

Agent Semantics, Semantic Spacetime, and Graphical Reasoning

Mark Burgess

June 16, 2025

Abstract

Some formal aspects of the Semantic Spacetime graph model are presented, with reference to its use for directed knowledge representations and capacity for process modelling. A finite $\gamma(3, 4)$ representation is defined to form a closed scaffolding set of operations that can scale to any degree of semantic complexity, with a highly simple rule set and a universal ontology. The Semantic Spacetime postulates bring predictability when reasoning, with minimal constraints on following pathways in graphs. The ubiquitous appearance of absorbing states in any partial graph means that certain graph processes leak information and represent entropy changing processes. The issue is closely associated with the issue of division by zero, which signals a loss of closure and the need for manual injection of remedial information. The Semantic Spacetime model (and its Promise Theory) origins help to clarify how such absorbing states are associated with boundary information where intentionality can enter.

Contents

1	Introduction	2
1.1	Spacetime characteristics	3
1.2	Euclidean vs graph	4
1.3	Symmetries	5
2	Semantics of spacetime and spacetime semantics	5
2.1	The purpose of Semantic Spacetime	6
2.2	The 4 relationship types	6
2.3	Resolving link type ambiguities with 3 types: <i>events</i> , <i>things</i> , and <i>concepts</i>	8
2.4	The combined $\Gamma(N, L) \mapsto \gamma(3, 4)$ representation	8
2.5	Strategy in semantic graph representation	10
2.6	Directional inference in the $\gamma(3, 4)$ representation	14
3	Graph-algebraic Semantic Spacetime	15
3.1	Proper names as semantic coordinates	15
3.2	Graph structures: appointments and loops	16
3.3	Propagation and terminating absorption of the $\gamma(3, 4)$ types	17
3.4	Allowed type transitions of $\gamma(3, 4)$	18
3.5	Inference rules and symmetries	19
3.6	Absorption by blind alley and by statistical aggregation: scales and degrees of freedom	20

4	Matrix representations of graphs	22
4.1	Matrices for $\gamma(3, 4)$ skeleton	23
4.2	Stable regions of a graph, information, emission and absorption	24
4.3	Sources and sinks	25
4.4	Frobenious-Perron eigenvector theorem	27
5	Graph semantics of operational arithmetic computation	28
5.1	Arithmetic	30
5.2	Special arithmetic states 1 and 0	30
5.3	Addition as a graphical process	31
5.4	Subtraction as a graphical process	33
5.5	Multiplication as a graphical process	34
5.6	Division	34
5.7	Division by zero semantics	36
6	Conclusions	38
A	Examples	42

1 Introduction

Semantic Spacetime (SST) is a discrete, graph theoretic ‘agent’ representation of configurations and process phenomena, used for modelling scenarios that include knowledge representations, in the form of labelled *directed graphs* [1–4]. It enables both qualitative and quantitative interpretations of processes by combining physical and virtual concepts (from physics and information science) into a Promise Theoretic agent model [5]. Promise Theory principles emphasize the autonomy or *locality* of causal behaviour, so there are clear motivations for modelling phenomena in this way. As a graph theoretical structure, a Semantic Spacetime is a collection of *nodes* (agents) joined by *links* (channels for process information), both of which may have annotations and numerical values associated with them.

A key application for Semantic Spacetime in artificial systems is to represent ‘knowledge’ (in its simplified sense) and process structures, such as those normally associated with indexing methods or Semantic Webs, like the triple store approaches of the Resource Description Framework (RDF) [6]. Further, one can explore how conceptual meanings and themes arise from these ground states [7, 8]. These applications are the subject of many papers spanning many decades, from the beginnings of taxonomies in the biosciences to database schemas to tuple spaces with supporting ontologies. The persistent difficulties and logical inconsistencies of these early methods, along with their lack of scalability, can be traced back to the adoption of an entity-centric modelling of the world in terms of ‘nouns’ and ‘verbs’ on a single scale—an affectation that probably stems from modern philosophy. The use of ontologies, in particular, to impose constraints on meaning at the level of logic is fraught with rigidity and consistency issues, because ontologies do not employ principles rooted in the processes of the world.

