A Bicategorical Semantics of Civic Explanations and Vertical Domains

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Abstract

The Civic Exchange Protocol (CEP) provides a canonical substrate for civic entities, relationships, and exchanges. The Contextual Evidence and Explanations (CEE) layer extends this substrate with structured, evidence-based explanations of why particular civic decisions, alerts, or highlights occur. This paper develops a geometric and categorical perspective on the combined CEP+CEE architecture. We introduce the notion of a *vertical domain* as a structured slice through the global civic graph, and show how verticals compose into a stack with a bicategorical semantics: objects as entities, 1-morphisms as relationships plus provenance, and 2-morphisms as explanations that relate one morphism-level interpretation to another. The resulting framework is mathematically tidy yet operationally useful: it supports interoperable modeling across domains such as SME-friendly procurement, community asset access, environmental compliance, education access, and disaster resilience, while making the logic of explanations explicit and reusable.

Keywords: Civic data; category theory; functorial data modeling; interoperability; canonicalization; identifiers; provenance.

1 Introduction

The development of the Civic Exchange Protocol (CEP) was motivated by a familiar problem in civic data work: heterogeneous sources, idiosyncratic schemas, and ad hoc integrations make it difficult to reason about civic processes in a principled way.

CEP responds by providing a uniform, deterministic substrate for civic entities, relationships, exchanges, and their provenance Case [2025b]. The entity model partitions civic data into six disjoint kinds—Actors (A), Sites (S), Instruments (I), Events (E), Jurisdictions (J), and Observations (O)—as defined by the Civic Accountable Entities (CAE) ontology Case [2025a]. CAE establishes *what kinds of things exist* in civic systems; CEP establishes *how they are represented and exchanged.* At this level the primary concerns are canonicalization, identity, and graph-level normalization.

Contextual Evidence and Explanations (CEE) extend this substrate in a different direction. Where CAE defines *what* civic entities are, and CEP defines *how* they flow, CEE addresses *why* particular nodes, edges, or subgraphs matter in a given context. CEE expresses evidence sets, attribution chains, and explanation bundles that make model behavior, rule-based systems, and analytic pipelines *legible* to both humans and machines.

Taken together, CAE, CEP, and CEE invite a higher-order point of view. Instead of treating each use case—SME-friendly procurement, community asset access, environmental compliance, education access, or disaster resilience—as a separate system, we view each as a *vertical domain*: a structured slice through the same canonical civic universe. Each vertical selects a subset of CAE entity kinds, a family of CEP adapters from raw data into canonical form, and a set of CEE explanation types that capture domain-specific notions of accountability and impact.

Once this perspective is made explicit, a striking simplification occurs. Apparent complexity collapses into a small number of recurring patterns. Different verticals reuse the same canonicalization machinery, the same identity semantics, and the same explanation contracts. They differ mainly in which portions of the civic landscape they highlight and which explanations they attach.

The central claim of this paper is that this simplification reflects deeper mathematical structure. We argue that the combined CAE+CEP+CEE architecture admits a natural *bicategorical* interpretation:

- **0-cells** (objects) correspond to canonical entities classified by CAE kinds,
- **1-morphisms** correspond to relationships and exchanges governed by CEP semantics, and
- **2-morphisms** correspond to CEE explanations that relate one interpretation of a relationship to another.

Vertical domains then appear as structured slices or fibres in this bicategorical space, and maps between verticals correspond to functors and natural transformations that transport both data and explanations.

This paper makes three contributions:

1. We formalize *vertical domains* as fibered structures over the CEP base category, showing how domain-specific semantics layer cleanly atop the shared canonical substrate.
2. We develop a *bicategorical semantics* for CEE explanations, capturing how evidence chains, attributions, and narratives compose across

accountability relationships.

3. We demonstrate the framework on two concrete verticals (SME-friendly procurement and community asset access), showing that diverse policy domains instantiate the same structural patterns.

The remainder of the paper is organized as follows. Section 2 reviews the Civic Accountable Entities (CAE) ontology and the categorical semantics of the Civic Exchange Protocol (CEP) on which Contextual Evidence and Explanations (CEE) is built. Section 3 summarizes the categorical structure of CEP as the canonical exchange layer. Section 4 introduces vertical domains at an operational level, emphasizing how domain-specific entities, relationships, and exchanges are composed. Section 5 describes the layered stack architecture relating CAE, CEP, jurisdictional adapters, and CEE. Section 6 develops the bicategorical semantics underlying vertical composition and interaction. Section 7 explores functorial and oplax maps between vertical domains. Section 8 instantiates the framework on concrete verticals, including SME-friendly public procurement and community asset access. Section 9 discusses related work. Section 10 outlines a forward-looking research agenda. Section 13 concludes.

In addition, the appendices collect background material and supporting exposition intended to improve accessibility without interrupting the main narrative. Appendix A reviews category-theoretic background required for the paper. Appendix B presents worked examples illustrating vertical composition and explanation generation. Appendix C provides proof sketches for key semantic claims, including bicategorical coherence conditions. Appendix D offers diagrammatic intuition using commutative and bicategorical diagrams. Appendix E introduces categorical concepts specific to CEE, including observations, explanations, and attribution. Appendix F provides additional plain-language explanations of categorical terminology used throughout the paper for non-specialist readers.

