# A monad for Latent Semantic Analysis workflows

## Version 0.9

Anton Antonov

MathematicaForPrediction at WordPress

MathematicaForPrediction at GitHub

MathematicaVsR at GitHub

September 2019

## Introduction

In this document we describe the design and implementation of a (software programming) monad, [Wk1], for Latent Semantic Analysis workflows specification and execution. The design and implementation are done with Mathematica / Wolfram Language (WL).

What is Latent Semantic Analysis (LSA)?: A statistical method (or a technique) for finding relationships in natural language texts that is based on the so called Distributional hypothesis, [Wk2, Wk3]. (The Distributional hypothesis can be simply stated as "linguistic items with similar distributions have similar meanings"; for insightful an philosophical and scientific discussion see [MS1].) LSA can be seen as the application of Dimensionality reduction techniques over matrices derived with the Vector space model.

The goal of the monad design is to make the specification of LSA workflows (relatively) easy and straightforward by following a certain main scenario and specifying variations over that scenario.

The monad is named LSAMon and it is based on the State monad package "StateMonadCodeGenerator.m", [AAp1, AA1], the document-term matrix making package "DocumentTermMatrixConstruction.m", [AAp4, AA2], the Non-Negative Matrix Factorization (NNMF) package

"NonNegativeMatrixFactorization.m", [AAp5, AA2], and the package "SSparseMatrix.m", [AAp2, AA5], that provides matrix objects with named rows and columns.

The data for this document is obtained from WL's repository and it is manipulated into a certain ready-to-utilize form (and uploaded to GitHub.)

The monadic programming design is used as a Software Design Pattern. The LSAMon monad can be also seen as a Domain Specific Language (DSL) for the specification and programming of machine learning classification workflows.

Here is an example of using the LSAMon monad over a collection of documents that consists of 233 US state of union speeches.

```
LSAMonUnit[aStateOfUnionSpeeches] ⇒ Uplift text data into the monad.

LSAMonMakeDocumentTermMatrix ⇒ Make the document-term contingency matrix.

LSAMonEchoDocumentsStatistics ⇒ Echo the documents collection statistics.

LSAMonApplyTermWeightFunctions["IDF", "None", "Cosine"] ⇒ Apply LSI weight functions.

LSAMonExtractTopics[24, "MaxSteps" → 20, Method → "NNMF"] ⇒ Extract topics using NNMF.

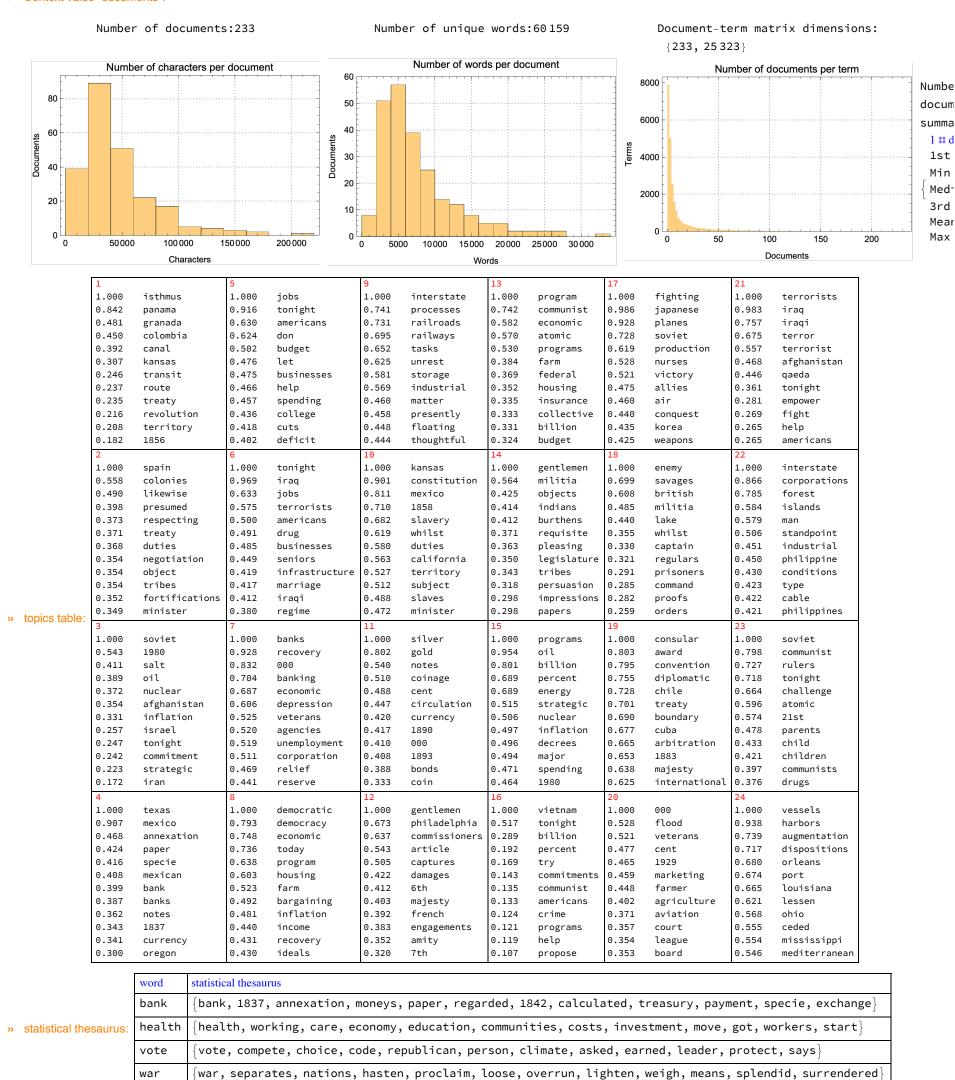
LSAMonEchoTopicsTable["NumberOfTableColumns" → 6] ⇒ Echo topics table.

LSAMonExtractStatisticalThesaurus[{"vote", "bank", "health", "war"}, 12] ⇒ Extract statistical theasurus.

LSAMonEchoStatisticalThesaurus

Echo thesaurus entries.
```

#### » Context value "documents":



The table above is produced with the package "MonadicTracing.m", [AAp2, AA1], and some of the explanations below also utilize that package.

As it was mentioned above the monad LSAMon can be seen as a DSL. Because of this the monad pipelines made with LSAMon are sometimes called "specifications".

**Remark:** With "term" we mean "a word, a word stem, or other type of token".

**Remark:** LSA and Latent Semantic Indexing (LSI) are considered more or less to be synonyms. I think that "latent semantic analysis" sounds more universal and that "latent semantic indexing" as a name refers to a specific Information Retrieval technique. Below we refer to "LSI functions" like "IDF" and "TF-IDF" that are applied within the generic LSA workflow.

#### Contents description

The document has the following structure.

- The sections "Package load" and "Data load" obtain the needed code and data.
  - (Needed and put upfront from the "Reproducible research" point of view.)

- The sections "Design consideration" and "Monad design" provide motivation and design decisions rationale.
- The sections "LSAMon overview", "Monad elements", and "The utilization of SSparseMatrix objects" provide technical descriptions needed to utilize the LSAMon monad.
  - (Using a fair amount of examples.)
- The section "Unit tests" describes the tests used in the development of the LSAMon monad.
  - (The random pipelines unit tests are especially interesting.)
- The section "Future plans" outlines future directions of development.
  - (The most interesting and important one is the "conversational agent" direction.)
- The section "Implementation notes" just says that LSAMon's development process and this document follow the ones of the classifications workflows monad ClCon, [AA6].

**Remark:** One can read only the sections "Introduction", "Design consideration", "Monad design", and "LSAMon overview". That set of sections provide a fairly good, programming language agnostic exposition of the substance and novel ideas of this document.

## Package load

The following commands load the packages [AAp1--AAp7, AAp11]:

## Data load

In this section we load data that is used in the rest of the document. The text data was obtained through WL's repository, transformed in a certain more convenient form, and uploaded to GitHub.

The text summarization and plots are done through LSAMon, which in turn uses the function RecordsSummary from the package "MathematicaForPredictionUtilities.m", [AAp7].

#### Hamlet

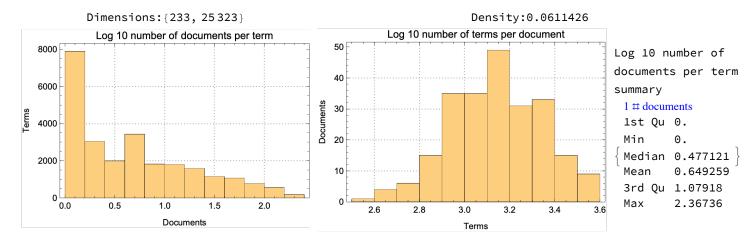
Dimensions: {223, 4440} Density:0.0111603 Number of terms per document Number of documents per term 140 Number of 2500 120 documents per term summary 100 2000 1 ♯ documents 80 » Context value "documentTermMatrix": 1500 1st Ou 1 60 Median 1 1000 40 Min 1 500 3rd Qu 2 20 Max 69 20 200 Documents

#### USA state of union speeches

#### In[\*]:= lsa0bj =

LSAMonUnit[aStateOfUnionSpeeches] ⇒ LSAMonMakeDocumentTermMatrix ⇒ LSAMonEchoDocumentTermMatrixStatistics["LogBase" → 10];

» Context value "documentTermMatrix":



#### log[\*]:= TakeLargest[ColumnSumsAssociation[lsaObj $\Rightarrow$ LSAMonTakeDocumentTermMatrix], 12]

```
\textit{Out[*]=} \hspace{0.2cm} \langle \hspace{0.1cm} \big| \hspace{0.1cm} \text{government} \rightarrow 7106, \hspace{0.1cm} \text{states} \rightarrow 6502, \hspace{0.1cm} \text{congress} \rightarrow 5023, \hspace{0.1cm} \text{united} \rightarrow 4847, \hspace{0.1cm} \text{people} \rightarrow 4103, \\ \text{year} \rightarrow 4022, \hspace{0.1cm} \text{country} \rightarrow 3469, \hspace{0.1cm} \text{great} \rightarrow 3276, \hspace{0.1cm} \text{public} \rightarrow 3094, \hspace{0.1cm} \text{new} \rightarrow 3022, \hspace{0.1cm} 000 \rightarrow 2960, \hspace{0.1cm} \text{time} \rightarrow 2922 \hspace{0.1cm} \big| \rangle
```

### Stop words

In some of the examples below we want to explicitly specify the stop words. Here are stop words derived using the built-in functions DictionaryLookup and DeleteStopwords.

stopWords = Complement[DictionaryLookup["\*"], DeleteStopwords[DictionaryLookup["\*"]]];

#### In[\*]:= Short[stopWords]

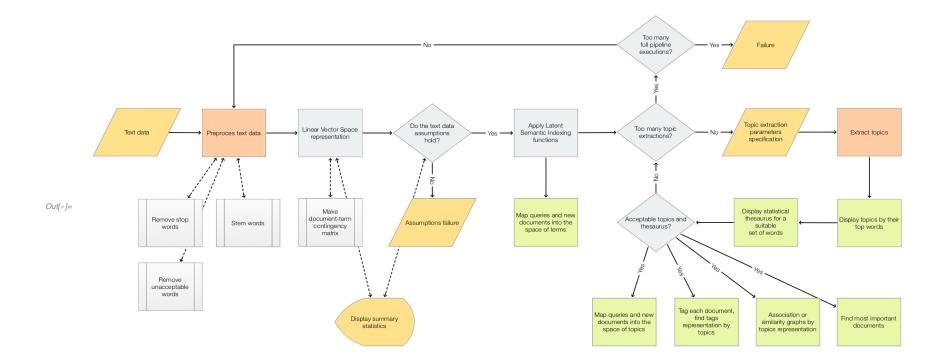
Out[\*]//Short= {a, about, above, across, add-on, after, again, <<290>>>, you'll, your, you're, yourself, yourselves, you've}

## Design considerations

The steps of the main LSA workflow addressed in this document follow.

- 1. Get a collection of documents with associated ID's.
- 2. Create a document-term matrix.
  - **2.1.** Here we apply the Bag-or-words model and Vector space model.
    - **2.1.1.** The sequential order of the words is ignored and each document is represented as a point in a multi-dimensional vector space.
    - **2.1.2.** That vector space axes correspond to the unique words found in the whole document collection.
  - **2.2.** Consider the application of stemming rules.
  - **2.3.** Consider the removal of stop words.
- 3. Apply matrix-entries weighting functions.
  - **3.1.** Those functions come from LSI.
  - 3.2. Functions like "IDF", "TF-IDF", "GFIDF".
- 4. Extract topics.
  - **4.1.** One possible statistical way of doing this is with Dimensionality reduction.
  - 4.2. We consider using Singular Value Decomposition (SVD) and Non-Negative Matrix Factorization (NNMF).
- 5. Make and display the topics table.
- 6. Extract and display a statistical thesaurus of selected words.
- 7. Map search queries or unseen documents over the extracted topics.
- 8. Find the most important documents in the document collection. (Optional.)