Processes are sequences of steps, akin to algorithmic data transformations, consisting of states unfolding over time as a set of ‘states’ in the most general sense. Processes embody everything from manufacturing (operations on raw materials) to simple arithmetic (operations on numbers). Our goal in representing knowledge is to model the changes to every volume of spacetime we care about in terms of its role in a cohesive semantic process that we can use as a model for reasoning. Semantic Spacetime aims to be sufficiently formal to be precise and yet sufficiently flexible to be widely useful.

1.1 Spacetime characteristics

A spacetime has certain regular concepts and ‘functional’ behaviours such as distance, time, scale, containment, direction: forwards, backwards and so on. In addition, other physical concepts such as symmetry and order can be applied to spacetime ‘states’ and the operations involving them.

The concept of ‘functional semantics’ [9] is not a topic normally associated with natural sciences like physics, except perhaps for a small number of category theorists [10, 11]. In linguistics, on the other hand, semantics are closely allied with semiotics of meaning [12, 13]. In Computer Science, semantics have a more pragmatic meaning: related to the outcomes of algorithms, i.e. what behaviour or response is implied by certain concepts and types. In addition to this, a meaning closer to the linguistic one is used in connection with knowledge representations and ontology [14].

Although the terminology of ‘semantics’ may be somewhat frowned upon and do not apparently play a prominent role in discussions of natural science, they are very much in evidence just below the surface in the form of roles and representations. In the traditional mathematics of rings, fields, and groups, for example, sometimes x is a quantity of something, with mundane semantics (what one would call a number), but in other cases x can be part of a generating function—an obstable used as part of, say, a differential operator scheme for algorithmically for representing certain patterns and process fragments, e.g. as in the partition functions of thermodynamics and quantum field theory, or the ladder operations of groups and graphs. Graphs therefore play a natural bridging role between disciplines and we should include some discussion of how visual imagery is used to give meaning to even abstract concepts and procedures.

Spacetime can certainly be described in terms of concepts then, but here we are equally interested in the converse. If we turn this association around, we can explore the way those concepts are perhaps derived from spacetime concepts. After all, earlier in our evolutionary journey there was nothing else to describe except where things were, how to find them, and basic process impetus like survival. There are likely insights that can be applied to other areas like knowledge representation and reasoning. The chief reason for wanting to do this is the idea that human concepts ultimately derive from concepts about space and time. Consider the following phrases, each steeped in spacetime metaphors and yet apparently representing quite different meanings:

Christmas is fast approaching.
We are getting close to an answer. We are approaching a result.
A long time coming.
The conclusion is just around the corner.
It’s a far far better thing.
A sufficiently large contribution.
Time flies like an arrow.
The moment of truth approaches.
The subject is heavy reading.
Just north of 200 dollars.
Within the bounds of reason.
What makes it tick?
I would point out.
At this point of the proceedings.
It then took a left turn.
That’s close enough to a win.
A long running affair.
On some level.
Just in time, within acceptable limits.
How long will you be?

We are moving ahead in spite of setbacks.
Our position is clear.

These are examples of how we use spacetime semantics at the very root of meaning. In addition, the gradual erosion of literal meanings and normalization of metaphors that are later forgotten is a plausible model of how language evolves in the first place [7, 8, 15]. All this is a good reason to explore the idea that descriptions of events and the knowledge associated with them can be simplified by appealing to the underlying spacetime types and how they are used to express meaning. It requires a more expansive and open minded approach to meaning than is common in either physics or linguistics, yet this is what we attempt to give a more formal substantial meaning to here.

1.2 Euclidean vs graph

The Euclidean-Cartesian view of space is the most prevalent in science. Space is associated with translations in directions x, y, z etc. The Minkowski space generalizes this to include coordinates for an observer's local time too. This view stems from the ballistic origins of physics as a way of predicting the trajectories of weapons fire. Occasionally, one might use radial coordinates from a localized point when considering perimeter boundaries, orbits, and Gaussian enclosures and so forth. However, the concept of translational invariance of space is engineered so deeply into our tools that any deviation from symmetry group thinking feels like a major deviation from common sense. In the Newtonian tradition, we separate what happens in space from the theatrical enclosure of abstract space itself.