2 Background

This section summarizes the ingredients on which our framework rests: the Civic Exchange Protocol (CEP), the Contextual Evidence and Explanations (CEE) layer, and the categorical tools we draw upon.

2.1 The Civic Exchange Protocol (CEP)

CEP is a schema- and rewriting-based specification for civic data. At a high level, CEP defines:

- *Entity schemas* for units such as municipalities, facilities, tenders, lots, contracts, programs, shelters, and so on;
- *Relationship schemas* for links such as buyer–issues–tender, asset–located-in–area, facility–has–permit;
- *Exchange and record-envelope schemas* that bundle entities and relationships with provenance and validation artifacts;
- *Vocabularies* for shared codes (procedure types, violation types, status codes, etc.);
- *Canonicalization and identity rules* that normalize data and compute stable fingerprints such as SNFEI.

The operational specification and reference implementation are maintained in the open-source *Civic Interconnect* project [Case and Group, 2025], which also defines concrete schemas, vocabularies, and adapters for multiple domains (campaign finance, municipalities, environment, education, and procurement). We refer the reader to the companion CEP theory paper for the underlying theory of rewriting, canonical forms, graph normalization, and identity fingerprints, which builds on standard accounts of term rewriting systems [Baader and Nipkow, 1998, Terese, 2003].

2.2 Contextual Evidence and Explanations (CEE)

CEE is a nascent but structurally constrained layer that sits above CEP. Conceptually, CEE introduces three core artifacts:

Evidence sets summarizing the facts, metrics, and features that support a particular decision or classification.

Attribution sets describing which models, rules, pipelines, and agents are responsible for that decision.

Explanation bundles packaging evidence, attribution, and narrative fields for a specific subject entity or graph.

An explanation bundle is always anchored to one or more CEP entities or relationships, and to an explicit *explanation type*, such as:

- SME-friendly procurement lot;
- Priority neighborhood for community asset investment;
- Elevated environmental compliance risk facility;
- High-value education program for a given learner profile;
- Critical shelter or route in a resilience plan.

In this way, CEE does not operate over arbitrary raw data, but over canonicalized CEP graphs. This dependency is central to the semantic structure developed later: explanations are layered *on top of* canonical entities and relationships, and they inherit their invariants from the underlying canonicalization and identity rules.

2.3 Category Theory and Categorical Semantics

At a technical level, we appeal to standard notions from category theory and bicategory theory. Good general references include Mac Lane's classic treatment of categories [Mac Lane, 1998], modern introductions by Awodey [Awodey, 2010], Leinster [Leinster, 2014], and Riehl [Riehl, 2017], and applied accounts such as Spivak's work on functorial data modeling and migration [Spivak, 2012, 2014] and Fong and Spivak's "Seven Sketches" [Fong and Spivak, 2019]. We assume familiarity with:

- categories, functors, and natural transformations;
- monoidal structure and composition of morphisms;
- bicategories, 2-morphisms, and coherence conditions.

Appendix 13 provides a brief, self-contained review of the categorical notions we rely on, and Appendix 13 offers a glossary-oriented view for non-specialists.

Informally, we will use the following guiding intuition:

- CEP canonical graphs form (at least) a category: objects are canonical entities or graphs; morphisms are relationships and validated transformations between them.
- CEE explanations behave like 2-morphisms: they "sit between" morphisms, refining or annotating one morphism with respect to another.

This is enough to motivate a bicategorical semantics without committing to a fully formalized construction in this paper. The remainder of the paper builds on this background to develop the notion of vertical domains and their bicategorical structure.

3 Recap of the CEP Canonical Layer

The first paper in this series develops CEP as a rewriting-based, categorical framework for civic data. Here we briefly recap the aspects that are most relevant for vertical domains and civic explanations.

3.1 Canonical Entities and Relationships

Let $\mathcal{E}$ denote the collection of CEP entity schemas, and $\mathcal{R}$ the collection of relationship schemas. Each entity instance is normalized by a strategycontrolled rewriting system that:

- canonicalizes lexical forms (names, addresses, codes);
- aligns fields with schema-defined structures;
- resolves identifiers and computes SNFEI-style fingerprints.

The result is a set of canonical entities and relationships that can be treated as objects and morphisms in a category we denote by **CEP**.

3.2 Graphs, Envelopes, and Provenance

CEP records live not in isolation but as graphs: entities linked by relationships, wrapped in record envelopes that carry provenance and validation evidence. The canonical encoding specification (CEC) and graph normalization specification (GNS) ensure that equivalent graphs normalize to identical representations and hashes.

For the purposes of this paper, it is enough to view:

- *Objects* of **CEP** as canonical entity-graph components (possibly with envelopes attached);
- *Morphisms* as (typed) relationships, exchanges, or validated graph transformations between such components.

3.3 Adapters as Functorial Bridges

Adapters map raw data from external sources into the canonical CEP universe. Each adapter:

1. canonicalizes the input (lexical and semantic normalization);
2. aligns it to CEP schemas;
3. computes identities and attaches attestations.

Operationally, an adapter behaves like a functor from a "source category" of raw records to the target category **CEP**. In the first paper, this is treated primarily at the level of rewriting and graph semantics; here we build on that intuition to organize vertical domains.

4 Vertical Domains

Informally, practitioners already speak of “verticals”: procurement verticals, health verticals, education verticals, and so on. In most software settings these labels refer to application silos or product lines. In the CEP+CEE setting we adopt a more structural definition.

