RSparseMatrix

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Introduction

This notebook has the function implementations for manipulating objects with head RSparseMatrix that behave like SparseArray objects but have the added functionalities to use row names and column names in a manner similar to that of the sparse arrays objects from the base library Matrix [2] for the programming language R [1].

The idea is fairly simple: we can use associations or replacement rules to map row names and column names into integers. Similarly to how it is done in R, RSparseMatrix handles only strings as row names and column names.

Note that the package does not use RLink -- it has purely Mathematica language implementations.

The following function signatures are implemented:

```
RowNames[ RSparseMatrix]
ColumnNames[ RSparseMatrix]
SetRowNames[ RSparseMatrix, { String..}]
SetColumnNames[ RSparseMatrix, { String..}]
DimensionNames[ RSparseMatrix]
Dimensions[ RSparseMatrix]
RowsCount[ RSparseMatrix]
ColumnsCount[ RSparseMatrix]
RowSums[ RSparseMatrix]
ColumnSums[ RSparseMatrix]
Total[ RSparseMatrix, ]
ArrayRules[ RSparseMatrix]
Transpose[ RSparseMatrix]
MatrixForm[ RSparseMatrix]
MatrixPlot[ RSparseMatrix]
Times[ RSparseMatrix, RSparseMatrix]
Times[ , RSparseMatrix]
Times[ RSparseMatrix, ]
Plus[ RSparseMatrix, RSparseMatrix]
Plus[ , RSparseMatrix]
Plus[ RSparseMatrix, _]
Dot[ RSparseMatrix, RSparseMatrix]
Dot[ , RSparseMatrix]
```

```
Dot[_RSparseMatrix, _]
Part[_RSparseMatrix, _String | {_String ...}, __]
Part[_RSparseMatrix, _,_string | {_String ...}]
Part[_RSparseMatrix, _,_string | {_String ...}, _String | ...}, _String | ...]
Part[_RSparseMatrix, _RSparseMatrix]
ColumnBind[_RSparseMatrix,_RSparseMatrix]

Note that assignment (with Set[__]) is not implemented.

The package can be loaded from GitHub [3]:

In[1]:= Import["https://raw.githubusercontent.com/antononcube/MathematicaForPrediction/master/Misc/RSparseMatrix.m"]
In[2]:= (*Import["~/MathFiles/MathematicaForPrediction/Misc/RSparseMatrix.m"]*)
```

Exposition functions

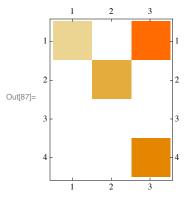
Tests and experiments

■ SparseArrays for comparisons

```
In[84]:= mat = SparseArray[{{1, 1} → 1, {2, 2} → 2, {4, 3} → 3, {1, 4} → 4, {3, 5} → 2}];
MatrixPlot[mat]

Out[85]=
3
4
4
```

ln[86]:= mat2 = SparseArray[{{1, 1} \rightarrow 1, {2, 2} \rightarrow 2, {4, 3} \rightarrow 3, {1, 3} \rightarrow 4}]; MatrixPlot[mat2]



This illustrates row binding and column binding corresponding to R's function cbind and rbind. Here it is done over sparse arrays below it is done with RSparseMatrix objects.

Creation

```
 \begin{aligned} & \text{Im} \  \  \, \text{Im} \  \, \text{Im} \  \  \, \text{Im} \  \  \, \text{Im} \  \  \, \text{Im} \  \, \text{Im} \  \, \text{Im} \  \  \, \text{Im} \
```

Note that the output is for mated to look like sparse array object. The package has this definition:

```
Format[RSparseMatrix[obj ]] := obj["sparseArray"];
```

The function MatrixForm shows the RSparseMatrix objects with their row and column names:

```
In[91]:= rmat // MatrixForm
```

Out[91]//MatrixForm=

$$\texttt{Out} \texttt{[92]= RSparseMatrix} \left[\left. \left\langle \right. \right| \texttt{sparseArray} \rightarrow \texttt{SparseArray} \left[\right. \right. \right] \\ \texttt{Dimensions} \left. \left\{ \textbf{4, 5} \right\} \right. \right] \textbf{,}$$

