Dissipation and Decoherence in Quantum Systems: The Role of Anharmonic Environments

Cristobal I. Vallejos [1] Ricardo C. Heitzer [1] Jorge O. Sofo [1, 2]

[1]Department of Physics, Penn State [2]Department of Materials Science and Engineering, and Materials Research Institute, Penn State

Overlook

Quantum systems are never completely isolated—they constantly interact with their surroundings (the bath). This coupling, especially through vibrations, can limit the performance of emergent technologies such as quantum sensors. In a purely harmonic bath, where phonons have infinite lifetimes, energy transfer is restricted by a spectral gap. Introducing bath anharmonicities (nonlinear effects) enables phonon scattering processes that allow low-frequency transitions. We study a qubit embedded in a crystal and show how these processes influence both its energy dissipation and decoherence.

Physical Motivation

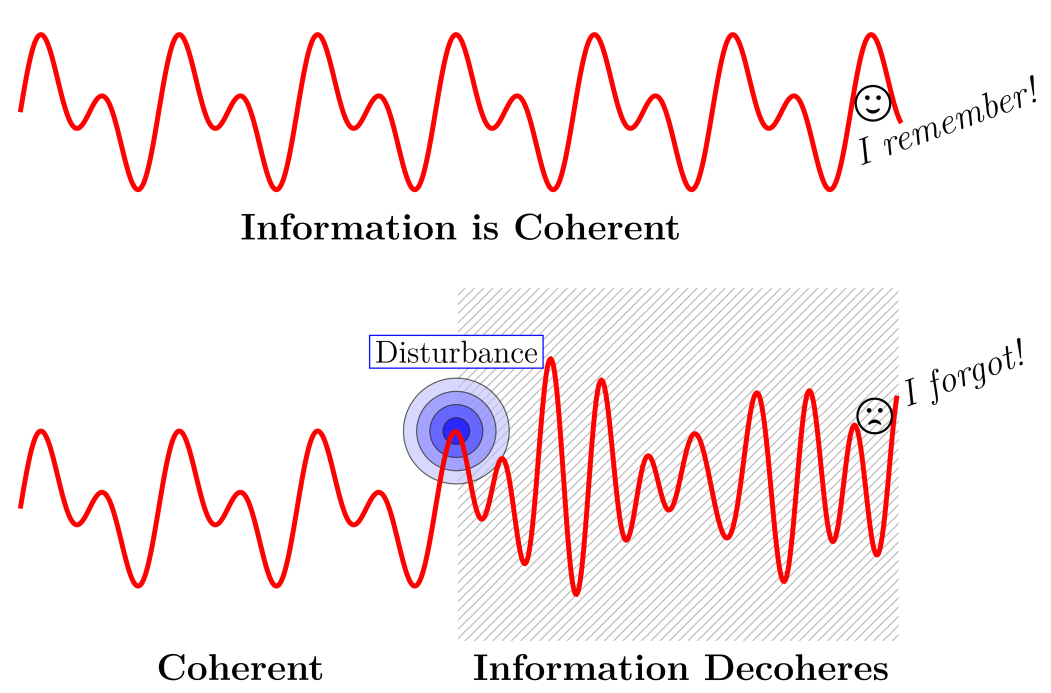

Figure 1. Quantum phase coherence: discernible, repeating pattern (top). Decoherence: loss of a clear pattern (bottom).