Definition 4.1 (Vertical domain)**.** A *vertical domain* V consists of three interlocking components:

(a) A *CEP scope* C_V: a selected subset of CEP schemas, vocabularies, and identifier schemes, together with the induced rewriting and identity rules on the corresponding entity and relationship objects.

(b) An *adapter scope* A_V: a set of deterministic adapters that map one or more raw data sources into canonical CEP entities and relationships within C_V.

(c) A *CEE scope* E_V: a family of explanation types, evidence-set schemas, and attribution-set schemas that attach structured explanations to entities and relationships in C_V.

Intuitively, C_V specifies *what in the civic universe we are looking at*; A_V specifies *how raw data enters that universe*; and E_V specifies *why certain parts of that universe are highlighted as important.* A vertical is therefore not merely an application domain but a *sustained lens* on the canonical civic graph.

Examples.

- A **SME-friendly procurement** vertical might include entities for buyers, tenders, lots, contracts, and suppliers; relationships describing which buyer issues which tender and which contract is awarded to which supplier; adapters from OCDS-style procurement releases into that schema; and CEE explanation types that answer questions such as "Why is this lot SME-friendly?".
- A **community asset access** vertical might include entities for public assets (parks, libraries, recreation centres), neighbourhoods, and jurisdictions; relationships placing assets into neighbourhoods and neighbourhoods into jurisdictions; adapters from city open data portals and population grids; and explanations that answer "Why is this neighbourhood prioritized for new assets?".

In both cases the vertical is defined by a *coherent pattern of use* across CEP, adapters, and CEE, not by any single file or schema.

Design constraints. Vertical domains are subject to several design constraints that distinguish them from one-off analyses:

(i) **Reusability.** The CEP scope C_V should reuse existing canonical schemas and vocabularies wherever possible, rather than proliferating bespoke types.

(ii) **Determinism.** Adapters in A_V must respect CEP's canonicalization and identity rules so that repeated runs produce stable entity identifiers and graph structure.

(iii) **Legibility.** Explanations in E_V must satisfy CEE's evidence and attribution constraints, so that their semantics are inspectable, testable, and comparable across instances and time.

(iv) **Locality.** Each vertical should be cancellable or extensible with minimal disruption to others: one can remove the community asset vertical without affecting the SME procurement vertical, and vice versa.

These constraints are reflected in the repository structure: each vertical has its own *about* file, examples, adapters, and tests, but they all sit on top of the same CEP and CEE foundations. They ensure that verticals compose cleanly into a larger civic data ecosystem.

5 The Stack of Vertical Domains

Although vertical domains can be developed independently, they share a common architectural pattern. We describe this pattern as a *stack* layered over the CEP base.

5.1 Layers of the Stack

Conceptually, the stack has four layers:

(1) **Canonical base (CEP).** Schemas, vocabularies, identity, canonicalization, and graph normalization.
(2) **Adapters.** Deterministic mappings from raw sources into CEP entities and relationships.
(3) **Explanations (CEE).** Explanation types, evidence sets, attribution sets, and narratives over canonical graphs.
(4) **Vertical domains.** Slices that select subsets of the above three layers for specific civic questions.

The first three layers are domain-agnostic: they apply to any civic context expressible in CEP. The fourth layer instantiates them for particular questions of interest.

5.2 Verticals as Fibers over the CEP Universe

Let $\mathcal{U}_{\text{CEP}}$ denote the "universe" of CEP schemas and vocabularies. Each vertical V chooses a subset $\mathcal{U}_V \subseteq \mathcal{U}_{\text{CEP}}$ together with adapters and explanations.

The collection of all verticals can be arranged as a family of fibers over $\mathcal{U}_{\text{CEP}}$:

$$\pi : \mathsf{Vert} \to \mathcal{U}_{\text{CEP}},$$

where the fiber over a given subset of schemas is the set of verticals that use exactly those schemas (up to specified equivalence).

We do not formalize this as a fibration in the categorical sense, but the analogy is useful: moving along the base corresponds to changing which parts of CEP are in scope; moving within a fiber corresponds to changing explanations or adapters while keeping the same schemas.

5.3 Reusability Across Verticals

A major benefit of the stack perspective is reusability. Because verticals share the CEP base, several components can be reused:

- entity schemas (e.g., municipalities, regions, facilities) may appear in multiple verticals;
- identity schemes (e.g., SNFEI) provide cross-vertical linking;
- vocabularies (e.g., procedure types, violation types) can be shared;
- explanation types may be reused or adapted across verticals.

This manifests as maps between vertical domains, a topic we return to in Section 6 and Section 8.

6 A Bicategorical Semantics of Civic Explanations

We now sketch a bicategorical semantics that integrates CEP and CEE. The goal is not a fully formal construction, but a coherent picture that makes vertical domains and their interconnections mathematically transparent.

6.1 CEP as a Category of Canonical Civic Graphs

Let **CEP** be the category whose:

- objects are canonical CEP graphs (entities, relationships, envelopes, identity, and provenance) modulo graph normalization;
- morphisms are well-typed graph homomorphisms and schema-respecting transformations, composed by ordinary function composition.

Adapters are then functors $F : \mathcal{C} \to \mathbf{CEP}$ from source categories $\mathcal{C}$ of raw records.

For a vertical V, we restrict to a subcategory $\mathbf{CEP}_V \subseteq \mathbf{CEP}$ as in Section 4.

