Piano Key Frequency Mapping on the University of North Georgia Dahlonega Campus using Digital Signal Processing

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Abstract

Various pianos are provided for the students of the University of North Georgia's (UNG) Dahlonega's Nix Cultural Center for reasons of both leisure and practice. The quality of each piano varies greatly due to a myriad of reasons which could include type of piano, age, material, tuning, location, etc. The objective of this paper is to quantify the differences in pianos around campus using a metric of how in tune the pianos are as recorded by a microphone using digital signal processing (DSP).

Keywords

Piano, MATLAB, Digital Signal Processing (DSP), Butterworth Filter, Fast Fourier Transform (FFT)

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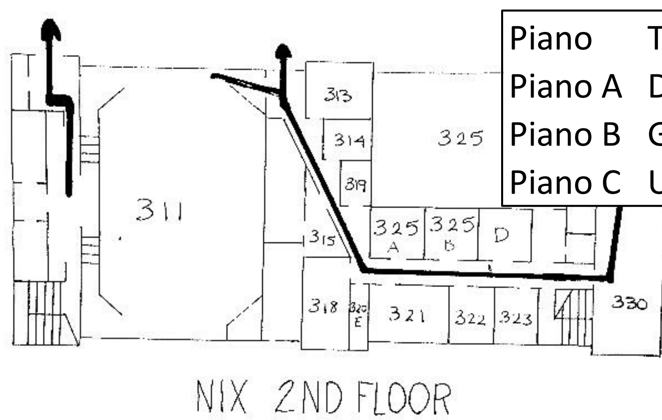
1. Introduction

For both amateur and professional piano players on campus the lack of availability of quality pianos to play is a distasteful inconvenience, and to audiences and passer-byes it can be a nuisance. Many pianists on campus traditionally vie for the "better" pianos that are available, leaving an unlucky few with an unsatisfactory piano performance. I am using digital filters to identify the key that was pressed on the piano, and map it against a preordained standard for in-tune pianos. Using MATLAB I will apply a Butterworth filter to remove unwanted noise and the purposeful harmonization's of the piano to identify the exact frequency of a note played by real pianos in the Nix Center by recording them through a microphone. After finding the frequency of the real piano's keys, I will compare them to the A440 standard [1] used in the United States. With the effects of this research, both an objective analysis of the quality of the Nix Center's pianos will be made, and show the differences between them and the A440 standard.

2. Recording

There are several pianos openly available in the Nix Center to play.

Piano	Type of Piano	Location
Piano A	Digital Piano	Nix Center Room 320
Piano B	Grand Piano	Nix Center Room 325A
Piano C	Upright Piano	Nix Center Room 325B



In order to quantify how out of tune the pianos in the Nix Center were, I played the entire 3rd and 4th octave one key at a time on each of the pianos. To convert the analog signal to a digital record, I used my laptop's webcam microphone and MATLAB's audiorecorder function to record to audio signal for 1 second at a sampling rate of 8000Hz (which is sufficient for the 3rd and 4th Octaves I am sampling). I allowed for a 1 second pause in between recordings, to avoid any echoing sound from the previously played note.

3. Key Mapping

Pianos throughout the United States and many other places in the world are tuned according to a standard known as A440, meaning that the note A above Middle C has a pitch that equals to 440Hz. From here, A above Middle C is tuned and all other keys are tuned according to the Pythagorean Scale, which allows the pitches of other notes to be mathematically derived from one another[2]. Based off this, all the keys in a piano have the following frequencies:

Octave													
0	Note	A	A#	B									
	Hz	27.5	29.14	30.87									
1	Note	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	32.7	34.65	36.71	38.89	41.2	43.65	46.25	49	51.91	55	58.27	61.74
2	Note	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	65.41	69.3	73.42	77.78	82.41	87.31	92.5	98	103.83	110	116.54	123.47
3	Note	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	130.81	138.59	146.83	155.56	164.81	174.61	185	196	207.65	220	233.08	246.94
4	Note	Middle C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	261.63	277.18	293.66	311.13	329.63	349.23	369.99	392	415.3	440	466.16	493.88
5	Note	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	523.25	554.37	587.33	622.25	659.26	698.46	739.99	783.99	830.61	880	932.33	987.77
6	Note	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	1046.5	1108.7	1174.7	1244.5	1318.5	1396.9	1480	1568	1661.2	1760	1864.7	1975.5
7	Note	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	Hz	2093	2217.5	2349.3	2489	2637	2793.8	2960	3136	3322.4	3520	3729.3	3951.1
8	Note	C											
	Hz	4186.01											

Table 1. A440 Standard with 3rd and 4th Octaves highlighted[1]

By matching the frequency of a note played by the pianos in the Nix Center to the A440 standard, I will be able to find out how out or in tune a particular piano is. Each key is recorded using a typical laptop microphone through MATLAB. After changing the domain from time to frequency using the Fast Fourier Transform (FFT), I am able to observe the frequencies of the audio signal. Once the prospective frequency has been found for each key in the 3rd and 4th octave (my sampling data for this research), the difference between those frequencies and the A440 standard will be quantified using subtraction, and the results will be displayed individually for each key as they could potentially not be uniformly out of tune.