The classical view of spacetime attributes all phenomena to the exterior space of points. Nothing happens 'within' points. Only in the quantum theory do we encounter interior states, such as wave functions, field values, spin, and other quantum numbers. The idea of hidden dimensions amounts to interior degrees of freedom at each point. Together with agent models, quantum theory sees each location in spacetime as having intrinsic or 'interior' properties that may result from unobservable processes within, but may also engage in more classical cooperative processes between agents in an exterior ballistic view.

A graph $\Gamma = (N, L)$ is a pair of sets N of nodes (also called vertices) representing locations and L of links (also called edges). Graph oriented spacetime has rather different challenges as compared to the more familiar continuum spacetime we are taught in school. In a continuum, everything is about the continuity. The concept of infinity lies embedded in nearly every assumption, without a clear sense of how. In a continuum, there is no reason for one location (node) to point explicitly to another as in a graph. One doesn't need a reason for the next point to be there—indeed, one rather needs a reason why it might *not* be connected and ready to receive some material body. There are many aspects of Newtonian age physics that one takes for granted that fail to make certain sense on closer examination. The continuum limit enables these slights of hand in a way which is both elegant and audacious [16].

Arrows may be used arbitrarily in as labelling devices, but in the world of causal events there are different kinds of graph. Petri Nets, bigraphs, and Labelled Transition Systems are examples of attempts to model process order as graphs, for instance. On the other hand, the standard Entity-Relation model of SQL databases uses arrows to represent attributes, and uses counters to represent order. In Semantic Spacetime one makes no rules about this; instead, some simple principles keep the meanings of arrows and attributes clear with a weak implicit typing.

The relationship between graphs and continuum spaces in the literature may lead to confusion. Embedding of a graph within a Euclidean space is how one often gives meaning to concepts like distance and dimension in graphs. The lattice approximations to Euclidean space uses regular graphs with long range order to emulate Euclidean space with graphs. Triangulation graphs can form approximate coverings of general manifolds. These ideas are not how we are to understand Semantic Spacetime: we are not trying to construct a skeletal embedding of trajectories in a Euclidean theatre.

Similarly, we are not looking to yield a classification tree such as a taxonomy, which is a hierarchical tree for organizing membership of sets, subsets, collections, or grouping as a form of containment. There,

parent nodes branch out, forming a semantic branching process [17]. The fact that all instances express a conceptual similarity gives meaning to parent-child relationships. A taxonomy is an interior spanning tree for internal concepts. Containment of things is a way of using a hierarchy because partitioning of space follows a fractal branching rule. Graphs therefore unify loosely topological and strictly geometrical concepts. Semantic spacetime has aspects of both conceptual decomposition and spanning of geometry. It is not a taxonomy or an ontology, though we may find those embedded within it. It is neither a roadmap embedded within Euclidean space with many redundant points in between. One has only nodes and links with interior values.

Embeddings interior and exterior to nodes are used widely in knowledge engineering. This is how feature vectors are introduced as interior spaces in artificial neural network models (see figure 6) [18]. In the world of ANNs, there are two underlying spaces: the network of the ANN and the feature vector space that is trained to represent concepts. This is almost the opposite of the model of physics where Euclidean space is the embedding for processes. There are many attempts to formulate a mathematical description of the dynamics of these representations, but they are inscrutable property models [19]. In SST we take a more symbolic algebraic approach to semantics, closer to language encoding than a statistical model of meaning.

1.3 Symmetries

The role of symmetry in spacetime is to seek a minimalism of description by emphasizing regular properties like homogeneity and isotropy. Symmetry in Euclidean space is represented by translation and rotation group transformations. A discrete graph, on the other hand, has too little structure to allow uniformity to form long range order. Nonetheless, there are processes in graphs that render dissimilar actions symmetrical. These include process equivalence, redundancy of nodes and paths (also called degeneracy in statistical physics), as well as the existence of ‘fixed points’ or absorbing regions of the graph that transmute any values into a ‘zero’ value. The semantics of how we deal with these are related to the efforts to give meaning to division by zero in rings and fields [20] and we’ll comment on these at the end.

Any kind of process from start to finish is basically isomorphic to some kind of journey in some kind of space measure by some kind of time. Time in this sense is the Aristotelian concept of proper time as countable changes, as observed by the agent concerned. These ideas have been discussed at length in a variety of works [21]. The virtual or imagined space of concepts has no obvious structure of its own. It can be imagined in any shape we like. For example, any neural associative memory leads to the existence of ‘hubs’ or ‘appointed nodes’ that join together many others in different ways because of shared attributes or common containers. Without the ability to express semantic symmetries, such as belonging to the same class or type of information, it would be impossible to reduce the entropy of graphs and create orderly information.