6.2 CEE Explanations as 2-Morphisms

Explanations in CEE relate not only objects but *morphisms*. Informally:

- A relationship, classification, or decision is represented by a morphism $f : X \to Y$ in $\mathbf{CEP}_V$.
- An explanation bundle describes why f holds, which evidence it depends on, and which agents or models are responsible.

Given two morphisms $f, g : X \to Y$ (for example, two different ways of classifying the same lot or facility), an explanation can be understood as a 2-morphism

$$\alpha : f \Rightarrow g$$

witnessing how f is obtained from g under a particular explanatory contract, or how f is justified relative to a baseline.

This motivates the following heuristic construction.

Conjecture 6.1. *There exists a bicategory* **Civ** *whose:*

- *objects are canonical civic graphs (or families of entities);*

- *1-morphisms are CEP relationships and graph transformations with provenance;*
- *2-morphisms are CEE explanations relating 1-morphisms under explanation types and contracts.*

We do not claim to have constructed **Civ** in full generality, but the vertical domains we study can be understood as fragments of such a bicategory.

6.3 Verticals as Sub-bicategories

For each vertical V, we may consider the full sub-bicategory $\mathbf{Civ}_V$ of **Civ** whose:

- objects are those graphs built from $\mathcal{E}_V$ and $\mathcal{R}_V$;
- 1-morphisms are CEP morphisms in $\mathbf{CEP}_V$;
- 2-morphisms are explanations in Ξ_V attaching to those morphisms.

This aligns with the architectural view of Section 5: each vertical is a slice of the full bicategory, with its own adapters and explanation types.

6.4 Maps Between Vertical Domains

Once we acknowledge this bicategorical structure, maps between verticals become semantically meaningful. For verticals V and W we can study:

- *Entity alignment functors* $F : \mathbf{CEP}_V \to \mathbf{CEP}_W$ that identify shared entity types (e.g., municipalities, regions, facilities).
- *Evidence transport* that lifts metrics or indices from one vertical to another (e.g., deprivation indices used in both health and community asset explanations).
- *Explanation composition* where 2-morphisms in $\mathbf{Civ}_V$ and $\mathbf{Civ}_W$ compose to yield a joint explanation (e.g., combining SME-friendly procurement with local economic impact).

These constructions are sketched in more concrete form in Section 8.

7 Maps Between Vertical Domains

Once vertical domains are regarded as structured slices of a common bicategorical space, it becomes natural to ask how they relate to one another. In this section we sketch three classes of "maps between verticals", each corresponding to a different way of transporting structure.

7.1 Entity-Alignment Functors

The most basic maps are functors that align entities across verticals. For instance, the same municipality may appear in both the community asset vertical and the environmental compliance vertical. Both verticals use the same CEP entity schema and the same SNFEI identifier, but they emphasise different relationships and explanations.

An *entity-alignment functor* between verticals V and W is a functor $F : \mathcal{C}_V \to \mathcal{C}_W$ between the underlying entity-relationship categories such that:

- on objects, F identifies shared entities (e.g. a municipality in V with the same municipality in W);
- on morphisms, F preserves those relationships that are meaningful in both verticals (e.g. jurisdictional embeddings).

These functors formalize the intuition that verticals live in the same universe; they differ in focus, not in ontological commitment.

7.2 Evidence-Transport Maps

A richer kind of map transports *evidence semantics* from one vertical to another. For example, a deprivation index used in the community asset vertical may also be relevant in a health outcomes vertical, or risk metrics computed in an environmental compliance vertical may inform a resilience vertical.

Given verticals V and W, an *evidence-transport map* associates to each explanation type in V a compatible explanation type or evidence schema in W, together with transformation rules for metric names, scales, and interpretations. At the bicategorical level, such a map can be viewed as a 2-functor that acts on explanation 2-morphisms.

7.3 Explanation-Composition Maps

Finally, explanations from different verticals can sometimes be composed to yield higher-level narratives. For instance:

- An SME-friendly procurement explanation may be composed with a community asset explanation to yield an account of why a particular procurement decision supports community economic resilience.
- An environmental risk explanation may be composed with a shelter-criticality explanation to express how facility risk impacts evacuation planning.

We refer to such constructions as *explanation-composition maps*. Formally, they appear as higher-order 2-morphisms that combine explanations from different verticals along shared objects and relationships.

These maps are central to the idea of a civic *stack*: they show how verticals remain modular while still participating in a coherent global semantics.

8 Illustrative Case Studies of Vertical Domains

We briefly illustrate how the preceding ideas manifest in concrete verticals. Each case study is treated schematically; detailed schemas and examples are available in the accompanying implementation.

8.1 SME-Friendly Procurement

The SME procurement vertical V_{SME} selects:

- entities: buyers, tenders, lots, contracts, suppliers;
- relationships: buyer–issues–tender, tender–has–lot, contract–awarded-to–supplier, and so on;
- adapters: mappings from OCDS-style releases to CEP entities;
- explanation types: SME-friendly procurement explanations attached to lot entities.

The primary question is:

> Why is this lot considered SME-friendly?

Evidence includes lot value relative to a threshold, procedure type, lot structure, and possibly historical SME participation. Attribution identifies the rule or model version and responsible agents.

In bicategorical terms, SME-friendly explanations are 2-morphisms in $\mathbf{Civ}_{\mathrm{SME}}$ that refine classifications of lots. They enable functorial comparison across jurisdictions and time.

