Consider the following learning set:

1 4 1
2 6 1
3 4 1
2 6 1
2 5 1
2 2 2
1 0 2
0 1 2
1 0 2
1 1 2
4 4 3
4 3 3
5 2 3
3 4 3
4 3 3

The numbers in red represents each patterns class

Requirements:

a) Find the class of the following patterns using type 3 classifier:
    1 3
    4 5
    0 0

b) Solve the problem from the 5th laboratory using type 3 classifier

Type 3 classifier means calculating some discriminant functions for each distinct class from the learning set. The class of our pattern will be given by the function that returns the highest value

$$\varphi_i = w_i \cdot \overline{x}$$

$$1 \le i \le m$$

$$W = \begin{pmatrix} w_{11} & w_{12} & \ldots\ldots & -\dfrac{1}{2}\displaystyle\sum_{k=1}^{p} w_{1k}^{2} \\ \\ w_{m1} & w_{m2} & \ldots\ldots & -\dfrac{1}{2}\displaystyle\sum_{k=1}^{p} w_{mk}^{2} \end{pmatrix}$$

$\overline{x}$ - the pattern for which we want to find the class

m – nr of classes

p – nr of features

w_{ij} - the average of feature j for calculated for the patterns from class i

If we have m number of classes, we will have to calculate m φ functions (one for each class). The highest value returned will identify the function corresponding to the searched class

Example:

Let's consider the following learning set (classes are in red):

1	2	1
2	3	1
11	12	2
12	13	2
21	22	3

25	26	3
4	5	?

For finding the class for pattern (4   5), we will first have to calculate the W matrix

We have 3 distinct classes in the learning set, so the W matrix will have 3 rows

Each pattern has 2 features. This means that the W matrix will have 2+1 columns

W11 = (1+2)/2 = 1.5 (average of first feature calculated for the patterns from class 1)

W12 = (2+3)/2 = 2.5 (average of second feature calculated for the patterns from class 1)

W13 = (-1/2) * ($1.5^2 + 2.5^2$) = -4.25 (free term)

W21 = (11+12)/2 = 11.5 (average of first feature calculated for the patterns from class 2)

W22 = (12+13)/2 = 12.5

W23 = (-1/2) * ($11.5^2 + 12.5^2$) = -144.25

W31 = (21+25)/2 = 23

W32 = (22+26)/2 = 24

W33 = (-1/2) * ($23^2 + 24^2$) = -552.5

Now, we have to calculate the discriminant functions using the formula:

$$\varphi_i = w_i \cdot \overline{x}$$

Since we have 3 different classes, we will have to calculate the function for each class

$\varphi_{1=w_1 \cdot \overline{x}} = (1.5 \quad 2.5 \quad -4.25) \cdot \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} = 1.5*4 + 2.5*5 + (-4.25 * 1)$

$= 14.25$

$\varphi_{2=w_2 \cdot \overline{x}} = (11.5 \quad 12.5 \quad -144.25) \cdot \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} = 11.5*4 + 12.5*5 + (-144.25 * 1)$

$= -35.75$

$\varphi_{2=w_2 \cdot \overline{x}} = (23 \quad 24 \quad -552.5) \cdot \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} = 23*4 + 25*5 + -552.5 * 1)$

$= -340.5$

We notice that the biggest return value is corresponding to the function for the first class , so the class we are searching is 1.