In any knowledge representation, which is neither complete nor undirected graph, there will be hubs that lead to absorbing states and information loss under certain transformations. The effect is to erase information from the interior states of the nodes, which can only be replaced with new boundary data from outside the graph, such as outside policy choices. Absorbing states are non-conserving of information.

2 Semantics of spacetime and spacetime semantics

We define the Semantic Spacetime model first in terms of the kinds of links that connect nodes, and then in terms of the kinds of nodes that make sense with those connections.

In a *process* view of the world, each point is a process event that needs to be justified rather than postulated as a generalized container for motion. This view is behind the famous relationship between Feynman diagrams and the algebraic and differential generating functional representations of Schwinger

and Tomonaga in Quantum Field Theory [22–24]. Further, both in Quantum Field Theory and in Group Theory one has ladder stepping operators (creation and annihilation) generating graphs by the action of an algebra of operators over abstract states. Underpinning all those variants are a mathematical description of rings and fields that underpin everything else practically as axioms. Questioning these feels unproductive, yet graph representations in particular have interpretations of arithmetic that poke holes in the rules we take for granted. One has to be clear about whether returning to the same state is actually the same node in a graph or a new version of a corresponding node which is similar but distinguishable. These notions have been long understood in statistical mechanics and thermodynamics [25]. In this regard, a semantic spacetime is akin to causal sets [26–28].

2.1 The purpose of Semantic Spacetime

Semantic Spacetime can be used in a number of areas including agent modelling, collaboration networks like supply chains, service mapping, formal reasoning, and narrative representations. The role of a knowledge representation is to capture experiences and to abstract and generalize them, associate and collect them into meaningful buckets that makes the knowledge easy to access and interpret. For example, the flow of processes through a general graph can represent:

- Scene description and forensic reconstruction.
- Flow of utilities (e.g. electrical, water) through grids etc.
- Communications networks like the telephone and Internet networks.
- Dependency networks, like supply chains, ownership and market structures.

Any amount of detail can be added to a graph of meaning, in principle. Recent developments in Large Language Models has placed much emphasis on generating fluent language, which is a vast area of subtle meanings. Before we get as far as semiotics and linguistic considerations, we need to look into the constraints on meaning imposed by topology and conservation of information. Dynamical considerations will always trump semantic considerations, because flow is a dynamical phenomenon semantics piggyback on quantitative amounts.

2.2 The 4 relationship types

The Semantic Spacetime model is basically a graph structure in which nodes and links are used to represent and expose the compressed meaning normally described in more fluent natural language. This is in the spirit of formal languages and state machines; however, it operates on a coarse level from the assumption that anything that happens is a process than can be expressed in terms of some kind of spacetime journey.

The four types arise from the model of agents. In an agent model of causality, which is fully localized, processes and information only happen inside agents unless the agents cooperate to represent larger structures. However, it is clear that activity only happens by agent states changing. Ideas and structure cannot be imposed upon them from outside without violating their local autonomy. This is a strong form of locality as expressed in physics. Based on the semantics of agents, and the model of intent developed by Promise Theory, the semantic spacetime model settles on four basic arrows between nodes in a graph that are postulated to be sufficient for any semantic description [1–3]. This remains a hypothesis for now, but it is not a particularly original one. Various authors have suggested that spacetime concepts underpin natural language, on the basis that they are the only objective concepts an organism has to bootstrap meaning from. Promise Theoretically, one derives four kinds of promise or relation between nodes in order to represent spacetime processes. These are called (see table 1):

ST TYPE	FORWARD	REVERSE	SPACETIME STRUCTURE
ST 0 = ‘N’ ‘NEAR’	is close to is similar to sounds like is correlated with	is close to is similar to sounds like to is correlated with	PROXIMITY “near” Semantic symmetrization similarity
ST 1 = ‘L’ ‘LEADS TO’	enables causes precedes to the left of	depends on is caused by follows to the right of	ordering GRADIENT/DIRECTION “follows”
ST 2 = ‘C’ ‘CONTAINS’	contains surrounds generalizes	is a part of / occupies inside is an aspect of / exemplifies	boundary perimeter AGGREGATE / MEMBERSHIP “contains” / coarse graining
ST 3 = ‘E’ ‘EXPRESS PROPERTY’	has name or value has property expresses attribute promises has approximation	is the value of property is a property of is an attribute expressed by approximates	qualitative attribute DISTINGUISHABILITY “expresses” Asymmetrizer

