

ECE 445 (ML for ENGG): Homework #4

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1. Given a random variable X , express its variance $\text{Var}(X)$ in terms of its mean, $\mathbb{E}[X]$, and its second moment, $\mathbb{E}[X^2]$.
2. Consider a sequence of independent and identically distributed (iid) real-value random variables $\{X_i\}_{i=1}^M$ whose PDF is given by $p_X(x)$.
 - What is $\mathbb{E}[\prod_{i=1}^M X_i]$?
 - What is $\mathbb{E}[\prod_{i=1}^M (X_i - \mathbb{E}[X_i])^2]$?
 - Given a set $\mathcal{A} \subset \mathbb{R}$, what is $\Pr(X_1 \in \mathcal{A}, \dots, X_M \in \mathcal{A})$?
3. Specify whether the following functions have (i) unique global minimum and no local minima, (ii) multiple global minima and no local minima, (iii) local minima, or (iv) none of the above.
 - $f(x) = x^4$
 - $f(x) = \cos(\pi x) + \sin(\frac{\pi}{2}x)$
 - $f(x) = |x|$
 - $f(x) = \tan(x)$
 - $f(x) = \sin(x)$
4. Consider a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ that is defined as follows:

$$f(\mathbf{w}) = (w_1 + 2w_2 - 7)^2 + (2w_1 + w_2 - 5)^2,$$

where w_1 and w_2 denote the first and second entry of \mathbf{w} , respectively. Analytically, find $\mathbf{w}^* = \arg \min_{\mathbf{w} \in \mathbb{R}^2} f(\mathbf{w})$.