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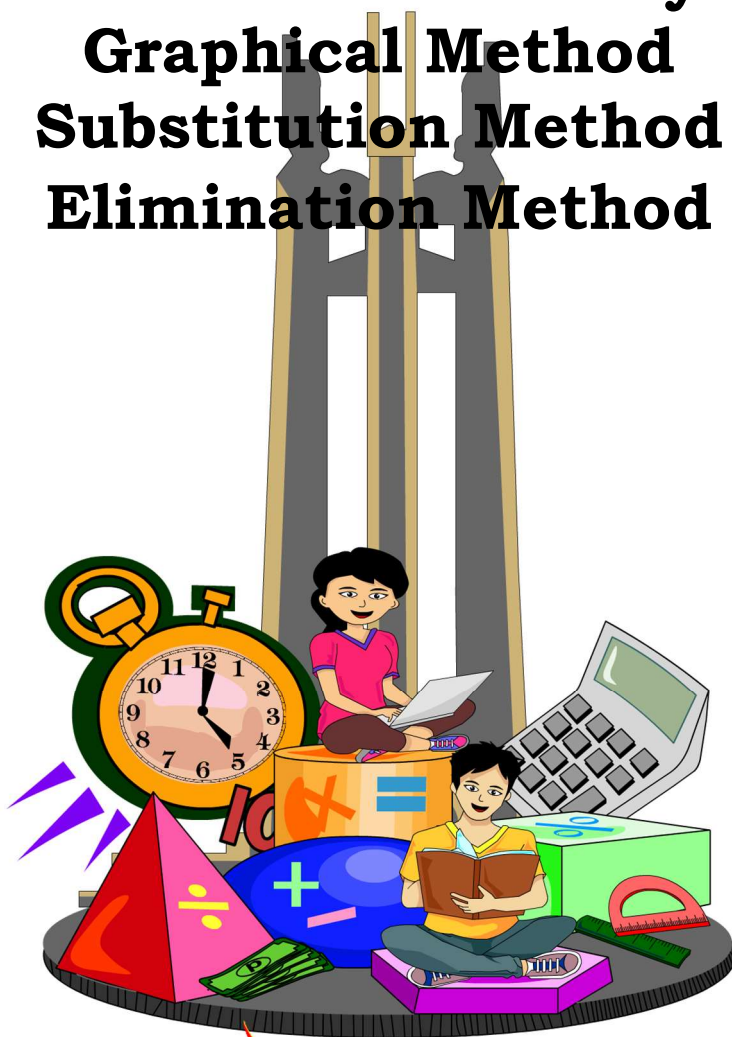
Mathematics

Quarter 1 –Module 9

Solving Problems Involving Systems of Linear Equations in Two Variables by Graphical Method Substitution Method Elimination Method

**Mathematics – Grade 8
Alternative Delivery Mode
Quarter 1 – Module 9: Solving Problems Involving System of Linear Equations
                        In Two Variables by Graphical, Substitution, and Elimination
First Edition, 2020**

Mathematics 8
Quarter 1 – Module 9: Week 9

**Lesson 1: Solving Systems of
Linear Equations In
Two Variables by Graphing**

**Lesson 2: Solving Systems of
Linear Equations In
Two Variables by Substitution**

**Lesson 3: Solving Systems of
Linear Equations In
Two Variables by Elimination**

**Lesson 4: Solving Word Problems
Involving Systems of
Linear Equations
In Two Variables**

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DEPARTMENT OF EDUCATION

Introductory Message

For the facilitator:

Welcome to the __Mathematics 8__ Alternative Delivery Mode (ADM) Module on Solving Systems of Linear Equations in Two Variables by Graphing, Solving Systems of Linear Equations by Substitution, Solving Systems of Linear Equations by Elimination. Solving Word Problems Involving a Systems of Linear Equations in Two Variables

This module was collaboratively designed, developed, and reviewed by educators both from public and private institutions to assist you, the teacher, or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners in guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st-century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Mathematics 8 Alternative Delivery Mode (ADM) Module on Solving Systems of Linear Equations in Two Variables by Graphical Method, Substitution Method, and by Elimination Method and Solving Word Problems Involving Systems of Linear Equation In Two Variables.

The hand is one of the most symbolized parts of the human body. It is often used to depict skill, action, and purpose. Through our hands, we may learn, create, and accomplish. Hence, the hand in this learning resource signifies that you as a learner are capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

	What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
	What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
	What's In	This is a brief drill or review to help you link the current lesson with the previous one.
	What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity, or a situation.
	What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	What I Have Learned	This includes questions or blank sentences/paragraphs to be filled in to process what you learned from the lesson.
	What I Can Do	This section provides an activity that will help you transfer your new knowledge or skill in real-life situations or concerns.
	Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends to the retention of learned concepts.
	Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain a deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written to help you learn in the easiest possible way. Aside from learning lessons from the internet and any other learning materials, it will not only serves as one of your tools to learn in solving systems of linear equations in two variables by graphical, substitution, and elimination method but also will help you to familiarize and master the concept of solving problems involving linear equations. The scope of this module provides application in many different learning situations and can also be applied to a daily life situation. The arrangement of the lessons follows the standard sequence of the course as provided by the Department of Education.

The module is divided into four lessons, namely:

Lesson1 - Solving Systems of Linear Equations in Two Variables
by Graphical Method
Lesson 2 - Solving Systems of Linear Equations in Two Variables
by Substitution Method
Lesson 3 – Solving Systems of Linear Equations in Two Variables
by Elimination Method
Lesson 4 – Solving Word Problems Involving Systems of Linear Equations in
Two Variables

After going through this module, you are expected to:

1. solve systems of linear equations in two variables by graphical method;
2. solve systems of linear equations in two variables by substitution method;
3. solve systems of linear equations in two variables by the elimination method;
4. solve word problems involving systems of linear equations in two variables; and
5. show interest in the lesson by doing your assigned activities.

<table>
<tr><td>**Lesson**

1</td><td>**Solving Systems of Linear Equations in Two Variables by Graphical Method**</td></tr>
</table>

In this lesson, solving systems of linear equations in two variables by the graphical method will enhance your talent in sketching straight lines because you can solve the solutions of the systems not by algebraically, but it can be done by using your hand on a graph paper.

What I Know

Write the letter of the correct answer on a separate answer sheet.

1. Which is a solution of the equation $x + 4y = 6$?

    A. (1, 2)   B. (2, 1)   C. (2, -1)   D. (-2, 1)

2. Solve $\begin{cases} 2x + y = 8 \\ x - 3y = 4 \end{cases}$ graphically.

    A. (0, 8)   B. (0, -4)   C. (4, 0)   D. (-4, 0)

3. Which of the following ordered pairs satisfy the $\begin{cases} x - y = -1 \\ 2x - y = 2 \end{cases}$ system?
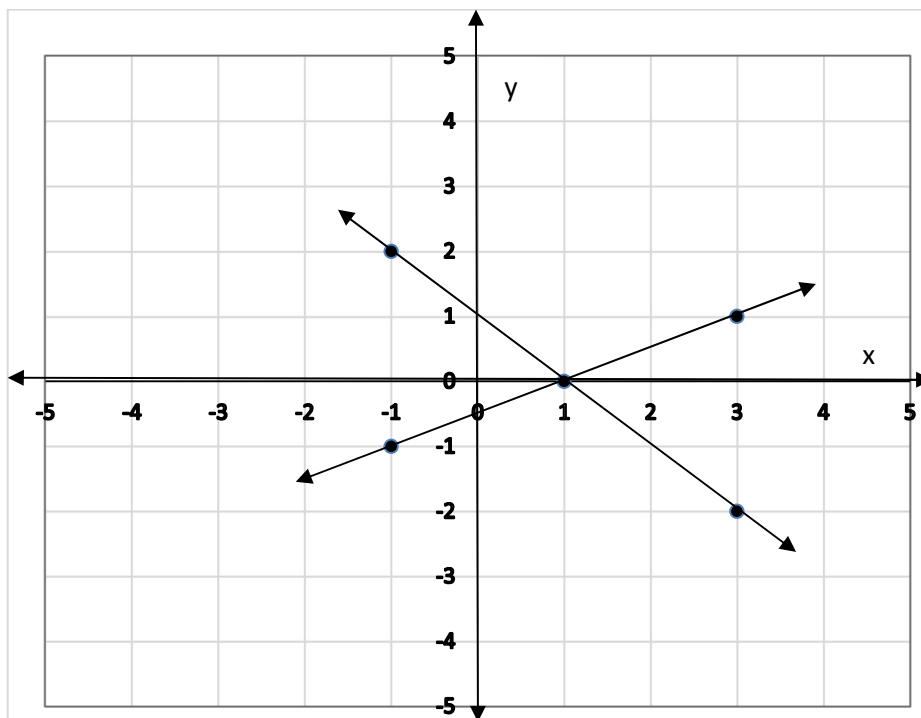
    A.   (3, 4)   B. (3, -4)   C. (-3, 4)   D. (4, 3)

4. The sum of the two numbers is 24. The second number is 7 times the first number? If x is the first number, how will you represent the second number?