Out[93]//MatrixForm=

(а			d	
Α	1	0	0	4	0
A B C D	0	2	0	0	0
С	0	0	0	0	2
D	0	0	3	0	0

Setting names

In[94]:= rmat2 = rmat

 $\texttt{rownames} \rightarrow \, <\mid \texttt{A} \rightarrow \texttt{1, B} \rightarrow \texttt{2, C} \rightarrow \texttt{3, D} \rightarrow \texttt{4} \mid > \text{, colnames} \rightarrow \, <\mid \texttt{a} \rightarrow \texttt{1, b} \rightarrow \texttt{2, c} \rightarrow \texttt{3, d} \rightarrow \texttt{4, e} \rightarrow \texttt{5} \mid > \text{, dimnames} \rightarrow \, <\mid \texttt{U} \rightarrow \texttt{1, V} \rightarrow \texttt{2} \mid > \ \ \Big| \, \rangle \, \Big| \, \rangle$

In[95]:= MatrixForm[rmat2]

Out[95]//MatrixForm=

(а	b	С	d	е	1
Α	1	0	0	4	0	
A B C D	0	2	0	0	0	
С	0	0	0	0	2	
D	0	0	3	0	0	J

In[96]:= SetRowNames[rmat2, ToString /@ Range[RowsCount[rmat]]]



 $\texttt{rownames} \rightarrow \ \, \langle | \ 1 \rightarrow 1 \text{, } 2 \rightarrow 2 \text{, } 3 \rightarrow 3 \text{, } 4 \rightarrow 4 \ | \ \, \rangle \text{ colnames} \rightarrow \ \, \langle | \ a \rightarrow 1 \text{, } b \rightarrow 2 \text{, } c \rightarrow 3 \text{, } d \rightarrow 4 \text{, } e \rightarrow 5 \ | \ \, \rangle \text{ dimnames} \rightarrow \ \, \langle | \ U \rightarrow 1 \text{, } V \rightarrow 2 \ | \ \, \rangle \ \Big| \ \, \rangle \ \Big| \ \, \rangle \ \Big| \ \, \rangle$

In[97]:= MatrixForm[rmat2]

Out[97]//MatrixForm=

In[98]:= SetColumnNames[rmat2, ToString /@ Range[ColumnsCount[rmat]]]

Out[98]= RSparseMatrix [⟨ sparseArray → SparseArray | Dimensions {4, 5}



 $\texttt{rownames} \rightarrow \langle | \, 1 \rightarrow 1 \,, \, 2 \rightarrow 2 \,, \, 3 \rightarrow 3 \,, \, 4 \rightarrow 4 \, | \, \rangle \,, \, \\ \texttt{colnames} \rightarrow \langle | \, 1 \rightarrow 1 \,, \, 2 \rightarrow 2 \,, \, 3 \rightarrow 3 \,, \, 4 \rightarrow 4 \,, \, 5 \rightarrow 5 \, | \, \rangle \,, \, \\ \texttt{dimnames} \rightarrow \langle | \, 0 \rightarrow 1 \,, \, 0 \rightarrow 2 \, | \, \rangle \, \left| \, \right| \, \langle \, 0 \rightarrow 1 \,, \, 0 \rightarrow 1 \,$

In[99]:= MatrixForm[rmat2]

Out[99]//MatrixForm=

(1	2	3	4	5	1
1	1	0	0	4	0	
2	0	2	0 0 3	0	0	
3	0	0	0	0	2	
4	0	0	3	0	0	

In[100]:= MatrixForm[rmat]

Out[100]//MatrixForm=

(а		С		е
Α	1	0	0	4	0
В	0	2	0	0	0
С	0	0	0	0	2
D	0	0	3	0	0

Query functions

These functions can be used to names of rows, columns, and dimensions. They correspond to R's functions rownames, colnames, dimnames.

```
In[101]:= RowNames[rmat]
           ColumnNames[rmat]
           DimensionNames[rmat]
Out[101]= \{A, B, C, D\}
Out[102]= {a, b, c, d, e}
Out[103]= \{U, V\}
  Functions that can be applied to sparse arrays follow.
 In[104]:= Dimensions[rmat]
Out[104]= \{4, 5\}
 In[105]:= ArrayRules[rmat]
\text{Out[105]= } \{ \{1\text{, }1\} \rightarrow 1\text{, } \{1\text{, }4\} \rightarrow 4\text{, } \{2\text{, }2\} \rightarrow 2\text{, } \{3\text{, }5\} \rightarrow 2\text{, } \{4\text{, }3\} \rightarrow 3\text{, } \{\_\text{, }\_\} \rightarrow 0 \}
```

Visualization

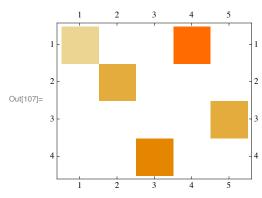
The redefinitions of MatrixForm and MatrixPlot are very useful for visualizing the RSparseMatrix objects.