Table 1: Examples of the four irreducible association types, characterized by their spacetime origins, from [3]. In a graph representation, ‘has attribute’ and ‘contains’ are clearly not independent, so implementation details can still compress the number of types.

- 0 NEAR: a directionless assertion of equivalence or proximity between two nodes
- ± 1 LEADS TO: a temporally or causally ordered arrow denoting a sequence of events
- ± 2 CONTAINS: a spatially ordered collection of containment regions
- ± 3 EXPRESSES: a locally promised attribute of a node or distinguishing mark

The directionless type 0 (proximity) may be interpreted as an equivalence, approximately equal to, close to, etc. Type 1 (linear sequential order) may represent time, or unidirectional ordering, causation, dependency, etc. Type 2 (containment) may represent membership in a group, generalization of a collection of concepts, location inside or outside a perimeter, etc.

See table 1, which originally appeared in earlier papers [3,4] with some errors. An alternative way of listing the 4 types is in a tabular form (see table 2). Some of the relations address physical material attributes, while others represent virtual or conceptual attributes used to describe scenarios.

- Physical is also assumed to refer to properties that are external to the agent, and are therefore ‘tangible*, ‘material’, or even ‘real’.
- Virtual is assumed to refer to properties that are internal to the agent (or ‘somewhere else’ in a hidden dimension), and are therefore intangible or purely informational.

Any number of aliases or alternative interpretations of the four spatial relationships are possible; indeed, they are encouraged in effective communication for expressivity and qualification. However, the more specialized kinds of arrow we introduce, the harder it is to reason about them directly. However, we should also be cautious that informal association of linguistic metaphor also leads to confusions about the appropriate classification of meaning under the irreducible types, as interpretation by metaphor is fluid in human language [15]. What the four types enable is a basic generic form of reasoning about the meaning of a graph based on what kind of subject it describes.

DISTINCTION INFO	EQUIVALENCE	DISCRIMINATION
(PHYSICAL) SITUATION	CONTAINS spatial	LEADS-TO temporal
(VIRTUAL) INFORMATION	NEAR similarity/distance	EXPRESS PROPERTY attribute/name

Table 2: A alternative tabulation of the arrow types as belonging to physical (exterior) and virtual (interior) realms. Expressions may either denote equivalence of neighbours or group roles, or discriminants of nodes or what forms sequences.

ROLE/MEANING	NODE TYPE	PROPERTIES	space/direction
ACTIVITY/ACTION	event	ephemeral/realized	timelike (process) agent
SUBJECT/OBJECT	thing	invariant/realized	spacelike (snapshot) agent
SUBJECT/OBJECT	concept	persistent/unrealized	virtual (role/intention) agent

Table 3: Agent node types in semantic spacetime. Virtual agents may be thought of as being situated or ‘interior’ attributes of spacelike or timelike agents.

2.3 Resolving link type ambiguities with 3 types: *events*, *things*, and *concepts*

There is potentially some freedom in how to represent information given the foregoing link types. Even with the four SST categories there are some potential modelling ambiguities, so from a computational perspective it’s helpful to eliminate these with an additional formality. The residual ambiguity can be resolved by recognizing nodes as belonging to one of three meta-types, denoted with small Latin letters (e, t, c), which are induced by the four link types. The result is a generic $\gamma(3, 4)$ representation, where:

- e *Events* (e) An event is a temporary or ephemeral phenomenon. Such happenings may persist or change in time, by LEADS-TO which is a time like transition vector.
- t *Things* (t) are persistent phenomena, physical or realized (manifest or reified) agents in what one would call classical space. A thing is a persistent phenomenon but may be created or destroyed. They are agents that behave like matter.
- c *Concepts* (c) A concept is an invariant notion which cannot be created or destroyed. It belongs to a virtual space of ‘unrealized’ or ‘potential’ characteristics that can only be materialized by attaching to a physical material agent.