4. Filtering

There are several problems that must be faced when analyzing the audio signal of a note played by a real piano. The conditions for playing of course vary, but most significantly background noise and the harmonization of notes cause the biggest problems for frequency analysis. Filtering out background noise, while maintaining interesting frequencies poses a unique problem when recording notes played in real life. Often, the magnitude of the interesting frequencies can be very low, while the frequencies of the harmonization of notes may have a significantly higher magnitude than the actual frequency of the note itself. To solve this problem, I designed an algorithm that generates multiple Butterworth filters that allow me to sample only frequencies at the frequency with the highest magnitude and any frequencies that exist in lower octaves of that note. Because notes played by a piano will never harmonize with one from a lower octave, by picking up the first high magnitude from the filtered signal after it has been cast into the frequency domain, I am able to identify the exact frequency of the note played by a particular key even if the harmonization of that note has an even high magnitude than the note's frequency, and without sacrificing the flexibility to be able to analyze any of the other 88 notes using the same algorithm.

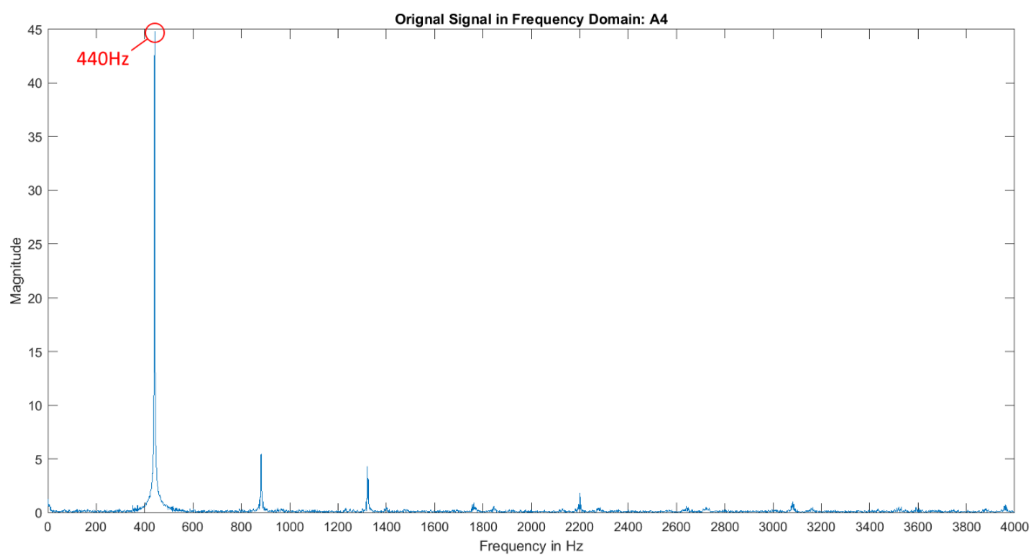


Figure A. A above Middle C in Frequency Domain with highest magnitude circled

Results like those in Figure A. are typical and expected. The highest peak for A above Middle C on Piano A is recorded at 440Hz, which is the same as the A440 standard, and the subsequent harmonizations of that note are also recorded. So the process of identifying the real frequency that the piano is playing is as simple as finding the frequency of the highest magnitude (which is just Voltage^2). While this is pretty straight-forward, not every piano, and not every note, displayed similar results.

