = \frac{(x+y)(x-y)}{x+y}$

Again, x^2-y^2 is a difference of two squares and $\frac{x-y}{x-y} = 1$

$\frac{x^2-y^2}{x+y} = \frac{\cancel{(x+y)}(x-y)}{\cancel{x+y}} = x-y$

c. $\frac{4n+12}{n+3} = \frac{4(n+3)}{n+3}$

Factor out 4 in 4n + 12, and then cancel out factors equivalent to 1

$\frac{4n+12}{n+3} = \frac{4\cancel{(n+3)}}{\cancel{n+3}} = 4$

d. $\frac{x+y}{x-y}$

The numerator and denominator are prime polynomials since each expression has no polynomials factors with integral coefficients other than itself and 1.

Hence the expression that can no longer be simplified is $\frac{x+y}{x-y}$.

2. What is the simplest form of $\frac{3c-6}{c-2}$?

Solution: $\frac{3c-6}{c-2} = \frac{3(c-2)}{c-2}$

Factor out 3 in the numerator and then cancel out factors equivalent to 1

$\frac{3c-6}{c-2} = \frac{3\cancel{(c-2)}}{\cancel{c-2}} = 3$

3. Express $\frac{4a^2+4ab+b^2}{2a+b}$ in simplest form.

Solution: $\frac{4a^2+4ab+b^2}{2a+b} = \frac{(2a+b)(2a+b)}{2a+b}$

Factor $4a^2 + 4ab + b^2$ and cancel out $\frac{2a+b}{2a+b}$ since it is equal to 1

$$\frac{4a^2+4ab+b^2}{2a+b} = \frac{\cancel{(2a+b)}(2a+b)}{\cancel{2a+b}} = 2a + b$$

4. Simplify $\frac{3n-n^2}{n^2-2n-3}$.

Solution: $\frac{3n-n^2}{n^2-2n-3} = \frac{n(3-n)}{(n-3)(n+1)}$

Factor and look for opposites

3 – n = –1 (– 3 + n) or –1(n – 3)

write n (3 – n) as n(–1)(n – 3) and cancel out $\frac{n-3}{n-3}$

$$\frac{3n-n^2}{n^2-2n-3} = \frac{-1(n)(n-3)}{(n-3)(n+1)}$$

$$\frac{3n-n^2}{n^2-2n-3} = \frac{\cancel{-1(n)(n-3)}}{\cancel{(n-3)}(n+1)} = \frac{-n}{n+1}$$

What's More

A. Simplify the following rational algebraic expressions. None of the denominators is zero.

1. $\frac{12xy^3}{28x^3y^2}$
2. $\frac{25b^4c^5}{15b^4c^3}$
3. $\frac{32x^3}{16x^2-8x}$
4. $\frac{d+2}{5d^2+7d-6}$
5. $\frac{m^2+5m+6}{m^2-4}$

What I Have Learned

Rational Algebraic Expression is in the simplest form if the numerator and the denominator are relatively prime which means that they have no common factor except 1.

To simplify a rational algebraic expression,

a. factor the numerator and the denominator and then
b. cancel out factors equal to 1.

What I Can Do

A. Simplify each of the following rational expressions.

1. $\frac{(x+1)^2}{x^2-1}$
2. $\frac{m^2-9}{(m-3)^2}$
3. $\frac{a^3-4a}{a^3-4a^2+4a}$
4. $\frac{x-x^4}{x+x^3}$
5. $\frac{3x^2-3y^2}{6(x+y)^2}$

Assessment

A. Direction: Choose the letter of the best answer and write it on a separate sheet of paper.

1. Which of the following expressions CANNOT be simplified?

A. $\frac{w-1}{w+1}$ B. $\frac{h^2-k^2}{h+k}$ C. $\frac{4a+12}{a+3}$ D. $\frac{m+n}{m^2-n^2}$

2. What is the simplest form of $\frac{2a-6}{a-3}$?

A. $a-2$ B. 2 C. $a+2$ D. -2

3. Express $\frac{c^2+5c+6}{c+3}$ in simplest form.

A. $c+2$ B. $c+1$ C. $c-2$ D. $c-1$

4. Simplify $\frac{5n-n^2}{n^2-2n-15}$.

A. $\frac{5}{n+3}$ B. $\frac{n}{n+3}$ C. $-\frac{n}{n+3}$ D. $-\frac{5}{n+3}$

5. Which of the following will give 2 when simplified?

A. $\frac{x+1}{2x+2}$ B. $\frac{2b-10}{b-5}$ C. $\frac{4c}{8c}$ D. $\frac{2p+4}{8+4p}$

Lesson 3 Multiplication and Division of Rational Algebraic Expressions

You learned from the previous lesson that simplifying rational algebraic expressions is basically the same as simplifying rational numbers. We use basic theorems on multiplication and division of rational numbers to multiply and divide rational algebraic expressions.