    A. second number = 7   C. second number = 7x
    B. second number = x + 7   D. second number = 7^2

5. Given the graph of the system of linear equations below, determine its solution.


A. (-1, 2)   B. (0, 1)   C. (-1, -1)   D. (1, 0)

What's In

From our previous lesson, you have learned the three kinds of systems of linear equations and their characteristics, one of which is the independent and consistent system. This system contains one solution because the graph of the lines intersects at only one point. The illustration below shows an independent and consistent system.
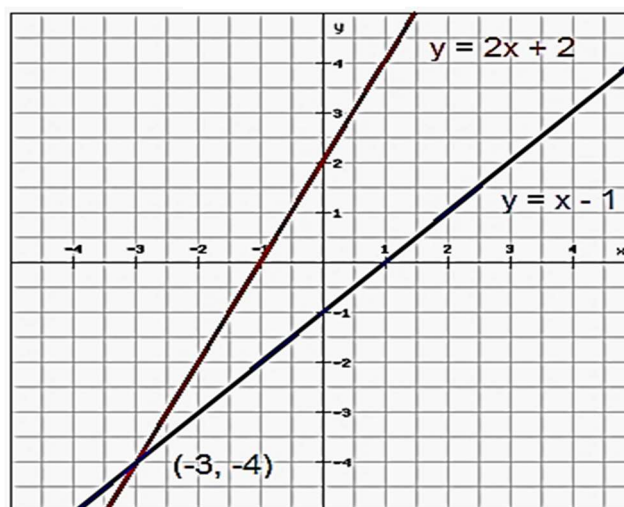

$y = 2x + 2$

$y = x - 1$

$(-3, -4)$
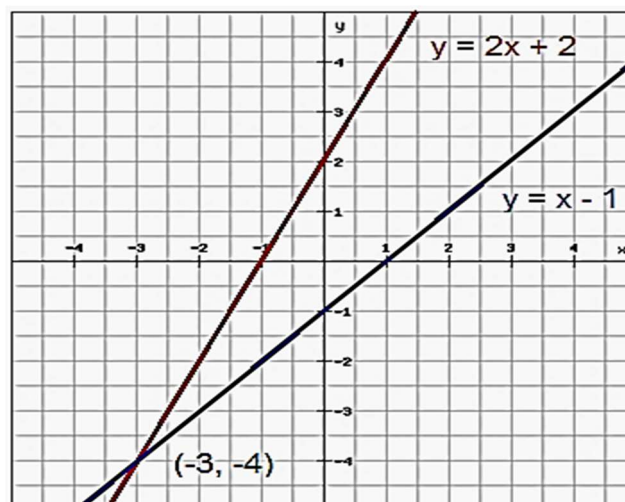
Independent and Consistent system

What's New

Consider the graph of the system below. How do we solve the system, graphically? 1. Use the slope, y-intercept method; 2. Use the x-intercept and y-intercept method. Show how in illustration, the 2 methods in graphing the linear equations in one coordinate plane.

$$\left. \begin{array}{l} y = 2x + 2 \\ y = x - 1 \end{array} \right\}$$

Since the graph of the system intersects at only one point, then the system has one solution. The point of intersection (-3, -4) is the solution of the system, and it is called the solution set. How do you know that (-3, -4) is the solution set of the system?

To check whether (-3, -4) is a solution of the system, use the algebraic method in checking. The solution set: (-3, -4) must satisfy the two equations.

Using equation 1:   y = 2x + 2   ; sol. set: (-3, -4)

y = 2x + 2,   solution set: (-3, -4), meaning x = -3 and y = -4
-4 = 2(-3) + 2   by substitution property
-4 = -6 + 2   by multiplication
-4 = -4 ✓   therefore (-3, -4) is a solution of the equation

Using equation 2: y = x – 1   ; sol. set (-3, -4)
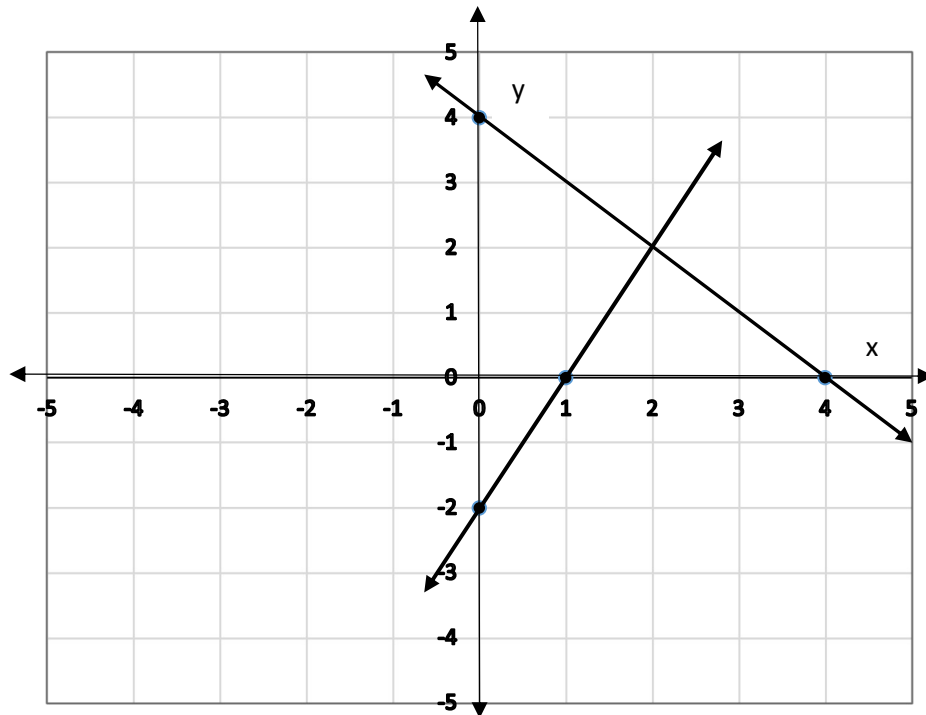
y = x - 1   solution set: (-3, -4), meaning x = -3 and y = -4
-4 = -3 – 1   by substitution property
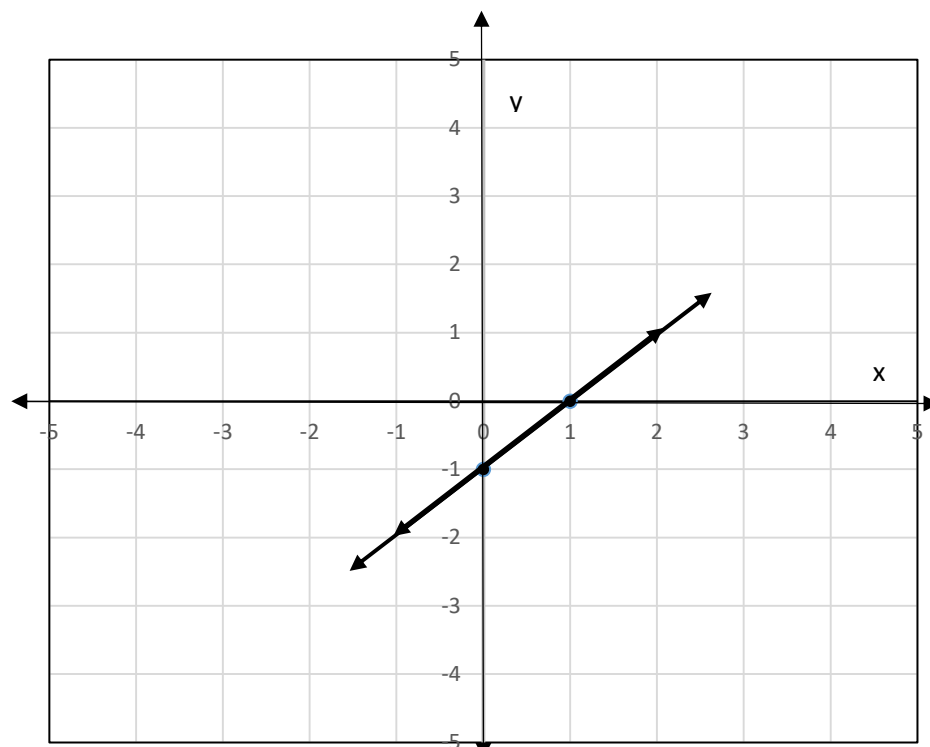-4 = -4 ✓   therefore (-3, -4) is a solution of the equation

Hence, (-3, -4) is a solution of the system $\left. \begin{array}{l} y = 2x + 2 \\ y = x - 1 \end{array} \right\}$ because it makes the two equations true.

