Advanced Computational Methods in Geotechnical Engineering

Lecture

April 18, 2008

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Consolidation Settlement & Average Degree of Consolidation → FD Solution

$$\boxed{S = m_v s H - m_v \int_0^H u dz} \quad \text{(A)}$$

$$S_c = m_v s H$$ → When the pore pressures is completely dissipated.

Average degree of consolidation →

$$U(\%) = 1 - \frac{\int_0^H u_{t=t_i} dz}{\int_0^H u_{t=0} dz}$$

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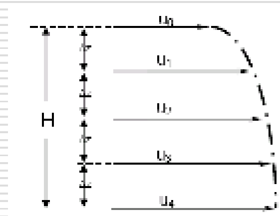
ü In Eqn. *(A)*, the integral of the excess pore pressure cannot be evaluated exactly because the excess pore pressure is only at the grid points.

ü However, the integral can be evaluated approximately using some numerical techniques. The simplest approach is to use the Trapezoidal Method.

$$\boxed{\int_0^H u\,dz = \left[\left(\frac{u_0+u_n}{2}\right)+\sum_{i=1}^{n-1} u_i\right]\Delta z}$$



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Variable Loading

ü In reality, a loading is never placed instantaneously and the problem can be easily handled through finite difference formulation.

ü Variable loading usually occurs when there is a structure being built on top of the soil layer. As the construction of the structure proceeds, the load induced onto the ground gradually increases with time.

ü Considering that the construction is a time consuming process, we cannot assume a formulation of instantaneous load to be used in the calculation of settlement.

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Abrupt Change of Load

ü When the existing soil layers are too weak, it has to be loaded in stages. Loading in stages will increase the shear strength and bearing capacity of the soil as it consolidates.

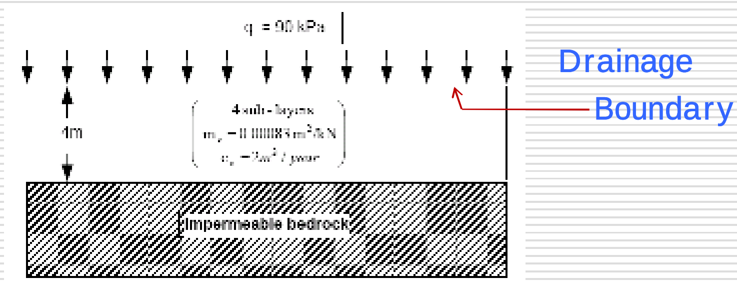
Variable Coefficient of Consolidation

ü The coefficient of consolidation is usually considered as constant in consolidation analysis although it is a variable quantity.

ü It is known empirically that C_V changes during consolidation as the void ratio of the soil changes causing a decrease in permeability and compressibility of the soil.

$$\boxed{C_V = \frac{k}{m_v g_w}}$$

Example Problem (Assignment #3)



- Analyze the pore water pressure, settlement and average degree of consolidation of the soil layer after 2 years of consolidation using FDM.

Loading Scenarios

- Case 1: *constant load scenario:* a uniform load of 90 kPa is applied onto the soil layer at once.

Settlement (mm)												
t(yrs)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.4	1.6	1.8	2.0
q(kPa)	90	90	90	90	90	90	90	90	90	90	90	90
z=0 m	0											
z=1 m	90											
z=2 m	90											
z=3 m	90											
z=4 m	90											
U_{av}(%)	0.0											

Loading Scenarios

ü Case 2: *variable load scenario:* a fill having a unit weight 15 kN/m^3 is placed at a rate of 0.5 m/month during the first year and the filling stops after one year.