What I Know

Direction: Choose the letter of the correct answer and write it on a separate of paper.

1. Find the product of $\frac{x^2-1}{x}$ and $\frac{2x}{x+1}$.

A. $2x(x-1)$ B. $2(x-1)$ C. $(x-1)(x+1)$ D. $\frac{x-1}{x+1}$

2. Multiply and simplify: $\frac{2a}{5b}$ and $\frac{10b^2}{a^2}$.

A. $\frac{20ab^2}{5a^2b}$ B. $\frac{4b}{a}$ C. $4ab$ D. $\frac{4a}{b}$

3. What is the quotient when $\frac{3y+15}{y}$ is divided by $\frac{y+5}{y}$?

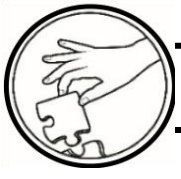
A. 3 B. $\frac{3}{y+5}$ C. $\frac{3y+15}{y+5}$ D. $\frac{3y}{y+5}$

4. Divide: $\frac{5x^3}{4} \div \frac{15x}{8}$.

A. $2x^2$ B. $\frac{2x^2}{3}$ C. $\frac{x^2}{3}$ D. $\frac{x^2}{6}$

5. The area of a rectangle is $\frac{2x+1}{4x^2-1}\ cm.^2$. If the length of the rectangle is $\frac{2}{2x-1}cm$, then the width is __ cm.

A. $\frac{2}{2x-1}$ B. $\frac{1}{2}$ C. $\frac{2}{2-x}$ D. $2x$



What's In

Recall: Answer the following.

1. What is the factored form of $a^2 - 25$?
2. What is the simplest form of $\frac{3z-6}{4z-8}$?
3. What is the greatest common factor of $(3c^2 + 15c)$ and $(3c^3 - 75c)$?
4. What is the reciprocal of $\frac{x+y}{x^2+xy}$?
5. What is the greatest common factor of $y^2 - 2y - 3$ and $y^2 - 7y + 12$?

What's New

Activity 1: Fraction Caption Game

Rules:

a. Multiply as indicated in each box.

b. Write the word which corresponds to your answer in the answer box. The first is done for you.

Ex. $\frac{7}{8} \cdot 8 = 7$ Find 7 in the answer box and write math.

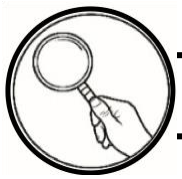
c. Decode the hidden message.

Problem Box

	$\frac{7}{8} \cdot 8$	$\frac{3}{4} \cdot \frac{6}{9}$	$\frac{1}{2} \cdot \frac{4}{7}$	$\frac{15}{2} \cdot \frac{10}{3}$	$9 \cdot \frac{2}{6}$
	MATH	MIND	CAN	AN	NEW
$\frac{65}{100} \cdot \frac{72}{96}$	$6 \cdot \frac{1}{4}$	$\frac{24}{32} \cdot 18$	$5 \cdot \frac{10}{25}$	$\frac{3}{8} \cdot 2$	$\frac{55}{121} \cdot \frac{33}{125}$
HOW	DO	OLD	WITH	MATH	YOU

Answer Box

	$\frac{39}{80}$	$\frac{2}{7}$	$\frac{3}{25}$	$\frac{3}{2}$	3
7 MATH	2	25	$\frac{27}{2}$	$\frac{3}{4}$	$\frac{1}{2}$



What is It

How do you multiply fractions? You get your answer by finding the quotient of the product of the numerators and the product of the denominators. Rational algebraic expressions are also multiplied in the same manner. In symbols if m, n, p and q are polynomials, such that n ≠ 0 and q ≠ 0, then

$$\frac{m}{n} \bullet \frac{p}{q} = \frac{mp}{nq}.$$

Examples:

