Questo notebook contiene lo svolgimento della prova di esame; tutte le funzioni necessarie sono state riportate all'interno del notebook (non è necessario l'uso di altri file per eseguire questo)

In [ ]:
```
#functions to implement the hit-or-miss method
import numpy as np
import math
import random

def uniform_range(minimum: float,
                  maximum: float) -> float:
    """
    Generation of a pseudo-casual number distributed accordingly to uniform dist
    [minimum, maximum)

    Args:
        minimum: lower limit of the range (included)
        maximum: upper limit of the range (excluded)

    Returns:
        A pseudo-casual numbers generated according to uniform distribution betw
    """

    return minimum + (random.random() * (maximum - minimum))

def list_uniform_range(minimum: float,
                       maximum: float,
                       n: int,
                       seed: float = 0.) -> list[float]:
    """
        Generation of a list of n pseudo-casual number distributed accordingly
        to uniform distribution between [minimum, maximum) starting from an opti
        different from 0.

        Args:
            minimum: lower limit of the range (included)
            maximum: upper limit of the range (excluded)
            n: number of pseudo-casual numbers to generate
            seed: starting seed for the random generation (optional)

        Returns:
            A list of n pseudo-casual numbers generated according to uniform dis
        """

    if seed != 0.:
        random.seed(seed)
    random_list = []
    for i in range(n):
        # Return the next random floating point number in the range 0.0 <= X < 1
        random_list.append(uniform_range(minimum, maximum))
    return random_list

def hom(function,
        xmin: float,
        xmax: float,
        ymax: float,
        n_evt: int = 100000) -> tuple[float, float]:
```

```
    """
    Calculation of a defined integral of a function using the hit-or-miss method

    Args:
        function: function whose integral has to be calculated [must be expresse
        xmin: lower limit of the integral
        xmax: upper limit of the integral
        ymax: maximum value of the function in the interval
        n_evt: number of points generated to calculate the integral (optional, d

    Returns:
        The defined integral and the uncertainty of the value obtained
    """

    x_coord = list_uniform_range(xmin, xmax, n_evt)
    y_coord = list_uniform_range(0., ymax, n_evt)

    points_under = 0
    for x, y in zip(x_coord, y_coord):
        if function(x) > y:
            points_under = points_under + 1

    area_rect = (xmax - xmin) * ymax
    frac = float(points_under) / float(n_evt)
    integral = area_rect * frac
    integral_unc = area_rect ** 2 * frac * (1 - frac) / n_evt
    return integral, math.sqrt(integral_unc)
```

In [ ]:
```
#because max of cos^2=1, max of A*cos^2=A
#first, calculation of the area of cos^2
area_1, sigma_area_1 = hom(lambda x: np.cos(x)*np.cos(x), 0, ((3/2)*math.pi), 1,
print('Value of the area of the function with A=1:' + str(area_1) + ' +- ' + str
```

```
Value of the area of the function with A=1:2.3561473663025407 +- 0.00074509411978
56888
```

In [ ]:
```
#the function is normalized if A=1/area(1*cos^2)
#definition of the normalized pdf
def pdf(x):
    if (0<x<((3/2)*math.pi)):
        return (1/area_1)*np.cos(x)*np.cos(x)
    else:
        return 0
```

In [ ]:
```
#check of the normalization
area_norm, sigma_area_norm = hom(pdf, 0, ((3/2)*math.pi), 1, 10000000)
print('Value of the area of the pdf:' + str(area_norm) + ' +- ' + str(sigma_area

area_control = (abs(1-area_norm))/sigma_area_norm
print('True value at ' + str(area_control) + ' standard deviations')
```

```
Value of the area of the pdf:1.0004703398251442 +- 0.0006093984331883285
True value at 0.7718100335168785 standard deviations
```

In [ ]:
```
#implementation of try-and-catch method

def tac_box(function,
            x_minimum: float,
            x_maximum: float,
            y_minimum: float,
```

```
            y_maximum: float,
            seed: float = 0.) -> float:
    """
    Generation of a pseudo-casual number distributed accordingly to a function
    with the try-an-catch algorithm, into a "box" delimited by x_minimum and x_m
    horizontal axis and y_minimum and y_maximum for the vertical axis starting f
    different from 0.

    Args:
        function: the function which rules the distribution of numbers
        x_minimum: lower limit of the range for the horizontal axis
        x_maximum: upper limit of the range for the horizontal axis
        y_minimum: lower limit of the range for the vertical axis
        y_maximum: upper limit of the range for the vertical axis
        seed: starting seed for the random generation (optional)

    Returns:
        A single number generated with the try-an-catch algorithm distributed ac
    """

    if seed != 0.:
        random.seed(seed)
    x = uniform_range(x_minimum, x_maximum)
    y = uniform_range(y_minimum, y_maximum)
    while y > function(x):
        x = uniform_range(x_minimum, x_maximum)
        y = uniform_range(y_minimum, y_maximum)
    return x
```