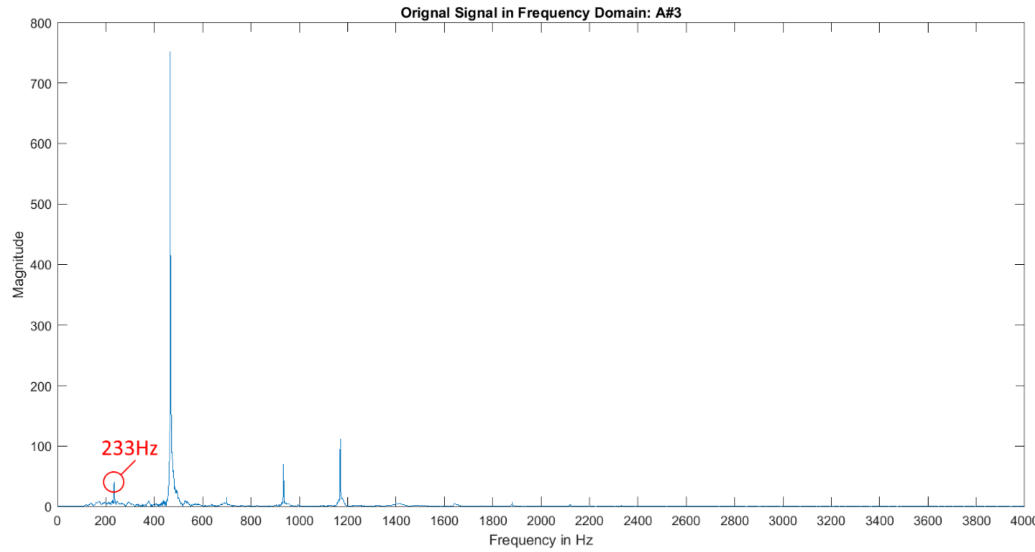


Figure B. A# in the 3rd Octave for Piano A, with real note frequency circled

In Figure B. the highest magnitude recorded is 466Hz, which matches to A# in the octave above the key that was actually pressed. So a more robust filter was needed, this time gathering all relatively large magnitudes, and identifying the first "large" magnitude as of the interesting frequency. This was effective since a note will never harmonize with the same note in a lower octave. This was effective at handling the vast majority of notes played by Piano A.

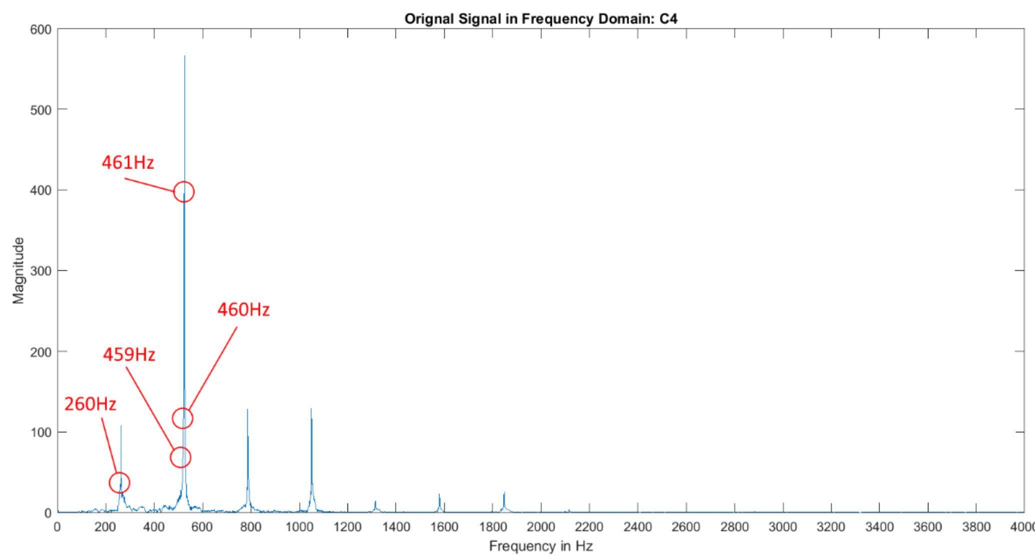


Figure C. Piano A, C in the 4th Octave with aliasing errors circled

But by using this technique, aliasing errors began to interfere with identifying the real frequency of the note. Many times, as in Figure C, aliasing errors that occurred during the FFT caused large magnitudes to be found at frequencies extremely close to the ones I was trying to identify. By picking up the first frequency with a relatively large magnitude, I was opening myself up to cases where values from aliasing were being identified, and not the real frequency, with no consistency across samples to offset this problem.