The following flow-chart corresponds to the list of steps above.



In order to address:

- the introduction of new elements in LSA workflows,
- workflows elements variability, and
- workflows iterative changes and refining,

it is beneficial to have a DSL for LSA workflows. We choose to make such a DSL through a functional programming monad, [Wk1, AA1].

Here is a quote from [Wk1] that fairly well describes why we choose to make a classification workflow monad and hints on the desired properties of such a monad.

[...] The monad represents computations with a sequential structure: a monad defines what it means to chain operations together. This enables the programmer to build pipelines that process data in a series of steps (i.e. a series of actions applied to the data), in which each action is decorated with the additional processing rules provided by the monad. [...]

Monads allow a programming style where programs are written by putting together highly composable parts, combining in flexible ways the possible actions that can work on a particular type of data. [...]

**Remark:** Note that quote from [Wk1] refers to chained monadic operations as "pipelines". We use the terms "monad pipeline" and "pipeline" below.

## Monad design

The monad we consider is designed to speed-up the programming of LSA workflows outlined in the previous section. The monad is named LSAMon for "Latent Semantic Analysis Monad".

We want to be able to construct monad pipelines of the general form:

$$\mathsf{LSAMon}\left[\_\right] \xrightarrow[\mathsf{LSAMonBind}\left[\mathsf{LSAMon}\left[\_\right],\mathsf{f}\_\right]]{} f_1 \xrightarrow[\mathsf{LSAMonBind}\left[\mathsf{LSAMon}\left[\_\right],\mathsf{f}\_\right]]{} f_2 \xrightarrow[\mathsf{LSAMonBind}\left[\mathsf{LSAMon}\left[\_\right],\mathsf{f}\_\right]]{} \cdots \xrightarrow[\mathsf{LSAMonBind}\left[\mathsf{LSAMon}\left[\_\right],\mathsf{f}\_\right]]{} f_k \tag{1}$$

LSAMon is based on the State monad, [Wk1, AA1], so the monad pipeline form (1) has the following more specific form:

$$LSAMon[pval\_, context\_] \xrightarrow[QRMonBind[m\_, f\_]]{} .... \left\{ \begin{array}{ll} f_i[\$LSAMonFailure] & m \equiv \$LSAMonFailure \\ f_i[x\_, c\_Association] & m \ \textit{is} \ LSAMon[x\_, c\_Association] \\ \$LSAMonFailure & otherwise \\ \end{array} \right\} \xrightarrow[LSAMonBind[m\_, f\_]]{} .... (2)$$

This means that some monad operations will not just change the pipeline value but they will also change the pipeline context.

In the monad pipelines of LSAMon we store different objects in the contexts for at least one of the following two reasons.

- 1. The object will be needed later on in the pipeline, or
- 2. The object is (relatively) hard to compute.

Such objects are document-term matrix, Dimensionality reduction factors and the related topics.

Let us list the desired properties of the monad.

- Rapid specification of non-trivial LSA workflows.
- The monad works with associations with string values, list of strings.
- The monad use the Linear vector spaces model .
- The document-term frequency matrix is can be created after removing stop words and/or word stemming.
- It is easy to specify and apply different LSI weight functions. (Like "IDF" or "GFIDF".)
- The monad can do dimension reduction with SVD and NNMF and corresponding matrix factors are retrievable with monad functions.
- Documents (or query strings) external to the monad a easily mapped into monad's Linear vector space of terms and the Linear vector space of topics.
- The monad allows of cursory examination and summarization of the data.
- The pipeline values can be of different types. Most monad functions modify the pipeline value; some modify the context; some just echo results.

- It is easy to obtain the pipeline value, context, and different context objects for manipulation outside of the monad.
- It is easy to tabulate extracted topics and related statistical thesauri.
- It is easy to specify and apply re-weighting functions for the entries of the document-term contingency matrices.

The LSAMon components and their interactions are fairly simple.

The main LSAMon operations implicitly put in the context or utilize from the context the following objects:

- document-term matrix,
- the factors obtained by matrix factorization algorithms,
- extracted topics.

Note the that the monadic set of types of LSAMon pipeline values is fairly heterogenous and certain awareness of "the current pipeline value" is assumed when composing LSAMon pipelines.

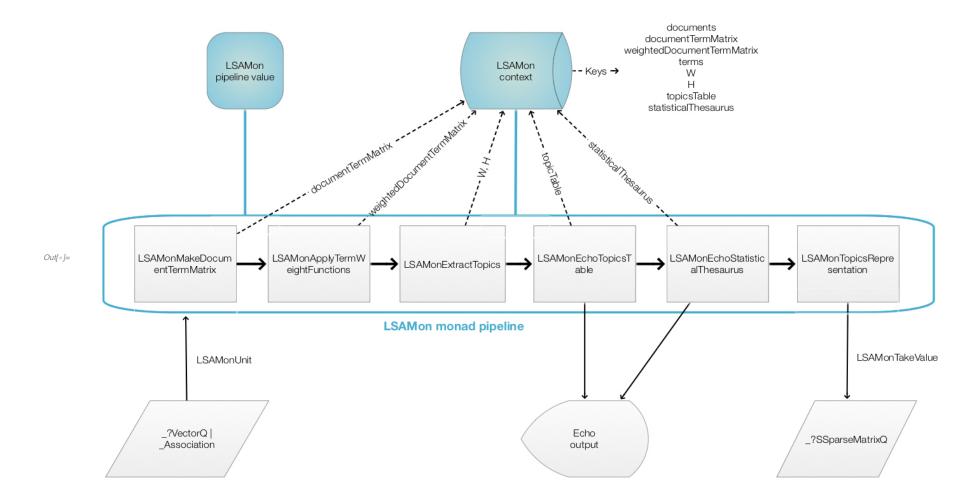
Obviously, we can put in the context any object through the generic operations of the State monad of the package "StateMonadGenerator.m", [AAp1].

## LSAMon overview

When using a monad we lift certain data into the "monad space", using monad's operations we navigate computations in that space, and at some point we take results from it.

With the approach taken in this document the "lifting" into the LSAMon monad is done with the function LSAMonUnit. Results from the monad can be obtained with the functions LSAMonTakeValue, LSAMonContext, or with the other LSAMon functions with the prefix "LSAMonTake" (see below.)

Here is a corresponding diagram of a generic computation with the LSAMon monad:



Remark: It is a good idea to compare the diagram with formulas (1) and (2).

Let us examine a concrete LSAMon pipeline that corresponds to the diagram above. In the following table each pipeline operation is combined together with a short explanation and the context keys after its execution.

Here is the output of the pipeline:

The LSAMon functions are separated into four groups:

- operations,
- setters and droppers,
- takers,
- State Monad generic functions.

### Monad functions interaction with the pipeline value and context

An overview of the those functions is given in the tables in next two sub-sections. The next section, "Monad elements", gives details and examples for the usage of the LSAMon operations.

#	name	echoes result	puts in context	uses from context	uses pipeline value
1	operations				

2	LSAMonApplyTermWeightFun ctions	no	<pre>{weightedDocumentTermMat\</pre>	{documentTermMatrix}	through LSAMonGetDocuments
3	LSAMonDocumentCollectionQ	no	none	none	no
	LSAMonEchoDocumentsStati:	yes	none	{documents, documentTermMatrix}	no
5	LSAMonEchoDocumentTermMax trixStatistics	yes	none	{documentTermMatrix}	no
6	LSAMonEchoStatisticalThe: saurus			no	
7	LSAMonEchoTopicsTable	yes	none	{topicsTable}	no
8	LSAMonExtractStatistical: Thesaurus	no	{statisticalThesaurus}	<pre>{terms, topicColumnPositions}</pre>	no
9	LSAMonExtractTopics	no	<pre>{W, H,   topicColumnPositions,   automaticTopicNames,   method}</pre>	<pre>{weightedDocumentTermMat\     rix}</pre>	no
10	LSAMonFindMostImportantD: ocuments	no	none	{weightedDocumentTermMat\ rix}	tries first
11	LSAMonGetDocuments	no	none	{documents}	if the context does not have "documents".
12	LSAMonInterpretBasisVect: or	no	none	{W, H}	no
13	LSAMonMakeDocumentTermMa: trix	no	{documentTermMatrix, stemmingRules, stopWords}	{documents}	no
14	LSAMonMakeGraph	no	none	<pre>{documentTermMatrix, weightedDocumentTermMat\     rix}</pre>	tries first
15	LSAMonMakeTopicsTable	no	{topicsTable}	{W, H}	no
16	LSAMonRepresentByTerms	no	none	<pre>{documentTermMatrix,   globalWeights,   localWeightFunction,   normalizerFunction,   stemmingRules, stopWords}</pre>	no
17	LSAMonRepresentByTopics	no	none	<pre>{documentTermMatrix,   globalWeights,   localWeightFunction,   normalizerFunction,   stemmingRules,   stopWords, H}</pre>	no
	LSAMonRepresentDocumentT agsByTopics	no	{docTopicIndices}	{documentTermMatrix, W, H}	no
	LSAMonSetAutomaticTopicN: ames	no	automaticTopicNames	none	no
	LSAMonSetContext	no	context	none	no
	LSAMonSetDocuments	no	documents	none	no
23	LSAMonSetDocumentTermMat: rix	no	documentTermMatrix	none	no
	LSAMonSetGlobalWeightFun ction	no	globalWeightFunction	none	no
	LSAMonSetGlobalWeights	no	globalWeights	none	no
	LSAMonSetH LSAMonSetLocalWeightFunc	no	H localWeightFunction	none	no no
	tion				
	LSAMonSetMethod	no	method	none	no
	LSAMonSetNormalizerFunct: ion	no	normalizerFunction	none	no
	LSAMonSetStatisticalThes: aurus	no	statisticalThesaurus	none	no
	LSAMonSetStemmingRules	no	stemmingRules	none	no
32	LSAMonSetStopWords	no	stopWords	none	no
	LSAMonSetTerms	no	terms	none	no
	LSAMonSetTopicColumnPosi : tions	no	topicColumnPositions	none	no
	LSAMonSetTopicsTable	no	topicsTable	none	no
	LSAMonSetValue	no	value	none	no
37	LSAMonSetW	no	W	none	no

38	LSAMonSetWeightedDocumen	no	weightedDocumentTermMatr	none	no
	tTermMatrix		ix		
39	droppers				
				automoti aTani aNama	
40	LSAMonDropAutomaticTopic:	no	no	automaticTopicNames	no
	Names				
41	LSAMonDropDocuments	no	no	documents	no
42	${\sf LSAMonDropDocumentTermMa} :$	no	no	documentTermMatrix	no
	trix				
43	LSAMonDropFromContext	no	no	fromContext	no
44	LSAMonDropGlobalWeightFu	no	no	globalWeightFunction	no
44	•	110	110	g toba twe igner unce for	110
	nction				
45	LSAMonDropGlobalWeights	no	no	globalWeights	no
46	LSAMonDropH	no	no	Н	no
47	LSAMonDropLocalWeightFun	no	no	localWeightFunction	no
	ction				
48	LSAMonDropMethod	no	no	method	no
49	LSAMonDropNormalizerFunc	no	no	normalizerFunction	no
	tion				
50	LSAMonDropStatisticalThe	no	no	statisticalThesaurus	no
	saurus				
51	LSAMonDropStemmingRules	no	no	stemmingRules	no
52	LSAMonDropStopWords	no	no	stopWords	no
53	LSAMonDropTerms	no	no	terms	no
54	LSAMonDropTopicColumnPos	no	no	topicColumnPositions	no
	itions				
55	LSAMonDropTopicsTable	no	no	topicsTable	no
56	LSAMonDropW	no	no	W	no
	·				-
57	LSAMonDropWeightedDocume	no	no	weightedDocumentTermMatr	no
	ntTermMatrix			ix	
58	takers				
59	LSAMonTakeAutomaticTopic	no	no	automaticTopicNames	no
	Names				
60	LSAMonTakeContext	no	no	context	no
61	LSAMonTakeDocuments	no	no	documents	no
62	LSAMonTakeDocumentTermMa	no	no	documentTermMatrix	no
	trix				
60				-1 -h -1 W - i -h + 5	
63	LSAMonTakeGlobalWeightFu:	no	no	globalWeightFunction	no
	nction				
64	LSAMonTakeGlobalWeights	no	no	globalWeights	no
65	LSAMonTakeH	no	no	Н	no
66	LSAMonTakeLocalWeightFun	no	no	localWeightFunction	no
	ction				
67	LSAMonTakeMatrix	no	no	matrix	no
68	LSAMonTakeMethod	no	no	method	no
69	LSAMonTakeNormalizerFunc	no	no	normalizerFunction	no
	tion				
70					
70	LSAMonTakeStatisticalThe	no	no	statisticalThesaurus	no
	saurus				
	8	no	no	stemmingRules	no
72	LSAMonTakeStopWords	no	no	stopWords	no
73	LSAMonTakeTerms	no	no	terms	no
74	LSAMonTakeTexts	no	no	texts	no
75	LSAMonTakeTopicColumnPos	no	no	topicColumnPositions	no
	itions				
76	LSAMonTakeTopicsTable	no	no	topicsTable	no
77	LSAMonTakeValue	no	no	value	no
78	LSAMonTakeW	no	no	W	no
79	LSAMonTakeWeightedDocume	no	no	weightedDocumentTermMatr.	no
. 3	ntTermMatrix			ix	
0.0		no	200		no
80	LSAMonTakeWeightedMatrix	no	no	weightedMatrix	no

## State monad functions

Here are the LSAMon State Monad functions (generated using the prefix "LSAMon", [AAp1, AA1].)