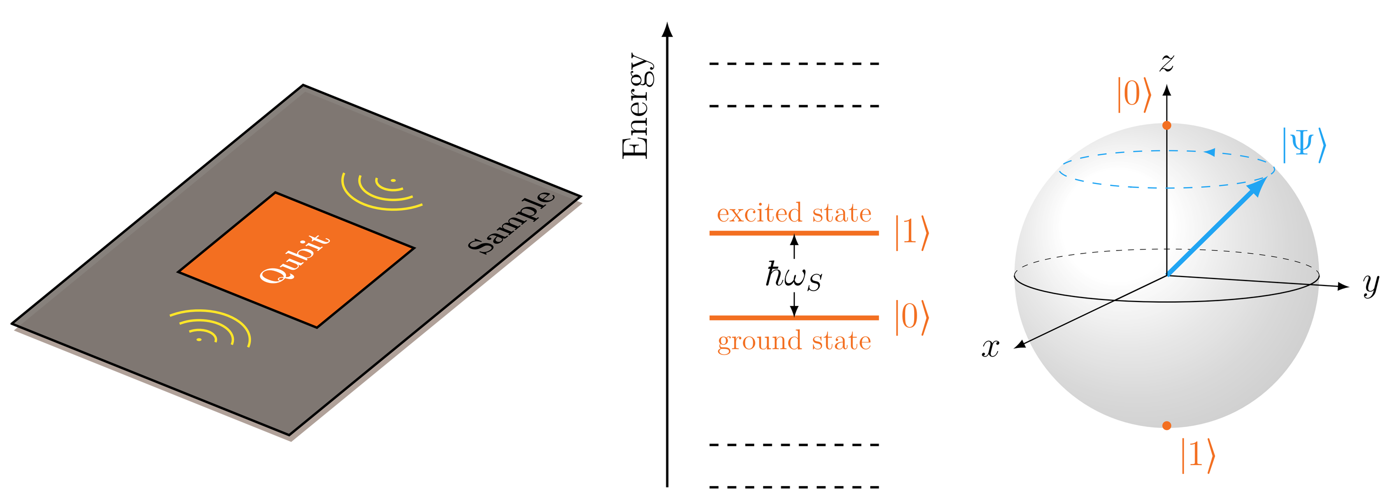

Figure 2. A qubit embedded in a solid affected by vibrations (left). Qubit seen as a two level system (middle). Bloch sphere representation of a qubit state and its (free) time evolution (right).

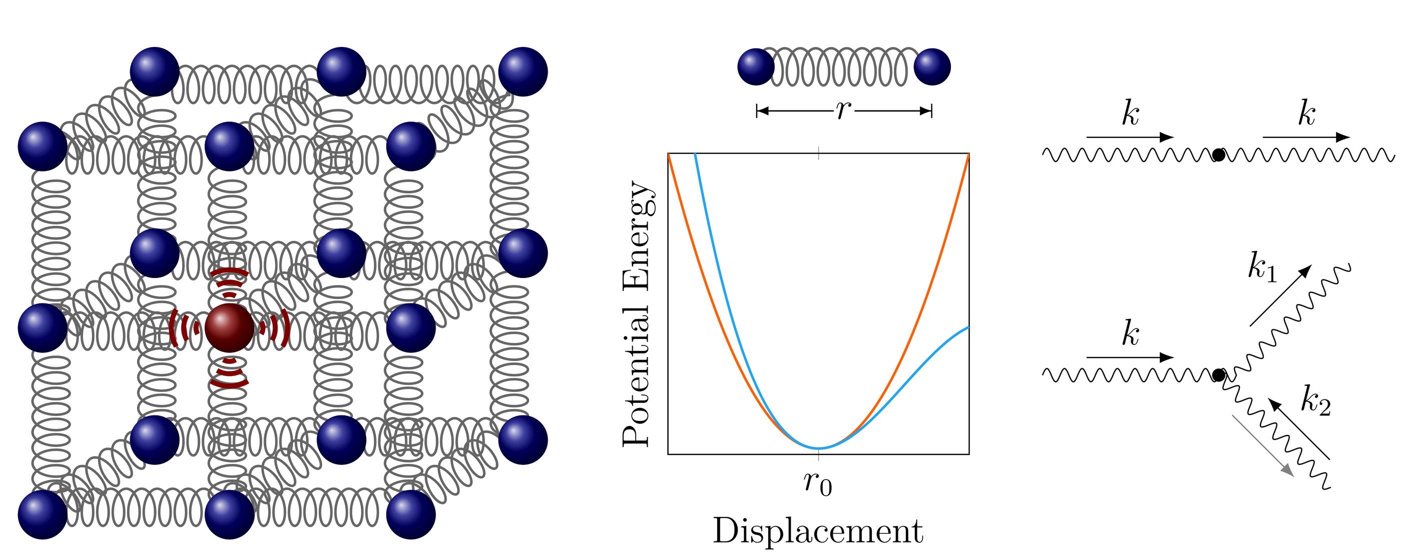

Figure 3. Pictorial representation of lattice vibrations in a solid (left). Interatomic potential energy: harmonic potential vs anharmonic potential, $U(r) = kr^2/2 - \lambda r^3$ (middle). Diagram of 2–phonon and 3–phonon processes (right).