8.2 Community Asset Access

The community asset vertical V_{Assets} selects:

- entities: assets (parks, libraries), areas, jurisdictions;
- relationships: asset–located-in–area, area–part-of–jurisdiction;
- adapters: from open data portals and population grids;
- explanation types: area access priority explanations.

The primary question is:

> Why is this neighborhood prioritized for investment in community assets?

Evidence aggregates population, distance to nearest assets, asset counts within thresholds, and equity indices. These explanations act as 2-morphisms over morphisms that connect areas to assets and jurisdictions.

8.3 Environmental Compliance, Education Access, and Resilience

Similarly structured verticals arise for environmental compliance, education access and value, and disaster resilience. In each case:

- a subset of CEP schemas defines the entities and relationships;
- adapters bring public data into canonical form;
- explanations attach to highlighted entities or relationships, answering a focused "why" question.

Across these verticals, common entities (e.g., municipalities) and indices (e.g., deprivation measures) appear repeatedly, enabling maps between sub-bicategories and composition of explanations.

These case studies illustrate how vertical domains instantiate the abstract framework of bicategories of explanations, providing structured, interpretable insights tailored to specific civic questions.

9 Related Work

We briefly situate this work within three strands of literature: interoperability specifications, explainable AI, and categorical semantics.

9.1 Interoperability Specifications

CEP is inspired in part by standards such as OCDS for procurement, FHIR for health data, and PROV for provenance. These specifications establish domain-specific schemas and sometimes limited notions of provenance and interpretation. CEP extends this tradition with a rewriting-based canonicalization layer and unified identity semantics.

The notion of vertical domains resonates with how profiles and extensions are used in these communities but adds an explicit semantic stack and categorical framing.

9.2 Explainable AI and Model Governance

CEE is aligned with work on model cards, data statements, and regulatory requirements for explanation in domains such as credit, employment, and public benefits. However, most existing approaches treat explanations as annotations of model artifacts, not as 2-morphisms over canonical civic graphs. Our approach suggests a more structural integration of explanations with data semantics.

9.3 Categorical Semantics and Applied Category Theory

Finally, this work connects to applied category theory in databases, open games, probabilistic programming, and causal inference. The idea of treating explanations as 2-morphisms is reminiscent of compositional approaches to lenses, rewrites, and double categories. We view this paper as an invitation to bring similar tools to civic data and governance contexts.

10 Research Agenda

The preceding sections outline a conceptual framework. Turning this into a mature theory and a robust body of practice requires sustained work along several fronts. Here we sketch a research agenda structured around three themes: formalization, empirical validation, and tooling.

10.1 Formalization

On the formal side, several questions arise naturally:

- **Bicategory structure.** Precisely characterize the bicategory of civic entities, relationships, and explanations. Specify identity and composition laws for 2-morphisms and prove coherence theorems.
- **Vertical domains as fibres.** Develop a fibration or indexed-category perspective in which each vertical domain appears as a fibre over a base of CEP schemas and vocabularies.
- **Transport of explanations.** Characterize when explanations in one vertical can be transported to another via functors and 2-functors between corresponding categories.
- **Universal properties.** Investigate whether certain verticals satisfy universal properties (limits, colimits, adjunctions) with respect to others, especially in multi-criteria decision settings.

These questions are not merely abstract. They capture when and how explanations can be safely reused, compared, or composed across domains.

10.2 Empirical Validation

The framework must also be tested against real civic data and realistic decision contexts.

- **Case studies.** Develop detailed case studies for a small set of verticals (such as SME procurement and community asset access), tracing the entire pipeline from raw data through CEP, adapters, and CEE into concrete decisions.

- **Identity robustness.** Evaluate the stability and robustness of SNFEI identifiers and other identity schemes under noisy and adversarial naming conditions, since identity is central to cross-vertical reasoning.
- **Explanation quality.** Assess whether CEE-style explanations improve human understanding, trust, and decision quality compared to baseline dashboards or opaque scores.
- **Cross-vertical scenarios.** Design scenarios where decisions depend on combining information from multiple verticals, and study how well the proposed mapping and composition mechanisms behave.

10.3 Tooling and Automation

Finally, there is a practical tooling agenda.

- **Vertical scaffolding.** Automate the creation of vertical scaffolds from compact `about.yaml` specifications, including directory layouts, adapter stubs, CEE helper modules, and example tests.
- **Schema- and vocab-aware IDE support.** Develop editor and CI tooling that keeps schemas, adapters, explanations, and vertical metadata in sync.
- **Visualization.** Create visualization tools that expose the geometric and categorical structure: civic graphs, explanation layers, and maps between verticals.
- **Reference implementations.** Maintain a small but representative library of open verticals that act as regression suites for both the theory and the tooling.

Together, these threads describe how the initial geometric and bicategorical picture can grow into a stable, shared foundation for civic data and civic explanations.

11 Limitations

This paper develops a structural and categorical perspective on civic explanations and vertical domains. Several limitations should be made explicit.

First, the bicategorical semantics proposed here is intentionally *schematic*. While the objects, 1-morphisms, and 2-morphisms are well motivated by CEP and CEE practice, we do not provide a fully formalized construction of the bicategory **Civ**, nor do we prove full coherence or universality results. The aim is conceptual clarity rather than maximal formal generality.