1. What is the product of $\frac{h^2}{2k} \bullet \frac{k^2}{2h}$?

Solution: $\frac{h^2}{2k} \bullet \frac{k^2}{2h} = \frac{h \bullet h}{2k} \bullet \frac{k \bullet k}{2h}$ Definition of exponent

$= \frac{\cancel{h} \bullet h}{2\cancel{k}} \bullet \frac{k \bullet \cancel{k}}{2\cancel{h}}$ Cancel out factors equal to 1

$= \frac{h}{2} \bullet \frac{k}{2}$

$\frac{h^2}{2k} \bullet \frac{k^2}{2h} = \frac{hk}{4}$ Multiply the numerators then multiply the denominators

2. What is the product of $\frac{2ab^2}{3c} \bullet \frac{9c^2}{4ab}$?

Solution: $\frac{2ab^2}{3c} \bullet \frac{9c^2}{4ab} = \frac{2ab \bullet b}{3c} \bullet \frac{3 \bullet 3c \bullet c}{2 \bullet 2ab}$ Definition of exponent

$= \frac{\cancel{2ab} \bullet b}{\cancel{3c}} \bullet \frac{3 \bullet \cancel{3c} \bullet c}{2 \bullet \cancel{2ab}}$ Cancel out factors equal to 1

$= b \bullet \frac{3c}{2}$ Multiply the numerators then multiply the denominators

$\frac{2ab^2}{3c} \bullet \frac{9c^2}{4ab} = \frac{3bc}{2}$

Do you understand the process? How about doing the next item?

3. What is the area of a square if each side is $\frac{2z}{6}$ cm long?

Solution: $A = (\frac{2z}{6})^2$ Take note that $A_{square} = s^2$.

$= \frac{2z}{6}\text{ cm} \bullet \frac{2z}{6}\text{ cm}$ Definition of exponent

$= \frac{2z}{2 \bullet 3}\text{ cm} \bullet \frac{2z}{2 \bullet 3}\text{ cm}$ Cancel out factors equal to 1

$= \frac{z}{3}\text{ cm} \bullet \frac{z}{3}\text{ cm}$

A $= \frac{z^2}{9}$ square cm Do we have the same answer?

4. Give the product of $\frac{y-4}{y} \bullet \frac{y^2}{y^2-16}$.

Express the numerator and denominator of the 2nd rational algebraic expression in factored form.

Solution: $\frac{y-4}{y} \bullet \frac{y^2}{y^2-16} = \frac{y-4}{y} \bullet \frac{y \bullet y}{(y+4)(y-4)}$

$= \frac{y-4}{y} \bullet \frac{y \bullet y}{(y+4)(y-4)}$ Cancel out factors equal to 1.

$\frac{y-4}{y} \bullet \frac{y^2}{y^2-16} = \frac{y}{y+4}$

Did you get the correct answer? If not, review your solution. Try the next question.

5. How much should you pay for $\frac{2}{3c^2+15c}$ eggs if each egg costs P$(3c^3-75c)$?

What is your first step? Express $3c^2 + 15c$ and $3c^3 - 75c$ in factored form. Since $3c^2 + 15c = 3c(c + 5)$ and $3c^3 - 75c = 3c(c^2 - 25) = 3c(c + 5)(c - 5)$, we have,

$\frac{2}{3c^2+15c} \bullet (3c^3-75c) = \frac{2}{3c(c+5)} \bullet 3c(c+5)(c-5)$

$\frac{2}{3c^2+15c} \bullet (3c^3-75c) = 2(c-5)$

Hence, you should pay 2(c – 5) pesos. Is your answer the same? If not, review your solution. Did you get it this time? Very good!

The rule for division of fractions tells us that to divide by a real number, we should multiply by its reciprocal. The division of rational algebraic expressions follows the same rule, that is if $\frac{p}{q}$ and $\frac{v}{w}$ are rational algebraic expressions,

then $\frac{p}{q} \div \frac{v}{w} = \frac{p}{q} \bullet \frac{w}{v}$, the denominators are not equal to zero.

Examples:

1. Divide $\frac{s^2}{10}$ by $\frac{s}{15}$.

Solution: $\frac{s^2}{10} \div \frac{s}{15} = \frac{s^2}{10} \bullet \frac{15}{s}$ — Get the reciprocal of the denominator then multiply

$= \frac{s \bullet s}{5 \bullet 2} \bullet \frac{5 \bullet 3}{s}$ — Factor then simplify factors equal to 1

Hence the answer is $\frac{3s}{2}$.