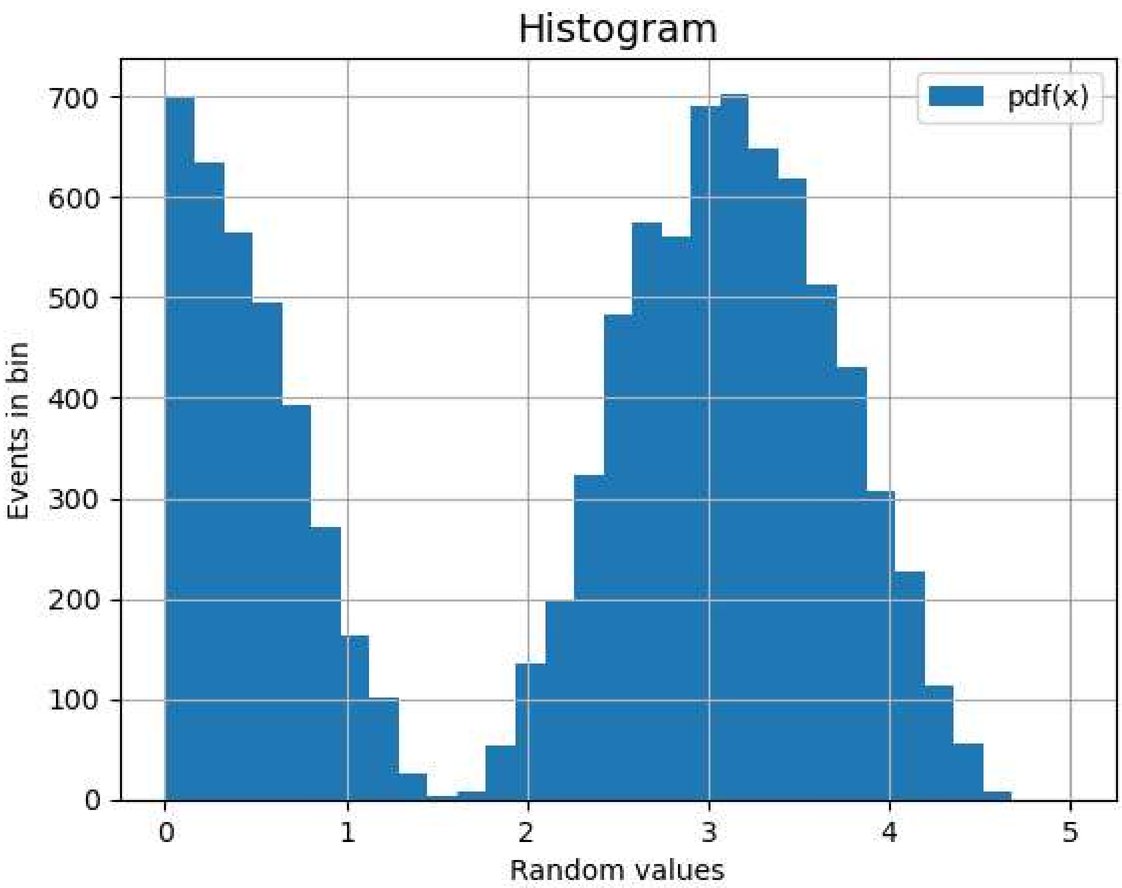
In [ ]:

```
#generation of 10000 pseudo-casual numbers
data = []
for i in range(10000):
    data.append(tac_box(pdf, 0, ((3/2)*math.pi), 0, (1/area_1)))
```