Second, the scope of this work is restricted to *canonical, revision-based civic records*. Streaming data, probabilistic estimates, continuously updated signals, and purely statistical aggregates are not modeled. Extending the framework to such settings may require enriched, temporal, or probabilistic categorical structures.

Third, explanations in CEE are treated as structured, inspectable artifacts, not as psychological or normative guarantees. This paper does not claim that explanations defined in this framework are sufficient for fairness, accountability, or regulatory compliance in all settings; rather, it provides a semantic substrate on which such claims can be evaluated.

Finally, while vertical domains are motivated by real civic use cases, the examples in this paper are illustrative rather than exhaustive. Production deployments raise additional concerns around governance, institutional incentives, data quality, and long-term maintenance that lie beyond the present treatment.

12 Future Work

The framework developed here opens several natural directions for extension.

On the theoretical side, future work includes completing the formal bicategorical construction sketched in Section 6, establishing coherence laws for explanation composition, and clarifying the precise categorical status of vertical domains as indexed or fibered substructures over the CEP base.

On the semantic side, richer explanation types may be developed, including counterfactual explanations, contrastive explanations, and policy-sensitive explanations that depend explicitly on normative or regulatory regimes. Integrating such explanations while preserving modularity and inspectability remains an open challenge.

On the systems side, further work is needed to connect the semantics to robust tooling. This includes automated validation of explanation bundles, regression testing of verticals across data updates, and visualization techniques that make the layered structure of entities, relationships, and explanations legible to practitioners.

Finally, the interaction between CEP+CEE and external governance frameworks—such as procurement law, environmental regulation, and AI oversight regimes—deserves sustained study. The categorical perspective developed here provides a promising foundation for such work, but its institutional implications remain to be explored.

13 Conclusion

This paper has proposed a way to see civic data and civic explanations as parts of a single geometric and categorical architecture. The Civic Exchange Protocol (CEP) supplies a canonical substrate for entities, relationships, identities, and provenance. The Contextual Evidence and Explanations (CEE) layer adds contracts for evidence, attribution, and narratives. Vertical domains then appear as structured slices through this shared universe: they select particular schemas, adapters, and explanation types to answer concrete civic questions.

By interpreting entities as objects, relationships (with provenance) as 1-morphisms, and explanations as 2-morphisms in a bicategory, we obtain a language for reasoning both *within* and *across* domains. A single municipality, facility, or program can participate in multiple verticals without duplication; explanation patterns can be transported, compared, and composed; and new verticals can be added without destabilizing existing ones. In this view, civic explanations are not merely annotations on a graph but first-class morphisms between morphisms: disciplined ways of saying *why* a relationship holds, under what assumptions, and how it relates to other relationships.

From a practical standpoint, the framework offers a disciplined path for building end-to-end pipelines that are simultaneously interoperable, explainable, and mathematically coherent. It suggests how civic technologists, regulators, and communities might coordinate around shared abstractions—schemas, adapters, and explanation types—rather than bespoke one-off systems.

From a theoretical standpoint, it opens a research program at the intersection of category theory, graph rewriting, and AI transparency: formalizing the bicategory of civic entities and explanations, characterizing verticals via universal properties, and studying when and how explanations can be transported between domains.

Perhaps the most striking feature is the simplification effect. What begins as an apparently intractable landscape of heterogeneous systems resolves, under the right abstractions, into a small collection of reusable patterns and maps between them. This is often the signature of a durable theory: once the correct level of description is found, what was impossibly complicated can suddenly appear almost self-evident.

The work ahead—including refining the formal semantics, validating verticals on real data, and stress-testing explanation transport across domains—is substantial but tractable. More importantly, it is grounded in concrete civic questions: who is affected, what decisions are being made, and how those decisions can be explained and improved. In that sense, the stack of vertical domains is not simply an abstract construction; it is a proposed blueprint for civic infrastructure that can reason about itself in ways that are legible and justifiable to the communities it serves.

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Appendix A. Category-Theoretic Background

This appendix summarizes the categorical notions used in the bicategorical semantics of vertical domains and civic explanations. It parallels Appendix A in the CEP semantics paper but extends it to the 2-categorical setting required for CEE.

A.1 Categories

A *category* $\mathbf{C}$ consists of:

- a collection of *objects*,
- a collection of *morphisms* (arrows) between objects,
- a composition law for morphisms,
- identity morphisms for every object,

satisfying associativity and identity axioms.

In the combined CEP+CEE setting:

- objects correspond to canonical entity states,
- morphisms correspond to relationships and provenance-preserving transformations between entities.

A.2 Functors

A *functor* $F : \mathbf{C} \to \mathbf{D}$ maps:

- each object X to an object $F(X)$,
- each morphism $f : X \to Y$ to a morphism $F(f) : F(X) \to F(Y)$,

preserving identity and composition.

Adapters in CEP behave functorially under normalization and provenance.

A.3 Natural Transformations

Given two functors $F, G : \mathbf{C} \to \mathbf{D}$, a *natural transformation* $\eta : F \Rightarrow G$ assigns to each object X a morphism $\eta_X : F(X) \to G(X)$ such that the usual naturality square commutes.

In our setting:

- natural transformations model attestations,
- they ensure that processing and validating commute coherently.

A.4 Monoidal Categories

A *monoidal category* adds a binary tensor product $\otimes$ and a unit object I satisfying coherence identities.