2. What is the quotient when $\frac{2}{x^2 - 16}$ is divided by $\frac{4}{x-4}$?

Solution: $\frac{2}{x^2-16} \div \frac{4}{x-4} = \frac{2}{(x-4)(x+4)} \cdot \frac{2 \cdot 2}{x-4}$ — Get the factors of $x^2 - 16$ and 4

$= \frac{2}{(x-4)(x+4)} \bullet \frac{(x-4)}{2 \bullet 2}$ — Get the reciprocal of the second rational expression then multiply

$= \frac{2}{(x-4)(x+4)} \bullet \frac{(x-4)}{2 \bullet 2}$ — Divide factors equal to 1.

$= \frac{1}{x+4} \bullet \frac{1}{2}$

The final answer is $\frac{1}{2(x+4)}$. Do you have the same answer? The answer may be left in factored form.

3. Give the quotient of $\frac{v}{v+3} \div \frac{3v}{3v+9}$.

Solution: Following the same steps we did in the previous examples, we have:

$$\frac{v}{v+3} \div \frac{3v}{3v+9} = \frac{v}{v+3} \div \frac{3v}{3(v+3)}$$

$$= \frac{v}{v+3} \bullet \frac{3(v+3)}{3v} = \frac{v}{v+3} \bullet \frac{3(v+3)}{3v}$$

$$\frac{v}{v+3} \div \frac{3v}{3v+9} = 1$$

Do we have the same answer? Good!

4. How much is one mango if $\frac{y^2-2y-3}{y^2+4y+3}$ mangos cost $\frac{y^2-7y+12}{y+3}$ pesos?

Let us divide the total cost by the number of mangos and then do the steps we did in the first three examples.

$$\frac{y^2-7y+12}{y+3} \div \frac{y^2-2y-3}{y^2+4y+3} = \frac{y^2-7y+12}{y+3} \bullet \frac{y^2+4y+3}{y^2-2y-3}$$

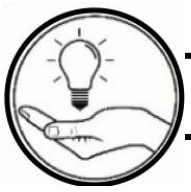
$$= \frac{(y-4)(y-3)}{y+3} \bullet \frac{(y+3)(y+1)}{(y-3)(y+1)}$$

If you'll cancel out factors equal to 1, the final answer is y - 4. Is this also your answer? That's good to hear

What's More

A. Perform the indicated operations.

1. $\frac{ay-by}{(a-b)^2} \cdot \frac{ax-bx}{5a-5b}$
2. $\frac{2a+2b}{a^2+ab} \div \frac{4}{a}$
3. $\frac{x^2-x-20}{x^2-10x+25} \cdot \frac{x^2-25}{x^2+7x+12}$
4. $\frac{x^2+2x+1}{x^2+4x+3} \div \frac{(x+1)^2}{x^2-1}$
5. $\frac{81xz^3}{36y} \div \frac{27x^2z^2}{12xy}$



What I Have Learned

- To multiply rational algebraic expressions:
  a. rewrite each numerator and denominator in prime factored form;
  b. cancel out factors equivalent to 1 or divide by the common factor/s; and then
  c. multiply the remaining numerators and denominators.
- To divide rational algebraic expressions:
  a. multiply the first rational algebraic expression by the reciprocal of the divisor;
  b. factor the polynomials whenever possible; and
  c. divide factors that are common to the numerator and denominator

What's More

A. Answer the following.

1. Find the area of the rectangle if the width is $\frac{x+4}{3x^2-2x-5}$ and the length is $\frac{6x-10}{2x^2+11x+12}$.
2. The area of the parallelogram is $\frac{13xy^2}{x^2+3x-18}$.Its height is $\frac{26x^4y^2}{x^2-9}$.Find its base.
3. The area of the rectangle is $\frac{x^2+7x+12}{x-5}$.Its width is $\frac{x+3}{2x-10}$.Find the length of the rectangle.