Activity 1: Determine if each of the following graphs of systems of linear equations has one
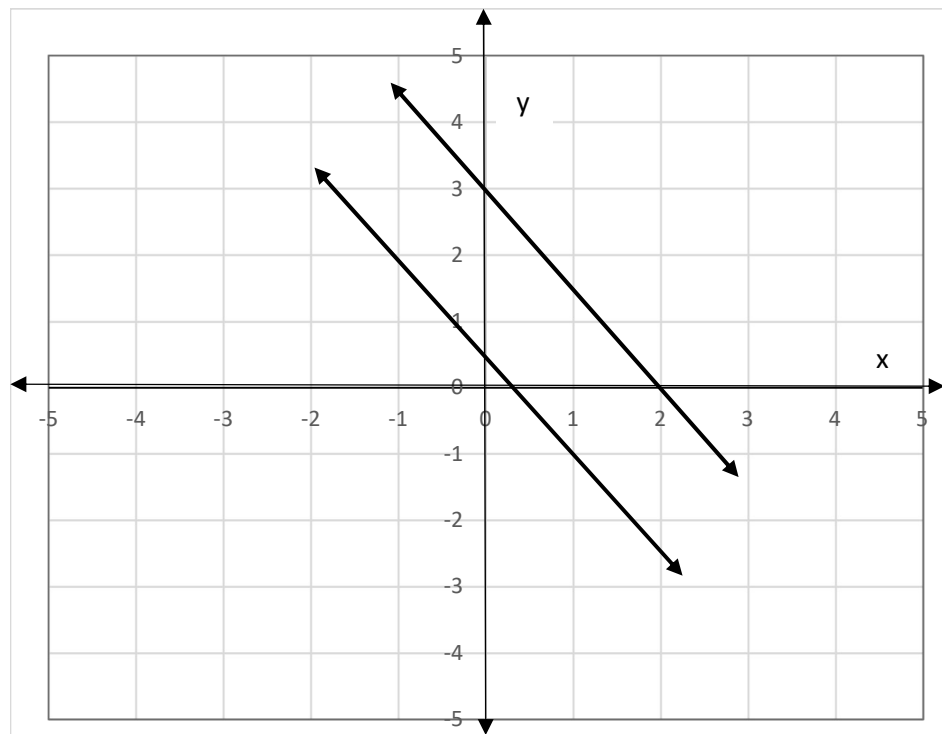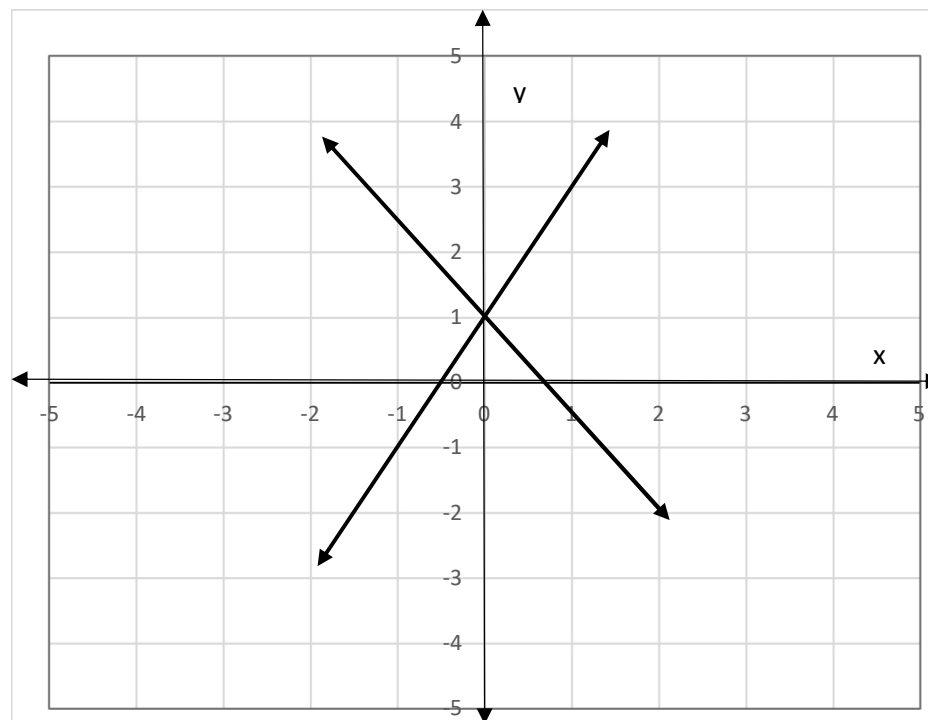solution, no solution, or infinitely many solutions.

1.

2.

3.

4.

What is It

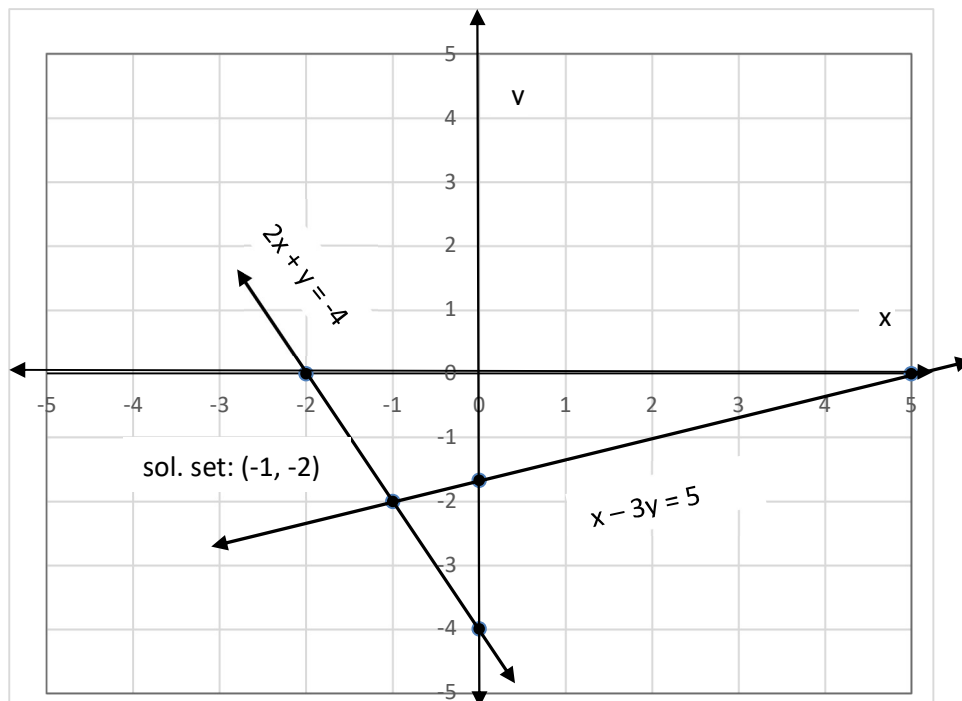**How to solve systems of linear equations by graphical method?**

Example 1.