Canonicalization in CEP is monoidal: it combines subcomponents into canonical strings in a deterministic order.

A.5 Bicategories

A *bicategory* is a weakening of a strict 2-category. It has:

- objects,
- 1-morphisms between objects,
- 2-morphisms between 1-morphisms.

Composition is associative only up to coherent isomorphism.

CEE explanations naturally form 2-morphisms in a bicategory whose 1-morphisms are provenance-preserving relationships.

A.6 Fibrations and Fibers

A *fibration* describes a category whose objects carry context-sensitive structures above a base. CEE's CTags and explanation types live in fibers above canonical entities.

A.7 Universal Properties

SNFEI exhibits a universal property: its canonical string is terminal among structures satisfying invariance, determinism, and rewrite stability.

Understanding verticals often hinges on universal constructions across domains.

Appendix B. Worked Examples

This appendix presents two concrete worked examples of bicategorical interpretation: one for SME-friendly procurement and one for community asset access.

B.1 SME-Friendly Procurement

Consider a procurement lot with noisy inputs: multiple spellings of the supplier's name, ambiguous CPV codes, and missing procedure-type metadata.

Step 1: Canonicalization (CEP base).

1. Normalize the entity fields (legal name, jurisdiction, value).
2. Produce a canonical string in fixed order.
3. Compute SNFEI via SHA-256.

Step 2: Adapter semantics. Jurisdictional quirks (missing CPV codes, inconsistent currencies) are handled via oplax functorial rules.

Step 3: CEE explanation. Evidence: low estimated value, open procedure type, minimal documentation. Attribution: rule-based model “sme-rule-v1”. Narrative: “This lot appears SME-friendly because . . . ”.

Here the explanation is a 2-morphism refining the procurement relationship.

B.2 Community Asset Access

Consider a neighborhood polygon and a dataset of parks and libraries.

Step 1: Construct CEP entities:

- area entity (neighborhood),
- asset entities (parks, libraries),
- relationships linking assets to areas.

Step 2: Compute evidence layers and metrics: population-served, distance-to-assets, equity index.

Step 3: Perform CEE prioritization: Based on computed metrics and attribution model, the vertical outputs a bundle with AREA_ACCESS_PRIORITY. This bundle is a 2-morphism living above the area’s incoming and outgoing relationships.

B.3 Composition Across Verticals

A municipality appearing in both verticals supports functorial maps aligning their CEP entities and enabling interoperable explanations.

Appendix C. Proof Sketches

This appendix outlines informal sketches of several propositions stated in the main text. Full formalization is left for future work.

C.1 SNFEI Uniqueness up to Canonical Equivalence

Claim. If two entities normalize to the same canonical string, then they receive the same SNFEI, and any allowed CEP morphism preserves SNFEI.

Sketch. Normalization is a strict monoidal functor from raw records to canonical strings. SHA-256 is a deterministic function. Thus SNFEI is invariant under any morphism commuting with normalization.

C.2 Explanation Bundles Form 2-Morphisms

Claim. CEE explanations satisfy the axioms of 2-morphisms in a bicategory.

Sketch. Given relationships $f, g : X \to Y$, an explanation bundle assigns contextual evidence relating them. Composition of explanations corresponds to sequential evidence refinement. Identity explanations come from trivial evidence sets. Associativity holds up to narrative equivalence.

C.3 Vertical Domains as Fibers

Claim. Each vertical domain forms a fiber over the CEP base category.

Sketch. Fix an entity type E. All CEE explanations whose subject is E form a fiber above E. Domain boundaries merely restrict fibers to selected schemas and explanation types.

Appendix D. Diagrammatic Intuition

This appendix translates the bicategorical semantics into diagrammatic intuition using string diagrams and geometric metaphors.

D.1 1-Morphisms as Flows

Entities are points; relationships are arrows connecting them. Adapters bend and reshape arrows while preserving alignment.

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, dashed, "N"'] & Y \arrow[d, dashed, "N"] \\ \mathsf{Canon}(X) \arrow[r, "\mathsf{Canon}(f)"'] & \mathsf{Canon}(Y) \end{tikzcd}$$

D.2 Explanations as Surfaces

A 2-morphism $E : f \Rightarrow g$ becomes a surface spanning two arrows. Narratives correspond to labeling this surface with evidence and attribution.

D.3 Vertical Columns

Each vertical domain is a translucent column through the CEP+CEE stack. Horizontally, arrows represent intra-domain semantics. Curved ribbons between columns represent maps of verticals.

D.4 The Stack Visualization

The geometric visualization comprises:

- a hexagonal foundational plane (CEP),
- crystalline pipelines (adapters),

- floating iridescent polyhedra (explanations),
- vertical translucent columns (domains),
- light ribbons between columns (functors and 2-morphisms).

Appendix E. Categorical Concepts for CEE

This appendix extends the categorical glossary in CEP Appendix E to cover concepts specific to the bicategorical semantics of CEE. For foundational definitions of *category*, *functor*, *natural transformation*, *monoidal category*, *oplax functor*, *pullback*, and *fibered category*, see CEP Appendix E.

E.1 Bicategory (New in CEE)

Formally, CEE is modeled as a bicategory whose hom-categories capture explanatory variation over CEP morphisms. A *bicategory* generalizes a category by allowing transformations between morphisms, called *2-morphisms*.