2.4 The combined $\Gamma(N, L) \mapsto \gamma(3, 4)$ representation

With the selections described above, we have a compact set of organizing meta-types that allow certain basic inferences to be made, relating to process semantics. The typing immediate implies a few constraints

that tighten up modelling. We can summarize the design implications for these choices in a few rules.

1. Things may be contained but not expressed.
2. Concepts may be expressed but not contained.
3. Concepts become realized by anchoring them to things or events.
4. Verbs are dangling concepts without a subject or object to instantiate them.
5. Verbs that are anchored to subjects or objects (things) are events.
6. A state of being which is realized is an event (john is/was happy, there was a moment of happiness).
7. A state of being which is unrealized is a concept (happiness).
8. A type of thing which is realized is a thing (a specific animal).
9. A type of thing which is unrealized is a concept (a general animal).

Example 1 (*e, t, c* examples) *We shouldn't think of objects as having a single representation in a knowledge space. Different aspects of a phenomenon have to be represented differently to obtain functional semantics:*

- *Mark's life is an event.*
- *Mark' body is a thing.*
- *Mark's character is a concept.*
- *Mark's appearance is a series of events expressing observations of descriptive concepts.*

Example 2 (What is real?) *Which part of semantic spacetime does an idea belong to? Physical or virtual? In the debates over the meaning of quantum physics, many authors have questioned what 'being real' means, e.g. whether we can say electrons are real if they don't behave like large scale material bodies. The use of 'real' is too loaded with ideological connotations to be useful. Adopting this terminology of $\gamma(3, 4)$ avoids these ambiguous stumbling blocks. One can be more precise about what is 'real', 'beable', or conceptual etc.*

Example 3 (Collectives or collective histories?) *Our ability to group events and concepts into a single namable entity "all bicycles" (a concept) from the evidence of many bicycle instances (things) is powerful but misleading because it exists only at a snapshot in time. In general, a process or history is a storyline, a conical structure of causal development rather than an encapsulation of things. We only reduce it to a single node in order to give it a proper name. Proper naming induces "nodality", but generalizations may be time dependent.*

In the spacetime sense, things or concepts can be invariants. Concepts may be said to exist yet be unrealized, i.e. they exist in the imagination, as a model or idea. Events are ephemeral but still realized by their happening. The distinction between ideas that are 'realized' and 'unrealized' is a simple clarification of the implicit metaphorical indirection we use all too frequently in natural language. Are we talking about the actual object or event X or are we talking about the idea of X in some context, or all possible X ? Invariants can't be contained except by other invariants

Example 4 (Many roles and contexts need different types) *Consider the word 'library', which we might use quite liberally in language:*

- *A library is a concept.*
- *The library is a thing.*
- *Library is a role for a building.*
- *Library is a location attribute for an event.*
- *The opening of the library is an event.*
- *A library contains books is a statement of concept.*
- *The library contains books is a statement of things.*
- *Library expresses attributes, old, well stocked, damp, at centre of city.*

When there are multiple paths, the selection of a particular value in a multi-valued inverse is affected by whatever semantics are applied to the arrows.

2.5 Strategy in semantic graph representation

Graphs as knowledge representations are typically understood either as classifications or ordered concepts or as social networks:

- Taxonomy: a hierarchy of proper naming and sub-classification.
- Ontology: a network of proper naming, classification and rules.
- Ad hoc property graphs: an unstructured network of atomic items and their relationships. All items are assumed to describe entities on a similar scale or level.

In recent years, the proliferation of graph models and Semantic Web technologies has encouraged users to model nodes in a graph by arbitrary linguistic atoms. Any simple conceptual meanings are presumed ‘atomic’. The Semantic Web is best known example of a social network in the stigmergic sense, with embedded order [6, 29, 30].

Nodes are given meaning with supplementary ‘ontologies’ [14]. The ontology movement is rooted in the primacy of first order logic for computer scientists. The focus of an ontology is to specify and share meaning, whereas the focus for a database schema is to describe data [31]. A relational database schema has a single purpose: to structure a set of instances for efficient storage and querying. The structure is specified as tables and columns. An ontology can also be used to structure a set of instances in a database. However, the instances would usually be represented in a [possibly virtual] triple store, or deductive database rather than directly in a relational database. Ontology acts as a formal straitjacket because it only works if everyone agrees to it.