Settlement (mm)											
t(yrs)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
q(kPa)	0	9	18	27	36	45	54	63	72	81	90
z=0 m	0	0	0								
z=1 m	0	9	15.19								
z=2 m	0	9									
z=3 m	0	9									
z=4 m	0	9									
U_{av}(%)	0	9									

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Loading Scenarios

ü Case 2: *variable load scenario (continued):*

Settlement (mm)										
t(yrs)	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
q(kPa)	90	90	90	90	90	90	90	90	90	90
z=0 m										
z=1 m										
z=2 m										
z=3 m										
z=4 m										
U_{av}(%)										

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Loading Scenarios

- Case 3: *abrupt change of load scenario:* In the abrupt change of load scenario, the soil layer will be loaded with fill of 60 kPa and after one year, an additional fill of 30 kPa will be laid on the existing fill.

Settlemnt (mm)											
t(yrs)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
q(kPa)	60	60	60	60	60	60	90	90	90	90	90
z=0 m	0.0										
z=1 m	60										
z=2 m	60										
z=3 m	60										
z=4 m	60										
U_{av}(%)	0.0										

Loading Scenarios

- Case 4: *constant Load with Variable C_v:* the C_v is assumed to decrease with a rate of 0.1 m^2/year.

Settlemnt (mm)											
C_V	-	2.0	1.98			1.9					1.8
t(yrs)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
q(kPa)	90	90	90	90	90	90	90	90	90	90	90
z=0 m	0.0										
z=1 m	90										
z=2 m	90										
z=3 m	90										
z=4 m	90										
U_{av}(%)	90										

3. FD Solution for Two-dimensional Flow

Flow of Water through soils is governed by Laplace's equation:

$$\boxed{k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0}$$

where h is total head, k_x & k_z coefficients of permeability in x & z directions.

- If the soil is isotropic $k_x = k_z$. Therefore:

$$\boxed{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$

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3.1 FD Equation for 2D flow

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = \frac{k_x\left(h_{i+1,j} - 2h_{i,j} + h_{i-1,j}\right)}{\Delta x^2} + \frac{k_z\left(h_{i,j+1} - 2h_{i,j} + h_{i,j-1}\right)}{\Delta z^2} = 0$$

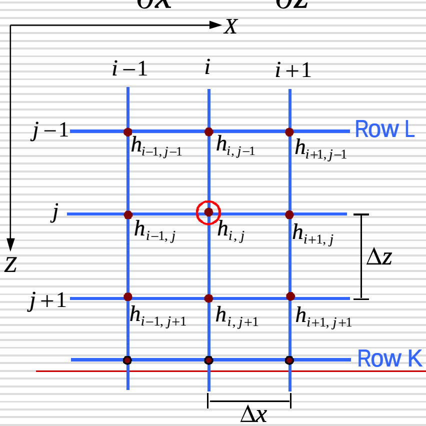

Fig. 3.1: Partial grid of the flow domain

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Let a = k_x / k_z and Dx = Dz (square grid), then:

$$h_{i,j} = \frac{\left(a h_{i+1,j} + a h_{i-1,j} + h_{i,j+1} + h_{i,j-1}\right)}{2(1+a)}$$

For isotropic condition a = 1 ($k_x = k_z$), hence:

$$h_{i,j} = \frac{\left(h_{i+1,j} + h_{i-1,j} + h_{i,j+1} + h_{i,j-1}\right)}{4} \qquad (3.1)$$

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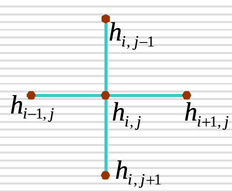
This equation shows that, in a square grid, the total head at every grid is the average of the total heads at the four adjacent grids.



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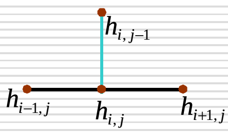
3.2 Boundary Conditions

Flow can not cross impermeable boundaries, therefore, for a horizontal impermeable surface:



$$\frac{\partial h}{\partial z} = 0 \Rightarrow$$ Neumann Boundary Condition

FDE for the boundary conditions is:

$$\frac{\partial h}{\partial z} = \frac{(h_{i,j+1} - h_{i,j-1})}{2\Delta z} = 0$$