- **0-cells**: objects (civic entities)
- **1-morphisms**: relationships between entities
- **2-morphisms**: transformations between relationships

In CEE, explanations live as 2-morphisms: they relate one interpretation of a relationship to another, providing the "why" behind civic claims. This is the key structural innovation that distinguishes CEE from CEP.

E.2 Fibered Category (Extended)

CEP Appendix E introduces fibered categories as "contextual layers." CEE extends this with multiple fibered layers over the CEP base:

$$p_{\mathrm{CTag}} : \mathrm{CTag} \to \mathrm{CEP} \qquad p_V : V \to \mathrm{CEP} \qquad p_{\mathrm{CEE}} : \mathrm{CEE} \to \mathrm{CEP}$$

Each projection sends annotations, vertical semantics, or explanations to the CEP record they describe. The fiber over a record r is the collection of all structures "about" r. This layered architecture allows CEE to extend CEP without modifying the base protocol.

E.3 Universal Property (Clarified)

CEP Appendix E describes universal properties as “uniqueness by optimality.” In CEE, we clarify the role of SNFEI within this construction:

- The *stable verifiable ID* is the universal construction.
- SNFEI is the *canonical fallback* when no external registry identifier (LEI, SAM UEI, etc.) is available.
- Any deterministic identification scheme respecting CEP invariants factors uniquely through the verifiable ID scheme.

E.4 Summary Table

Concept	**Intuition**	**CEE Role**
Category	Things, allowed changes	CEP record layer (see CEP)
Functor	Structure-preserving map	Adapters (see CEP)
Natural transformation	Coherent comparison	Attestations (see CEP)
Monoidal category	Combine objects	Canonicalization (see CEP)
Bicategory	Morphisms between morphisms	**CEE explanations**
Oplax functor	Weak preservation	Jurisdiction adapters (see CEP)
Pullback	Consistent join	Data fusion (see CEP)
Fibered category	Contextual layers	CTags, verticals, CEE layers
Universal property	Optimal construction	Verifiable ID (SNFEI fallback)

E.5 Closing Note

CEE's bicategorical structure provides the vocabulary for civic explanations: evidence chains, attribution, and narratives become first-class mathematical objects. Combined with CEP's categorical foundation, this enables explanations that are coherent, auditable, and interoperable across domains.

For additional categorical terminology used in CEE (Cartesianness, compositionality, lifting, quotienting, etc.), see Appendix F.

Appendix F. Supporting Categorical Terminology

This appendix defines categorical terms used in CEE that extend beyond the foundational concepts in CEP Appendix E. Readers unfamiliar with category theory should consult CEP's glossary first for definitions of *functor*, *natural transformation*, *monoidal category*, *pullback*, and *fibered category*.

Cartesianness
: A property ensuring structured data can be "pulled back" along relationships without losing information. Cartesian structure guarantees that restricting a federal dataset to a state jurisdiction preserves all relevant relationships. See *pullback* in CEP Appendix E.

Compositionality
: The principle that complex structures are built from simpler parts, with the meaning of the whole determined by the parts and how they combine. CEE's compositional design means evidence chains can be assembled from individual attestations.

Definedness
: Whether an operation or relationship is valid for given inputs. Partial functions may be undefined for some inputs; CEE explicitly tracks definedness to distinguish "no data" from "not applicable."

Endofunction
: A function from a set to itself. Update operations on entity state are endofunctions: they take a state and return a new state of the same type.

Fibered Category
: A category equipped with a structure-preserving projection to a base category, allowing objects and morphisms to be indexed by elements of that base. In CEE, vertical domains are modeled as fibered categories over the CEP base category, enabling domain-

specific semantics layered atop a shared canonical infrastructure. See *fibered category* in CEP Appendix E.

Fibered Structure

The organizational pattern induced by a fibered category, in which objects are grouped into fibers over a base category according to context or domain. In CEE, fibered structure allows vertical domains to specialize interpretation and evidence while remaining coherently anchored to the same underlying CEP entities and morphisms.

Functoriality

The property of preserving structure across transformations. A mapping is *functorial* if it respects composition and identity—translating data between systems without corrupting accountability relationships. See *functor* in CEP Appendix E.

Lifting Given a relationship at one level, finding a corresponding relationship at a higher level that projects down to it. Lifting allows inference of jurisdiction-level patterns from entity-level exchanges.

Monoidality

Structure allowing associative combination with a neutral element. Monetary values are monoidal under addition (zero is neutral). A mapping is *monoidal* if it preserves this combining structure. See *monoidal category* in CEP Appendix E.

Naturality

A transformation is *natural* if it works uniformly across all instances without depending on arbitrary choices. Natural mappings are canonical rather than ad hoc. See *natural transformation* in CEP Appendix E.

Oplaxity

A relaxed form of structure preservation where equations become directed inequalities. Oplax mappings allow "preservation up to a coherence map" rather than strict equality, accommodating real-world data where composition may lose information. See *oplax functor* in CEP Appendix E.

Permutability

The property that certain operations can be performed in any order with the same result. Permutable operations simplify parallel processing and reduce coordination requirements in distributed systems.

Preordered

A set equipped with a reflexive and transitive relation (not necessarily antisymmetric). Time stamps and version numbers form preorders, generalizing "earlier than" or "less detailed than."

Quotienting

Collapsing distinctions by treating related elements as equivalent. Quotienting by jurisdiction aggregates city-level data into state-level summaries, treating individual cities as interchangeable for certain analyses.

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