Different Filtering Approaches

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Introduction

Filters are standard methods for solving state space models. A state space model contains two sequences of random variables.

1. A latent (hidden) state, X_t, which forms a Markov chain, so

$$p(x_t|x_{0:t-1}) = p(x_t|x_{t-1})$$

2. A series of observations, Y_t, which depends only on, X_t. Hence, given X_t, Y_t is independent of X_s, Y_s with $s \neq t$.

The filtering problem of interest is how to associate all observations to estimate hidden states. We may look for either $p(x_{0:t}|y_{0:t})$ or $p(x_t|y_{0:t})$.

This problem has a recursive nature which allows online calculations. There are several established methods in the literature for the filtering problem.

In realistic scenarios measurements could be nonlinear such as the distance between a race car which is driving around a circular track to a sensor or the distance between a maneuvering airplane to a radar station.

Different Techniques of Filtering

Discrete Bayes filter: It is multimodal, can handle nonlinear measurements, and can be extended to work with nonlinear behaviour. However, it is discrete and univariate.

Kalman filter: The Kalman filter produces optimal estimates for unimodal linear systems with Gaussian noise.

Unscented Kalman filter: The UKF handles nonlinear, continuous, multivariate problems. However, it is not multimodal nor does it handle occlusions. Not well with distributions that are very non-Gaussian or problems that are very nonlinear.

Extended Kalman filter: The EKF has the same strengths and limitations as the UKF, except that is it even more sensitive to strong nonlinearities and non-Gaussian noise.

Particle Filtering

- We are interested in marginal or joint distribution of the latent variables at time t given all observations up to that point.
- The intractability of the normalising constant, $p(y_t|y_{0:t-1})$, often prohibits direct calculation.
- Importance Sampling must be used.
- Sequentially update the posterior distribution at time t without modifying the previous states $x_{0:t-1}$.
- The unnormalized weights are:

$$\tilde{w}_t^{(i)} = \tilde{w}_{t-1}^{(i)} \frac{p(y_t|x_t)p(x_t|x_{t-1})}{q(x_t|x_{0:t-1}, y_{0:t})}, \text{ for i = 1, . . . , N}$$

- q is the proposal distribution

$$q(x_{0:t}|y_{0:t}) = q(x_{0:t-1}|y_{0:t-1})\, q(x_t|x_{0:t-1}, y_{0:t})$$

- q updates recursively in time when the next observation becomes available
- choice of q is very critical.

Unscented Transform

UKF uses unscented transform to capture the posterior mean and covariance to the higher orders. The purpose is to calculate the statistics of a random variable which undergoes a nonlinear transformation **y = g(x)** where **g:** $\mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_y}$ and n_x, n_y are the dimensions of x *and* y respectively.

$$\boldsymbol{X}_0 = \bar{x} \qquad W_0 = \mathrm{k}/(n_x + \mathrm{k}) \qquad i = 0$$

$$\boldsymbol{X}_i = \bar{x} + \left(\sqrt{(n_x + k)\boldsymbol{P}_x}\right)_i \qquad W_i = 1/\{2(n_x + \mathrm{k})\} \qquad i = 1, \dots, n_x$$

$$\boldsymbol{X}_i = \bar{x} - \left(\sqrt{(n_x + k)\boldsymbol{P}_x}\right)_i \qquad W_i = 1/\{2(n_x + \mathrm{k})\} \qquad i = n_x + 1, \dots, 2n_x$$