State of the art: The problem can be solved exactly for the harmonic bath case [2]. The anharmonic bath has been extensively studied on their own [3]. Numerical fits to anharmonic baths exist [4]. This work provides a microscopic framework to understand how anharmonic baths affect qubit dynamics.

Theoretical Framework

The dynamics of the qubit is governed by the Lindblad equation [1, 5]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S(t) = -\frac{i}{\hbar}[H_S, \rho_S(t)] + \mathcal{D}[\rho_S(t)] \quad \text{with} \quad \rho_S(t) = \begin{pmatrix} \rho_{00}(t) & \rho_{01}(t) \\ \rho_{10}(t) & \rho_{11}(t) \end{pmatrix}. \tag{1}$$

- ρ_S: density matrix (populations + coherences).
- $H_S = \hbar\omega_S\sigma_z$: qubit Hamiltonian.
- $\mathcal{D}[\rho_S(t)]$: dissipative bath contributions.

The bath influences the qubit through:

1. Frequency shift: renormalized frequency $\tilde{\omega}_S = \omega_S + \Delta$.
2. Effective damping coefficient: exponential decay $e^{-\gamma t}$.

Both Δ and γ depend on the qubit splitting frequency ω_S and the bath spectral density $J(\omega)$, which encodes temperature, characteristic bath frequencies, anharmonicities, and system–bath coupling.

Assumptions: Weak system–bath coupling and Markovian bath.