In [ ]:

```
#data visualization

import matplotlib.pyplot as plt

def histogram(sample: list[float],
              title: str = 'Histogram',
              xlabel: str = 'Random values',
              ylabel: str = 'Events in bin',
              label: str = 'pdf(x)',
              sturges: bool = True):
    """
    Plots a histogram of samples, with optional title and x-label and y-label an
    saves the histogram as a png image

    Args:
        sample: list of floats representing data
        title: title of the histogram
        xlabel: label of the x-axis
        ylabel: label of the y-axis
        label: title of the histogram in the legend
        sturges: if it is true, bins in the histogram are divided accordingly to

    Returns:
        The plot of the histogram of the sample
    """
```

```
    fig, ax = plt.subplots(nrows=1, ncols=1)
    if sturges is True:
        ax.hist(sample, label=label, bins=np.linspace(math.floor(min(sample)), m
                                                      math.ceil(1 + 3.322 * np.l
    else:
        ax.hist(sample, label=label)
    ax.set_title(title, size=14)
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)
    ax.grid(True)
    ax.legend()

    plt.show()
    return

histogram(data)
```



In [ ]:

```
#implementation of the statistics functions

def mean(sample: list[float]) -> float:
    """
    Calculation of the mean of the sample passed as argument

    Args:
        sample: list of floats representing data

    Returns:
        The mean of the sample
    """

    summ = sum(sample)
```

```python
    n = len(sample)
    return summ / n

def variance(sample: list[float],
             bessel: bool = True) -> float:
    """
    Calculation of the variance of the sample present in the object

    Args:
        sample: list of floats representing data
        bessel: applies the bessel correction (optional, default: True)

    Returns:
        The variance of the sample
    """

    summ = 0.
    sum_sq = 0.
    n = len(sample)
    for elem in sample:
        summ += elem
        sum_sq += elem * elem
    var = sum_sq / n - summ * summ / (n * n)
    if bessel:
        var = n * var / (n - 1)
    return var

def stddev(sample: list[float],
           bessel: bool = True) -> float:
    """
    Calculation of the standard deviation of the sample present in the object

    Args:
        sample: list of floats representing data
        bessel: applies the bessel correction (optional, default: True)

    Returns:
        The standard deviation of the sample
    """

    return math.sqrt(variance(sample, bessel))

def skewness(sample: list[float]) -> float:
    """
    Calculation of the skewness of the sample passed as argument

    Args:
        sample: list of floats representing data

    Returns:
        The skewness of the sample
    """

    mean_sample = mean(sample)
    skew = 0.
    for x in sample:
        skew = skew + math.pow(x - mean_sample,  3)
    skew = skew / (len(sample) * math.pow(stddev(sample), 3))
    return skew
```

```python
def kurtosis(sample: list[float]) -> float:
    """
    Calculation of the kurtosis of the sample passed as argument

    Args:
        sample: list of floats representing data

    Returns:
        The kurtosis of the sample
    """

    mean_sample = mean(sample)
    kurt = 0.
    for x in sample:
        kurt = kurt + math.pow(x - mean_sample,  4)
    kurt = kurt / (len(sample) * math.pow(variance(sample), 2)) - 3
    return kurt
```

In [ ]:
```python
#calculation of mean, standard deviation, skewness and kurtosis of the data set

mean_data = mean(data)
stddev_data = stddev(data)
skew_data = skewness(data)
kurt_data = kurtosis(data)

print('Statistics of the data set: \n mean: ' + str(mean_data) + '\n standard de
```