$$\mathbf{Y}_i = \mathrm{g}(X_i) \qquad \mathrm{i} = 0, \dots, 2n_x$$

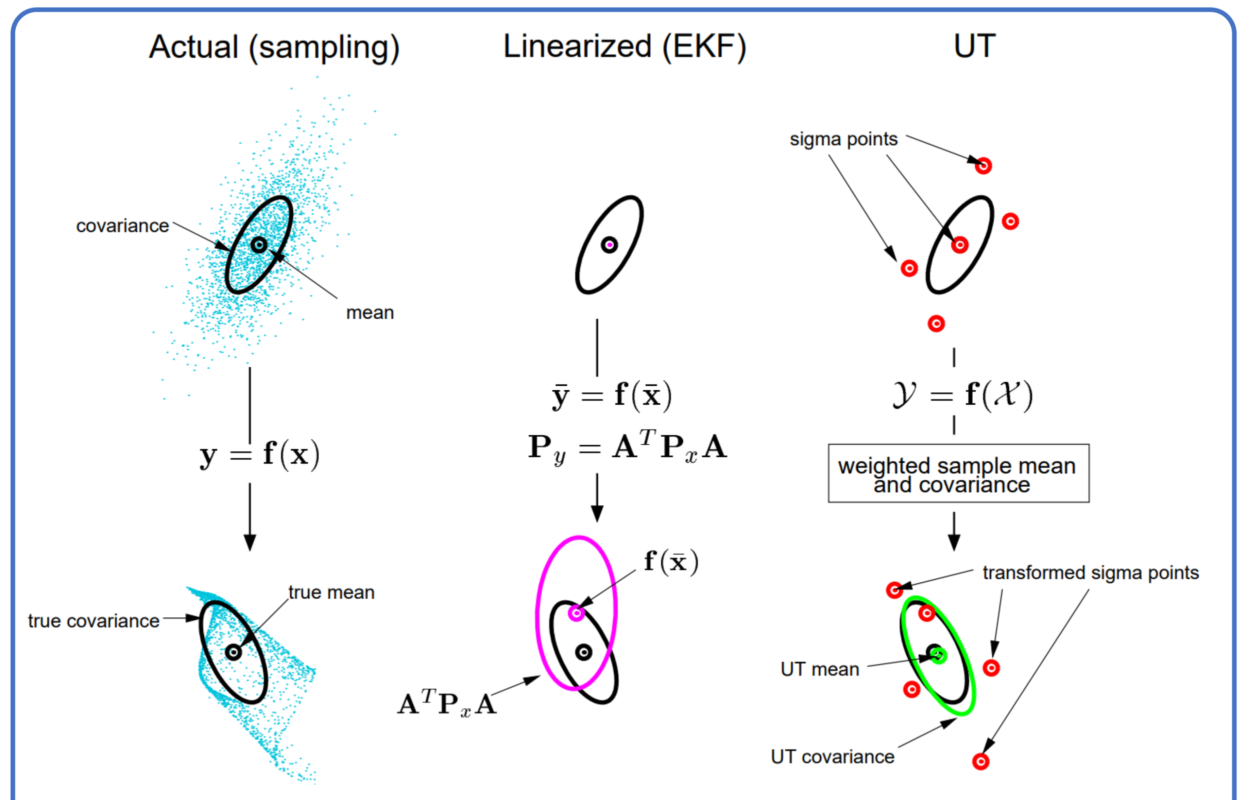
$$\bar{y} = \sum_{i=0}^{2n_x} W_i Y_i\,, \qquad \boldsymbol{P}_y = \sum_{i=0}^{2n_x} W_i (Y_i - \bar{y})\,(Y_i - \bar{y})^T$$

Unscented Particle Filter

Importance Sampling Step is modified

- Update the particles with the UKF
  - Calculate sigma points
  - Propagate particle into future (time update)
  - Incorporate new observation (measurement update)
- Sample $\hat{x}_t^{(i)} \sim q\left(x_t^{(i)} \middle| x_{0:t-1}^{(i)}, y_{1:t}\right) = \mathcal{N}(\bar{x}_t^{(i)}, \hat{P}_t^{(i)})$
- Set $\hat{x}_{0:t}^{(i)} \leftarrow (x_{0:t-1}^{(i)}, \hat{x}_t^{(i)})$ and $\widehat{\boldsymbol{P}}_{0:t}^{(i)}(\boldsymbol{P}_{0:t-1}^{(i)}, \widehat{\boldsymbol{P}}_t^{(i)})$

$$w_t^{(i)} \propto \frac{p\left(y_t \middle| \hat{x}_t^{(i)}\right) p\left(\hat{x}_t^{(i)} \middle| x_{t-1}^{(i)}\right)}{q\left(\hat{x}_t^{(i)} \middle| x_{0:t-1}^{i}, y_{1:t}\right)}, \text{ for i = 1, . . . , N}$$



Experiment

A time-series was generated by the following process model

$$x_{t+1} = 1 + \sin(\omega\pi t) + \phi_1 x_t + v_t$$

Where v_t is a Gamma $\mathcal{G}(3, 2)$ random variable modelling the process noise, and ω = 4e-2 and ϕ_1 = 0.5 are scalar parameters.

A non-stationary observation model,

$$y_t = \begin{cases} \phi_2 x_t^2 + n_t & t \leq 30 \\ \phi_3 x_t - 2 + n_t & t > 30 \end{cases}$$

is used, with ϕ_2 =0.2 and ϕ_3 =0.5 The observation noise, n_t , is drawn from $\mathcal{N}(0, 0.00001)$.

Algorithm	MSE	
	mean	var
Extended Kalman Filter	0.374	0.015
Unscented Kalman Filter	0.280	0.012
Particle Filter	0.424	0.053
Unscented Particle Filter	0.070	0.006

References

1. Van Der Merwe, R., Doucet, A., De Freitas, N., & Wan, E. A. (2001). The unscented particle filter. In *Advances in neural information processing systems* (pp. 584-590).
2. Julier, S. J., & Uhlmann, J. K. (1997, April). A new extension of the Kalman filter to nonlinear systems. In Int. symp. aerospace/defense sensing, simul. and controls (Vol. 3, No. 26, pp. 182-193).

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