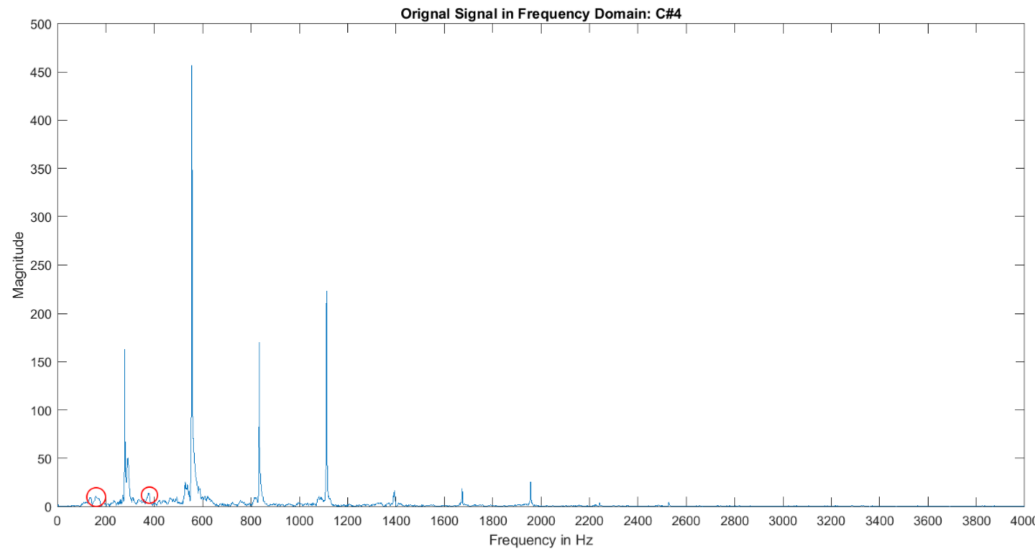


Figure D. Piano A, C# in the 4th Octave with high magnitude noise circled

Additionally, as in Figure D during the recording of Piano A's C# in the 4th Octave, noise sometimes created magnitudes that often where picked up by this method. And those frequencies weren't even associated with any frequency I was interested in. So to resolve this problem I redesigned my filter entirely, this time using an algorithm to generate multiple Bandpass Butterworth filters.

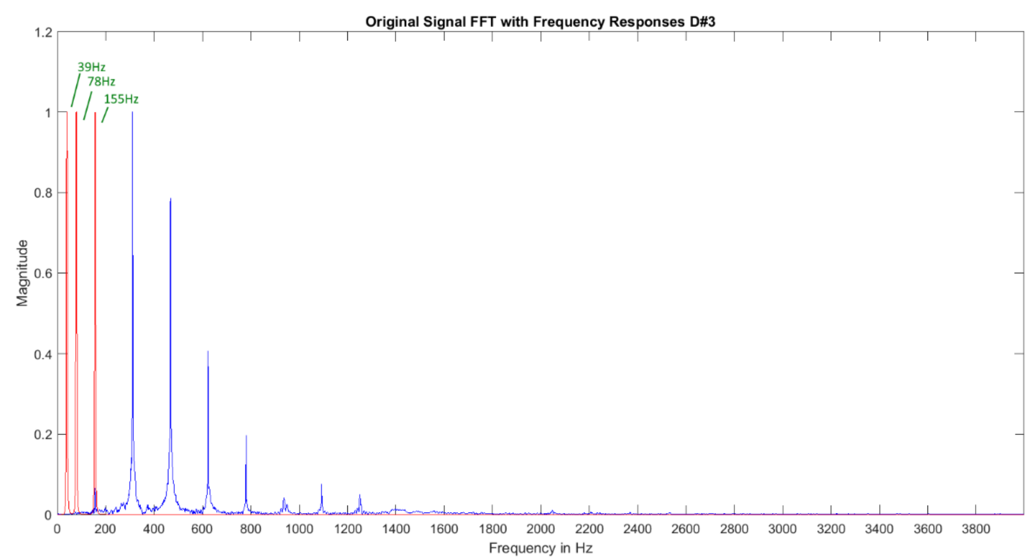


Figure E. Piano C, D# in the 3rd Octave. Bandpass Filters in red, and their frequency window midpoint in green