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Therefore:

$$h_{i,j-1} = h_{i,j+1}$$

Substituting into eqn. (3.1), we get:

$$\boxed{h_{i,j} = \frac{\left(h_{i+1,j} + h_{i-1,j} + 2h_{i,j-1}\right)}{4}} \quad \textbf{(3.2)}$$

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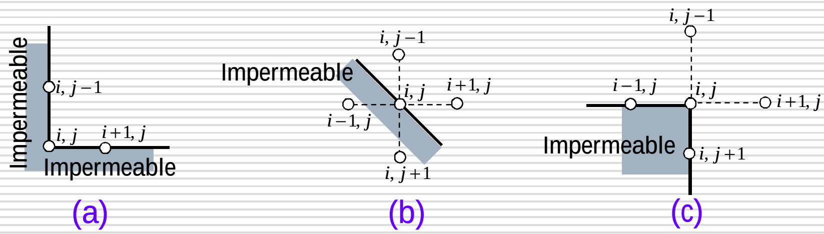
Various types of geometry of impermeable boundaries are encountered in practice. Following are three examples:



The FDE for Figures (a) & (b) is:

$$h_{i,j} = \tfrac{1}{2}\left(h_{i+1,j} + h_{i,j-1}\right) \qquad \textbf{(3.3)}$$

The FDE for Figure (c) is:

$$h_{i,j} = \tfrac{1}{3}\left(h_{i,j-1} + h_{i+1,j} + h_{i,j+1} + \tfrac{1}{2}h_{i-1,j} + \tfrac{1}{2}h_{i,j+1}\right) \qquad \textbf{(3.4)}$$

3.3 Pore Water Pressure

$$\boxed{h = h_p + z}$$

Where h is total head, h_p is pressure head and z is elevation head.

Pore water pressure is obtained by:

$$\boxed{u = g_w h_p = g_w (h - z)}$$

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Therefore:

$$\boxed{u_{i,j} = g_w \left(h_{i,j} - z_{i,j}\right)} \quad (3.5)$$

Contours of potential heads can be drawn from the discrete values of $h_{i,j}$.

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3.4 Velocity

The horizontal velocity of flow at any node is given by Darcy's law:

$$v_{i,j} = k_x i_{i,j}$$

Where $i_{i,j}$ is the hydraulic gradient expressed as:

$$i_{i,j} = \frac{(h_{i+1,j} - h_{i-1,j})}{2\Delta x}$$

Therefore:

$$v_{i,j} = \frac{k_x (h_{i+1,j} - h_{i-1,j})}{2\Delta x}$$

3.5 Flow rate

- The flow rate, q, is obtained by considering a vertical plane across the flow domain.

- Let L be the top row and K be the bottom row of a vertical plane defined by column i (Fig. 3.1 slide 14). Then the expression for q is:

$$q = \frac{k_x}{4}\left(h_{i+1,L} - h_{i-1,L} + 2\sum_{j=L+1}^{K-1}(h_{i+1,j} - h_{i-1,j}) + h_{i+1,K} - h_{i-1,K} \right) \tag{3.6}$$

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3.6 Procedure for using FDM in 2D flow in soils

Step 1:

- Divide the flow domain into a square grid.

- Generally, finer grids give more accurate solution than coarser grids.

- If the flow is symmetrical, you only need to consider one-half of the flow domain (below is an example).

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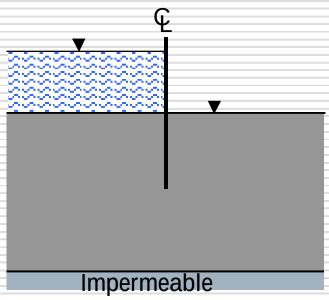
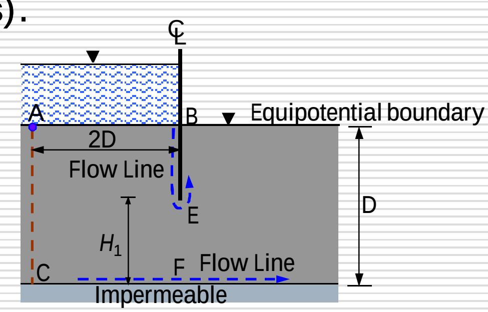


Fig. 3.2: A sheet pile wall.