Our Toy Model

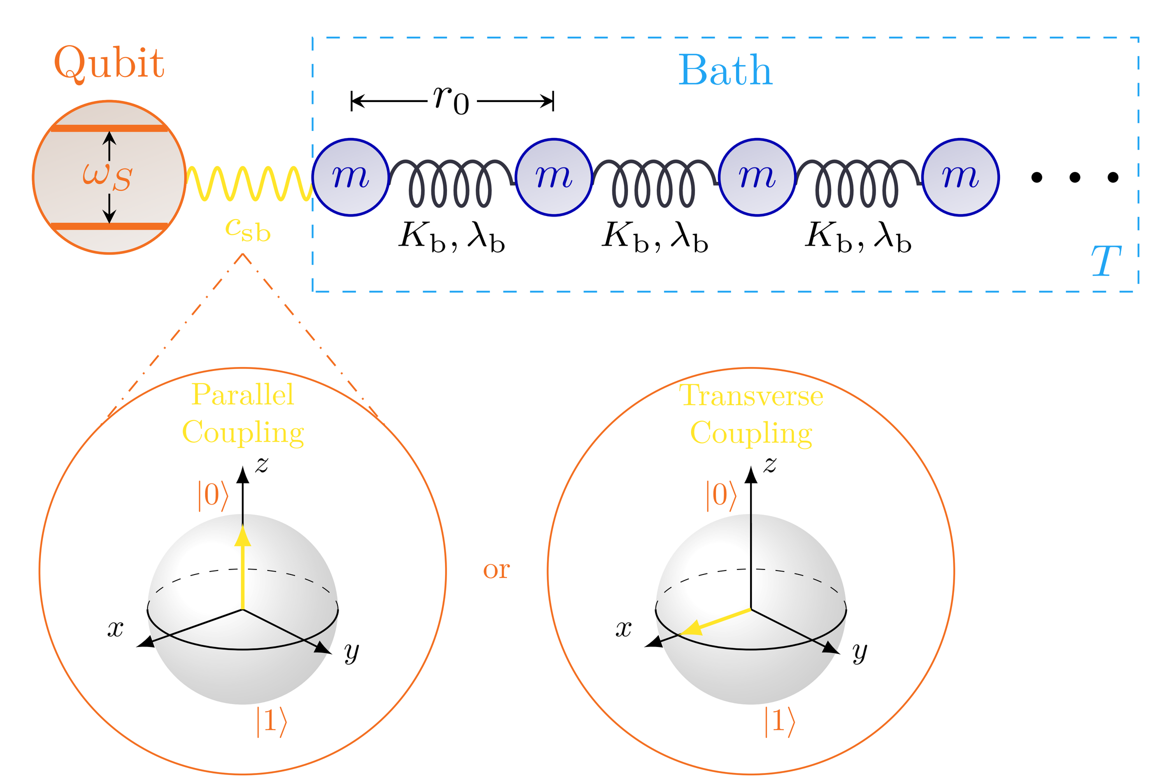

Figure 4. Pictorial representation of the one–dimensional model of a qubit coupled to an anharmonic crystal. The coupling can be parallel or transverse to the qubit states.

Numerical Results

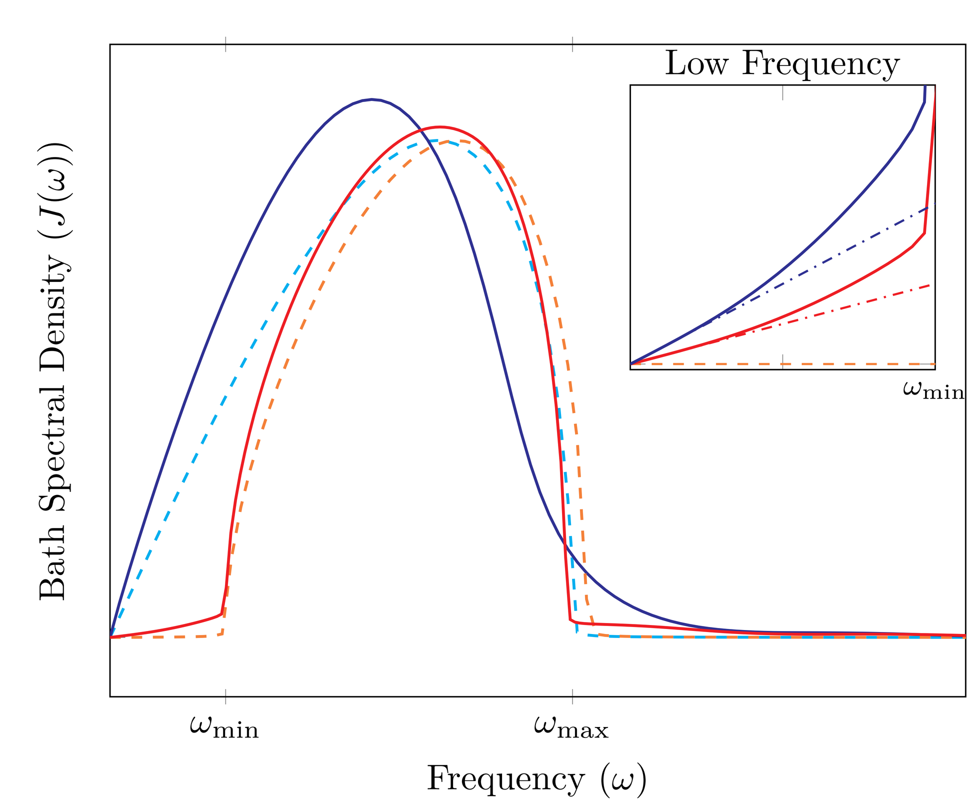

Figure 5. Bath spectral function in term of frequency for a gapped bath.

Interpretation of the Results

- The introduction of anharmonicites enhances the low–frequency regime.
- For low frequencies, we observe that: $J(\omega) = \chi\omega$, where $\chi = \chi(T, \omega_{\text{min}}, \omega_{\text{max}}, \lambda_b, c_{sb})$.
- For parallel coupling, the phase damping rate is given by:

$$\gamma_z = \lim_{\omega\to 0}\frac{J(\omega)}{\tanh(\hbar\omega/(2k_BT))} = \frac{2k_BT\chi}{\hbar} \propto T^2 c_{sb}^2 \lambda_b^2 . \tag{2}$$