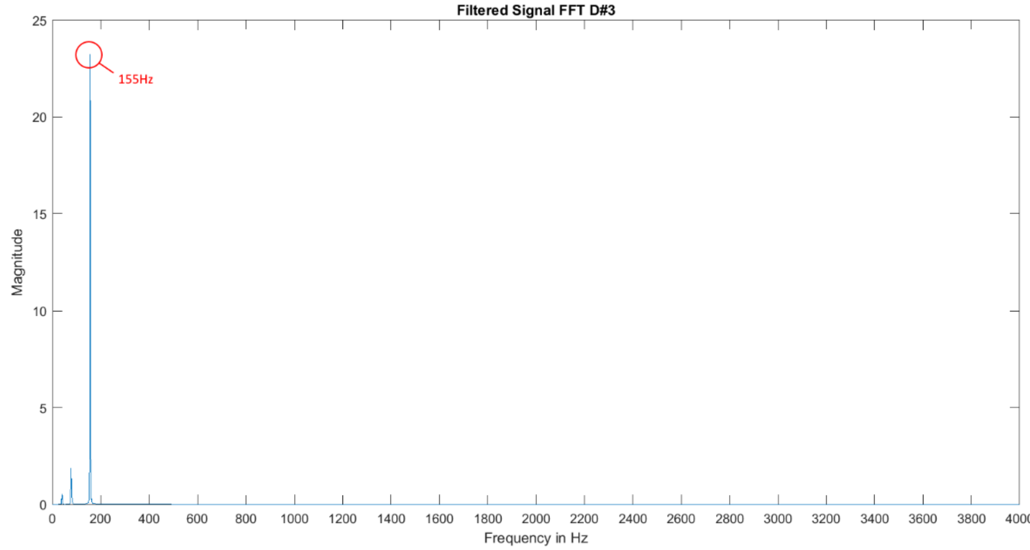


Figure F. Piano C, D# in the 3rd Octave, with highest magnitude's frequency circled

Because the notes follow the Pythagorean Scale, and can be derived from the same note in a higher octave by a factor of 2, my algorithm set up Bandpass filter up at every frequency that was the highest magnitude's frequency divided by 2^0, … 2^8. And since no note plays at a frequency below 27Hz, I was able to throw out any filters for frequencies below this number. Figure E. shows the frequency response of these newly designed filters, and Figure F. how I was able to use it to pick up the real frequency of the key using the highest magnitude of the FFT of the filtered signal, and not its harmonization.

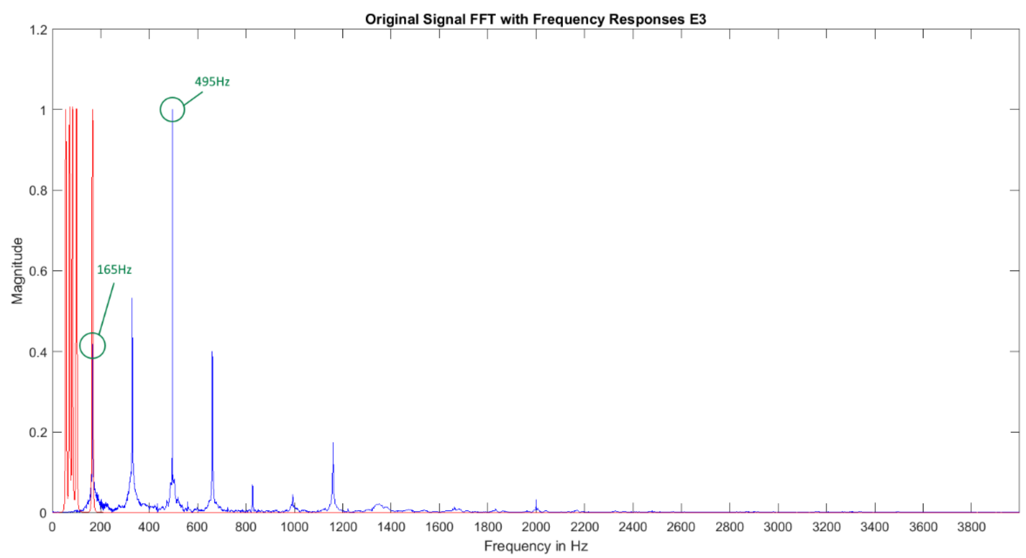


Figure G. Piano C, E in the 3rd Octave with frequency response of filters and highest magnitude's frequency and real key frequency circled.

But even this was not sufficient for filtering out complex signals from real acoustic pianos, as there were numerous problems regarding their deteriorated condition, such as the case in Figure G. Not every highest magnitude was a harmonization that occurred in a higher octave, sometimes it was the 3rd or 5th harmonization, an a check to handle this exception created Bandpass filters in a geometric scale for some basic numbers not factors of 2 (3, 5, 6, 7, and 9). This allowed me to use the same method for finding the real frequency of the key, picking up the highest magnitude of the filtered signal, as shown in Figure H.

