New York University Tandon School of Engineering

Biomedical Engineering
Applied Mathematics and Statistics for Biomedical Engineering

Fall 2021
Professor Mirella Altoe
Tuesday 5:00-7:30PM Rogers Hall 325

Computer Lab Assignment #5

QUESTION 1: The number of cases of Lyme disease, a tick-borne illness, more than doubled between 2001 and 2015 in North America. A vaccine for Lyme disease was approved by the U.S. Food and Drug Administration in 1998, but it was then removed from the market just 3 years later (*Nigrovic and Thompson 2007*). Did it work? A clinical trial of the efficacy of this vaccine compared 5469 people who received the vaccine to 5467 people who received a placebo (*Steere et al. 1998*). Of these, 15 people with the vaccine had developed Lyme disease after 2 years, and 65 people in the placebo control group had developed the disease. In the same study, side effects were also tracked. The most severe possible side effect was joint pain. Of the vaccinated group, 71 people had joint pain, while in the control group 6 people had joint pain.

A. **(0.50 pts.)** Calculate the relative risk of getting Lyme disease for patients who received the vaccine compared to those who did not.

(A) $\widehat{RR} = \frac{\hat{p}_1}{\hat{p}_2}$

$$\hat{p}_{vaccine} = \frac{\#_{Lyme}}{\#_{vaccine}} = \frac{15}{5469+15} = \underline{0.00273}$$

$$\hat{p}_{control} = \frac{\#_{Lyme}}{\#_{control}} = \frac{65}{5467+65} = \underline{0.0118}$$

$$\widehat{RR} = \frac{\hat{p}_{vaccine}}{\hat{p}_{control}} = \frac{0.00273}{0.0118} = \boxed{0.23}$$

* Those who received the vaccine had 0.23 times the relative ~~dose~~ risk of developing Lyme disease, compared to those that did not.

B. **(0.50 pts.)** Based on that relative risk calculation, by what amount does the vaccine reduce the rate of getting Lyme disease?

$1 - 0.23$

$= \boxed{0.77}$ ★ Vaccination reduces rate of Lyme disease by 77%

C. **(0.75 pts.)** Do these data provide evidence that the vaccine was effective in reducing the rate of contracting Lyme disease? Carry out an appropriate hypothesis test.

	Vaccine	control	total
lyme	15	65	80
healthy	5454	5402	10856
	5469	5467	10936

$\hat{P}[\text{vaccine}] = 0.50009144$

$\hat{P}[\text{control}] = 0.4999856$

$\hat{P}[\text{lyme}] = 0.007315$

$\hat{P}[\text{healthy}] = 0.992685$

$\hat{P}r[\text{vaccine - lyme}] \times \text{total} = 40.007$

$\hat{P}r[\text{vaccine - healthy}] \times \text{total} = 5428.993$

$\hat{P}r[\text{control - lyme}] \times \text{total} = 39.993$

$\hat{P}r[\text{control - healthy}] \times \text{total} = 5427.007$

$$\chi^2 = \sum \frac{(obs - exp)^2}{exp}$$

$$\left(\frac{(15 - 40.007)^2}{40.007}\right) + \frac{(65 - 39.993)^2}{39.993} + \frac{(5454 - 5428.993)^2}{5428.993} + \frac{5402 - 5427.007}{5427.007}$$

$\boxed{\chi^2 = 31.5}$, df = 1, crit value = 3.84

★ our observed χ^2 value of 31.5 is much greater than that of the test statistic (3.84). Therefore, we can reject H_0 and accept H_a - that probability of lyme disease is contingent on vaccination.