- The total flow domain should have a width of at least four times the thickness of the soil layer.

Step 2:

- Identify boundary conditions, example, impermeable boundaries (flow lines) and permeable boundaries (equipotential lines).



Step 3:

- Determine the heads at the permeable or equipotential boundaries. For example, the heads along the equipotential boundary AB (Fig. 3.2) is *DH*.

-Therefore, all the nodes along this boundary will have a constant head of *DH*.

-Because of symmetry, the head along nodes directly under the sheet pile wall (*EF*) is *DH/2*.

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Step 4:

-Apply the known heads to the corresponding nodes and assume reasonable initial values for the interior nodes.

Step 5:

-Apply Eqn. (3.1) if the soil is isotropic *(slide 15)* to each node except:

a) at impermeable boundaries – eqn. (3.2) *(slide 18)*

b) at corners – eqn. (3.3) & (3.4) *(slide 20)*

c) at nodes where the heads are known

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Step 6:

- Repeat item 5 until the new value at the node differs from the old value by a small numerical tolerance for example 0.001 m.

Step 7:

-Arbitrarily select a sequential set of nodes along a column of nodes and calculate the flow, q, using eqn. (3.6) (*slide 25*) . It is best to calculate $q' = q$ for a unit permeability value to avoid too many decimal points in the calculations.

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Step 8:

-Repeat items 1 to 6, to find the flow distribution by replacing heads by flow q'. For example, the flow rate, calculated in item 7 is applied to all nodes along *AC* and *CF* (*slide 28*).

Step 9:

-Calculate the pore water pressure distribution using eqn. (3.5) (*slide 22*).

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A spreadsheet program can be prepared to automatically carry out the above procedure. However, you should carry out hand calculations at selected nodes to verify that the spreadsheet calculation is correct.

EXAMPLE 3.1

Determine the flow under the sheet pile wall (Fig. E3.1) and the pore water pressure distribution using the Finite Difference Method.

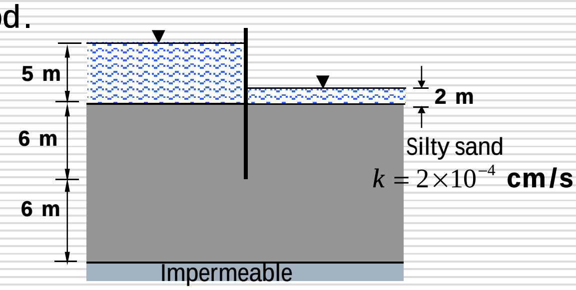

Fig. E3.1

Strategy Use a spreadsheet and follow the procedures.

SOLUTION 3.1

Step 1:

Divide the flow domain into a grid. See Fig. E3.1a

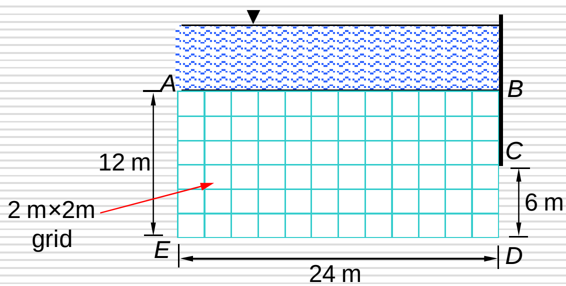

Fig. E3.1a

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Step 2:

Identify the boundary conditions.

Permeable boundaries: AB and CD are equipotential lines.

Impermeable boundaries: BC, AE and ED are flow lines.

Step 3:

Determine the heads at equipotential boundaries.

Along AB the head difference is 5 - 2 = 3 m.

Along CD the head difference is 3/2 = 1.5 m.