Determine the solution of the system $\begin{matrix} 2x + y = -4 \\ x - 3y = 5 \end{matrix} \Big\}$

Do the following steps to find the solution:

    a. Draw the graph of the line of equation (1) 2x + 4y = -4 by using the x and y-intercepts.

    b. Draw the graph of the line of equation (2) x – 3y = 5 by using x and y-intercepts.
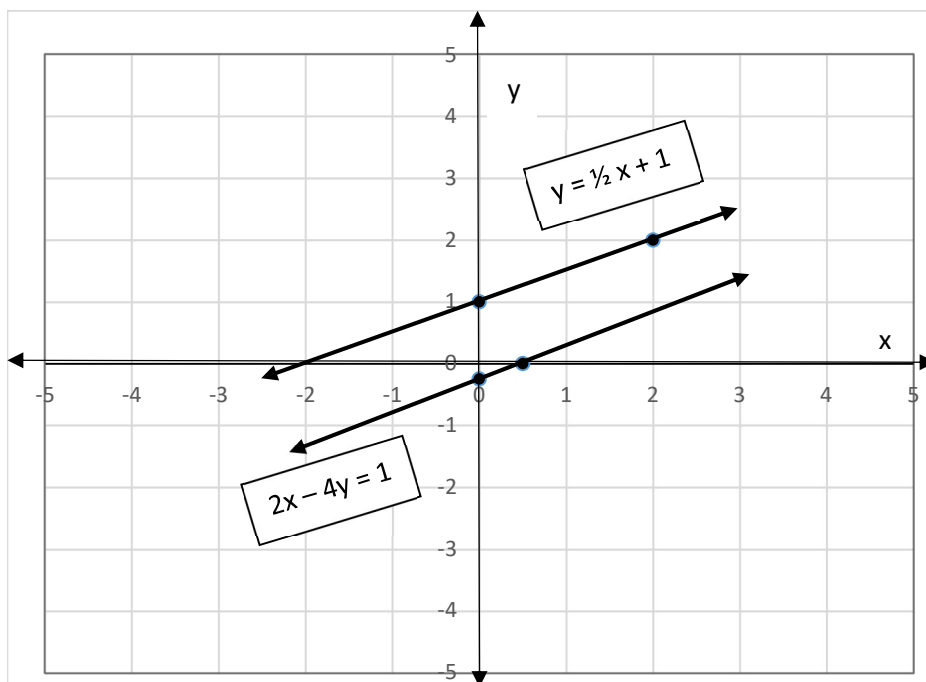

    c. Solution set: (-1, -2)

Example 2. Show the solution of the system $\begin{matrix} y = \dfrac{1}{2}x + 1 \\ 2x - 4y = 1 \end{matrix} \Big\}$ by graphical method.

Step 1. Draw the graph of the first equation by using the slope and the y-intercept.

Step 2. Draw the graph of the second equation by using the x and y-intercepts.

Step 3. The graph has no solution because the graph of the lines are parallel.

Example 3.   Find the solution by graphing.

$$\left.\begin{array}{l} y = 2x + 3 \\ 6x - 3y = -9 \end{array}\right\}$$

Step 1. Draw the graph of the first equation by using the slope and the y-intercept.

Step 2. Draw the graph of the second equation by using the x and y-intercepts.
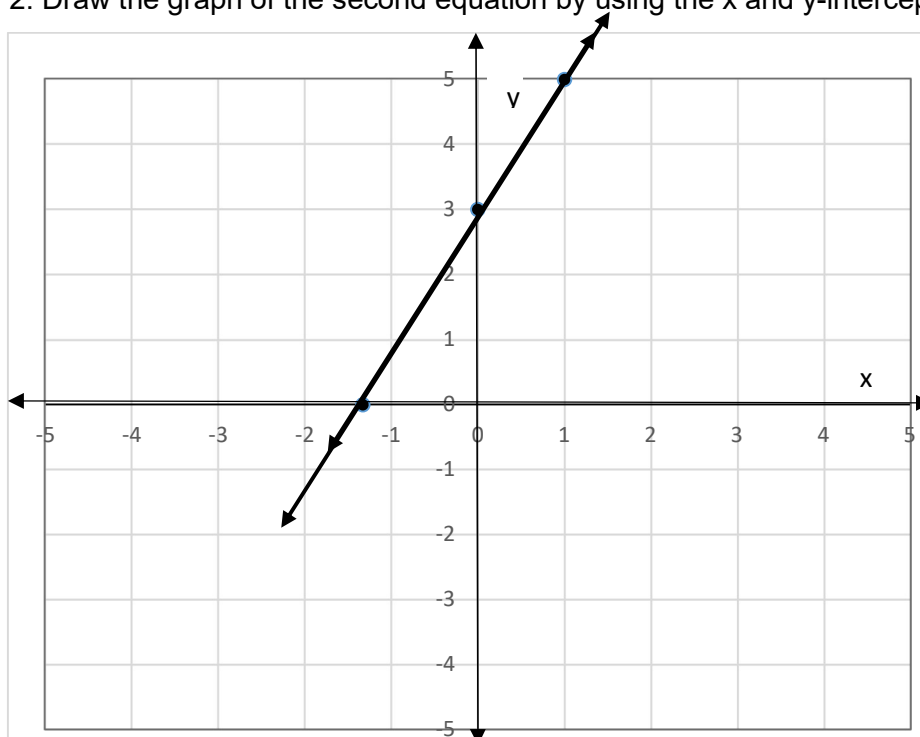
Step 3. There are infinitely many solutions because the graph is overlapping

Example 4.

The sum of the two numbers is 4. The larger number is thrice the smaller number. Find the numbers.

Solution:

Let x be the smaller number

Let y be the larger number

Make working equations:

$x + y = 4$ The sum of the two numbers is 4

$y = 3x$ The larger number is thrice the smaller number
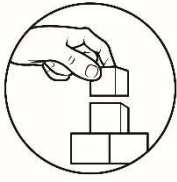
Then the two equations are $\left.\begin{array}{l} x + y = 4 \\ y = 3x \end{array}\right\}$

Solve the system $\left.\begin{array}{l} x + y = 4 \\ y = 3x \end{array}\right\}$ by graphical method
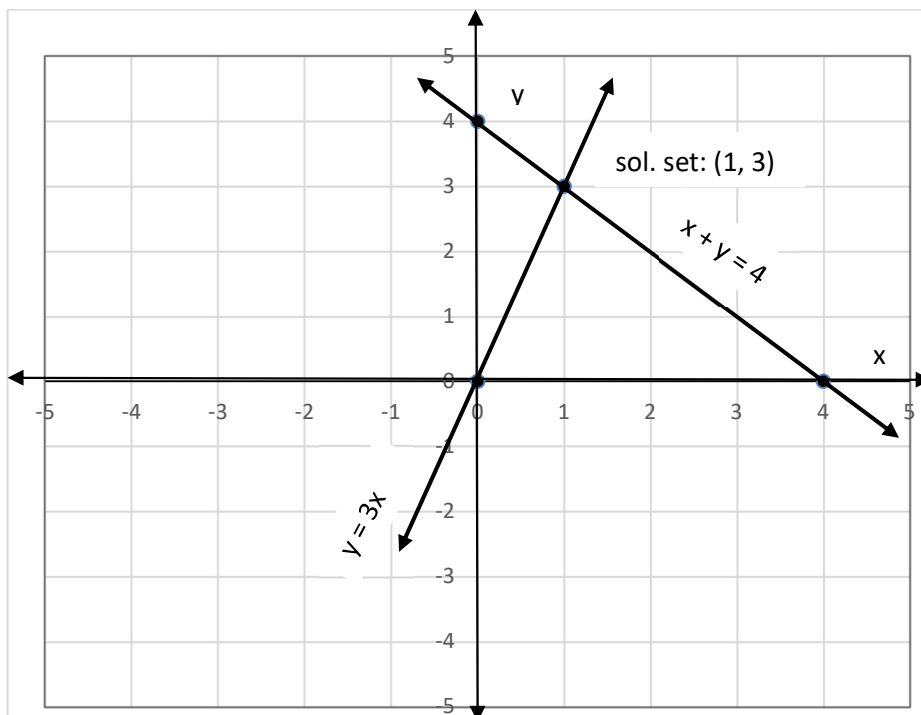
The solution set is (1, 3). Since x represents the smaller number while y represents the larger number, then the smaller number is 1 and the larger number is 3.

What's More

**Solve each system of linear equations using the graphical method.**

1. $\left.\begin{array}{l} x + y = 5 \\ x - y = 1 \end{array}\right\}$

2. $\left.\begin{array}{l} 2x - y = -2 \\ x + 4y = -1 \end{array}\right\}$

3. $\left.\begin{array}{l} 3x + 5y = -10 \\ 4x - y = 2 \end{array}\right\}$

4. $\left.\begin{array}{l} y = 2x + 3 \\ 2y = 4x + 6 \end{array}\right\}$

5. $\left.\begin{array}{l} 3x - 2y = 4 \\ 3x - 2y = 6 \end{array}\right\}$

What I Have Learned

To solve system of linear equations by graphical method:

- draw the graph of the line of the first equation.
- draw the graph of the line of the second equation.
- if the graphs of the lines intersect each other, then the ordered pair describes the point of the intersection is the solution set.
- if the graphs are parallel, then there will be no solution.
- if the graphs are overlapping, then there will be infinitely many solutions.
- ensure the accuracy in graphing because it is necessary to produce a graph that will indicate the exact solution.
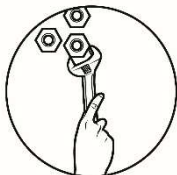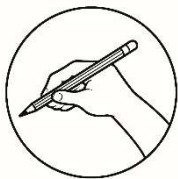
What I Can Do

Draw the graph of each of the following systems of linear equations and tell whether it has one solution, no solution, or infinitely many solutions. And if it has one solution, name its ordered pair.