#	name	description					
1	LSAMonAddToContext	LSAMonAddToContext[varName_String] adds to the monad context the monad value under key varName.  LSAMonAddToContext[arg_Association] joins the monad context with arg.  LSAMonAddToContext[] joins the monad context with the monad value.					
2	LSAMonBind	Monad binding function.					
3	LSAMonDropFromContext	Drop from the monad context elements withe specified keys.					
4	LSAMonEcho	Echoes the argument. If no argument is given the short print of the monad object is echoed.					
5	LSAMonEchoContext	Echoes the monad context.					
6	LSAMonEchoFunctionContext	Echoes function application over the monad context.					
7	LSAMonEchoFunctionValue	Echoes function application over the monad value.					
8	LSAMonEchoValue	Echoes the monad value.					
9	LSAMonFail	Failure.					
10	LSAMonIf	LSAMonIf[f_, fYes_, fNo_] executes fYes[LSAMonUnit[xs,context]] if f[LSAMonUnit[xs,context]] is True; fNo[LSAMonUnit[xs,context]] otherwise.					
11	LSAMonIfElse	LSAMonIfElse[testFunc_, fYes_, fNo_] executes fYes[xs, context] if TrueQ[testFunc[xs, context]]; otherwise fNo[xs, context].					
12	LSAMonModifyContext	LSAMonModifyContext[f] replaces the monad context f[context].					
13	LSAMonOption	If the application of the argument to the monad produces monad failure the monad is unchanged.					
14	LSAMonPutContext	Replaces the monad context with the argument.					
15	LSAMonPutValue	Replaces the monad value with the argument.					
16	LSAMonRetrieveFromContext	LSAMonRetrieveFromContext[varName_String] retrieves from the monad context the value of the key varName.					
17	LSAMonSetContext	Replaces the monad context with the argument.					
18	LSAMonSetValue	Replaces the monad value with the argument.					
19	LSAMonSucceed	Success.					
20	LSAMonTakeContext	Takes the monad context.					
21	LSAMonTakeValue	Takes the monad value.					
22	LSAMonUnit	LSAMon monad unit constructor.					
23	LSAMonUnitQ	LSAMon monad unit test.					
24	LSAMonWhen	Shorter version of LSAMonIfElse.					

### Main monad functions

Here are the usage descriptions of the main (not monad-supportive) LSAMon functions, which are explained in detail in the next section.

#	name	description
1	LSAMonApplyTermWeightFunctions	Apply term weight functions to entries of the document-term matrix.
2	LSAMonDocumentCollectionQ	Gives True if the argument is a text collection.
3	LSAMonEchoDocumentsStatistics	Echo statistics for the text collection.
4	LSAMonEchoDocumentTermMatrixStatistics	Echo document-term matrix statistics.
5	LSAMonEchoStatisticalThesaurus	Echo the statistical thesaurus entries for a specified list of words.
6	LSAMonEchoTopicsTable	Echo the a table with the extracted topics.
7	LSAMonExtractStatisticalThesaurus	Extract the statistical thesaurus for specified list of words.
8	LSAMonExtractTopics	Extract topics.
9	LSAMonFindMostImportantDocuments	Find the most important texts in the text collection.
10	LSAMonGetDocuments	Get monad's document collection.
11	LSAMonInterpretBasisVector	Interpret the a specified basis vector.
12	LSAMonMakeDocumentTermMatrix	Make the document-term matrix.
13	LSAMonMakeGraph	Make a graph of the document-term, document-document, or term-term relationships.
14	LSAMonMakeTopicsTable	Make a table of topics.
15	LSAMonRepresentByTerms	Find the terms representation of a matrix or a document.
16	LSAMonRepresentByTopics	Find the topics representation of a matrix or a document.
17	LSAMonRepresentDocumentTagsByTopics	Find the topic representation corresponding to a list of tags. Each monad document is expected to have a tag. One tag might correspond to multiple documents.

## Monad elements

In this section we show that LSAMon has all of the properties listed in the previous section.

## The monad head

The monad head is LSAMon. Anything wrapped in LSAMon can serve as monad's pipeline value. It is better though to use the constructor LSAMonUnit. (Which adheres to the definition in [Wk1].)

```
log_{i} = LSAMon[textHamlet, <||>] \Rightarrow LSAMonMakeDocumentTermMatrix[Automatic, Automatic] \Rightarrow LSAMonEchoFunctionContext[Short];
```

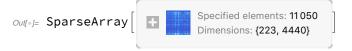
 $>\!\!\!> \ \, \langle \, \big| \, \text{documents} \, \rightarrow \, \langle \, \big| \, \text{id.0001} \, \rightarrow \, \text{1604, id.0002} \, \rightarrow \, \text{THE TRAGEDY OF HAMLET, PRINCE OF DENMARK,} \, \, \langle \langle 220 \rangle \rangle \,, \, \, \text{id.0223} \, \rightarrow \, \text{THE END} \, \big| \, \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \text{id.0223} \, \rightarrow \, \text{THE END} \, | \, \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \text{id.0223} \, \rightarrow \, \text{THE END} \, | \, \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \text{id.0223} \, \rightarrow \, \text{THE END} \, | \, \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \text{id.0223} \, \rightarrow \, \text{THE END} \, | \, \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \text{id.0223} \, \rightarrow \, \text{THE END} \, | \, \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,, \, \, \langle 220 \rangle \rangle \,, \, \, \langle \langle 220 \rangle \rangle \,,$  $<\!<$ 3>>> , stemmingRules  $\rightarrow$  Automatic |  $\rangle$ 

## Lifting data to the monad

The function lifting the data into the monad QRMon is QRMonUnit.

The lifting to the monad marks the beginning of the monadic pipeline. It can be done with data or without data. Examples follow.

 $m[\cdot]:= LSAMonUnit[textHamlet] \Rightarrow LSAMonMakeDocumentTermMatrix \Rightarrow LSAMonTakeDocumentTermMatrix$ 



 $log_{ij} = LSAMonUnit[] \Rightarrow LSAMonSetDocuments[textHamlet] \Rightarrow LSAMonMakeDocumentTermMatrix \Rightarrow LSAMonTakeDocumentTermMatrix$ 

```
Out[*]= SparseArray
                                  Dimensions: {223, 4440}
```

(See the sub-section "Setters, droppers, and takers" for more details of setting and taking values in LSAMon contexts.)

Currently the monad can deal with data in the following forms:

- vectors of strings,
- associations with string values.

Generally, WL makes it easy to extract columns datasets order to obtain vectors or matrices, so datasets are not currently supported in LSAMon.

### Making of the document-term matrix

As it was mentioned above with "term" we mean "a word or a stemmed word". Here is are examples of stemmed words.

```
ln[-s]:= <code>WordData[#, "PorterStem"] & /@ {"consequential", "constitution", "forcing", ""}</code>
Out[*]= {consequenti, constitut, forc, }
```

The fundamental model of LSAMon is the so called Vector space model (or the closely related Bag-of-words model.)

The document-term matrix is a linear vector space representation of the documents collection. That representation is further used in LSAMon to find topics and statistical thesauri.

Here is an example of ad hoc construction of a document-term matrix using a couple of paragraphs from "Hamlet".

```
ln[\bullet]:= inds = {10, 19};
       aTempText = AssociationThread[inds, textHamlet[inds]]
 out_{0} = \langle 10 \rightarrow \mathsf{ACT} \ \mathsf{I.} \ \mathsf{Scene} \ \mathsf{I.} \ \mathsf{Elsinore.} \ \mathsf{A} \ \mathsf{platform} \ \mathsf{before} \ \mathsf{the} \ \mathsf{Castle.} | \rangle
  ln[*]:= MatrixForm@CrossTabulate[Flatten[KeyValueMap[Thread[{#1, #2}] &, TextWords /@ToLowerCase[aTempText]], 1]]
Out[ • ]//MatrixFc
             a act before castle elsinore i. ii in of platform room scene state the
         10 | 1
                                                         0
                                                                                   0
                                                                                                  0
                                                                                                         1
         19 1
```

When we construct the document-term matrix we (often) want to stem the words and (almost always) want to remove stop words. LSAMon's function LSAMonMakeDocumentTermMatrix makes the document-term matrix and takes specifications for stemming and stop words.

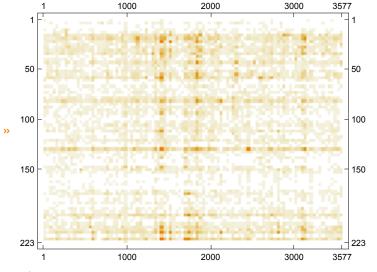
```
In[*]:= lsa0bj =
```

LSAMonUnit[textHamlet] ⇒

LSAMonMakeDocumentTermMatrix["StemmingRules" → Automatic, "StopWords" → Automatic] ⇒

LSAMonEchoFunctionContext[MatrixPlot[#documentTermMatrix] &] ⇒

LSAMonEchoFunctionContext[TakeLargest[ColumnSumsAssociation[#documentTermMatrix], 12] &];



»  $\langle | \text{ham} \rightarrow 359, \text{lord} \rightarrow 232, \text{king} \rightarrow 204, \text{come} \rightarrow 138, \text{queen} \rightarrow 121,$  $\verb| shall \rightarrow 114, \verb| hamlet \rightarrow 112, \verb| good \rightarrow 111, \verb| let \rightarrow 109, \verb| hor \rightarrow 109, \verb| thou \rightarrow 107, \verb| like \rightarrow 89 \>| \>\rangle$ 

We can retrieve the stop words used in a monad with the function LSAMonTakeStopWords.

```
In[⊕]:= Short[lsa0bj ⇒ LSAMonTakeStopWords]
```

out black of the variety of the vari

We can retrieve the stemming rules used in a monad with the function LSAMonTakeStemmingRules.

```
In[•]:= Short[lsaObj⇒LSAMonTakeStemmingRules]
```

Out[\*]//Short= Automatic

The specification Automatic for stemming rules uses WordData[#, "PorterStem"] &.

Instead of the options style signature we can use positional signature.

LSAMonMakeDocumentTermMatrix[StemmingRules → {}, StopWords → Automatic] Positional style: LSAMonMakeDocumentTermMatrix[{}}, Automatic]

#### LSI weight functions

After making the document-term matrix we will most likely apply LSI weight functions, [Wk2], like "GFIDF" and "TF-IDF". (This follows the "standard" approach used in search engines for calculating weights for document-term matrices; see [MB1].)

#### Frequency matrix

We use the following definition of the frequency document-term matrix *F*.

Each entry  $f_{ij}$  of the matrix F is the number of occurrences of the term j in the document i.

#### Weights

Each entry of the weighted document-term matrix M derived from the frequency document-term matrix F is expressed with the formula  $m_{ij} = g_j l_{ij} d_i,$ 

where

 $g_i$  -- global term weight;

*l<sub>i i</sub>* -- local term weight;

 $d_i$  -- normalization weight.

Various formulas exist for these weights and one of the challenges is to find the right combination of them when using different document collections.

Here is a table of weight functions formulas.

	weight type	name	formula
	global	None	1
	global	Inverse document frequency (IDF)	$\log\left(\frac{n}{\sum_{j}\chi\left(f_{ij}\right)}\right)$
	global	Global frequency inverse document frequency (GFIDF)	$\frac{\sum_{j} f_{ij}}{\sum_{j} \chi \left( f_{ij} \right)}$
Out[•]=	global	Normal	$rac{1}{\sqrt{\sum_{i}f_{ij}^{2}}}$
	local	Binary	$\chi(\mathbf{f_{ij}})$
	local	Logarithmic	$\log(f_{ij}+1)$
	local	Term frequency	$f_{ij}$
	normalization	None	1
	normalization	Cosine	$rac{1}{\sqrt{\sum_{j} g_{j} \; l_{i  j}}}$

m[v]:= lsaHamlet = LSAMonUnit[textHamlet]  $\Longrightarrow$  LSAMonMakeDocumentTermMatrix;

#### **Computation specifications**

LSAMon function LSAMonApplyTermWeightFunctions delegates the LSI weight functions application to the package "DocumentTermMatrixConstruction.m", [AAp4].