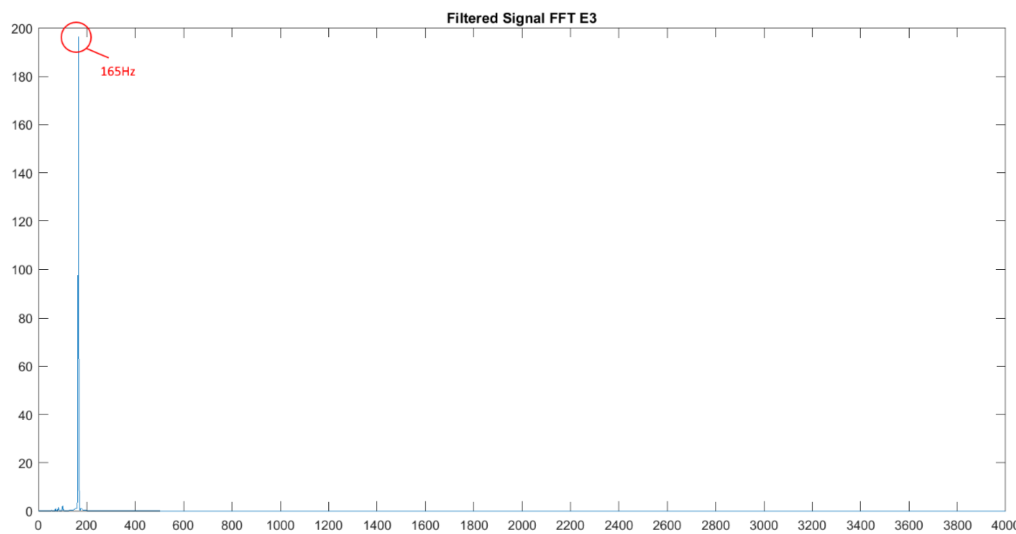


Figure H. Piano C, E in the 3rd Octave with highest magnitude's frequency circled

There was one case I was not able to map. Piano B's C key in the 3rd Octave was so complex I was not able to ascertain one specific frequency for the real key, even though its harmonizations were clear. The highest magnitude recorded is the 3rd harmonization of the 'real' key's frequency. Figure I. shows this key's sound in the frequency domain, and even after repeated recordings, the signal remained fairly complex. Even though I could manually sample the sampled frequency, how my filter designates whether or not the highest found magnitude is a harmonization was not able to tell its signal from the other high magnitudes around it.

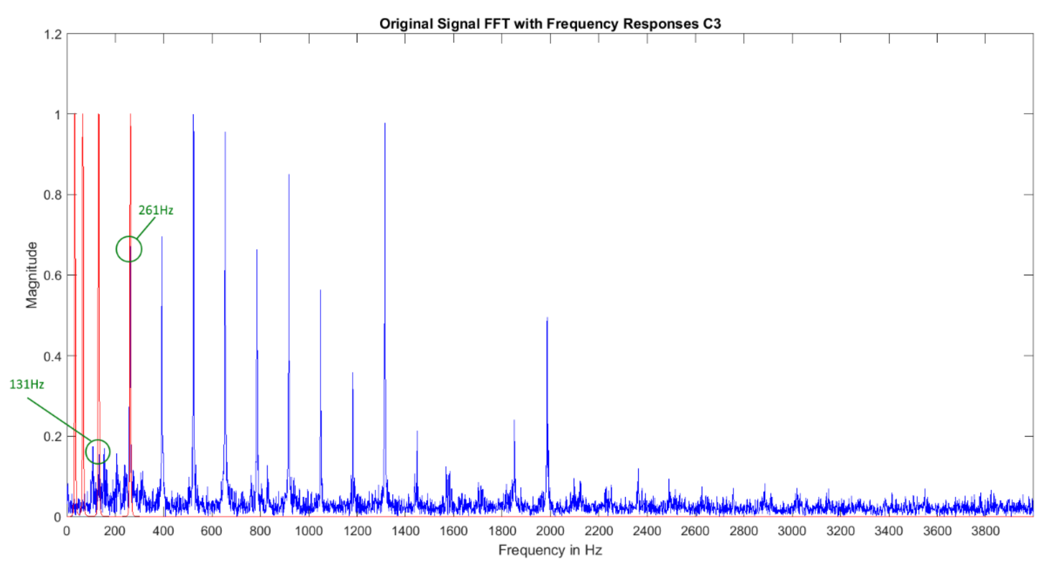


Figure I. Piano B's C in the 3rd Octave with frequency responses of filters in red, and the 'real' key and detected frequency circled

5. Results

The pianos were found to be surprisingly in tune for the most part, though their signals differed considerably. All results show a graph of the recorded keys frequency (blue circles) mapped against the A440 standard (red *) and a table showing the exact calculated difference. The y-axis is labeled with the interesting frequency of each note, identified as the actual frequency the key played. Since all my frequencies where recorded as integer values of Hz, and the A440 standard is accurate to decimal places, I consider a difference of under, but not equal to, 1Hz to be in tune.

Piano A:

As a digital piano, Piano A's keys were found to be perfectly in tune as expected, albeit the strength of its harmonizations were alarmingly all over the place.