In[106]:= MatrixForm[rmat]

Out[106]//MatrixForm=

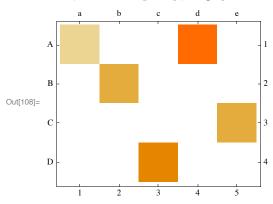
(а	b	С	d	е
A	1	0	0	4	0
В	0	2	0	0	0
С	0	0	0	0	2
D	0	0	3	0	0

In[107]:= MatrixPlot[rmat]



Here is another version of the MatrixPlot call that uses the row and column names as ticks.

In[108]:= MatrixPlot[rmat, FrameTicks → {{Transpose[{Range[RowsCount[rmat]], RowNames[rmat]}}], All}, {All, Transpose[{Range[ColumnsCount[rmat]], ColumnNames[rmat]}]}}]



■ Transpose

In[109]:= MatrixForm[Transpose[rmat]]

Out[109]//MatrixForm=

$$\begin{pmatrix} & A & B & C & D \\ \hline a & 1 & 0 & 0 & 0 \\ b & 0 & 2 & 0 & 0 \\ c & 0 & 0 & 0 & 3 \\ d & 4 & 0 & 0 & 0 \\ e & 0 & 0 & 2 & 0 \\ \end{pmatrix}$$

In[110]:= DimensionNames[Transpose[rmat]]

Out[110]= $\{V, U\}$

■ Sums

In[111]:= Total[rmat, 2]

Out[111]= 12

In[112]:= RowSums[rmat]

Out[112]= $\{1, 2, 3, 4, 2\}$

In[113]:= ColumnSums[rmat]

Out[113]= $\{5, 2, 2, 3\}$

■ Dot product

■ Matrix by vector

	expr	rmat	$Transpose[rmat[{1}, All]]$	<pre>rmat.Transpose[rmat[{1}, All]]</pre>
Out[114]=	result	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	(A A D D D D D D D D	(A 17 B 0 C 0 D 0
	head	RSparseMatrix	RSparseMatrix	RSparseMatrix

expr rmat rmat[1, All] rmat.rmat[1, All] b c d e 0 0 4 0 0 В 0 result 0 0 0 0 Out[116]= С 0 С 0 0 0 0 2 4 D 0 D 0 0 3 0 0 0 head RSparseMatrix SparseArray RSparseMatrix

■ Matrix by matrix

First we look into a dot product to the right of RSparseMatrix with a sparse array and a dot product to the left of RSparseMatrix with a sparse array.

	expr	rmat	mat	rmat.Transpose[mat]	Transpose[mat].rmat
Out[118]=	result	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\left(\begin{array}{cccccc} 1 & 0 & 0 & 4 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 & 0 \end{array}\right)$	$ \left(\begin{array}{c cccc} A & 17 & 0 & 0 & 0 \\ B & 0 & 4 & 0 & 0 \\ C & 0 & 0 & 4 & 0 \\ D & 0 & 0 & 0 & 9 \end{array} \right) $	$ \left(\begin{array}{cccccccccccccccccccccccccccccccccccc$
_	head	RSparseMatrix	SparseArray	RSparseMatrix	RSparseMatrix

This creates another RSparseMatrix object with no row and column names:

| In[120]:= rmat2 = ToRSparseMatrix[SparseArray[RandomInteger[{0, 4}, {ColumnsCount[rmat], RowsCount[rmat]}]]];

Next we look into two dot products of two RSparseMatrix objects.