1.
$$3x - y = -1$$
$$5x + 2y = 13$$

2.
$$3x - y = 6$$
$$y = 3x + 1$$

3.
$$2x = 3y + 1$$
$$-6y = -4x + 2$$

Assessment

Choose the letter of the correct answer. Write the letter of your choice on a separate sheet of paper.

1. Solve by graphical method.
$$2x + y = 10$$
$$x - y = 2$$
?

   A. (2, 4)   B. (4, 2)   C. (-2, 4)   D. (4, -2)

2. Which of the following ordered pairs will make the two equations true?

$$3x - y = -3$$
$$2x + y = -2$$

   A. (0, -1)   B. (1, 0)   C. (0, 1)   D. (-1, 0)

3. Find the solution to the system
$$4x + y = 14$$
$$x - 3y = -3$$
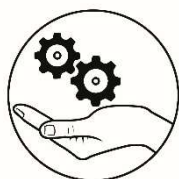
   A. (3, 2)   B. (3, -2)   C. (-3, 2)   D. (2, 3)

4. Find the value of x in the solution of the system
$$2x + y = -4$$
$$x - 2y = -2$$

   A. 0   B. -2   C. 2   D. -4

5. Describe the graph of the system
$$y = 4x - 1$$
$$4x - y = 3$$

   A. intersecting   C. parallel
   B. coincide or overlapping   D. can not be described

Additional Activity

Solve each of the following by graphical method.

1.
$$\left.\begin{array}{l} y = \dfrac{1}{2}x + 3 \\[2em] y = -\dfrac{1}{4}x + \dfrac{9}{2} \end{array}\right\}$$

2. Find two numbers such that thrice the smaller minus twice larger is 2, and six times the smaller plus thrice the larger is 39.

Lesson 2	Solving Systems of Linear Equations in Two Variables by Substitution Method

The graphical method is a good way of finding a solution. However, the graphical method of solving system of equations is not always a convenient method to use for the following reasons: the coordinates may involve large numbers or fractions that are difficult to plot, the non-integer solutions cannot always be read accurately, and it takes a great deal of time. Another way is an algebraic method called the substitution method.

What I Know

Solve what is being asked. Write the letter on a separate sheet of paper.

1. Which of the following is a solution of a system of linear equations $\begin{cases} y = x - 2 \\ x = 6 \end{cases}$

    a. (2, 4)   b. (4, 4)   c. (6, 4)   d. (8, 4)

2. In the system of $\begin{cases} y = 4x, \\ x + y = 10 \end{cases}$ What is the value of x?

    a. 2   b. 4   c. 6   d. 8

3. Which of the following systems of linear equations has a solution of (1, 3)?

    a. $\begin{cases} 2x + 3y = 11 \\ 3x + 2y = 9 \end{cases}$ c. $\begin{cases} 2x + 3y = 11 \\ 3x - 2y = 9 \end{cases}$

    b. $\begin{cases} 3x - 2y = 9 \\ 2x + 3y = 11 \end{cases}$ d. $\begin{cases} 3x - 2y = 9 \\ 2x - 3y = 11 \end{cases}$

4. What is the value of a, when b is substituted to equation 2?

    $\begin{cases} b = 4a \\ a + b = 15 \end{cases}$ Equation 1
                     Equation 2

    a. 1   b. 3   c. 5   d. 7

5. Solve the system $3x + 5y = 7$; $4x - y = -6$ by substitution method.

    a. (-1, 2)   b. (1, 2)   c. (-2, 1)   (2, -1)

What's In

Let us recall.

1. If $y = 3x - 1$ and $x = 1$, what is y? Replace x with 1, you should get $y = 2$.

2. What is $x + 4x$? Is your answer equal to 5x?

3. Solve for y in terms of x in $4x - y = -6$.

  My solution is

    $-y = -4x - 6$ dividing both sides by negative 1

y = 4x + 6

Do we have the same answer?

4. Solve for x in 5y – x = 8. Is your answer x = 5y – 8?

5. Solve for y in 5y – x = 8. Is your answer y = $\frac{x+8}{5}$?

What's New

Read and Solve:

Grade 8 - SPA ordered 5 classic pizza and 3 overload pizza for their closing ceremony party, costing P 795.00. An overloaded pizza cost P65 more than the classic pizza. How much is the cost of each classic and overload pizza?

What is It

1. How do you solve the word problem above?
    Let us solve the given problem using the problem-solving guidelines.
    (Understand, Develop and carry out the PLAN, Find the Answer and Check)

* Understand the problem:

Question: How much is the cost of each classic and overload pizza?	Clarifying the question
Data: 5 classic and 3 overload pizzas costs P795.00.	Finding the relationship

An overloaded pizza cost P65 more than the classic pizza.

* Develop and carry out the PLAN:

There are two statements in the problem. You translate each into an equation.

Let x represents the cost of a classic pizza.
Let y represents the cost of overload pizza.

Translating statement no. 1

<u>5 times the cost of a classic pizza</u> plus <u>3 times the cost of an overloaded pizza</u> is P795
  $5x$ + $3y$ = 795

Translating statement no. 2

<u>The cost of an overloaded pizza</u> is P65 <u>more than</u> <u>the cost of a classic pizza</u>
   y = x   +   65

We now have a system of equations:

$5x + 3y = 795$ } equation 1
$y = x + 65$ } equation 2

*Find the Answer and Check

Then we solve the system by substitution method.

$5x + 3y = 795$ } equation 1
$y = x + 65$ } equation 2

Solution:

a. Since y is already solved in terms of x, replace y in equation 1 with 65 + x.

$$5x + 3y = 795$$
$$5x + 3(x + 65) = 795$$
$$5x + 3x + 195 = 795$$
$$8x = 795 - 195$$
$$x = \frac{600}{8}$$
$$\textbf{x = 75}$$

b. Substitute this value for x in y = 65 + x, and solve for y.

$$y = x + 65$$
$$y = 75 + 65$$
$$\textbf{y = 140}$$

c. Check in both equations.

First Equation	**Second Equation**
5 x + 3y = 795	y = 65 + x
5(75) + 3(140) = 795	140 = 65 + 75
375 + 420 =795	140 = 140 ✓
795 = 795 ✓	

Therefore, the **P75** cost of each classic pizza and **P140** cost of each overload pizza.

Now, how do you solve a system of linear equations by substitution method?
Here are the following examples of solving system of linear equations by substitution
Method. It's an easy process to follow.

1. Solve   y = x - 2 } equation 1
                 x = 6   } equation 2

Solution:

a. Since the value of x is already given,   y = x - 2
   replace x in y = x -2 with 6   y = 6 – 2
                                              y = 4
b. Check in equation 1.   y = x – 2,
                                              4 = 6 – 2
                                              4 = 4 ✓

                The solution is **(6, 4)**

2.   Solve   b = 4a   } equation 1
                a + b = 15 } equation 2

Solution:

a. Since b is already solved   a + b = 15
   in terms of a, replace b with 4a   a + (4a) = 15
   in equation 2.   5a = 15
                                           a = 3

b. Substitute this value for a   b = 4a
   in b = 4a, and solve for b.   b = 4(3)
                                           b = 12

c. Check in both equations.

$$b = 4a, \text{ but } a = 3 \ \& \ b = 12$$
$$12 = 3(4)$$
$$12 = 12 \ \checkmark$$
$$a + b = 15$$
$$3 + 12 = 15$$
$$15 = 15 \ \checkmark$$

The solution is **a=3, b=12**

3. Solve $3x + 5y = 7$ }   equation 1
              $4x - y = -6$ }   equation 2

a. Solve for one variable. Since the coefficient of y is -1 in equation 2 it easier to solve for y

$$4x - y = -6$$
$$-y = -4x - 6$$
$$-1(-y = -4x - 6)$$
$$y = 4x + 6$$

b. Substitute the value of y in the equation 1 and solve for x.

$$3x + 5y = 7$$
$$3x + 5(4x + 6) = 7$$
$$3x + 20x + 30 = 7$$
$$23x + 30 = 7$$
$$23x = 7 - 30$$
$$23x = -23$$
$$\mathbf{x = -1}$$

c. Substitute this value for x in the equation obtained in step a.

$$y = 4x + 6$$
$$y = 4(-1) + 6$$
$$y = -4 + 6$$
$$\mathbf{y = 2}$$

d. Check in both equations. By substituting the values of x and y to check your answer

$$3x + 5y = 7, \qquad 4x - y = -6$$
$$3(-1) + 5(2) = 7 \qquad 4(-1) -2 = -6$$
$$-3 + 10 = 7 \qquad -4 - 2 = -6$$
$$7 = 7 \ \checkmark \qquad -6 = -6 \ \checkmark$$

The solution is **(-1, 2)**
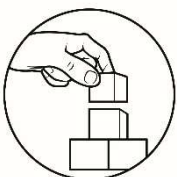
What's More

Consider the system: $x - 3y = 8$
                        $2x + 8y = -12$ $\Big\}$

How do you solve this system by the substitution method?