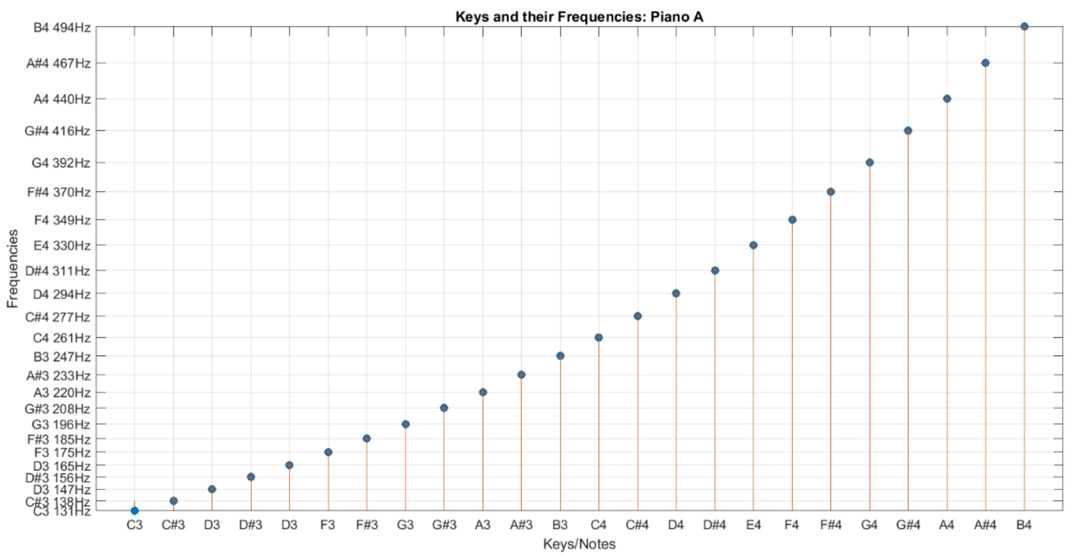


Figure J. The frequencies of Piano A after filtering, and A440 standard

In Figure J. Piano A's notes matched up nearly identical, but due to how I sampled the audio signal, I was unable to be more precise than integer values, while the A440 is plotted by its real decimal values, and hence many notes don't match up exactly. But the main point is that the keys do in fact play their intended frequency, just some of them with unusual harmonizing. Table 2. contains the differences between Piano A's identified values and corresponding frequency from Table 1.

Piano A

C3	C#3	D3	D#3	E3	F3	F#3	G3	G#3	A3	A#3	B3
0.19	0.59	0.17	0.44	0.19	0.39	0	0	0.35	0	0.08	0.06
C4	C#4	D4	D#4	E4	F4	F#4	G4	G#4	A4	A#4	B4
0.63	0.18	0.34	0.13	0.37	0.23	0.01	0	0.7	0	0.84	0.12

Table 2. Frequency differences calculation = |Piano A-A440|

Piano B:

Piano B was expected to be more out of tune than it was, being a much older grand piano, but for the most part it was hitting the notes fairly well, though this piano was certainly the worst of the three. C in the 3rd Octave was an outlier for my filter, the real key signal was so polluted by something, it was not distinguishable from other signals, and I was not able to identify the source of the noise, if it was noise at all and not just some awful sound the piano was making.

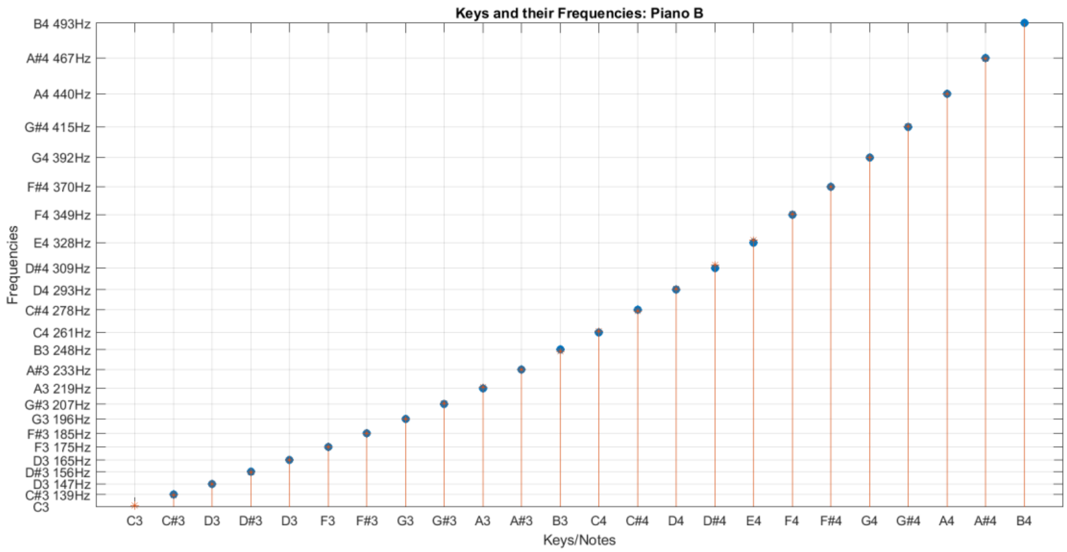


Figure K. The frequencies of Piano B after filtering, and A440 standard (C3 is missing)