	expr	rmat	rmat2	rmat.rmat2	rmat2.rmat	rmat.rmat2.rmat
=	result	$ \begin{pmatrix} & a & b & c & d & e \\ \hline A & 1 & 0 & 0 & 4 & 0 \\ B & 0 & 2 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 0 & 2 \\ D & 0 & 0 & 3 & 0 & 0 \\ \end{pmatrix} $	$\left(\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 4 & 0 & 1 \\ 1 & 2 & 2 & 3 \\ 1 & 0 & 2 & 4 \end{array}\right)$	$\left(\begin{array}{c cccc} A & 5 & 8 & 10 & 13 \\ B & 4 & 4 & 4 & 2 \\ C & 2 & 0 & 4 & 8 \\ D & 9 & 12 & 0 & 3 \end{array}\right)$	$ \left(\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{pmatrix} & a & b & c & d & e \\ \hline A & 5 & 16 & 39 & 20 & 20 \\ B & 4 & 8 & 6 & 16 & 8 \\ C & 2 & 0 & 24 & 8 & 8 \\ D & 9 & 24 & 9 & 36 & 0 \\ \end{pmatrix} $
	head	RSparseMatrix	RSparseMatrix	RSparseMatrix	RSparseMatrix	RSparseMatrix

Out[121]=

12 | RSparseMatrix.nb

Here Associations "swallows" the second value "U" because they are the same. (This is a bug.)

In[123]:= DimensionNames[rmat.Transpose[rmat]]

Out[123]= $\{U\}$

Arithmetic operations

The first three tables in this sub-section should be self-explanatory.

rmat + 1 rmat - 2 rmat 10 10 rmat + 2.33 rmat expr abcde b С d е а b С d е а С d е A 2 1 1 5 1 A - 1 - 2 - 2 2 - 2 A 10 0 0 40 0 A 12.33 0. 0. 49.32 0. result - 2 0 В 20 0. Out[124]= В 1 3 1 1 1 В -2 -2- 2 0 0 0 В 24.66 0. 0. 0. С С C 1 1 1 1 3 - 2 -2 -2 -20 С 0 0 0 0 20 0. 0. 0. 0. 24.66 D D 1 1 4 1 1 -2 -2 1 -2 -2D 0 0 30 0 0 D 0. 0. 36.99 0. 0. RSparseMatrix RSparseMatrix RSparseMatrix RSparseMatrix head

rmat + Transpose[rmat2] rmat Transpose [rmat2] expr rmat Transpose[rmat2] abcde 1 2 3 1 1 2 2 3 5 1 1 0 0 4 0 A 1 0 0 4 0 0 2 4 2 0 0 4 4 2 0 0 4 0 0 0 Out[126]= result В 0 2 0 0 0 2 2 0 2 2 2 2 0 2 4 0 0 0 0 4 С 0 0 0 0 2 1 1 1 3 4 1 1 4 3 4 0 0 3 0 0 D 0 0 3 0 0 head RSparseMatrix RSparseMatrix RSparseMatrix RSparseMatrix

In[128]:= rmat3 = rmat2;

In[129]:= SetRowNames[rmat3, ColumnNames[rmat]]; SetColumnNames[rmat3, RowNames[rmat]];

expr rmat Transpose [rmat3] rmat + Transpose[rmat3] rmat Transpose [rmat3] abcde abcde abcde abcde A 1 0 0 4 0 A 1 2 3 1 1 A 2 2 3 5 1 A 1 0 0 4 0 Out[131]= result В 0 2 0 0 0 B 0 2 4 2 0 В 0 4 4 2 0 В 0 4 0 0 0 C 0 0 0 0 2 C 2 2 0 2 2 C 2 2 0 2 4 C 0 0 0 0 4 D 0 0 3 0 0 D 1 1 1 3 4 D 1 1 4 3 4 D 0 0 3 0 0 head RSparseMatrix RSparseMatrix RSparseMatrix RSparseMatrix

When an arithmetic operation can be performed on the underlying sparse arrays but the row names or column names do not coincide the names are dropped.

In[133]:= SetRowNames[rmat3, "s." <> # & /@ ColumnNames[rmat]]; SetColumnNames[rmat3, RowNames[rmat]];

	expr	rmat	Transpose[rmat3]	rmat + Transpose[rmat3]	rmat Transpose[rmat3]
=	result	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\left(\begin{array}{cccccc} 2 & 2 & 3 & 5 & 1 \\ 0 & 4 & 4 & 2 & 0 \\ 2 & 2 & 0 & 2 & 4 \\ 1 & 1 & 4 & 3 & 4 \end{array}\right)$	$\left(\begin{array}{ccccc} 1 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 & 0 \end{array}\right)$
Ī	head	RSparseMatrix	RSparseMatrix	RSparseMatrix	RSparseMatrix

Out[135]=

■ Part

A major useful feature is to have Part work with row and column names. The implementation of that additional functionality for Part is demonstrated below.