- For transverse coupling, the amplitude damping rate is given by:

$$\gamma_x = J(\omega = \omega_S) = \chi\omega_S . \tag{3}$$

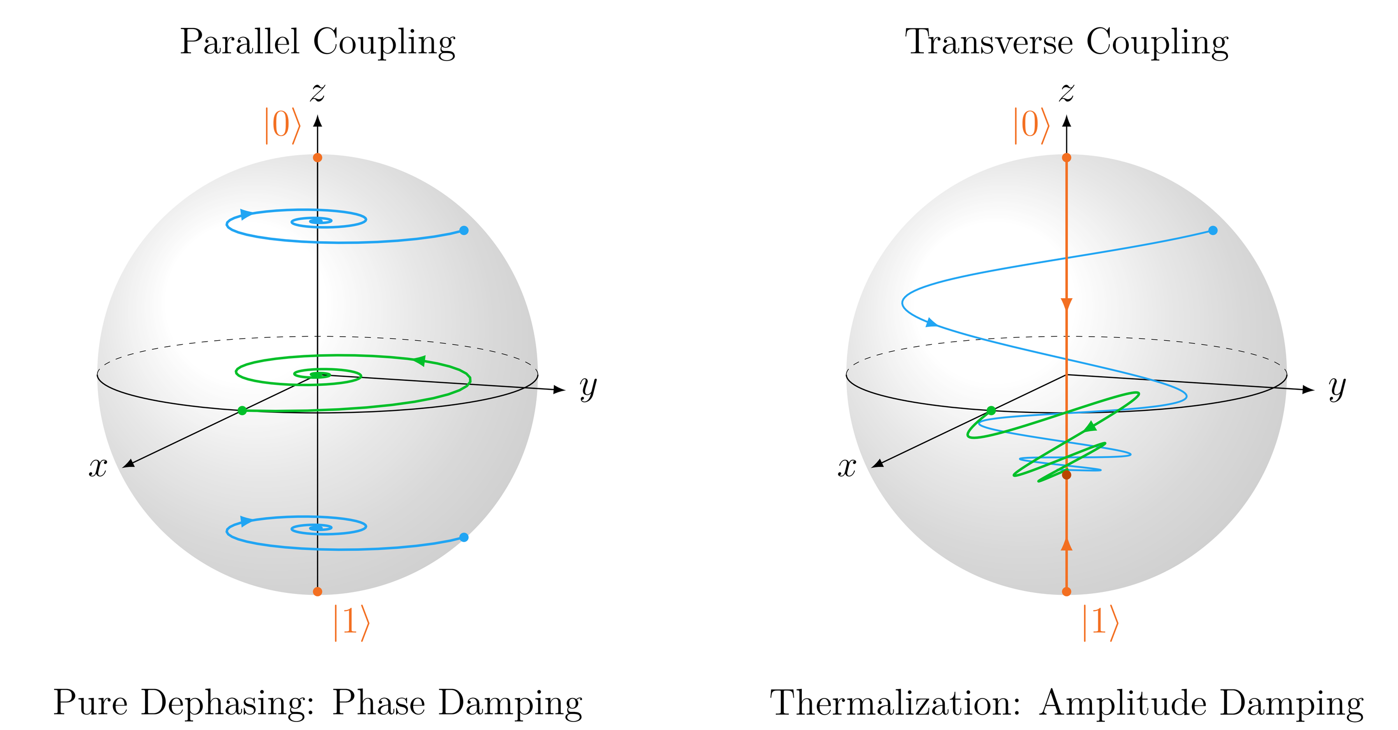

Figure 6. Bloch–sphere flow followed by the qubit for different initial states under the parallel coupling (left) and transverse coupling (right).

Conclusions and Future Work

Conclusions

1. Qubit–bath interaction in the toy model shows a linear low–frequency dependence of the bath spectral density.
2. Damping coefficient depends on temperature and bath anharmonicities: quadratic in temperature for parallel coupling and linear for transverse coupling.

Future Work and Applications

1. Extend the formalism to physically realizable systems such as nitrogen–vacancy (NV) centers in diamond (see Ricardo's poster).
2. Apply this framework to quantum computing, with the goal of reducing decoherence in qubits embedded in solid materials.

Take-Home Message

By better understanding how environmental vibrations affect quantum systems, we can move closer to designing materials and devices that preserve quantum coherence—an essential step toward practical quantum technologies.

References

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