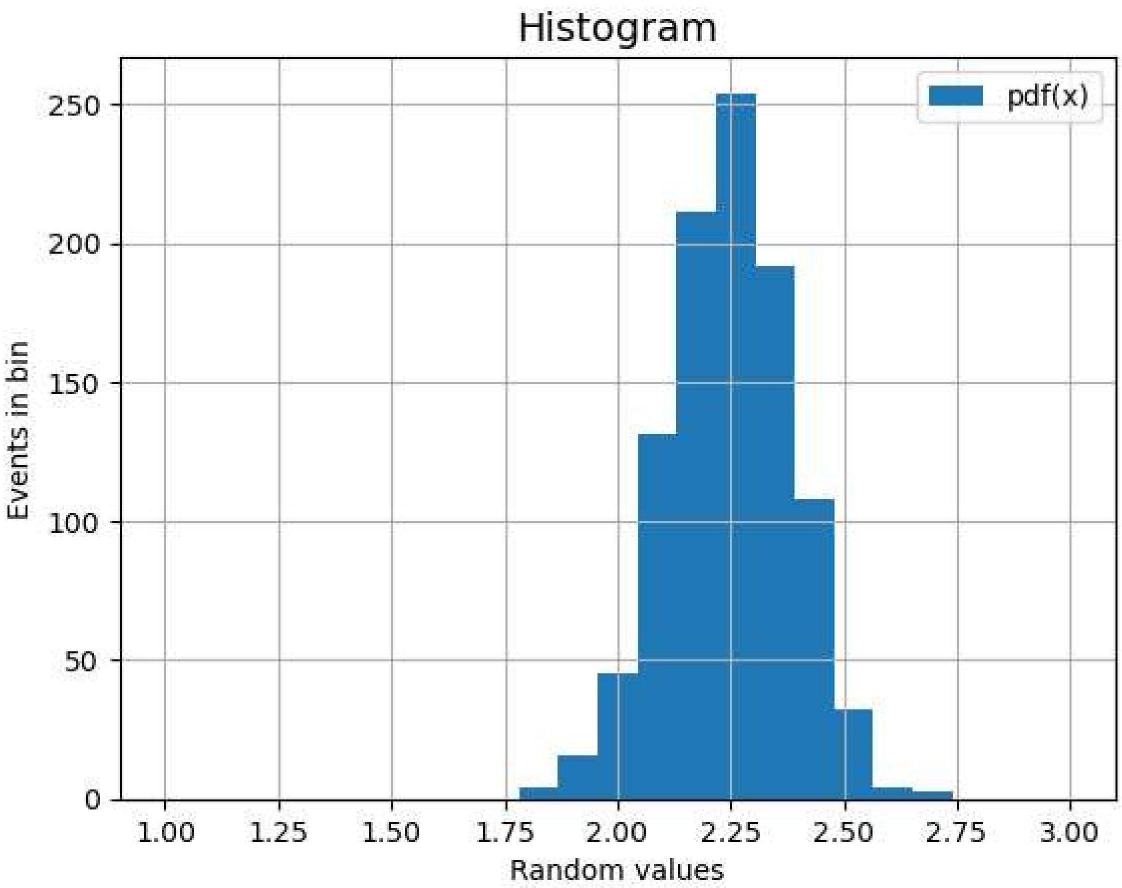
```
Statistics of the data set:
 mean: 2.2432374324304267
 standard deviation: 1.3543394025910938
 skewness: -0.4063307545928531
 kurtosis: -1.3444544976918944
```

In [ ]:
```python
#implementation of the functions for the central limit theorem method

def clt_minmax(minimum: float,
               maximum: float,
               n_sum: int = 10) -> float:
    """
    Generation of a pseudo-casual number with the central limit theorem algorith
    between [minimum, maximum)

    Args:
        minimum: lower limit of the range (included)
        maximum: upper limit of the range (excluded)
        n_sum: number of repetitions used in the algorithm (optional, default: 1

    Returns:
        A pseudo-casual numbers generated with the central limit theorem algorit
        between [minimum, maximum)
    """

    y = 0.
    for i in range(n_sum):
        y += tac_box(pdf, minimum, maximum, 0, (1/area_1))
    y /= n_sum
    return y

def list_clt_minmax(minimum: float,
                    maximum: float,
```

```
                    n: int,
                    n_sum: int = 10,
                    seed: float = 0.) -> list[float]:
    """
    Generation of a list of n pseudo-casual numbers distributed between [minimum
    with the central limit theorem algorithm starting from an optional seed
    different from 0.

    Args:
        minimum: lower limit of the range (included)
        maximum: upper limit of the range (excluded)
        n: length of the list
        n_sum: number of repetitions used in the algorithm (optional, default: 1
        seed: starting seed for the random number generator (optional, default:

    Returns:
        A list of pseudo-casual numbers generated according to gaussian distribu

    """

    if seed != 0.:
        random.seed(seed)
    random_list = []
    for i in range(n):
        random_list.append(clt_minmax(minimum, maximum, n_sum))
    return random_list
```

In [ ]:

```
#generation of 100000 events with central limit theorem method

clt_data = list_clt_minmax(0, ((3/2)*math.pi), 1000, n_sum=100)
histogram(clt_data)
```