Here is an example.

```
In[•]:= wmat =
              lsaHamlet⇒
                LSAMonApplyTermWeightFunctions["IDF", "TermFrequency", "Cosine"] ⇒
                LSAMonTakeWeightedDocumentTermMatrix;
 In[*]:= TakeLargest[ColumnSumsAssociation[wmat], 6]
\textit{Out[*]=} \hspace{0.2cm} \langle \hspace{0.1cm} | \hspace{0.1cm} \text{enter} \rightarrow 21.7403, \hspace{0.1cm} \text{ham} \rightarrow 9.8253, \hspace{0.1cm} \text{hamlet} \rightarrow 9.16829, \hspace{0.1cm} \text{polonius} \rightarrow 8.79678, \hspace{0.1cm} \text{scene} \rightarrow 8.56075, \hspace{0.1cm} \text{king} \rightarrow 8.1436 \hspace{0.1cm} | \hspace{0.1cm} \rangle
```

Instead of using the positional signature of LSAMonApplyTermWeightFunctions we can specify the LSI functions using options.

```
In[*]:= wmat2 =
           lsaHamlet⇒
             LSAMonApplyTermWeightFunctions["GlobalWeightFunction" → "IDF",
               "LocalWeightFunction" → "TermFrequency", "NormalizerFunction" → "Cosine"] ⇒
             LSAMonTakeWeightedDocumentTermMatrix;
In[*]:= TakeLargest[ColumnSumsAssociation[wmat2], 6]
\textit{Out[e]} = \langle \mid \texttt{enter} \rightarrow \texttt{21.7403}, \; \texttt{ham} \rightarrow \texttt{9.8253}, \; \texttt{hamlet} \rightarrow \texttt{9.16829}, \; \texttt{polonius} \rightarrow \texttt{8.79678}, \; \texttt{scene} \rightarrow \texttt{8.56075}, \; \texttt{king} \rightarrow \texttt{8.1436} \mid \rangle
```

Here we are summaries of the non-zero values of the weighted document-term matrix derived with different combinations of global, local, and normalization weight functions.

```
1 IDF, Binary, Cosine
                                                   1 GFIDF, None, None
                                                                             1 Binary, None, Cosine
                                                                                                      1 None, Log, RowStochastic
                                                                                                                                    1 ColumnStochastic, Log, None
                         1 IDF, None, RowStochastic
     0.0150041
                              0.000811524
                                                   1st Qu 1.
                                                                                  0.0181339
                                                                                                           0.00204538
                                                                                                                                         0.00193617
1st Qu 0.0573778
                        1st Qu 0.00332414
                                                   Median 1.
                                                                            1st Qu 0.0342393
                                                                                                      1st Qu 0.00362849
                                                                                                                                    1st Qu 0.0462098
Median 0.0760402
                        Median 0.00613544
                                                   Min 1.
                                                                            Median 0.057735
                                                                                                      Median 0.00655579
                                                                                                                                    Median 0.173287
                         3rd Qu 0.0114367
                                                   3rd Qu 1.5
                                                                            3rd Qu 0.090167
 Mean 0.102338
                                                                                                      3rd Qu 0.0113535
                                                                                                                                    Mean 0.271695
                                                                                                                                    3rd Qu 0.549306
3rd Ou 0.107593
                        Mean 0.020181
                                                   Mean 2.24491
                                                                            Mean 0.091407
                                                                                                      Mean 0.020181
                                                                                                                                           0.693147
Max
                                                          237.5
                                                                                                      Max
                                                                                                                                    1 ColumnStochastic,
                        1 GFIDF, Binary, Cosine
1 IDF, Binary, None
                                                   1 GFIDF, None, RowStochastic
                                                                             1 Binary, None, None
                                                                                                      1 None, None, Cosine
                                                                                                                                      Log, RowStochastic
                                                                             1st Qu 1
 Min
      1.15868
                              0.0336221
                                                          0.00082284
                                                                                                      Min
                        Min
                                                                                                           0.0181339
                                                                                                                                           0.0000298676
1st Qu 2.84222
                        1st Qu 0.048543
                                                   1st Qu 0.00205098
                                                                             3rd Qu 1
                                                                                                      1st Qu 0.0342393
                                                                                                                                    1st Ou 0.00128507
Mean 3.56758
                        Median 0.0642267
                                                   Median 0.00444174
                                                                            Median 1
                                                                                                      Median 0.057735
                                                                                                                                    Median 0.00425429
Median 3.79773
                        3rd Qu 0.093449
                                                   3rd Ou 0.00933166
                                                                            Min
                                                                                                      3rd Qu 0.090167
                                                                                  1
                                                                                                                                    3rd Qu 0.0112668
                        Mean 0.0965935
3rd Qu 4.71402
                                                   Mean 0.020181
                                                                            Mean
                                                                                  1.32217
                                                                                                      Mean 0.091407
                                                                                                                                    Mean 0.020181
       4.71402
                         Max
                                                                             Max
                                                                                   27
 Max
                               1.
                                                   Max
                                                          1.
                                                                                                      Max
                                                                                                             1.
                                                                                                                                    Max
                        1 GFIDF, Binary, None
1 IDF, Binary, RowStochastic
                                                   1 Binary, Binary, Cosine
                                                                            1 Binary, None, RowStochastic
                                                                                                      1 None, None, None
                                                                                                                                    1 ColumnStochastic, None, Cosine
                        1st Qu 1.
     0.000762208
                                                                                                      1st Qu 1
Min
                                                   Min 0.0494468
                                                                            Min 0.00168634
                                                                                                                                    Min
                                                                                                                                         0.000411024
1st Qu 0.00373532
                                                                            1st Ou 0.002849
                        Median 1.
                                                   1st Qu 0.0614295
                                                                                                      3rd Qu 1
                                                                                                                                    1st Qu 0.0124279
Median 0.0061038
                        Min
                              1.
                                                   Median 0.0811107
                                                                            Median 0.00595238
                                                                                                      Median 1
                                                                                                                                    Median 0.0396686
                        Mean 1.32217
                                                                                                                                    Mean 0.0780288
3rd Ou 0.0115251
                                                   3rd Ou 0.105409
                                                                            3rd Qu 0.0106383
                                                                                                      Min
                                                                                                            1
 Mean 0.020181
                        3rd Qu 1.33333
                                                         0.106594
                                                                                                            1.32217
                                                                                                                                    3rd Qu 0.101774
                                                                             Mean 0.020181
                                                                                                      Mean
                                                   Max
                                                                                                             27
Max
                        Max
                              13.
                                                                            Max
                                                                                                      Max
                                                                                                                                    Max
1 IDF, Log, Cosine
                         1 GFIDF, Binary, RowStochastic
                                                   1 Binary, Binary, None
                                                                            1 None, Binary, Cosine
                                                                                                                                    1 ColumnStochastic, None, None
                                                                                                      1 None, None, RowStochastic
Min 0.0164353
                        Min 0.00198353
                                                   1st Qu 1
                                                                            Min 0.0494468
                                                                                                      Min 0.00168634
                                                                                                                                    Min 0.0027933
1st Qu 0.0560995
                        1st Qu 0.00338414
                                                   3rd Qu 1
                                                                            1st Qu 0.0614295
                                                                                                      1st Qu 0.002849
                                                                                                                                    1st Qu 0.0769231
                                                   Max
                                                                                                                                    Median 0.25
Median 0.0738618
                        Median 0.00610167
                                                                            Median 0.0811107
                                                                                                      Median 0.00595238
                                                   Mean
                                                         1
 Mean 0.101692
                         3rd Qu 0.0105852
                                                                            3rd Qu 0.105409
                                                                                                      3rd Qu 0.0106383
                                                                                                                                    Mean 0.40181
                                                   Median 1
3rd Qu 0.107561
                                                                            Mean 0.106594
                        Mean 0.020181
                                                                                                      Mean 0.020181
                                                                                                                                    3rd Qu 1.
Max
     1.
                                                         1
                                                                                                                                    1 ColumnStochastic,
1 IDF, Log, None
                        1 GFIDF, Log, Cosine
                                                                             1 None, Binary, None
                                                                                                      1 ColumnStochastic, Binary, Cosine
                                                   1 Binary, Binary, RowStochastic
                                                                                                                                      None, RowStochastic
                              0.0132742
                                                        0.00244499
                                                                             1st Qu 1
Min 0.803133
                        Min
                                                                                                           0.000246694
                                                   Min
                                                                                                      Min
                                                                                                                                           0.0000294977
1st Qu 2.15194
                        1st Qu 0.0262098
                                                   1st Qu 0.00377358
                                                                            3rd Qu 1
                                                                                                      1st Qu 0.0104195
                                                                                                                                    1st Ou 0.00135327
Mean 2.723
                                                                            Max 1
                        Median 0.0436552
                                                   Median 0.00657895
                                                                                                      Median 0.036084
                                                                                                                                    Median 0.0042479
Median 2.78706
                                                                            Mean 1
                                                                                                      Mean 0.0766439
                        3rd Qu 0.0748374
                                                   3rd Ou 0.0111111
                                                                                                                                    3rd Ou 0.0111001
                                                                            Median 1
                        Mean 0.0810793
3rd Qu 3.26751
                                                   Mean 0.020181
                                                                                                      3rd Qu 0.101229
                                                                                                                                    Mean 0.020181
                        Max
 Max
       12.9073
                                                                            Min
                               1.
                                                   Max
                                                          1.
                                                                                                      Max
                                                                                                                                    Max
                                                                                                                                           1.
                         1 GFIDF, Log, None
                                                   1 Binary, Log, Cosine
                                                                             1 None, Binary, RowStochastic
1 IDF, Log, RowStochastic
                                                                                                      1 ColumnStochastic, Binary, None
                        1st Qu 0.693147
                                                       0.0386884
                                                                                 0.00244499
     0.000952831
                                                                                                            0.0027933
Min
                                                   Min
                                                                            Min
                                                                                                      Min
1st Qu 0.00360635
                        Median 0.693147
                                                   1st Qu 0.0502109
                                                                            1st Qu 0.00377358
                                                                                                      1st Qu 0.0625
Median 0.00633097
                        Min 0.693147
                                                   Median 0.0752375
                                                                            Median 0.00657895
                                                                                                      Median 0.2
                                                   Mean 0.102772
                                                                                                      Mean 0.381212
3rd Qu 0.0111849
                        3rd Qu 1.03972
                                                                            3rd Qu 0.0111111
 Mean 0.020181
                         Mean 1.15397
                                                   3rd Qu 0.102919
                                                                            Mean 0.020181
                                                                                                      3rd Qu 0.5
Max
       1.
                        Max
                               37.4467
                                                   Max 1.
                                                                                                      Max 1.
                                                                            Max
                                                                                   1.
                                                   1 Binary, Log, None
                                                                                                      1 ColumnStochastic,
                        1 GFIDF, Log, RowStochastic
1 IDF, None, Cosine
                                                                             1 None, Log, Cosine
                                                   Log[2] 9355
                                                                                                        Binary, RowStochastic
Min 0.0115357
                        Min 0.00144179
                                                                            Min 0.0386884
                                                   Log[3] 1031
                                                                                                           0.0000163371
1st Qu 0.0440683
                        1st Qu 0.00270264
                                                                            1st Qu 0.0502109
                                                   Log[4] 323
                                                                                                      1st Qu 0.00115157
Median 0.065596
                        Median 0.00517701
                                                                            Median 0.0752375
                                                   Log[5]
                                                           127
                                                                                                      Median 0.00419756
 Mean 0.0958
                                                                            Mean 0.102772
                        3rd Qu 0.0099619
                                                   Log[6]
                                                           76
                                                                                                      3rd Qu 0.0112038
3rd Qu 0.0993633
                        Mean 0.020181
                                                                            3rd Qu 0.102919
                                                                                                      Mean 0.020181
                                                   Log[7] 40
 Max
      1.
                        Max
                                                                            Max
                                                                                  1.
                                                                                                      Max
                                                                                                             1.
                                                   (Other) 98
                                                                             1 None, Log, None
                        1 GFIDF, None, Cosine
1 IDF, None, None
                                                   1 Binary, Log, RowStochastic
                                                                                                      1 ColumnStochastic, Log, Cosine
                                                                             Log[2] 9355
Min 1.15868
                        Min 0.00335833
                                                   Min 0.00204538
                                                                                                      Min 0.000413022
                                                                            Log[3] 1031
1st Qu 3.20995
                        1st Qu 0.0113981
                                                   1st Qu 0.00362849
                                                                                                      1st Qu 0.0115956
                                                                            Log[4] 323
Median 4.02088
                        Median 0.026682
                                                   Median 0.00655579
                                                                                                      Median 0.0380513
                                                                             Log[5]
                                                                                    127
 Mean 4.36558
                                                                                                      Mean 0.0775075
                        3rd Qu 0.0548158
                                                   3rd Ou 0.0113535
                                                                            Log[6]
                                                                                     76
3rd Qu 4.71402
                        Mean 0.06707
                                                   Mean 0.020181
                                                                                                      3rd Qu 0.102593
                                                                            Log[7]
                                                                                    40
Max 81.8626
                        Max
                               1.
                                                   Max
                                                          1.
                                                                                                      Max
                                                                             (Other) 98
```

#### **Extracting topics**

Streamlining topic extraction is one of the main reasons LSAMon was implemented. The topic extraction correspond to the so called "syntagmatic" relationships between the terms, [MS1].