D. **(0.50 pts.)** What is the relative risk of joint pain for vaccinated patients compared to the controls?

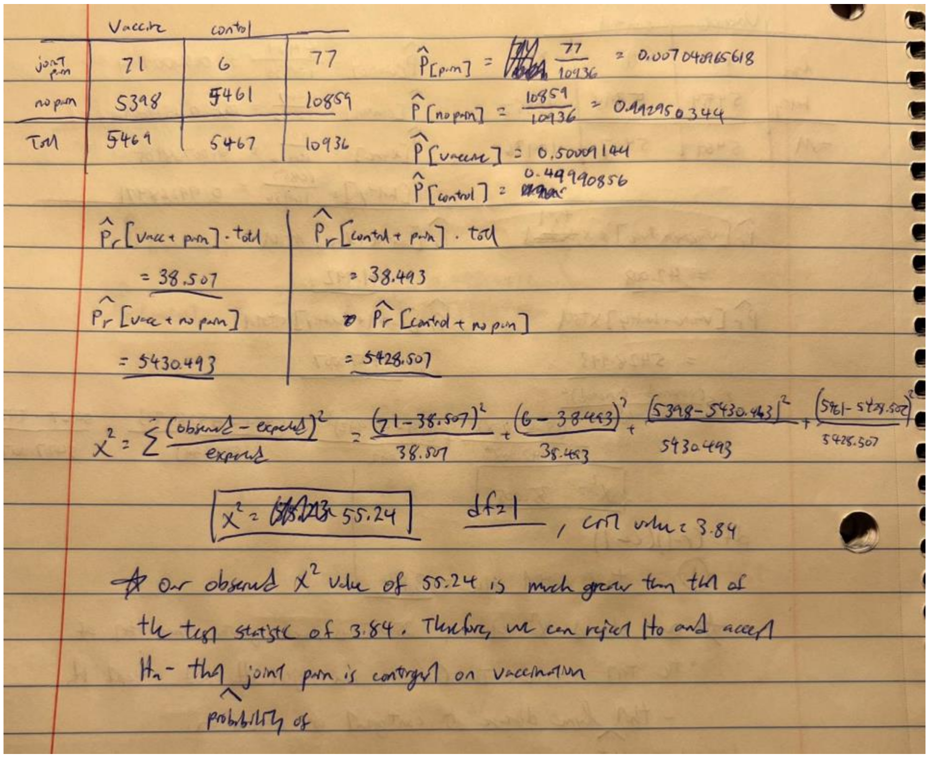
$\hat{P}_1 = \frac{\text{Pain}}{\text{Vaccine}} = \frac{71}{5469} = 0.0130$

$\hat{P}_2 = \frac{\text{Pain}}{\text{Vaccine}} = \frac{6}{5467} = 0.0011$

$= \boxed{11.83}$

★ those who got the vaccine have a 11.83 times the relative risk of developing joint pain compared to ~~control~~ those who did not.

E. **(0.75 pts.)** Is there evidence that the probability of joint pain is different between vaccinated and control groups? Do a hypothesis test.



	Vaccine	control	
joint pain	71	6	77
no pain	5398	5461	10859
Total	5469	5467	10936

$\hat{P}[pain] = \frac{77}{10936} = 0.00704096618$

$\hat{P}[no\ pain] = \frac{10859}{10936} = 0.99295\,0344$

$\hat{P}[vaccine] = 0.50009144$

$\hat{P}[control] = 0.49990856$

$\hat{P}r[vacc + pain] \cdot total = 38.507$

$\hat{P}r[control + pain] \cdot total = 38.493$

$\hat{P}r[vacc + no\ pain] = 5430.493$

$\hat{P}r[control + no\ pain] = 5428.507$

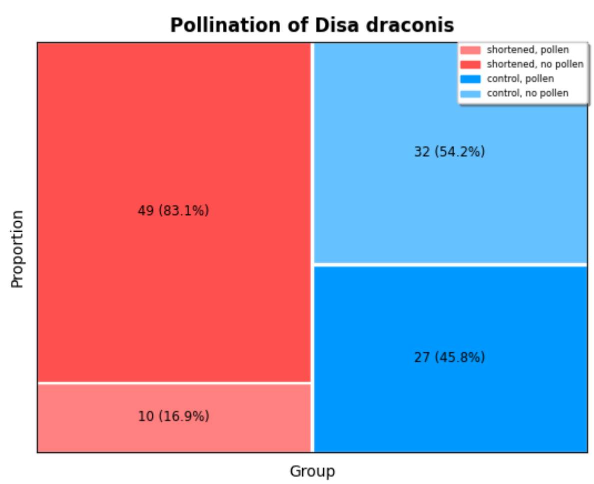
$$\chi^2 = \sum \frac{(observed - expected)^2}{expected} = \frac{(71-38.507)^2}{38.507} + \frac{(6-38.493)^2}{38.493} + \frac{(5398-5430.493)^2}{5430.493} + \frac{(5461-5428.507)^2}{5428.507}$$