Assessment

A. Read the following questions and write your answers on a separate sheet of paper.

1. Which of the following is the product of $\frac{g^2}{3h} \bullet \frac{h}{2g}$?

A. $\frac{hg}{6}$ B. $\frac{g^2}{6}$ C. 6g D. $\frac{g}{6}$

2. What is the product of $\frac{2mn^2}{3c} \bullet \frac{9c^2}{mn^2}$?

A. $6c$ B. $\frac{3c}{2}$ C. $\frac{2m}{3c}$ D. $\frac{6}{c}$

3. What is the area of a square if each side is $\frac{3p}{10}$ cm long?

A. $\frac{6p}{5}\ cm.$ B. $\frac{3p}{100}\ cm.^2$ C. $\frac{9p^2}{100}\ cm.^2$ D. $\frac{9p}{100}\ cm.^2$

4. Give the product of $\frac{b-7}{c} \bullet \frac{c^2}{b^2-49}$.

A. $c(b+7)$ B. $\frac{c}{b+7}$ C. $\frac{b+7}{c}$ D. $\frac{c}{b-7}$

5. How much should you pay for $\frac{1}{2c^2+2c}$ watermelons if each watermelon costs $(2c^3-2c)$ pesos?

A. $(c-1)\ pesos$ B. $(c+1)\ pesos$ C. $2c(c-1)\ pesos$ D. $c(c+1) pesos$

6. Divide $\frac{p^2}{7}$ by $\frac{p}{14}$.

A. $\frac{p}{2}$ B. $\frac{2}{p}$ C. $2p$ D. $\frac{1}{2p}$

7. What is the quotient when $\frac{3}{x^2-9}$ is divided by $\frac{6}{x-3}$?

A. $2(x+3)$ B. $x+3$ C. $\frac{2}{x+3}$ D. $\frac{1}{2(x+3)}$

8. Give the quotient of $\frac{w}{w+5} \div \frac{5w}{5w+25}$.

A. 1 B. $\frac{5}{w}$ C. $\frac{w}{5}$ D. $\frac{w}{w+5}$

9. Divide $\frac{4d-2}{2d}$ by $\frac{2d-1}{2}$.

A. $2d$ B. $\frac{2}{d}$ C. $\frac{d}{2}$ D. $\frac{1}{2d}$

10. How much is one banana if $\frac{x^2+3x-10}{x+3}$ bananas cost $\frac{x^2-6x+8}{x^2-x-12}$ pesos?

A. $\frac{1}{x+5}\ pesos$ B. $x+5$ pesos C. $\frac{x+5}{x+3}$ pesos D. $\frac{1}{x+3}\ pesos$

Answer key

Lesson 1:

What I Know

1. D
2. D
3. C
4. D
5. B

What's In

1. YES
2. $(x+2)(x+3)$
3. -2
4. YES
5. NO

What's More

A. 1. YES
   2. NO
   3. YES

B. 1. 0
   2. 2 and 3
   3. -2

What I Can Do

1. A
2. a. 0
   b. -4 and 3

Assessment

1. A
2. B
3. B
4. B
5. B

Lesson 2:

What I Know

1. D
2. D
3. A
4. A
5. B

What's In

1. $(x+y)(x-y)$
2. $3(c-4)$
3. $\frac{1}{3}$
4. $m-1$
5. NO

What's More

1. $\frac{3y}{7x^2}$
2. $\frac{5c^2}{3}$
3. $\frac{4x}{2x-1}$
4. $\frac{1}{5d-3}$
5. $\frac{m+3}{m-2}$

What I Can Do

1. $\frac{x+1}{x-1}$
2. $\frac{m+3}{m-3}$
3. $\frac{a+2}{a-2}$
4. $\frac{1-x^3}{1+x^2}$
5. $\frac{x-y}{2(x+y)}$

Assessment

1. A
2. B
3. A
4. C
5. B

Lesson 3:

What I Know

1. B
2. B
3. A
4. B
5. B

What's In

1. $(a+5)(a-5)$
2. $\frac{3}{4}$
3. $3c(c+5)$
4. $\frac{x^2+xy}{x+y}$
5. $y-3$

What's More

1. $\frac{xy}{5(a-b)}$
2. $\frac{1}{2}$
3. $\frac{x+5}{x+3}$
4. $\frac{x-1}{x+3}$
5. z

What I Can Do

1. $A = \frac{2}{(x+1)(2x+3)}\ sq.unit$
2. $b = \frac{x+3}{2x^3(x+6)}\ units$
3. $l = 2(x+4)\ units$

Assessment

1. D
2. A
3. C
4. B
5. A
6. C
7. D
8. A
9. B
10. A

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