## Theoretical outline

The original weighed document-term matrix *M* is decomposed into the matrix factors *W* and *H*.

$$M \approx W \cdot H$$
,  $W \in \mathbb{R}^{k m}$ ,  $H \in \mathbb{R}^{k n}$ .

The *i*-th row of *M* is expressed with the *i*-th row of *W* multiplied by *H*.

The rows of *H* are the topics. SVD produces orthogonal topics; NNMF does not.

The i-the document of the collection corresponds to the i-th row W. Finding the Nearest Neighbors (NN's) of the i-th document using the rows similarity of the matrix W gives document NN's through topic similarity.

The terms correspond to the columns of H. Finding NN's based on similarities of H's columns produces statistical thesaurus entries.

The term groups provided by H's rows correspond to "syntagmatic" relationships. Using similarities of H's columns we can produce term clusters that correspond to "paradigmatic" relationships.

## Computation specifications

Here is an example using the play "Hamlet" in which we specify additional stop words.

11

```
In[@]:= SeedRandom[2381]
   lsaHamlet =
    LSAMonUnit[textHamlet] ⇒
     LSAMonMakeDocumentTermMatrix["StemmingRules" → Automatic, "StopWords" → Join[stopWords, stopWords2]] ⇒
     {\tt LSAMonApplyTermWeightFunctions["GlobalWeightFunction"} \rightarrow {\tt "IDF"},
      "LocalWeightFunction" \rightarrow "None", "NormalizerFunction" \rightarrow "Cosine"] \Longrightarrow
     LSAMonEchoTopicsTable["NumberOfTableColumns" \rightarrow 6, "NumberOfTerms" \rightarrow 10];
```

		-		<b>J</b>		J		1		9		11	
		1.000	player	1.000	laert	1.000	state	1.000	hamlet	1.000	rosencrantz	1.000	answer
		0.063	plai	0.248	king	0.963	room	0.167	ghost	0.830	guildenstern	0.072	sir
		0.045	welcom	0.067	attend	0.915	castl	0.041	father	0.164	king	0.069	mother
		0.044	present	0.037	lord	0.781	elsinor	0.037	denmark	0.146	poloniu	0.031	mad
		0.042	tell	0.027	gertrud	0.040	fear	0.023	gentlemen	0.091	queen	0.027	night
		0.040	poloniu	0.026	shall	0.024	welcom	0.018	call	0.025	lord	0.025	good
		0.040	work	0.021	ophelia	0.019	mark	0.017	come	0.012	thank	0.017	bed
		0.039	hear	0.018	world	0.018	love	0.014	daughter	0.008	command	0.017	make
		0.039	queen	0.017	young	0.015	like	0.012	mother	0.008	gentlemen	0.017	look
	topics table:	0.032	make	0.017	come	0.015	make	0.009	rosencrantz	0.008	hath	0.016	matter
>>	topics table.												
	topico tabio:	2		4		6		8		10		12	
	100.00	2 1.000	ro	1.000	end	6 1.000	daughter	8 1.000	father	10 1.000	ophelia	12 1.000	horatio
		1.000 0.724	ro lord	•	end inde	-	daughter pass	ŭ	father thou		ophelia queen		horatio attend
			-	1.000		1.000	_	1.000		1.000	•	1.000	
		0.724	lord sir	1.000 0.111	inde	1.000 0.718	pass	1.000 0.816	thou	1.000 0.372	queen	1.000 0.146	attend
		0.724 0.364	lord sir bring	1.000 0.111 0.106	inde make king	1.000 0.718 0.604	pass love	1.000 0.816 0.815	thou king	1.000 0.372 0.200	queen poloniu	1.000 0.146 0.132	attend gentleman
		0.724 0.364 0.332	lord sir bring king	1.000 0.111 0.106 0.059	inde make king passion	1.000 0.718 0.604 0.561	pass love fair	1.000 0.816 0.815 0.702	thou king shall	1.000 0.372 0.200 0.122	queen poloniu mark nai	1.000 0.146 0.132 0.057	attend gentleman queen
		0.724 0.364 0.332 0.300	lord sir bring king	1.000 0.111 0.106 0.059 0.034	inde make king passion	1.000 0.718 0.604 0.561 0.226	pass love fair follow	1.000 0.816 0.815 0.702 0.685	thou king shall good	1.000 0.372 0.200 0.122 0.103	queen poloniu mark nai prai	1.000 0.146 0.132 0.057 0.021	attend gentleman queen night
		0.724 0.364 0.332 0.300 0.256	lord sir bring king bodi	1.000 0.111 0.106 0.059 0.034 0.028	inde make king passion fortun love	1.000 0.718 0.604 0.561 0.226 0.185	pass love fair follow nai	1.000 0.816 0.815 0.702 0.685 0.665	thou king shall good let	1.000 0.372 0.200 0.122 0.103 0.089	queen poloniu mark nai prai	1.000 0.146 0.132 0.057 0.021 0.017	attend gentleman queen night present
		0.724 0.364 0.332 0.300 0.256 0.179	lord sir bring king bodi shall	1.000 0.111 0.106 0.059 0.034 0.028	inde make king passion fortun love thought	1.000 0.718 0.604 0.561 0.226 0.185 0.184	pass love fair follow nai lord	1.000 0.816 0.815 0.702 0.685 0.665 0.584	thou king shall good let come	1.000 0.372 0.200 0.122 0.103 0.089 0.065	queen poloniu mark nai prai denmark lord	1.000 0.146 0.132 0.057 0.021 0.017	attend gentleman queen night present hast

Here is an example using the USA presidents "state of union" speeches.

```
In[*]:= SeedRandom[7681]
```

lsaSpeeches =

LSAMonUnit[aStateOfUnionSpeeches] ⇒

LSAMonMakeDocumentTermMatrix["StemmingRules" → Automatic, "StopWords" → Automatic] ⇒

LSAMonApplyTermWeightFunctions[

"GlobalWeightFunction" → "IDF", "LocalWeightFunction" → "None", "NormalizerFunction" → "Cosine"] ⇒

LSAMonExtractTopics["NumberOfTopics" → 36, "MinNumberOfDocumentsPerTerm" → 40, Method → "NNMF", "MaxSteps" → 12] ⇒ LSAMonEchoTopicsTable["NumberOfTableColumns" → 6, "NumberOfTerms" → 10];

	1.000	tonight	1.000	spain	1.000	arbitr	1.000	coloni	1.000	tonight	1.000	soviet
	0.816	tranquil	0.395	savag	0.878	cuba	0.767	slave	0.469	budget	0.190	oil
	0.748	indian	0.378	florida	0.851	award	0.729	emigr	0.379	let	0.186	help
	0.685	insurrect	0.361	presum	0.663	majesti	0.713	lake	0.343	america	0.172	korea
	0.639	murder	0.360	likewis	0.635	island	0.597	tribe	0.332	billion	0.164	aggress
	0.488	drug	0.334	articl	0.611	exposit	0.583	commenc	0.320	dream	0.155	challeng
	0.450	review	0.319	provinc	0.571	convent	0.572	indian	0.315	todai	0.155	threat
	0.398	subsist	0.290	ratifi	0.553	canal	0.552	slaveri	0.305	goal	0.150	nuclear
	0.360	emperor	0.233	minist	0.505	consular	0.548	pension	0.298	program	0.145	weapon
	0.353	matur	0.221	majesti	0.497	spain	0.543	000	0.274	help	0.126	world
	2	a ca.	8		14	ори	20		26	Поср	32	
	1.000	gold	1.000	todai	1.000	terror	1.000	bank	1.000	enemi	1.000	democraci
	0.742	silver	0.796	group	0.536	cruiser	0.529	slaveri	0.866	savag	0.308	recoveri
	0.486	circul	0.790	dictat	0.400	hundr	0.478	speci	0.720	british	0.243	problem
	0.451	note	0.783	slave	0.390	america	0.428	currenc	0.705	militia	0.193	modern
	0.406		0.765	uniti				slave				democrat
		currenc			0.382	weapon	0.351		0.483	command	0.176	
	0.307	coin	0.675	democrat	0.342	fifti	0.290	california	0.432	lake	0.162	religion
	0.295	bond	0.605	schedul	0.317	tonight	0.281	disburs	0.413	victori	0.156	unemploy
	0.269	000	0.599	slaveri	0.293	fight	0.265	note	0.326	prison	0.140	interpret
	0.257	metal	0.546	program	0.286	ideal	0.261		0.295	confin	0.139	billion
	0.242	speci	0.538	ahead	0.283	group	0.237	territori	0.263	captur	0.136	specul
	3		9		15		21		27		33	
	1.000	000	1.000	tonight	1.000	mexico	1.000	persuad	1.000	tribe	1.000	job
	0.416	cent	0.823	drug	0.897	texa	0.976	militia	0.706	indian	0.508	colleg
	0.412	method	0.709	job	0.522	mexican	0.884	expedi	0.605	provis	0.411	tonight
	0.408	veteran	0.679	parent	0.253		0.806	pleas	0.592	hitherto	0.395	student
	0.401	agricultur	0.668	help	0.228	territori	0.748	effectu	0.569	presum	0.376	clean
	0.376	consolid	0.650	child	0.211	steamer	0.745	requisit	0.551	tend	0.369	panama
	0.317	board	0.619	school	0.166	articl	0.739	legislatur	0.549	lessen	0.355	compani
	0.307	court	0.580	children	0.153	levi	0.738	uniform	0.541	calcul	0.344	oil
	0.289	flood	0.561	challeng	0.150	postag	0.682	paper	0.522	besid	0.315	recoveri 
» topics table:	0.281	bureau	0.541	colleg	0.147	minist	0.664	bless	0.512	ensu	0.268	crisi
	4		10		16	-	22		28		34	
	1.000	minist	1.000	militia 	1.000	goal	1.000	german	1.000	program	1.000	recoveri
	0.990	french	0.885	harbor	0.711	inflat	0.898	fight	0.534	billion	0.911	program
	0.726	franc	0.844	port	0.539	louisiana	0.666	air	0.496	percent	0.684	farm
	0.564	pari 	0.802	vessel	0.489	acquisit	0.661	germani	0.478	oil	0.554	econom
	0.544	chamber	0.718	amiti	0.466	achiev	0.658	win	0.457	job	0.511	group
	0.503	british	0.688	instruct	0.442	todai	0.596	enemi	0.450	spend	0.466	unemploy
	0.502	council	0.634	legislatur	0.412	level	0.557	victori	0.427	nuclear	0.466	relief
	0.440	instruct	0.625	boat	0.391	ohio	0.511	alli	0.409	inflat	0.431	bank
	0.413	belliger	0.616	shall	0.351	sensibl	0.406	•	0.376	technolog	0.422	democrat
	0.402	king	0.608	offend	0.334	help	0.399	partnership	0.367	help	0.418	veteran
	5		11		17		23		29		35	
	1.000	likewis	1.000	territori	1.000	forest	1.000		1.000	bank	1.000	job
	0.682		0.720		0.868	corpor	0.904		0.667	deposit	0.734	cut
	0.615	augment coloni	0.703	admiss	0.616	man	0.876	respectfulli california		surplu	0.477 0.358	deficit
	0.586 0.486		0.686 0.655	constitut	0.567 0.452	island	0.837 0.816		0.427 0.406	currenc distribut	0.338	spend percent
	0.437	suppress vessel	0.652	represent book	0.417	supervis type	0.655	-	0.377	specul	0.274	tonight
	0.433	spain	0.651	agit	0.417	railroad	0.648	-	0.370	paper	0.267	lot
	0.417	sentiment	0.618	nicaragua	0.355	mere	0.609		0.356	circul	0.258	tell
	0.417	vest	0.616	britain	0.340	moreov	0.603		0.331	note	0.256	program
	0.406	presum	0.598	assert	0.334	alaska	0.591	-	0.312	ration	0.245	invest
	6	pi couiii	12	45561 6	18	ataska	24	consucui	30	1461011	36	mvese
	1.000	railwai	1.000	domain	1.000	program	1.000	tariff	1.000	commission	1.000	silver
	0.480	concert	0.884	debat	0.279	feder	0.577		0.562	captur	0.381	coin
	0.472	railroad	0.842	thought	0.279	budget	0.487		0.551	majesti	0.373	indian
	0.360	commiss	0.780	readi	0.249	problem	0.448		0.498	articl	0.349	pension
	0.358	concili	0.778	ship	0.233	assist	0.423		0.458	award	0.290	fiscal
	0.338	arbitr	0.777	spent	0.232	studi	0.415		0.395	treati	0.287	cent
	0.324	postpon	0.764	task	0.232	mobil	0.392		0.361	appoint	0.265	june
	0.316	thorough	0.735	counsel	0.220	highwai	0.357		0.354	sick	0.247	citizenship
	0.304	hesit	0.733	alter	0.208	korea	0.350	-	0.342	damag	0.243	method
	0.303	eight	0.706	speak	0.202	employe	0.321		0.313	adjourn	0.234	statut
	<u> </u>	~	<u> </u>	•						-	<u> </u>	

Note that in both examples:

- 1. stemming is used when creating the document-term matrix,
- 2. the default LSI re-weighting functions are used: "IDF", "None", "Cosine",
- **3.** the dimension reduction algorithm NNMF is used.