$\boxed{\chi^2 = \cancel{55.213}\ 55.24}$ $df = 1$, crit value = 3.84

* Our observed χ^2 value of 55.24 is much greater than that of the test statistic of 3.84. Therefore, we can reject Ho and accept H_a - that probability of joint pain is contingent on vaccination

QUESTION 2: Darwin suggested that plants pollinated by long-tongued insects would benefit by having long flowers, because greater length would cause the insects to press themselves farther into the flower to reach the nectar, increasing deposition and removal of pollen. Several populations of the South African orchid, *Disa draconis*, evolved longer flowers after switching pollinators to long-tongued tanglewing flies. To measure the advantage of the long flowers, Johnson and Steiner (1997) experimentally shortened 59 of 118 flowers. The remaining 59 flowers were controls. One week later, 10 of the 59 shortened flowers had received pollen, whereas 27 of the 59 control flowers had received pollen.

A. (0.50 pts.) Illustrate these results in a mosaic plot.



B. **(0.50 pts.)** What is the estimated odds ratio of not receiving pollen after experimental shortening, as compared to control flowers? Provide a confidence interval for the population odds ratio.
Odds ratio = 0.242

experimental	control
10	27
49	32

$$\frac{ad}{bc} = \frac{10(32)}{(27)(49)} = 0.242$$

$$\ln(\widehat{OR}) = -1.42$$

$$se(\ln(\widehat{OR})) = \sqrt{\frac{1}{10} + \frac{1}{27} + \frac{1}{49} + \frac{1}{32}} = 0.434$$

$$-1.42 - 1.96(0.434) < \ln(\widehat{OR}) < -1.42 + 1.96(0.434)$$

$$= \boxed{-2.27 < \ln(\widehat{OR}) < -0.57}$$

Since confidence interval does not include 1, we can reject H_0 and conclude the length and pollination rates are associated.

QUESTION 3: In North America, between 100 million and 1 billion birds die each year by crashing into windows on buildings, more than any other human-related cause. This figure represents up to 5% of all birds in the area. One possible solution is to construct windows angled downward slightly, so that they reflect the ground rather than an image of the sky to a flying bird. An experiment by Klem et al. (2004) compared the number of birds that died as a result of vertical windows, windows angled 20 degrees off vertical, and windows angled 40 degrees off vertical. The angles were randomly assigned with equal probability to six windows and changed daily; assume for this exercise that windows and window locations were identical in every respect except angle. Over the course of the experiment, 30 birds were killed by windows in the vertical orientation, 15 were killed by windows set at 20 degrees off vertical, and 8 were killed by windows set at 40 degrees off vertical.

A. **(0.50 pts.) Clearly state an appropriate null hypothesis and an alternative hypothesis.**
H_0: Window incline and the rate of bird deaths are not related.
H_A: Probability of bird deaths were dependent of window incline.

B. **(0.25 pts.) What proportion of deaths occurred while the windows were set at a vertical orientation?**

$$\frac{D_v}{D_v + D_{20} + D_{40}} = \frac{30}{30+15+8}$$

$$= \boxed{0.57}$$

C. **(0.25 pts.)** What statistical test would you use to test the null hypothesis?
Chi squared test.

D. **(0.50 pts.)** Carry out the statistical test from part (c). Is there evidence that window angle affects the mortality rates of birds?

	Deaths	expected
vertical	30	17.67
20°	15	17.67
40°	8	17.67
total	53	

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(30-17.67)^2}{17.67} + \frac{(15-17.67)^2}{17.67} + \frac{(8-17.67)^2}{17.67}$$

$$= \boxed{14.3}, \quad \underline{df = 2}, \quad \text{critical value} = \underline{5.99}$$

★ Since our test statistic is greater than our critical value we can reject the null hypothesis. Bird deaths and window incline during this time interval were related.