In the cases when the dimension drops sparse arrays or numbers are returned. In R the operation "[" has the parameter "drop" -- the expression "smat[1,,drop=F]" is going to be a dense vector. The corresponding implementation is to have the option "Drop \rightarrow TruelFalse" for Part, but that does not seem a good idea. In the tables with examples below the last rows show the heads of the results.

■ Single row or column retrieval

	expr	rmat	${ t rmat} \llbracket { t A} rbracket$	<pre>rmat[All, a]</pre>	$\mathtt{rmat} \llbracket \{\mathtt{A}\} \rrbracket$	<pre>rmat[A, All]</pre>	<pre>rmat[All, a]</pre>	<pre>rmat[A, d]</pre>
Out[137]=	result	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\left(\begin{array}{c}1\\0\\0\\4\\0\end{array}\right)$	$\left(\begin{array}{c}1\\0\\0\\0\end{array}\right)$	$\left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\left(\begin{array}{c}1\\0\\0\\4\\0\end{array}\right)$	$\left(\begin{array}{c}1\\0\\0\\0\end{array}\right)$	4
	head	RSparseMatrix	SparseArray	SparseArray	RSparseMatrix	SparseArray	SparseArray	Integer

■ Permutation of both row names and column names

	expr			rm	at			r	mat	. [[{	С,	D,	Α,	B}]	rmat[{C, D, A, B}, {c, d, e, a, b}]	
			(a	b	С	d	e \		(a	b	С	d	e \	(cdeab)
		A	1	0	0	4	0		C	0	0	0	0	2	C 0 0 2 0 0	
Out[139]=	result	В	0	2	0	0	0		D	0	0	3	0	0	D 3 0 0 0 0	
		C	0	0	0	0	2		Α	1	0	0	4	0	A 0 4 0 1 0	
		D	0	0	3	0	0		В	0	2	0	0	0)	B 0 0 0 0 2	
	head	RS	рa	rse	eMa	tr	ix		RS	pa:	rse	eMa	tr	ix	RSparseMatrix	

■ Various subsets

	expr	rmat	rmat[{A, B}, {a, c, d}]	rmat[2;; 3, 1;; 2]	rmat[{A, B}, 1;; 2]	<pre>rmat[All, {a, c}]</pre>
Out[141]=	result	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \left(\begin{array}{c ccc} & a & c & d \\ \hline A & 1 & 0 & 4 \\ B & 0 & 0 & 0 \end{array}\right) $	$\left(\begin{array}{c cc} & a & b \\ \hline B & 0 & 2 \\ C & 0 & 0 \end{array}\right)$	$\left(\begin{array}{c cc} & a & b \\ \hline A & 1 & 0 \\ B & 0 & 2 \end{array}\right)$	$ \left(\begin{array}{c cc} & a & c \\ \hline A & 1 & 0 \\ B & 0 & 0 \\ C & 0 & 0 \\ D & 0 & 3 \end{array} \right) $
	head	RSparseMatrix	RSparseMatrix	RSparseMatrix	RSparseMatrix	RSparseMatrix

	expr	rmat	$rmat[{A}, 1]$	$rmat[{A}, {a}]$	rmat[2, 1;; 2]	rmat[C, All]	rmat[All, All]					
Out[143]=	result	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$	(1)	$\left(\begin{array}{c c} & a \\ \hline A & 1 \end{array}\right)$	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$	$\left(\begin{array}{c} 0\\0\\0\\0\\2\end{array}\right)$	$ \left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$					
	head	RSparseMatrix	SparseArray	RSparseMatrix	SparseArray	SparseArray	RSparseMatrix					

■ RowBind, ColumnBind

Row and column binding are useful in various data analysis scenarios.

When using row and column names there are couple of questions to be answered.