Things to keep in mind.

- **4.** The interpretability provided by NNMF comes at a price.
- **5.** NNMF is prone to get stuck into local minima, so several topic extractions (and corresponding evaluations) have to be done.
- 6. We would get different results with different NNMF runs using the same parameters. (NNMF uses random numbers initialization.)
- 7. The NNMF topic vectors are not orthogonal.
- **8.** SVD is much faster than NNMF, but it topic vectors are hard to interpret.
- 9. Generally, the topics derived with SVD are stable, they do not change with different runs with the same parameters.
- 10. The SVD topics vectors are orthogonal, which provides for quick to find representations of documents not in the monad's document collection.

The document-topic matrix W has column names that are automatically derived from the top three terms in each topic.

#### In[⊕]:= ColumnNames[lsaHamlet⇒LSAMonTakeW]

Out | Flayer-plai-welcom, ro-lord-sir, laert-king-attend, end-inde-make, state-room-castl, daughter-pass-love, hamlet-ghost-father, father-thou-king, rosencrantz-guildenstern-king, ophelia-queen-poloniu, answer-sir-mother, horatio-attend-gentleman}

Of course the row names of *H* have the same names.

#### In[⊕]:= RowNames[lsaHamlet⇒LSAMonTakeH]

Out | Player-plai-welcom, ro-lord-sir, laert-king-attend, end-inde-make, state-room-castl, daughter-pass-love, hamlet-ghost-father, father-thou-king, rosencrantz-guildenstern-king, ophelia-queen-poloniu, answer-sir-mother, horatio-attend-gentleman}

## Extracting statistical thesauri

The statistical thesaurus extraction corresponds to the "paradigmatic" relationships between the terms, [MS1].

Here is an example over the State of Union speeches.

LSAMonEchoStatisticalThesaurus;

```
<code>ln[v]:= entryWords = {"bank", "war", "economy", "school", "port", "health", "enemy", "nuclear"};</code>
In[•]:= lsaSpeeches⇒
       LSAMonExtractStatisticalThesaurus[
         "Words" \rightarrow Map[WordData[#, "PorterStem"] &, entryWords], "NumberOfNearestNeighbors" \rightarrow 12] \Longrightarrow
```

statistical thesaurus entries term bank  $\{\mathsf{bank},\,\mathsf{deposit},\,\mathsf{specul},\,\mathsf{currenc},\,\mathsf{speci},\,\mathsf{paper},\,\mathsf{distribut},\,\mathsf{charter},\,\mathsf{surplu},\,\mathsf{embarrass},\,\mathsf{credit},\,\mathsf{institut}\}$  $\{\mathsf{economi}\,,\,\mathsf{respons},\,\mathsf{strengthen},\,\mathsf{earn},\,\mathsf{elimin},\,\mathsf{higher},\,\mathsf{plan},\,\mathsf{better},\,\mathsf{creat},\,\mathsf{comprehens},\,\mathsf{save},\,\mathsf{dedic}\}$ economi enemi  $\{$ enemi, victori, command, savag, british, lake, prison, militia, superior, confin, warfar, volunt $\}$ {health, invest, coverag, work, middl, care, lower, hard, stai, economi, lot, start} health  $\{\mathsf{nuclear},\,\mathsf{technolog},\,\mathsf{ensur},\,\mathsf{strateg},\,\mathsf{energi},\,\mathsf{oil},\,\mathsf{middl},\,\mathsf{inflat},\,\mathsf{percent},\,\mathsf{basic},\,\mathsf{leader},\,\mathsf{proud}\}$ nuclear port {port, commenc, instruct, amiti, intercours, london, vessel, dispatch, extinguish, princip, arrang, harbor∤  $\{\mathsf{school},\,\mathsf{child},\,\mathsf{children},\,\mathsf{teacher},\,\mathsf{drug},\,\mathsf{parent},\,\mathsf{thank},\,\mathsf{famili},\,\mathsf{centuri},\,\mathsf{environ},\,\mathsf{student},\,\mathsf{medic}\}$ school {war, forc, peac, right, power, place, great, success, good, provid, continu, secur} war

» statistical thesaurus:

In the code above: (i) the options signature style is used, (ii) the statistical thesaurus entry words are stemmed first.

We can also call LSAMonEchoStatisticalThesaurus directly without calling LSAMonExtractStatisticalThesaurus first.

#### In[\*]:= lsaSpeeches⇒

LSAMonEchoStatisticalThesaurus["Words" → Map[WordData[#, "PorterStem"] &, entryWords], "NumberOfNearestNeighbors" → 12];

	term	statistical thesaurus entries
	bank	$\{ bank, deposit, specul, currenc, speci, paper, distribut, charter, surplu, embarrass, credit, institut \}$
	economi	$\{ ext{economi, respons, strengthen, earn, elimin, higher, plan, better, creat, comprehens, save, dedic}\}$
	enemi	$\big\{ \text{enemi, victori, command, savag, british, lake, prison, militia, superior, confin, warfar, volunt} \big\}$
3:	health	$\{ \text{health, invest, coverag, work, middl, care, lower, hard, stai, economi, lot, start} \}$
	nuclear	$\big\{ \text{nuclear, technolog, ensur, strateg, energi, oil, middl, inflat, percent, basic, leader, proud} \big\}$
	port	$\{ port,  commenc,  instruct,  amiti,  intercours,  london,  vessel,  dispatch,  extinguish,  princip,  arrang,  harbor \}$
	school	$\{ \texttt{school}, \texttt{child}, \texttt{children}, \texttt{teacher}, \texttt{drug}, \texttt{parent}, \texttt{thank}, \texttt{famili}, \texttt{centuri}, \texttt{environ}, \texttt{student}, \texttt{medic} \}$
	war	$\{$ war, forc, peac, right, power, place, great, success, good, provid, continu, secur $\}$

» statistical thesaurus:

#### Mapping queries and documents to terms

One of the most natural operations is to find the representation of an arbitrary document (or sentence or a list of words) in monad's Linear vector space of terms. This is done with the function LSAMonRepresentByTerms.

Here is an example in which a sentence is represented as a one-row matrix (in that space.)

```
In[*]:= obi =
```

lsaHamlet⇒

LSAMonRepresentByTerms["Hamlet, Prince of Denmark killed the king."] ⇒ LSAMonEchoValue;

```
Specified elements: 5
Dimensions: {1, 3569}
» value: SparseArray
```

Here we display only the non-zero columns of that matrix.

```
log[*]:= obj \Longrightarrow LSAMonEchoFunctionValue | Function[...] | + | ;

        denmark
        hamlet
        kill
        king
        princ

        1
        0.437157
        0.251741
        0.569795
        0.20823
        0.614405
```

### Transformation steps

Assume that LSAMonRepresentByTerms is given a list of sentences. Then that function performs the following steps.

1. The sentence is split into a list of words.

- 2. If monad's document-term matrix was made by removing stop words the same stop words are removed from the list of words.
- 3. If monad's document-term matrix was made by stemming the same stemming rules are applied to the list of words.
- **4.** The LSI global weights and the LSI local weight and normalizer functions are applied to sentence's contingency matrix.

#### Equivalent representation

Let us look convince ourselves that documents used in the monad to built the weighted document-term matrix have the same representation as the corresponding rows of that matrix.

Here is an association of documents from monad's document collection.

```
In[\bullet]:= inds = \{6, 10\};
    queries = Part[lsaHamlet⇒LSAMonTakeDocuments, inds];
    queries
out_{0} = \langle | id.0006 \rightarrow Getrude, Queen of Denmark, mother to Hamlet. Ophelia, daughter to Polonius.,
      id.0010 \rightarrow ACT I. Scene I. Elsinore. A platform before the Castle. |\rangle
/n[⊕]:= lsaHamlet⇒
       LSAMonRepresentByTerms[queries] ⇒
       LSAMonEchoFunctionValue Function[...] | ;
                       daughter denmark elsinor getrud
                                                               hamlet
                                                                         mother
                                                                                  ophelia platform poloniu
      id.0006
                        0.404674 0.331177
                                              0.
                                                     0.633921 0.190712 0.294272 0.294272
                                                                                              0.
                                                                                                     0.283923 0.185821
      id.0010 0.497202
                                            0.420682
                                                                                            0.758826
In[⊕]:= lsaHamlet⇒
       LSAMonEchoFunctionContext Function[...] | ;
                        daughter denmark elsinor
                                                      getrud
                                                               hamlet
                                                                         mother
                                                                                  ophelia platform poloniu
      id.0006
                        0.404674 0.331177
                                              0.
                                                     0.633921 0.190712 0.294272 0.294272
                                                                                              0.
                                                                                                     0.283923 0.185821
      id.0010 0.497202
                                            0.420682
                                                                                            0.758826
```

#### Mapping queries and documents to topics

Another natural operation is to find the representation of an arbitrary document (or a list of words) in monad's Linear vector space of topics. This is done with the function LSAMonRepresentByTopics.

Here is an example.

```
In[\bullet]:= inds = \{6, 10\};
    queries = Part[lsaHamlet⇒LSAMonTakeDocuments, inds];
    queries
\mathcal{O} of \mathcal{O} id.0006 	o Getrude, Queen of Denmark, mother to Hamlet. Ophelia, daughter to Polonius.,
      id.0010 
ightarrow ACT I. Scene I. Elsinore. A platform before the Castle. |\rangle
In[⊕]:= lsaHamlet⇒
       LSAMonRepresentByTopics[queries] ⇒
       LSAMonEchoFunctionValue Function[...] + ;
              player-plai-welcom state-room-castl daughter-pass-love hamlet-ghost-father rosencrantz-guildenstern-king ophelia-queen-polor
      id.0006
                   0.00295518
                                     -0.000452681
                                                          0.210033
                                                                              0.0889422
                                                                                                        0.0116542
                                                                                                                                    0.239248
      id.0010
                   0.0015422
                                       0.304837
                                                         -0.00184199
                                                                             0.000254639
                                                                                                       0.000101572
                                                                                                                                   0.000131886
In[•]:= lsaHamlet⇒
       LSAMonEchoFunctionContext | Function[...] + ;
               player-plai-welcom state-room-castl hamlet-ghost-father ophelia-queen-poloniu answer-sir-mother
                   0.00349852
                                     0.000671018
      id.0006
                                                          0.0919758
                                                                                 0.199069
                                                                                                     0.0182967
      id.0010
                                                                                    0.
```

#### Theory

In order to clarify what the function LSAMonRepresentByTopics is doing let us go through the formulas it is based on.

The original weighed document-term matrix M is decomposed into the matrix factors W and H.

```
M \approx W.H, W \in \mathbb{R}^{m \times k}, H \in \mathbb{R}^{k \times n}.
```

The *i*-th row of *M* is expressed with the *i*-th row of *W* multiplied by *H*.

For a query vector  $q_0 \in \mathbb{R}^m$  we want to find its topics representation vector  $x \in \mathbb{R}^k$ :

 $q_0 \approx x.H$ 

Denote with  $H^{(-1)}$  the inverse or pseudo-inverse matrix of H. We have:

```
q_0.H^{(-1)} \approx (x.H).H^{(-1)} = x.(H.H^{(-1)}) = x.I,
x \in \mathbb{R}^k
H^{(-1)} \in \mathbb{R}^{n \times k}
l \in \mathbb{R}^{k \times k}.
```

In LSAMon for SVD  $H^{(-1)} = H^T$ ; for NNMF is  $H^{(-1)}$  is the pseudo-inverse of H.

The vector x obtained with LSAMonRepresentByTopics.

## Tags representation

Sometimes we want to find the topics representation of tags associated with monad's documents and the tag-document associations are one-to-many. See [AA3].

Let us consider a concrete example -- we want to find what topics correspond to the different presidents in the collection of State of Union speeches.