What I Have Learned

How do you solve systems of linear equations using substitution method?

A. Solve one equation for one variable in terms of the other variable.

B. Substitute the expression obtained in STEP A into the other equation.

C. Solve the resulting equation in one variable.

D. Find the value of the other variable by substituting the solution found

   in STEP C into any equation containing both variables.

D. Check the solution in the original equations.

What I Can Do

I. Solve the following systems using substitution method:

1. $x + y = 15$ $\Big\}$ 2. $a = 6b$ $\Big\}$
       $x = 2y$ $2a - 3b = 27$

II. Read and Solve

My *alkansya* contains Php 1450 worth of coins in Php 5 and Php 10 coins. The number of Php 10 coins is 10 more than the number of Php 5 coins. How many Php 5 and Php 10 coins are there in my *alkansya*.

Assessment

Solve what is being asked. Write your answer on a separate sheet of paper.

1. If $3x + y = 8$ and $5x - 5y = 20$, what is the value of x?

a. 1   b. 3   c. 5   d. 10

2. Which of the following is the value of q in the system of linear equations $\left.\begin{array}{l} 3p + 4q = 5 \\ p = 3 \end{array}\right\}$

a. -1   b. 1   c. 2   d. 3

3. In this system of linear equations $\begin{array}{l} 4u + 5v = 30 \\ 5u = 25 \end{array}$. What is the value of v?

a. 2   b. 4   c. -2   d. -4

4. Solve the given system by the substitution method.

$$\left.\begin{array}{l} 5x + y = 15 \\ 6x - 4y = 18 \end{array}\right\}$$

a. (2, 0)   b. (2, 1)   c. (3, 0)   d. (3, 2)

5. By substitution method, find the solution of system of linear equations $\left.\begin{array}{l} 2a + 3b = 22 \\ b = 3a \end{array}\right\}$

a. (1, 5)   b. (1, 7)   c. (2, 4)   d. (2, 6)

Additional Activity

Read and follow the instructions.

In what city did coronavirus disease 2019, known as COVID 19 was first identified amid an outbreak of respiratory illness cases?

Find the solution of each system using the substitution method.

Write the letter of each solution on the top of the given answer in the boxes below to answer the question.

A	$3x + 2y = 3$ $y = x - 6$	**E**	$x = 3y$ $x + 2y = 5$
J	$2x + 2y = 52$ $x - 9y = -14$	**U**	$x - 5y = 0$ $2x + 5y = 15$
H	$x - 3y = -12$ $3x + 6y = 39$	**M**	$x - y = 11$ $x - 3y = 19$
N	$x = 28 - 5y$ $6x + y = 52$	**W**	$y = 2x$ $2x + y = 16$

(4, 8)	(5, 1)	(3, 5)	(3, -3)	(8, 4)

Lesson 3	**Solving Systems of Linear Equations in Two Variables by Elimination Method**

Another algebraic method of finding the solution of a system of linear equations in two variables is by elimination method. This is done by eliminating one variable to solve for the other.

The Addition Property of Equality and Multiplication Property of Equality are the properties most frequently used.
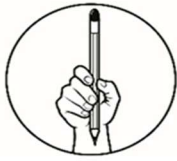
What I Know

Solved what is being asked. Write the letter on a separate sheet of paper.

1. In this system $\left.\begin{array}{l} 2x + y = 6, \\ 4x - y = 12 \end{array}\right\}$ what is the value of x?

     a. -1   b. 2   c. -3   d. 3

2. What values of *p* and *q* will make both of the following equations true if p is eliminated?

$$\left.\begin{array}{l} p - 4q = 14 \\ 3p + 2q = 28 \end{array}\right\}$$

     a. $p = 8$ and $q = -1$ c. $p = -10$ and $q = 2$

     b. $p = 10$ and $q = -1$ d. $p = -20$ and $q = 1$

3. When x be eliminated in the equation below what is the value of y?

$$\left.\begin{array}{l} 4x + 3y = 9 \\ 2x - y = 7 \end{array}\right\} \quad \begin{array}{l} \text{Equation 1} \\ \text{Equation 2} \end{array}$$

     a. 0   b. -1   c. 1   d. 2

4. What are the values of m and n in the equation $2m + 3n = 7$ if the sum of m and n is 5?

     a. m = 6, n = -1   c. m= 8, n= -3

     b. m= -6, n = 1   d. m = 8, n = 3

5. Solve $\left.\begin{array}{l} 5x - 3y = 7 \\ 3x + 3y = 9 \end{array}\right\}$ by elimination the solution is _____.

     a. (2, 1)   b. (2, 3)   c. (3,1)   d. (3, 2)

What's In

1. What is the numerical coefficient of 2a? Is it 2?
2. Give the opposite of 3. The sum of a number and its opposite is 0, thus the answer is -3.

3. What is – 2y – y? The answer is – 3y.
4. What is 2x + 4x? Is your answer equal to 6x?
5. What is the least common multiple of 3x and 6x? Your answer should be 6x.

What's New

You are going to guess;
How many marbles are in each can?
If there are 38 marbles in all and there are
10 more marbles on the 1st can than on the
2nd can.
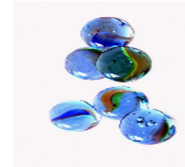
1st can   2nd can

What's your guess?

What is It

What are the clues in our guessing game above?

1st clue: There are 38 marbles in all.
2nd clue: There are 10 more marbles on the 1st can than on the 2nd can.

Let us represent x is the number of marbles on the 1st can
y is the number of marbles on the 2nd can

Using the given clues, we can write the system of linear equations.

$$x + y = 38 \quad \text{equation 1}$$
$$x - y = 10 \quad \text{equation 2}$$

Now we will use the elimination method to solve this system.

Add the two equations. We can eliminate y.

$$\begin{array}{r} x + y = 38 \\ \underline{x - y = 10} \\ 2x \quad = 48 \end{array}$$

Solve for x:

$$2x = 48$$

x = 24

Subtract the two equations. We can eliminate x.

$$x + y = 38$$
$$x - y = 10$$

$$2y = 28$$

$$\mathbf{y = 14}$$

Check: First Equation

$$x + y = 38$$
$$24 + 14 = 38$$
$$38 = 38 \quad ✓$$

Second Equation

$$x - y = 10$$
$$24 - 14 = 10$$
$$10 = 10 \quad ✓$$

The number of marbles on the 1st can is **24**

The number of marbles on the 2nd can is **14**

Examples of solving the system of linear equations using the elimination method.

Example No.1. $2x + y = 6$ equation 1
                 $4x - y = 12$ equation 2

Solution:

a. Look for the variable who's numerical
   coefficients are opposites. If this condition
   exists, add the equations

$$2x + y = 6 \quad \text{equation 1}$$
$$4x - y = 12 \quad \text{equation 2}$$
$$6x = 18$$

b. Solve the resulting equation

$$x = \frac{18}{6}$$
$$\mathbf{x = 3}$$

c. Solve for the other variable
   multiply equation 1 by -2 and
   add to equation 2 to eliminate x

$$-2(2x + y = 6)$$
$$-4x - 2y = -12$$
$$4x - y = 12$$
$$-3y = 0$$
$$\mathbf{y = 0}$$

d. Check in both original equations

First Equation	Second Equation
2x + 3y = 6,	4x – y = 12
2(3)+3(0) = 6	4(3) - 0 = 12
6 + 0 = 6	12 - 0 =12
6 = 6 ✔	12 = 12 ✔

Hence, the solution is **(3,0)**

Example No.2. $-x -2y = -10$ } equation 1

$x - y = 1$ } equation 2

Solution:

a. Look for the variable whose numerical coefficients are opposites. If this condition exists, add the equations

b. Solve the resulting equation

$-x -2y = -10$ equation 1

$x - y = 1$ equation 2

$-3y = -9$ since -x +x =0

$-3y = -9$

$$y = \frac{-9}{-3}$$

y = 3 substitutes to Eqn. 2

c. Solve for the other variable by substituting 3 for y in the second equation.