- 1. How duplication of row (column) names is handled?
- 2. How can we specify to ignore the row (column) names when the doing the binding?

```
In[145]:= MatrixForm[RowBind[rmat, rmat]]
```

Out[145]//MatrixForm=

(а	b	С	d	е	١
A.1	1	0	0	4	0	
B.1	0	2	0	0	0	
C.1	0	0	0	0	2	
D.1	0	0	3	0	0	
A.2	1	0	0	4	0	
B.2	0	2	0	0	0	
C.2	0	0	0	0	2	
D.2	0	0	3	0	0	,

 $\label{eq:local_$ MatrixForm[rmat2]

Out[147]//MatrixForm=

$$\begin{pmatrix} & a & b & c & d & e \\ \hline s.A & 1 & 0 & 0 & 4 & 0 \\ s.B & 0 & 2 & 0 & 0 & 0 \\ s.C & 0 & 0 & 0 & 0 & 2 \\ s.D & 0 & 0 & 3 & 0 & 0 \end{pmatrix}$$

| In[148]:= rmat3 = ToRSparseMatrix[rmat, "ColumnNames" → Map["t." <> # &, ColumnNames[rmat]]]; MatrixForm[rmat3]

Out[149]//MatrixForm=

(t.a	t.b	t.c	t.d	t.e
A	1	0	0	4	0
В	0	2	0	0	0
С	0	0	0	0	2
D	0	0	3	0	0 ,

				-										
Out[150]//MatrixForm=														
	(а	b	С	d	е	1							
	A	1	0	0	4	0								
	В	0	2	0	0	0								
	С	0	0	0	0	2								
	D	0	0	3	0	0								
	s.A	1	0	0	4	0								
	s.B	0	2	0	0	0								
	s.C	0	0	0	0	2								
	s.D	0	0	3	0	0	J							

In[151]:= {MatrixForm[ColumnBind[rmat, rmat2]], MatrixForm[ColumnBind[rmat, rmat3]]}

	(a.1	b.1	c.1	d. 1	e.1	a.2	b.2	c.2	d.2	e.2)	(а	b	С	d	е	t.a	t.b	t.c	t.d	t.e	1
r	Α	1	0	0	4	0	1	0	0	4	0		Α	1	0	0	4	0	1	0	0	4	0	1
Out[151]= {	В	0	2	0	0	0	0	2	0	0	0	,	В	0	2	0	0	0	0	2	0	0	0	}
Ĺ	С	0	0	0	0	2	0	0	0	0	2		С	0	0	0	0	2	0	0	0	0	2	ر
	D	0	0	3	0	0	0	0	3	0	0) '	D	0	0	3	0	0	0	0	3	0	0)

Profiling

```
In[73]:= smat = SparseArray[RandomReal[{0, 1}, {1000, 120}]];
|n[74]:= rmat = ToRSparseMatrix[smat, "RowNames" → Map["A" <> ToString[#] &, Range[Dimensions[smat][1]]]],
         "ColumnNames" → Map["b" <> ToString[#] &, Range[Dimensions[smat][2]]]]];
ln[75] := n = 100;
     tres =
       AbsoluteTiming[
        Do[sres = smat.Transpose[smat], {i, n}]
     tres[1] / n
Out[76]= \{5.4868, Null\}
Out[77]= 0.054868
In[78]:= tres =
       AbsoluteTiming[
        Do[rres = rmat.Transpose[rmat], {i, n}]
       ]
     tres[1] / n
Out[78]= \{5.50556, Null\}
Out[79]= 0.0550556
In[80]:= Norm[sres[1;; 120, 1;; 120]] - SparseArray[rres[1;; 120, 1;; 120]]]
Out[80]= 0.
In[81]:= Grid[{{
         MatrixPlot[rres[1; 120, 1; 120], ImageSize \rightarrow 350],
         MatrixPlot[rres["A" <> ToString[#] & /@ Range[120], 1;; 120], ImageSize → 350]}}]
```

References

- [1] The R Core Team, R Language Definition, (2015). URL: https://cran.r-project.org/doc/manuals/r-release/R-lang.pdf
- [2] D. Bates, M. Maechler, Sparse and Dense Matrix Classes and Methods, Package 'Matrix', (2015). URL: https://cran.r-project.org/web/packages/Matrix/Matrix.pdf.
- [3] A. Antonov, RSparseMatrix Mathematica packages, MathematicaForPrediction project at GitHub, (2015). $URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/Misc/RSparseMatrix.m \ . \\$