Here we find the document tags (president names in this case.)

```
In[*]:= tags = StringReplace[RowNames[lsaSpeeches⇒LSAMonTakeDocumentTermMatrix],
       RegularExpression[".\\d\\d\\d-\\d\\d"] → ""];
   Short[
    tags]
```

```
out[*]//Short= {George.Washington, George.Washington, George.Washington,
        George.Washington, <225>, Barack.Obama, Donald.Trump, Donald.Trump, Donald.Trump}
```

Here is the number of unique tags (president names.)

```
In[*]:= Length[Union[tags]]
```

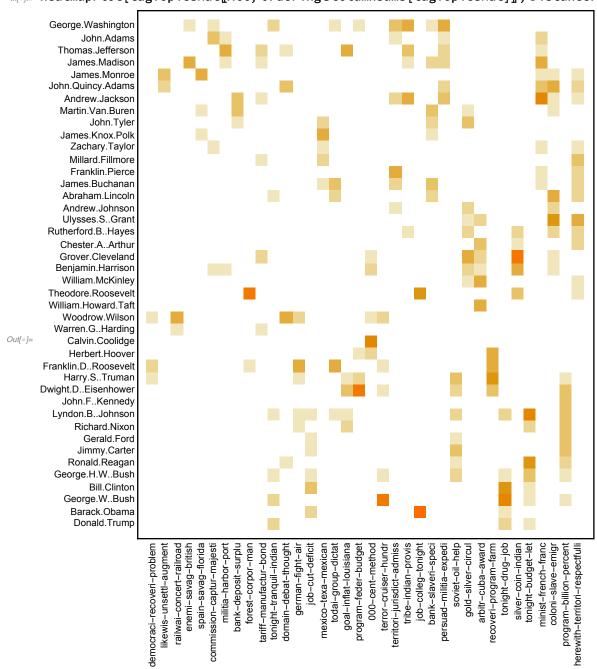
Out[•]= 42

Here we compute the tag-topics representation matrix using the function LSAMonRepresentDocumentTagsByTopics.

In[\*]:= tagTopicsMat = lsaSpeeches⇒ LSAMonRepresentDocumentTagsByTopics[tags] ⇒ LSAMonTakeValue Specified elements: 192 Out[ • ]= SparseArray Dimensions: {42, 36}

Here is a heatmap plot of the tag-topics matrix made with the package "HeatmapPlot.m", [AAp11].

<code>In[e]:= HeatmapPlot[tagTopicsMat[All, Ordering@ColumnSums[tagTopicsMat]], DistanceFunction → None, ImageSize → Large]</code>



## Finding the most important documents

There are several algorithms we can apply for finding the most important documents in the collection. LSAMon utilizes two types algorithms: (1) graph

centrality measures based, and (2) matrix factorization based. With certain graph centrality measures the two algorithms are equivalent. In this subsection we demonstrate the matrix factorization algorithm (that uses SVD.)

**Definition:** The most important sentences have the most important words and the most important words are in the most important sentences.

That definition can be used to derive an iterations-based model that can be expressed with SVD or eigenvector finding algorithms, [LE1].

Here we pick an important part of the play "Hamlet".

LSAMonEchoFunctionValue[GridTableForm];

```
<code>m[*]:= focusText = First@Pick[textHamlet, StringMatchQ[textHamlet, ___ ~~ "to be" ~~ __ ~~ "or not to be" ~~ ___, IgnoreCase → True]];</code>
    Short[focusText]
```

```
Out J/Short= Ham. To be, or not to be- that is the question: Whether 'tis nobler
        ... ecstasy. O, woe is me T' have seen what I have seen, see what I see!
```

Here we find the top 3 most important sentences from that part.

```
In[*]:= LSAMonUnit[StringSplit[ToLowerCase[focusText], {",", ".", ";", "!", "?"}]] \Rightarrow
      LSAMonMakeDocumentTermMatrix["StemmingRules" → {}, "StopWords" → Automatic] ⇒
      LSAMonApplyTermWeightFunctions⇒
      LSAMonFindMostImportantDocuments[3] ⇒
```

	#	1	2	3	4
	1	1.	30	id.0030	and enterprises of great pith and moment with
					this regard their currents turn awry and lose the name of action
>>	2	$\textbf{3.35433}\times\textbf{10}^{-8}$	3	id.0003	or not to be- that is the question: whether 'tis nobler in the mind to suffer
					the slings and arrows of outrageous fortune or to take arms against a sea of troubles
	3	$\textbf{4.95717}\times\textbf{10}^{-9}$	81	id.0081	for the power of beauty will sooner transform honesty from what it
					is to a bawd than the force of honesty can translate beauty into his likeness

### Setters, droppers, and takers

The values from the monad context can be set, obtained, or dropped with the corresponding "setter", "dropper", and "taker" functions as summarized in a previous section.

For example:

LSAMonUnit[textHamlet] => LSAMonMakeDocumentTermMatrix[Automatic, Automatic];

In[\*]:= p⇒LSAMonTakeMatrix

```
Out[*]= SparseArray
```

If other values are put in the context they can be obtained through the (generic) function LSAMonTakeContext, [AAp1]:

```
In[•]:= Short@ (p⇒QRMonTakeContext) ["documents"]
\textit{Out[*]//Short=} \hspace{0.2cm} \langle \hspace{0.1cm} | \hspace{0.1cm} \text{id.0001} \rightarrow \text{1604, id.0002} \rightarrow \text{THE TRAGEDY OF HAMLET, PRINCE OF DENMARK,} \\ \ll 220 \gg, \hspace{0.1cm} \text{id.0223} \rightarrow \text{THE END} \hspace{0.1cm} | \hspace{0.1cm} \rangle
```

Another generic function from [AAp1] is LSAMonTakeValue (used many times above.)

Here is an example of the "data dropper" LSAMonDropDocuments:

```
In[⊕]:= Keys[p⇒LSAMonDropDocuments⇒QRMonTakeContext]
Out[*]= {documentTermMatrix, terms, stopWords, stemmingRules}
```

(The "droppers" simply use the state monad function LSAMonDropFromContext, [AAp1]. For example, LSAMonDropDocuments is equivalent to LSAMonDropFromContext["documents"].)

## The utilization of SSparseMatrix objects

The LSAMon monad heavily relies on SSparseMatrix objects, [AAp6, AA5], for internal representation of data and computation results.

A SSparseMatrix object is a matrix with named rows and columns.

Here is an example.

P 0 0 0 0 0 0 G 0 0 0 0 1 0 X 0 0 0 0 0 0  $\mathsf{C} \ \mathsf{O} \ \mathsf{O} \ \mathsf{O} \ \mathsf{O} \ \mathsf{O} \ \mathsf{O} \ \mathsf{O}$ 

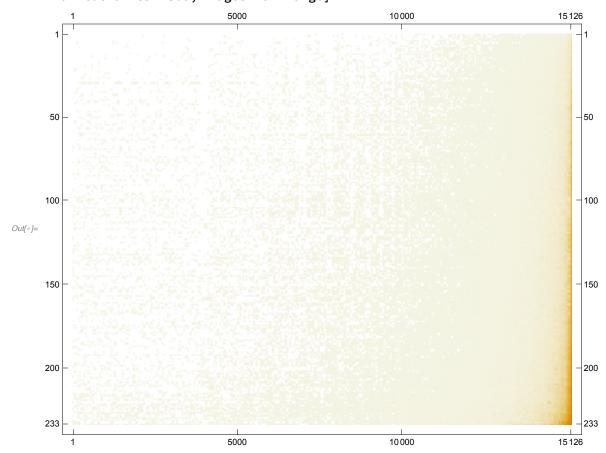
```
ln[\bullet] := n = 6;
       rmat = ToSSparseMatrix[SparseArray[{{1, 2} \rightarrow 1, {4, 5} \rightarrow 1}, {n, n}],
           "RowNames" → RandomSample[CharacterRange["A", "Z"], n], "ColumnNames" -> RandomSample[CharacterRange["a", "z"], n]];
       MatrixForm[
        rmat]
Out[ • ]//MatrixForm
          clavtf
        F 0 1 0 0 0 0
        S 0 0 0 0 0 0
```

In this section we look into some useful SSparseMatrix idioms applied within LSAMon.

#### Visualize with sorted rows and columns

In some situations it is beneficial to sort rows and columns of the (weighted) document-term matrix.

In[\*]:= docTermMat = lsaSpeeches⇒LSAMonTakeDocumentTermMatrix; MatrixPlot[docTermMat[Ordering[RowSums[docTermMat]], Ordering[ColumnSums[docTermMat]]]], MaxPlotPoints → 300, ImageSize → Large]



## Finding the most and least popular terms

The most popular terms in the document collection can be found through the association of the column sums of the document-term matrix.

```
m[v]:= TakeLargest[ColumnSumsAssociation[lsaSpeeches\LongrightarrowLSAMonTakeDocumentTermMatrix], 10]
```

```
\textit{Out[v]}= \ \langle \ | \ \text{state} \rightarrow 8852, \ \text{govern} \rightarrow 8147, \ \text{year} \rightarrow 6362, \ \text{nation} \rightarrow 6182,
                  \texttt{congress} \rightarrow \texttt{5040}, \, \texttt{unit} \rightarrow \texttt{5040}, \, \texttt{countri} \rightarrow \texttt{4504}, \, \texttt{peopl} \rightarrow \texttt{4306}, \, \texttt{american} \rightarrow \texttt{3648}, \, \texttt{law} \rightarrow \texttt{3496} \, \big| \, \rangle
```

Similarly for the lest popular terms.

n[v]:= TakeSmallest[ColumnSumsAssociation[lsaSpeeches ⇒ LSAMonTakeDocumentTermMatrix], 10]

```
\textit{Out[e]=} \  \  \langle |\  \, \textbf{036} \rightarrow \textbf{1,}\  \, \textbf{027} \rightarrow \textbf{1,}\  \, \underline{\qquad} \\ -1,\  \, \textbf{0211} \rightarrow \textbf{1,}\  \, \textbf{006} \rightarrow \textbf{1,}
              0000 \rightarrow 1, 0001 \rightarrow 1, ____ \rightarrow 1, ____ \rightarrow 1, ____
```

### Showing only non-zero columns

In some cases we want to show only columns of the data or computation results matrices that have non-zero elements.

Here is an example (similar to other examples in the previous section.)

In[⊕]:= lsaHamlet⇒

LSAMonRepresentByTerms[

{"this country is rotten", "where is my sword my lord", "poison in the ear should be in the play"}] ⇒ 

	(	countri	ear	lord	plai	poison	rotten	sword
	1	0.642798	Θ.	Θ.	0.	0.	0.766036	0.
~	2	0.	Θ.	0.367111	Ο.	0.	Ο.	0.930177
	3	ο.	0.529938	Θ.	0.454323	0.71607	Ο.	Ο.

In the pipeline code above: (i) from the list of queries a representation matrix is made, (ii) that matrix is assigned to the pipeline value, (iii) in the pipeline echo value function the non-zero columns are selected with by using the keys of the non-zero elements of the association obtained with ColumnSumsAssociation.

## Similarities based on representation by terms

Here is way to compute the similarity matrix of different sets of documents that are not required to be in monad's document collection.

```
In[*]:= sMat1 =
      lsaSpeeches⇒
       LSAMonRepresentByTerms[aStateOfUnionSpeeches[Range[-5, -2]]]] ⇒
       LSAMonTakeValue
Out[*]= SparseArray
                             Dimensions: {4, 15 126}
In[*]:= sMat2 =
      lsaSpeeches⇒
       LSAMonRepresentByTerms [aStateOfUnionSpeeches [Range[-7, -3]]]) \Rightarrow
        LSAMonTakeValue
                             Specified elements: 5831
Out[*]= SparseArray
```

In[\*]:= MatrixForm[sMat1.Transpose[sMat2]]

SSparseMatrix: The dimension names {1, 1} are the same; using {"1", "2"} instead.

Out[ • ]//MatrixForm=

	Barack.Obama.2013-02-12	Barack.Obama.2014-01-28	Barack.Obama.2015-01-20	Barack.Obama.2016-01-12 D
Barack.Obama.2015-01-20	0.435536	0.46657	1.	0.438214
Barack.Obama.2016-01-12	0.379678	0.404785	0.438214	1.
Donald.Trump.2017-02-28	0.263294	0.23298	0.218087	0.203749
Donald.Trump.2018-01-30	0.265816	0.25859	0.227366	0.217215

### Similarities based on representation by topics

Similarly to weighted Boolean similarities matrix computation above we can compute a similarity matrix using the topics representations. Note that an additional normalization steps is required.

```
In[*]:= sMat1 =
       lsaSpeeches⇒
        LSAMonRepresentByTopics[aStateOfUnionSpeeches[Range[-5, -2]]]] ⇒
        LSAMonTakeValue;
    sMat1 = WeightTermsOfSSparseMatrix[sMat1, "None", "None", "Cosine"]
Out[*]= SparseArray
In[*]:= sMat2 =
       lsaSpeeches⇒
        LSAMonRepresentByTopics[aStateOfUnionSpeeches[Range[-7, -3]]]] \Rightarrow
        LSAMonTakeValue;
    sMat2 = WeightTermsOfSSparseMatrix[sMat2, "None", "None", "Cosine"]
In[@]:= MatrixForm[sMat1.Transpose[sMat2]]
```

SSparseMatrix: The dimension names {1, 1} are the same; using {"1", "2"} instead.