$x - y = 1$ equation 2

$x - 3 = 1$

$x = 1 + 3$

x = 4

d. Check in both original equations

Equation 1	Equation 2
-x -2y = -10	x – y =1
-4 -2(3) = -10	4 - 3 = 1
-4 – 6 = -10	1 =1 ✔
-10 = -10 ✔	

Hence, the solution is **(4,3).**

Example No. 3.   2a + 4b = 3 } equation 1
                   a + 2b = 1 } equation 2

What do you observe about $\dfrac{a_1}{a_2}$ and $\dfrac{b_1}{b_2}$? Are they equal? Since the system is

inconsistent, it has no solution. Let us find out what will happen if the system will be solved by elimination.

Which variable has numerical coefficients that are opposites? None? Which equation should be multiplied? By what number should you multiply that equation? Why will you use -2 as your multiplier? 2 is the least common multiple of 2 and 1, the coefficients of an in equation 1 and equation 2, and – 2 is the opposite of 2.

Solution:

a. Multiply equation 2 by -2   -2 (a + 2b = 1)
                                     -2a - 4b = -2   equation 3
b. Add equations 1 and 3   2a + 4b = 3
                                     -2a - 4b = -2
                                           0 = 1 False Statement

Take note that both variables were eliminated and   0 ≠1. In this case the system has no solution.

Example No.4.   2x + 5y = 7 } equation 1
                  6x +15y = 21 } equation 2

What do you observe about $\dfrac{a_1}{a_2}$, $\dfrac{b_1}{b_2}$ and $\dfrac{c_1}{c_2}$? Are they equal? Since the system

is consistently dependent, it has infinitely many solutions. Let us find out what will happen if the system will be solved by elimination.

Which variable has numerical coefficients that are opposites? None? Which equation you're going to multiply? By what number should you multiply that equation? Why will you use -3 as your multiplier? 6 is the least common multiple of 2 and 6, the coefficients of an in equation 1 and equation 2, and – 6 is the opposite of 6.
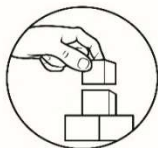
Solution:

a. Multiply equation 1 by -3   -3 (2x + 5y = 7)
                                     -6x – 15y = -21 equation 3

b. Add equations 3 and 2   -6x – 15y = -21

$6x + 15y = 21$ equation 2

$0 = 0$ True Statement

Take note that both variables were eliminated and $0 = 0$. In this case, the system has infinitely many solutions. Hence the solution set is $\left\{ (x,y) / y = \dfrac{-2x+7}{5} \right\}$

What's More

Consider the system: $\left.\begin{array}{l} 3x - 2y = 8 \\ 2x + 3y = -15 \end{array}\right\}$

How do you solve this system by elimination method?

What I Have Learned

How do you solve systems of linear equations using elimination method?

A. Choose which variable you want to eliminate.

B. Multiply one or both equations by an appropriate constant so that the variable that you want to eliminate becomes additive inverse of each other.

C. Add the resulting equations.

D. Solve the equation obtained in STEP C.

E. Substitute the value of the variable obtained in STEP D into one of the original equations and solve for another variable.
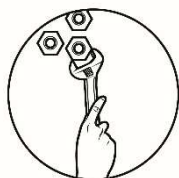
What I Can Do

I. Solve the following systems using elimination method:

1. $\left.\begin{array}{l} x + y = 14 \\ x - y = 6 \end{array}\right\}$

2. $\left.\begin{array}{l} 3x - 5y = 7 \\ 3x + 4y = -11 \end{array}\right\}$

II. Read and Solve

Marinette opens a new online clothing business to garner extra income. Her vintage shirts sell for Php.30 each and plain shirts for Php. 15 each. One month later, she sold a total of 18 shirts which cost Php.435. How many of each kind of shirt was sold?

Assessment

Directions: Choose the letter of the correct answer. Write the letter on a separate sheet of paper.

1.   If $x + y = 3$ and $4x - y = 12$, what is the value of x?

     a. 1   b. 3   c. 5   d. 10

2. Which of the following is the value of q in the system of linear equations

$$\left.\begin{array}{l} p + 2q = 4 \\ p + q = 3 \end{array}\right\}$$

     a. -1   b. 1   c. 2   d. 3

3. In this system of linear equations $\left.\begin{array}{l} 3x - 2y = 1. \\ 2x + 2y = 14 \end{array}\right\}$. What is the value of y?

     a. 2   b. 4   c. -2   d. -4

4. Solve the given system by the elimination method.

$$\left.\begin{array}{l} 3r + 2s = 13 \\ 3r - s = 7 \end{array}\right\}$$

     a. r =3, s =2   b. r = 3, s =1   c. r =4, s = 2   d. r = 4, s = 3

5. By elimination method the solution of system of linear equations
$$\left.\begin{array}{l} -x - y = -6 \\ x - y = 4 \end{array}\right\}$$

     a. (1, 5)   b. (1, 7)   c. (2, 4)   d. (5, 1)

Additional Activity

What is the welcome greeting of Filipino people?

To find out, solve each system of equations. Write the letter in the ANSWER BOX that corresponds to the answer.

O
$x + y = 3$
$3x - y = 1$

A
$3x - 5y = 1$
$4x + 5y = 13$

H
$3x + 2y = 26$
$x - y = 2$

Y
$x + y = 12$
$4x - y = -2$

U
$x + y = 16$
$x - y = 14$

W
$2x - y = 4$
$4x - 2y = 8$

B
$x - 2y = 5$
$4x + 3y = 9$

M
$3x - 5y = 4$
$3x + y = 10$

ANSWER BOX

|						
$x = 3$ $y = 1$	$x = 2$ $y = 1$	$x = 3$ $y = -1$	$x = 15$ $y = 1$	$x = 6$ $y = 4$	$x = 2$ $y = 1$	$x = 2$ $y = 10$

Lesson 4	**Solving Word Problems involving Systems of Linear Equations in Two Variables**

There's a lot of problems that can easily be solved and using one equation with one unknown, which can be solved even more easily; using two equations with two unknowns.

We shall learn this using some problems involving systems of linear equations in two variables.
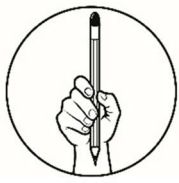
What I Know

Solve what is being asked. Write the letter on a separate sheet of paper

1. The sum of two numbers is 39 and their difference is 17. What is the larger number?

    a. 23   b. 28   b. 33   d. 36

2. Find 2 numbers whose sum is 105 and the larger number exceeds the smaller number by 25.

    a. 20 and 85   c. 40 and 65

    b. 30 and 75   d. 50 and 55

3. The length of the rectangle is 2 cm less than 3 times its width. If the perimeter of the rectangle is 20 cm, find its dimensions.

    a. ω= 3 cm and ℓ =7 cm.   c. ω= 3 cm and ℓ = 9 cm.

    b. ω= 5 cm and ℓ =7 cm.   d. ω= 5 cm and ℓ = 9 cm.

4. Mang Cardo wants to cut a 488 cm piece of wood into 2 pieces. The longer piece is to be 50 cm longer than twice the shorter piece. Find the length of the longer piece.

    a. 342 cm.   b. 324 cm.   c. 423 cm.   d. 432cm.

5. Allyn has a sum of money consisting of Php 20 and Php 50 bill amounting to Php.1500. If there are 48 pieces of bills in all, how many Php 20 bills does she has?

    a. 10   b. 20   c. 30   d. 40

What's In

1. What is the formula that you can use to find the perimeter of a rectangle?

    Is P = 2l + 2w correct?

2. If 2x = 52, what is the value of x? Dividing both sides by 2, you should get 26.

3. Translate into symbols: "The sum of 2 numbers, x, and y, is 41". My answer is

x + y = 41. Is this the same as your answer?

4. Translate into symbols: The longer piece of wood (l) is 30 cm longer than twice the shorter piece (s). My answer is l = 2s + 30. Do we have the same answer?