Trying to naively shoehorn natural language into a graph structure is unproductive. For example, suppose we try to express In common speech, we regularly transmute different aspects of a description into one another all the time without much caution. Linguistic inference makes the algebraic underpinning of natural language grammar difficult. The brain skips many pedantic typological conversion steps, flattening out sequences as if coarse-graining steps by aggregation. For example, consider:

The computer (saves data to) disk. (1)

This is the kind of relation one often sees in naive triple store models. The arrow ‘saves data to’ is bursting with assumptions. It is so specific that it is unusable outside of this example. In terms of the $\gamma(3, 4)$ types we can compare it to an SST graph (see figure 1).

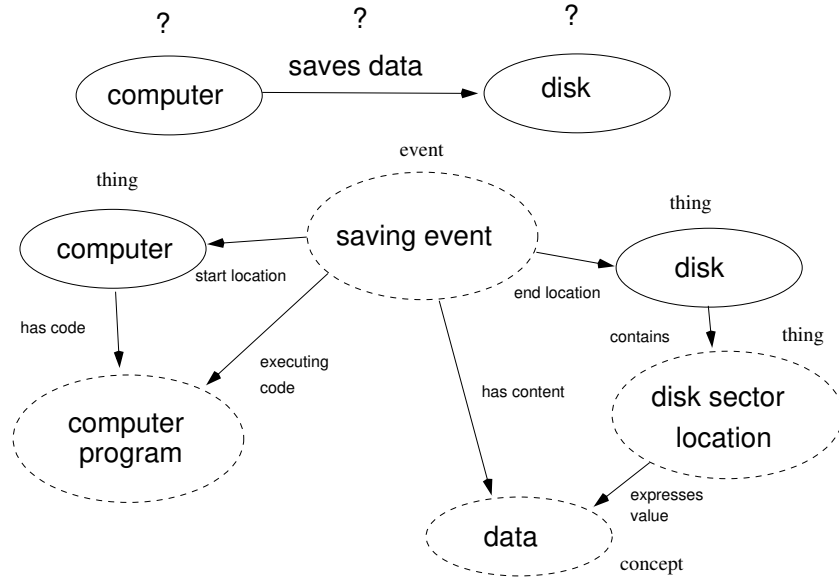


Figure 1: A triplet graph compared to an $\gamma(3, 4)$ graph.

One could argue that the ‘data’ implicit in the upper link of figure 1 should be provided as an input to the graph. Without a decomposition, this can only be done by extending notion of links to hyperlinks as in Milner’s bi-graphs [1, 32]. However, this introduces basic ambiguities that are easily resolved by the $\gamma(3, 4)$ decomposition.

The dotted lines indicate the subgraph representing the triple link. We see that it is not a simple substitution.

If we say a thing smells of perfume we are adding an implicit indirection from the concept of the smell of perfume to the liquid perfume itself. A smell is not a liquid, but there is only one natural interpretation of this muddling of proper names so no harm is done.

The cake (looks like) mark’s house (2)

What type is “looks like”? The cake is clearly a thing. Mark’s house is a thing, but “looks like” expresses a similarity. If we write “has the appearance of” is looks more like a concept attribute, which makes more sense. However now we have the rule that only concepts (not things) can be expressed as attributes. Mark’s house is not hanging off the cake, only the likeness or image of mark’s house, which is a conceptual representation.

These considerations are pedantic, like formal logic, because these are the typological distinctions that lead to precision. But then why not simply use logic? We are trying to avoid that level of specification by having only four types.

Concepts may express other concepts.

Stubbornness (means) explanatory concept (3)

A hammer (may be used for) hammering event/activity
 ” (may be used for) hammering activity

An example of something is an event or instance not a member of a group

How shall we decide the correct kind of link? When do we use express versus contains? The meanings of these concepts are rooted in spacetime ideas.

London (makes) cars
Tractors (are supplied by) Massey Ferguson
Apples (have colour) green
Apple (is a) fruit

In each of these cases, the kind of objects connected are wildly inhomogeneous. Without semantic homogeneity there cannot be homogeneous reasoning. Every new arrow relation. It's not difficult to see that many implicit steps are involved in each of these. Although we might say these abbreviated forms, we imply many more steps. Clear the city of London does not make cars: some factory located there contains a process of manufacturing and assembling. Making cars is a property of the factory. However, as the container of the factory, the city somehow inherits that promise of making cars. On the other hand, Sally is stubborn and Sally lives in London. This does not mean that London is stubborn.