In [ ]:

```
#check by fitting data to a gaussian using the binned likelihood in imiunit

from scipy.stats import norm, expon
from iminuit import Minuit
from iminuit.cost import BinnedNLL
from math import floor, ceil, log
from IPython.display import display
from scipy.stats import chi2


def cdf(bin_edges, mu, sigma):
    return norm.cdf(bin_edges, mu, sigma)


bin_content, bin_edges = np.histogram(clt_data, bins=int(ceil(1 + 3.322 * log(le

#inizialization of the parameters using appropriate values
sample_mean = np.mean(clt_data)
sample_sigma = np.std(clt_data)

my_cost_func = BinnedNLL(bin_content, bin_edges, cdf)

N_events = sum(bin_content)

my_minuit = Minuit(my_cost_func, mu=sample_mean, sigma=sample_sigma)

my_minuit.migrad()
my_minuit.minos()
display(my_minuit)

display(my_minuit.covariance.correlation())

for key in my_minuit.parameters:
    print('parameter ' + key + ': ' +
          str(my_minuit.values[key]) + ' +- ' +
          str(my_minuit.errors[key]))

p_value_fit = 1. - chi2.cdf (my_minuit.fval, df = my_minuit.ndof)

print ('associated p-value: ' + str(p_value_fit))

if p_value_fit > 0.05:
    print('Test passed')
```

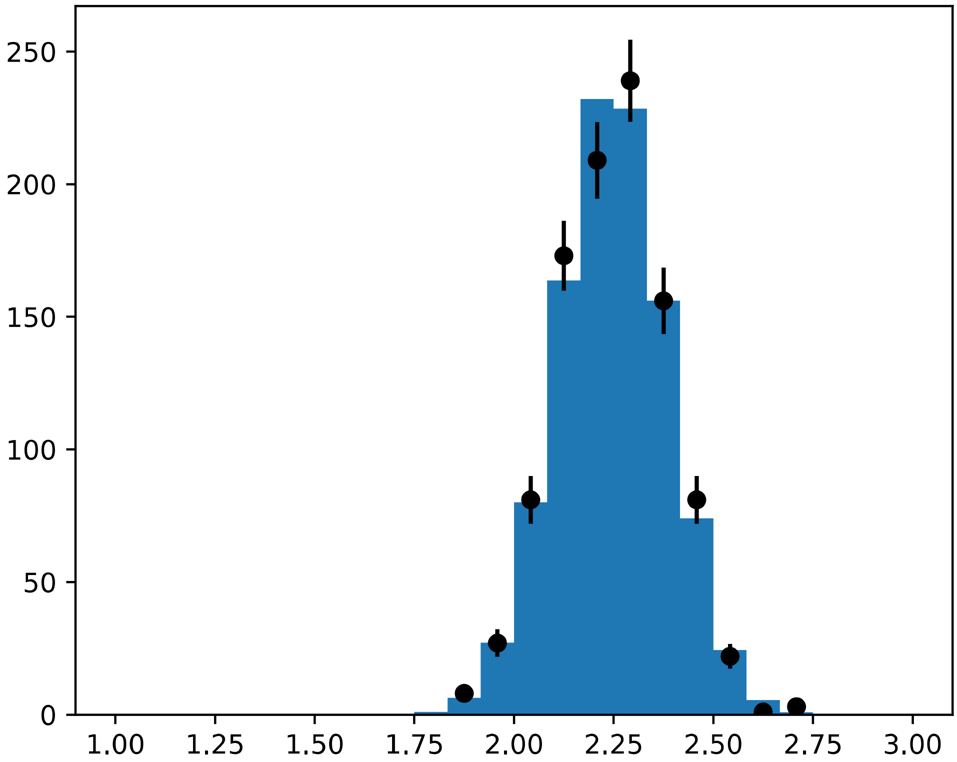
Migrad

FCN = 15.99 (χ^2/ndof = 0.7)	Nfcn = 72
EDM = 2.37e-05 (Goal: 0.0002)	
Valid Minimum	Below EDM threshold (goal x 10)
No parameters at limit	Below call limit
Hesse ok	Covariance accurate

	Name	Value	Hesse Error	Minos Error-	Minos Error+	Limit-	Limit+	Fixed
0	mu	2.246	0.004	-0.004	0.004			
1	sigma	0.1357	0.0031	-0.0031	0.0032			

	mu		sigma	
Error	-0.004	0.004	-0.0031	0.0032
Valid	True	True	True	True
At Limit	False	False	False	False
Max FCN	False	False	False	False
New Min	False	False	False	False

	mu	sigma
mu	1.9e-05	0e-6
sigma	0e-6	9.79e-06



	mu	sigma
mu	1	0
sigma	0	1

```
parameter mu: 2.246416248364238 +- 0.004358465515347127
parameter sigma: 0.13571123908219782 +- 0.0031287283995489004
associated p-value:  0.8161597024743836
```