5. If x + y = 41 and x = 26, what is the value of y? Is it 15?

 What's New

Read and solve:

One day I ask my daughter, Janna to go to Q-Mart to buy fruits. Janna bought apples at Php 35.00 each and Pomelos at Php120.00 each. She bought 9 fruits for a total cost of Php 570.00.

1. How many pomelos did she buy?
2. How many apples did she buy?

 What is It

How do you solve the given problem above? Can we use the system of linear equations in two variables to solve that problem? Can you use the substitution and elimination methods?

Now we will solve the given problem using our guide as our Example no. 1,

Solution:

a. Understand the problem. Decide   You are asked to find some.

what is asked for and what apples and pomelos that Janna's
information is given. bought.

b. Choose variables and use them   Let a = be the number of apples
with the given facts to represent   Let p = be the number of pomelos
each unknown described in the
problem.

c. Write equations that represent   a + p = 9   Eqn. 1
relationships stated in the problem.   35a + 120p = 570 Eqn. 2

d. Solve the equations. $a = 9 - p$ Eqn. 3
Using substitution method by $35(9 - p) + 120p = 570$
Solving a in terms of p in Equation 1 $315 - 35p + 120p = 570$
$$- 35p + 120p = 570 - 315$$
$$85p = 255$$
$$p = \frac{255}{85}$$
$$\mathbf{p = 3}$$
$$a = 9 - 3$$
$$\mathbf{a = 6}$$

e. Check your results with the words   Total Number of fruits 9
of the problems.   6 + 3 = 9

The cost of 9 fruits is Php. 5 ✔

$$6(35) + 3(120) = 570$$
$$210 + 360 = 570 \quad ✔$$

Hence, Janna bought **6** apples and **3** pomelos.

Example no. 2.

The sum of the two numbers is 56 and their difference is 18. Find the numbers

Solution:

a. Understand the problem. Decide   You are asked to find 2 numbers.
what is asked for and what
information is given

b. Choose variables and use them   Let x and y be the numbers.

with the given facts to represent
each unknown described in the
problem.

c. Write equations that represent $x + y = 56$ eqn. 1
   relationships stated in the problem. $x - y = 18$ Eqn. 2

d. Solve the equations. Using $x + y = 56$
   elimination method $\underline{x - y = 18}$
                                          $2x = 74$

$$x = \frac{74}{2}$$

x = 37   substitute to eqn. 1

$x + y = 56$ eqn. 1
$37 + y = 56$
$y = 56 - 37$
y = 19

e. Check your results with the words   The sum is 56: 37 + 19 = 56
   of the problems.   The difference is 18: 37 – 19

The numbers are **37** and **19.**

Example no. 3

A table maker wants to cut an 18-foot piece of wood for his project by 2 pieces. The longer piece is to be 3 feet longer than twice of the shorter piece. What is the length of each piece of his wood?

Solution:

a. Understand the problem. Decide   You are asked to find the length
   what is asked for and what of the 2 pieces of wood.
   information is given.

b. Choose variables and use them   Let s = the length of shorter piece

with the given facts to represent each unknown described in the problem.

ℓ = the length of longer piece

c. Write equations that represent relationships stated in the problem.

$s + \ell = 18$ eqn. 1
$\ell = 2s + 3$ Eqn. 2

d. Solve the equations using substitution method

$s + (2s + 3) = 18$
$s + 2s + 3 = 18$
$3s + 3 = 18$
$3s = 18$

$$S = \frac{15}{3}$$

s **= 5** substitutes to Eqn. 2

$\ell = 2(5) + 3$

$\ell = 10 + 3$

ℓ **= 13**

e. Check your results with the words of the problems.

the sum is 18: 13 + 15 = 18
The longer piece is 3 feet longer than twice of the shorter piece.
13 = 2(5) + 3
13 = 10 + 3
13 = 13

The longer piece is **13 feet** while
The shorter piece is **5 feet.**

Example no. 4

.   Two applicants for a job have ages that add up to 75 years. Twice the age of the younger man less the age of the older man is 30 years. Find their ages.

Solution:

a. Understand the problem. Decide what is asked for and what information is given.

You are asked to find the ages of the 2 applicants

b. Choose variables and use them

Let x = age of the older applicant

with the given facts to represent and y = age of the younger each unknown described in the applicant.

problem.

c. Write equations that represent relationships stated in the problem.

$x + y = 75$ equation 1

$2y - x = 30$ equation 2

d. Solve the equations.

$2y - x = 30$ can also be written as

$-x + 2y = 30$

$x + y = 75$
$-x + 2y = 30$
$\overline{ 3y = 105}$

$y = 35$ Substitute to eqn. 1

$x + y = 75$ eqn. 1
$x + 35 = 75$
$x = 75 - 35$
$x = 40$

e. Check your results with the words of the problems. Answer.

The sum of the ages is 75:

$40 + 35 = 75$

Twice the age of the younger man
Less the age of the older man is 30:

$= 2(35) - 40$
$= 70 - 40$
$= 30$

The older applicant is **40** years old while the younger one is **35** years old.

What's More

Try to solve more word problems

Problem 1.

Allyn and Ave are my friends. The sum of their ages is 80 years. Allyn's age is 8 years younger than Ave's age. Find their respective ages?
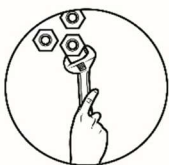
Problem 2.

The length of a rectangle is 5 more than twice its width. The perimeter is 70 meters. What is the area of a rectangle?

What I Have Learned

How do you solve word problems involving systems of linear equations in two variables?

  A. Understand the problem. Decide what are asked for and what information is given.

  B. Choose variables and use them with the given facts to represent each unknown, described in the problem.

  C. Write equations that represent relationships stated in the problem.

  D. Solve the system of linear equations formed.

  E. Check your results with the words of the problems. Give the answer in correct units of measure.

What I Can Do

Read and analyze the word problem below, and answer what is asked:

One day, I visited my grandmother's rice field. The rice field is rectangular, its perimeter is 414 meters. If its length is twice its width, what are the dimensions of the rice field?

  a. Represent the length and width using different variables.

  b. Write an equation for the perimeter of the garden.

  c. Write an equation for the relation of the length to the width.

d. Solve the system formed by the two equations.

e. What are the dimensions of the rice field?

Assessment

Refer to the given problem answer numbers 1 to 3

The sum of two numbers, x, and y, is 25 and their difference is 5.

1. Which of the following systems of equations describe the given problem?

A. $\begin{cases} x + y = 25 \\ x - y = 5 \end{cases}$
  B. $\begin{cases} x + y = 30 \\ x - y = 5 \end{cases}$
  C. $\begin{cases} x + y = 25 \\ x + y = 5 \end{cases}$
  D. $\begin{cases} x - y = 25 \\ x - y = 5 \end{cases}$

2. Which two numbers satisfy the given conditions?

    A. 25 and 5   B. 15 and 10   C. 20 and 10   D. 10 and 25

3. Which of the two numbers is the larger?

    A. 25   B. 15   C. 10   D. 5

Refer to the problem below answer numbers 4 and 5

Before the lockdown, Judith bought her food supplies for her kittens and puppies. She bought puppy's food at Php 225.00 a kilo and kitten's food at Php 185.00 a kilo. She bought 7 kilos for a total cost of Php 1415.00.

4. How many kilos did she buy for her kittens?

    A. 1kilos   B. 3kilos   C. 4kilos   D. 5kilos

5. How many kilos did she buy for her puppies?

    A. 1kilos   B. 2kilos   C. 3kilos   D.4kilos

Additional Activity

Read and answer what is asked in the situation.

The officers of the Student Supreme Government sold banana cakes and carrot cakes to the students as their fund-raising activity. They need to sell 250 slices and need to make Php.5525. How many slices of banana and carrot cakes do they need to sell if banana cakes cost Php. 20 per slice while the carrot cakes cost Php. 25 per slice.?

1. Let B the number of slices of banana and C is the number of slices of carrots.
2. Complete the table below.

	Banana Cake	Carrot Cake	Total
No. of Slices			
Amount in Php			

3. System of Equations: _____

           _____

4. Solution: Number of slices of banana cake   _____

                Number of slices of carrot cake   _____

Answer Key

	Banana Cake	Carrot Cake	Total
No. of Slices	145	105	250
Amount in Php	2900	2625	5525

$$\left\{\begin{array}{l} B + C = 250 \\ 20B + 25C = 5525 \end{array}\right.$$

Number of slices of banana cakes = 145
Number of slices of carrot cake = 105

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