Piano B

C3	C#3	D3	D#3	E3	F3	F#3	G3	G#3	A3	A#3	B3
130.19*	0.41	0.17	0.44	0.19	0.39	0	0	0.65	1	0.08	1.06
C4	C#4	D4	D#4	E4	F4	F#4	G4	G#4	A4	A#4	B4
0.63	0.82	0.66	2.13	1.63	0.23	0.01	0	0.3	0	0.84	0.88

Table 3. Frequency differences calculation = |Piano B-A440| (*C3 picked up the octave above it)

Piano C:

Piano C was an upright piano, that despite its age and condition, was surprisingly in tune as well. While a number of keys were 1Hz off, Piano C was more consistently out of tune than Piano B.

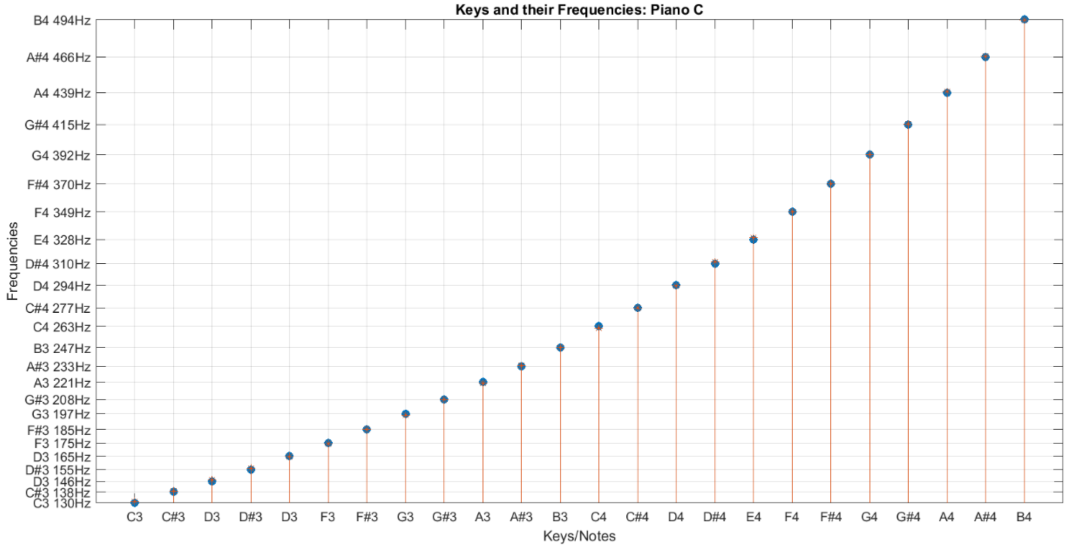


Figure L. The frequencies of Piano C after filtering, and A440 standard

Piano C

C3	C#3	D3	D#3	E3	F3	F#3	G3	G#3	A3	A#3	B3
0.81	0.59	0.83	0.56	0.19	0.39	0	1	0.35	1	0.08	0.06
C4	C#4	D4	D#4	E4	F4	F#4	G4	G#4	A4	A#4	B4
1.37	0.18	0.34	1.13	1.63	0.23	0.01	0	0.3	1	0.16	0.12

Table 4. Frequency differences calculation = |Piano C-A440|

6. Conclusions

In the end my research shows that the condition of the acoustic pianos in the Nix Center are largely adequate for practice, if the pianist can bare through a handful of cringes, while the digital piano if functioning decently. Additionally, my research shows that the live recording of piano signals from real pianos is actually rather complex, with harmonization's and their various strengths, nothing about this research was as expected. Precisely tuning a piano is exceedingly more complicated that just having it register according to the A440 standard, and as such the standard by itself is inadequate in providing guidance on piano tuning. Real world signals are complicated.

[1] B. H. Suits, *Physics of Music – Notes, Frequencies for equal-tempered scale, A4 = 440 Hz,*

http://www.phy.mtu.edu/~suits/notefreqs.html

[2] B. H. Suits, *Physics of Music – Notes, Pythagorean Scale*,

http://www.phy.mtu.edu/~suits/pythagorean.html