	(	Barack.Obama.2013-02-12	Barack.Obama.2014-01-28	Barack.Obama.2015-01-20	Barack.Obama.2016-01-12 D	)
	Barack.Obama.2015-01-20	0.950843	0.974615	1.	0.875923	-
	Barack.Obama.2016-01-12	0.902992	0.903844	0.875923	1.	
	Donald.Trump.2017-02-28	0.688586	0.696063	0.667645	0.531271	
	Donald.Trump.2018-01-30	0.599199	0.596653	0.591386	0.386631	

Note the differences with the weighted Boolean similarity matrix in the previous sub-section -- the similarities that are less than 1 are noticeably larger.

## Unit tests

The development of LSAMon was done with two types of unit tests: (i) directly specified tests, [AAp7], and (ii) tests based on randomly generated pipelines,

The unit test package should be further extended in order to provide better coverage of the functionalities and illustrate -- and postulate -- pipeline behavior.

### Directly specified tests

Here we run the unit tests file "MonadicLatentSemanticAnalysis-Unit-Tests.wlt", [AAp8].

### Random pipelines tests

Since the monad LSAMon is a DSL it is natural to test it with a large number of randomly generated "sentences" of that DSL. For the LSAMon DSL the sentences are LSAMon pipelines. The package "MonadicLatentSemanticAnalysisRandomPipelinesUnitTests.m", [AAp9], has functions for generation of QRMon random pipelines and running them as verification tests. A short example follows.

Generate pipelines:

```
SeedRandom[234]
pipelines = MakeLSAMonRandomPipelines[100];
Length[pipelines]
Out[*]= 100
```

Here is a sample of the generated pipelines:

```
pipeline
                                                     LSAMon[aStateOfUnionSpeeches, ⟨|⟩] ⇒LSAMonSetDocuments[
                                                                               hamletData] \Rightarrow LSAMonMakeDocumentTermMatrix[Automatic, Automatic] \Rightarrow LSAMonEchoDocumentsStatistics \Rightarrow
                                                                         LSAMonApplyTermWeightFunctions[IDF] \Longrightarrow LSAMonExtractTopics[NumberOfTopics \rightarrow 16,
                                                                               \texttt{Method} \rightarrow \texttt{SVD}] \Longrightarrow \texttt{LSAMonExtractStatisticalThesaurus} \ [ \{\texttt{life}, \texttt{countri}\}, \texttt{12}] \Longrightarrow \texttt{LSAMonEchoStatisticalThesaurus} \ [ \{\texttt{life}, \texttt{countri}\}, \texttt{12}] \ [ \{\texttt{life}, \texttt{countri}\}, \texttt{life}, \texttt{countri}\}, \texttt{life}, \texttt{c
                                                                               Words → {health, friend}]
                                                     LSAMon[aStateOfUnionSpeeches, <| \; | \; >] \Longrightarrow LSAMonSetDocuments[aStateOfUnionSpeeches] \Longrightarrow LSAMonMakeDocumentTermMatrix[\{\}, LSAMonMatrix[\{\}, LSAMonMatrix[\{\}, LSAMonMa
                                                                               stopWords] \Rightarrow LSAMonEchoDocumentsStatistics \Rightarrow LSAMonApplyTermWeightFunctions[] \Rightarrow LSAMonExtractTopics[
                                                                               NumberOfTopics \rightarrow -1, \ Method \rightarrow NNMF, \ MaxSteps \rightarrow 12] \Longrightarrow LSAMonFindMostImportantDocuments \Longrightarrow LSAMonEchoStatisticalThesaurus
                                                     LSAMon[None, ⟨|⟩] ⇒ LSAMonSetDocuments[
                                                                               hamletData] \Rightarrow LSAMonMakeDocumentTermMatrix[] \Rightarrow LSAMonEchoDocumentsStatistics \Rightarrow LSAMonApplyTermWeightFunctions[]
Out[1151]=
                                                                               NormalizerFunction \rightarrow Cosine] \Longrightarrow LSAMonExtractTopics[NumberOfTopics \rightarrow 16, Method \rightarrow SVD,
                                                                               LSAMon[None, \langle | \rangle] \Rightarrow LSAMonSetDocuments[hamletData] \Rightarrow LSAMonMakeDocumentTermMatrix[{},
                                                                               \verb|stopWords|| \Rightarrow \verb|LSAMonApplyTermWeightFunctions|| LocalWeightFunction|| \Rightarrow \verb|Binary||| \Rightarrow \verb|LSAMonExtractTopics||
                                                                               NumberOfTopics \rightarrow 16, Method \rightarrow SVD, MinNumberOfDocumentsPerTerm \rightarrow -12]
                                                     LSAMon[aStateOfUnionSpeeches, ⟨| |> ] ⇒LSAMonSetDocuments[
                                                                               a State Of Union Speeches \cite{AmonMakeDocumentTermMatrix} \Longrightarrow LSAMonEchoDocuments Statistics Stat
                                                                         LSAMonApplyTermWeightFunctions[
                                                                               {\tt IDF]} \Longrightarrow {\tt LSAMonExtractTopics[12]} \Longrightarrow {\tt LSAMonFindMostImportantDocuments} \Longrightarrow {\tt LSAMonTakeValue}
                                                     LSAMon[None, ⟨|⟩] ⇒ LSAMonSetDocuments[
                                                                               aStateOfUnionSpeeches] ⇒ LSAMonApplyTermWeightFunctions[GlobalWeightFunction → IDF] ⇒ LSAMonExtractTopics[12]
```

Here we run the pipelines as unit tests:

```
In[*]:= AbsoluteTiming[
    res = TestRunLSAMonPipelines[pipelines, "Echo" → False];
]
From the test report results we see that a dozen tests failed with messages, all of the rest passed.
In[*]:= rpTRObj = TestReport[res]
```

(The message failures, of course, have to be examined -- some bugs were found in that way. Currently the actual test messages are expected.)

## Future plans

## **Dimension reduction extensions**

It would be nice to extend the Dimension reduction functionalities of LSAMon to include other algorithms like Independent Component Analysis (ICA), [Wk5]. Ideally with LSAMon we can do comparisons between SVD, NNMF, and ICA like the image de-nosing based comparison explained in [AA8]. Another direction is to utilize Neural Networks for the topic extraction and making of statistical thesauri.

### Conversational agent

Since LSAMon is a DSL it can be relatively easily interfaced with a natural language interface.

Here is an example of natural language commands parsed into LSA code using the package [AAp13].

```
command: create the document term matrix
         parsed: MakeDocumentTermMatrix[{document, {term, matrix}}]
       residual: {}
        command: apply to the matrix entries idf
         parsed: LSICommand[LSIFuncsList[{LSIGlobalFunc[LSIGlobalFuncIDF[idf]]}]]
       residual: {}
Out[•]= 3
        command: extract 23 topics with the method NNMF and max steps 12
         parsed: TopicsExtractionCommand[{TopicsNumber[NumericValue[23]], TopicsParametersSpec[
                     Topics Parameters List \hbox{\tt [\{TopicsExtractionMethod[NNMF], \{TopicsMaxIterations[NumericValue[12]]\}\}]]\}]} \\
       residual: {}
        command: extract statistical thesaurus with 12 synonyms
         parsed: ThesaurusExtractionCommand[ThesaurusParametersSpec[ThesaurusNumberOfSynonyms[NumericValue[12]]]]
        residual: {}
```

## Implementation notes

The implementation methodology of the LSAMon monad packages [AAp3, AAp9] followed the methodology created for the ClCon monad package [AAp10, AA6]. Similarly, this document closely follows the structure and exposition of the ClCon monad document "A monad for classification workflows", [AA6].

A lot of the functionalities and signatures of LSAMon were designed and programed through considerations of natural language commands specifications given to a specialized conversational agent.

## References

#### **Packages**

[AAp1] Anton Antonov, State monad code generator Mathematica package, (2017), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/MonadicProgramming/StateMonadCodeGenerator.m.

[AAp2] Anton Antonov, Monadic tracing Mathematica package, (2017), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/MonadicProgramming/MonadicTracing.m.

[AAp3] Anton Antonov, Monadic Latent Semantic Analysis Mathematica package, (2017), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/MonadicProgramming/MonadicLatentSemanticAnalysis.m.

[AAp4] Anton Antonov, Implementation of document-term matrix construction and re-weighting functions in Mathematica, (2013), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/DocumentTermMatrixConstruction.m.

[AAp5] Anton Antonov, Non-Negative Matrix Factorization algorithm implementation in Mathematica, (2013), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/NonNegativeMatrixFactorization.m.

[AAp6] Anton Antonov, SSparseMatrix Mathematica package, (2018), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/SSparseMatrix.m.

[AAp7] Anton Antonov, MathematicaForPrediction utilities, (2014), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/MathematicaForPredictionUtilities.m.

[AAp8] Anton Antonov, Monadic Latent Semantic Analysis unit tests, (2019), MathematicaVsR at GitHub.

. URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/UnitTests/MonadicLatentSemanticAnalysis-Unit-Tests.wlt

[AAp9] Anton Antonov, Monadic Latent Semantic Analysis random pipelines Mathematica unit tests, (2019), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/UnitTests/MonadicLatentSemanticAnalysisRandomPipelinesUnitTests.m.

[AAp10] Anton Antonov, Monadic contextual classification Mathematica package, (2017), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/MonadicProgramming/MonadicContextualClassification.m.

[AAp11] Anton Antonov, Heatmap plot Mathematica package, (2017), MathematicaForPrediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/Misc/HeatmapPlot.m.

[AAp12] Anton Antonov, Independent Component Analysis Mathematica package, (2016), Mathematica For Prediction at GitHub.

URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/IndependentComponentAnalysis.m.

[AAp13] Anton Antonov, Latent semantic analysis workflows grammar in EBNF, (2018), Converasational Agents at GitHub.

URL: https://github.com/antononcube/Conversational Agents/blob/master/EBNF/English/Mathematica/Latent Semantic Analysis Workflows Grammar.m.

### MathematicaForPrediction articles

[AA1] Anton Antonov, "Monad code generation and extension", (2017), MathematicaForPrediction at GitHub, https://github.com/antononcube/Mathemati-

caForPrediction.

[AA2] Anton Antonov, "Topic and thesaurus extraction from a document collection", (2013), MathematicaForPrediction at GitHub.

[AA3] Anton Antonov, "The Great conversation in USA presidential speeches", (2017), MathematicaForPrediction at WordPress.

URL: https://mathematicaforprediction.wordpress.com/2017/12/24/the-great-conversation-in-usa-presidential-speeches.

[AA4] Anton Antonov, "Contingency tables creation examples", (2016), MathematicaForPrediction at WordPress.

[AA5] Anton Antonov, "RSparseMatrix for sparse matrices with named rows and columns", (2015), MathematicaForPrediction at WordPress.

[AA6] Anton Antonov, "A monad for classification workflows", (2018), MathematicaForPrediction at WordPress.

URL: https://mathematicaforprediction.wordpress.com/2018/05/15/a-monad-for-classification-workflows/.

[AA7] Anton Antonov, "Independent component analysis for multidimensional signals", (2016), MathematicaForPrediction at WordPress.

URL: https://mathematicaforprediction.wordpress.com/2016/05/23/independent-component-analysis-for-multidimensional-signals/.

[AA8] Anton Antonov, "Comparison of PCA, NNMF, and ICA over image de-noising", (2016), MathematicaForPrediction at WordPress.

URL: https://mathematicaforprediction.wordpress.com/2016/05/26/comparison-of-pca-nnmf-and-ica-over-image-de-noising/.

#### Other

[Wk1] Wikipedia entry, Monad,

URL: https://en.wikipedia.org/wiki/Monad\_(functional\_programming).

[Wk2] Wikipedia entry, Latent semantic analysis,

URL: https://en.wikipedia.org/wiki/Latent\_semantic\_analysis.

[Wk3] Wikipedia entry, Distributional semantics,

URL: https://en.wikipedia.org/wiki/Distributional\_semantics.

[Wk4] Wikipedia entry, Non-negative matrix factorization,

URL: https://en.wikipedia.org/wiki/Non-negative\_matrix\_factorization.

[Wk5] Wikipedia entry, Independent component analysis,

URL: https://en.wikipedia.org/wiki/Independent\_component\_analysis.

[LE1] Lars Elden, Matrix Methods in Data Mining and Pattern Recognition, 2007, SIAM. ISBN-13: 978-0898716269.

[MB1] Michael W. Berry & Murray Browne, Understanding Search Engines: Mathematical Modeling and Text Retrieval, 2nd. ed., 2005, SIAM. ISBN-13: 978-0898715811.

[MS1] Magnus Sahlgren, "The Distributional Hypothesis", (2008), Rivista di Linguistica. 20 (1): 33–53.

[PS1] Patrick Scheibe, Mathematica (Wolfram Language) support for IntelliJ IDEA, (2013-2018), Mathematica-IntelliJ-Plugin at GitHub. URL: https://github.com/halirutan/Mathematica-IntelliJ-Plugin.