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Go to Spreadsheet Program

Step 4:

Insert the heads at the nodes.

- Set up the initial parameters in column B, rows 3 to 9.
- In cells B12 to N12 copy cell B5.
- In cells N15 to N18 insert cell B5/2.
- Arbitrarily insert values in all other cells from B13 to M18, N13 & N14.

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Step 5:

Apply the appropriate equations.

- On the impermeable boundaries – cells B13 to B18, C18 to M18, and N13 to N14, apply eqn. (3.2) *(slide 18)*

- Apply eqn. (3.1) *(slide 15)* to all other cells except cells with known heads.

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Step 6:

Carry out the iterations.

-In excel, go to Tools … Options…Calculation. Select the following:

…Next Slide…

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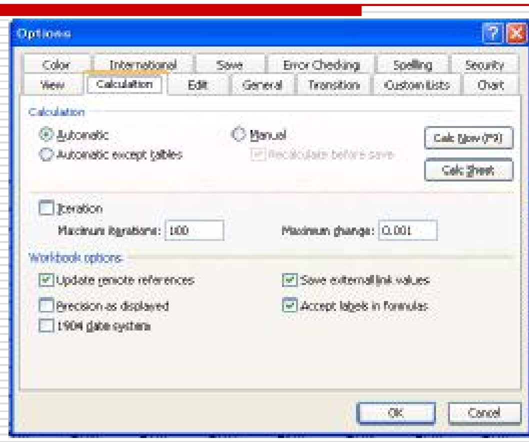

-You can then click on CalcNow (F9) or CalcSheet

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Step 7:

Calculate q.

- Use eqn. (3.6) *(slide 25)* to calculate q' for a unit value of permeability. In the spreadsheet for this example q' is calculated in cell C 20 as:

$$\{(B13 - B15 + N13 - N15 + 2 \times SUM\ (C13 : M13) - 2 \times SUM\ (C15 : M15))\}/4$$

- The actual value of q is

$$q = kq' = 1.63 \times 2 \times 10^{-4} \times 10^{-2} = 3.2 \times 10^{-6}\,\mathrm{m}^3/\mathrm{s}$$

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Step 8:

Calculate the flow for each cell.

-In cells B36 to B42, and C42 to N32, copy q'. The flow at the downstream end (cells N36 to N39) is zero.

-Apply eqn. (3.2) *(slide 18)* to cells C36 to M41 and N40 to N41.

-Apply eqn. (3.1) *(slide 4)* to all other cells except the cells with known values of q'. Carry out the iterations.

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Step 9:

Calculate the pore water pressure.

-From the potential heads, you can calculate the pore water pressure using eqn. (3.5) *(slide 22)* . A plot of the pore water pressure distribution is shown in Fig. E3.1b.

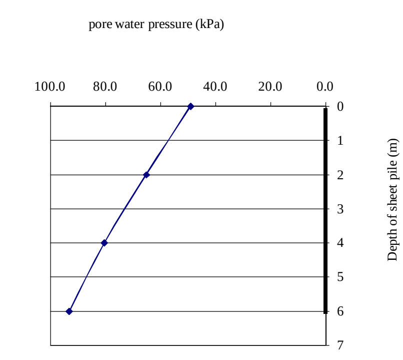

Fig. E3.1b

Assignment # 4

Determine the flow under the sheet pile wall (Fig. A3) and the pore water pressure distribution using the Finite Difference Method.

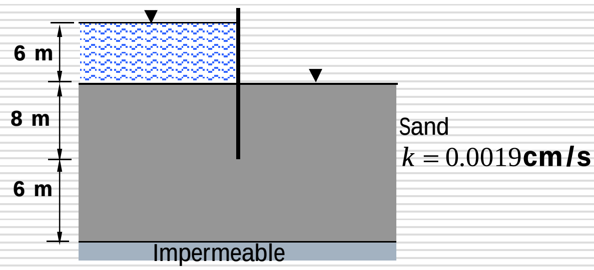

Fig. A3

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