How can we resolve these idiosyncratic anomalies in the reasoning? The key to unravelling these issues is to homogenize phenomena as generic processes. What process transmutes cities into cars, machinery into companies, fruit types into colours, and fruits into fruit types? The specific relations are not as important as whether something is a chain of events, an explanation of an attribute, or a part of a larger whole.

If several nodes are joined by a common association, typically a property expression or group container membership, then we can infer an implicit equivalence through correlation to the third party. If we have

$$\begin{array}{l} A \text{ (is located at) } X \\ B \text{ (is located at) } X \end{array} \quad (4)$$

we can assume that A and B are correlated somehow. However, this is likely lazy modelling. Is this really an invariant fact, or merely an ephemeral meeting? The database is describing a misleading snapshot of reality. Most likely A and B only met in a single event of a more dynamical scenario. X should not be a location but an event.

$$\text{John Williams (composed) Star Wars} \quad (5)$$

The use of a verb 'composed' lead to an ambiguous typing. Is Star Wars the film, the soundtrack, the brand, the franchise, etc. Is the composition a script, a piece of music, a clay model? Human inference can make a good guess and derive meaning from this (perhaps not without some confusion in general), but a machine algorithm relies on precise typing and cannot.

One could write explicitly

$$\text{John Williams (composer has musical composition) Star Wars Soundtrack} \quad (6)$$

This is then a reusable relation that implies two clear types: composer and musical composition. Written in this way as an invariant factoid, it misses the opportunity to be precise. We should, once again, turn to events as specific instances (analogous to objects instantiated from classes in computing).

$$\begin{array}{ll} \text{John Williams} & \text{composed the music for the movie Star Wars} \\ " & \text{(event has composer) John Williams} \\ " & \text{(event has composition) Star Wars Soundtrack} \end{array} \quad (7)$$

Star Wars the movie (has soundtrack) Star Wars Soundtrack (8)

[illegible]

We use the name of an object and the concepts and events it is part of loosely in natural language. Indeed, this is one of the strengths of natural language, which is likely optimized for the implicit functional capabilities human brain: many such connections can be left implicit and be filled in by inference. Part of the goal of semantic spacetime is to help understand these matters.

13

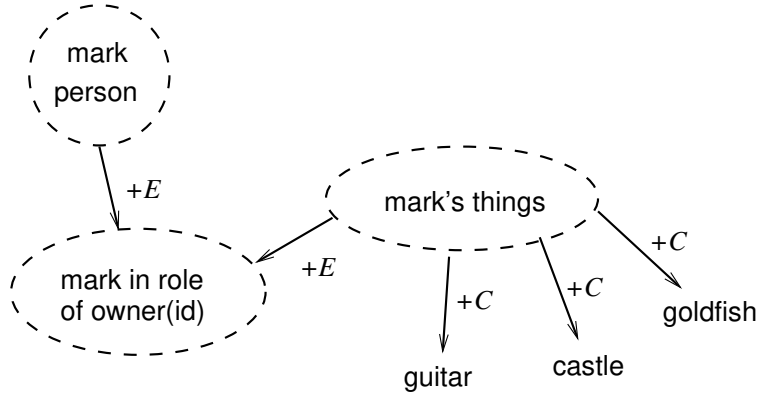


Figure 3: We use the name of an object and the concepts and events it is part of loosely in natural language. Ownership is a mixture of concepts and things. The person ‘mark’ is a thing so can only express a concept. To form a collection of things we need an entity that can contain them under a single umbrella. The estate of mark is thus a thing, and in order to express the ownership, we need to express the concept of ownership as a property of this collection. Since mark cannot (in a natural sense) contain many other items that are possibly far away from mark, we use the attribute of connecting ownership as an expression of mark’s identity (which is a concept).

2.6 Directional inference in the $\gamma(3, 4)$ representation

Spacetime operations are often associated with group transformations that form chains or closures. With a concept of graphical scale, and the type separations of $\gamma(3, 4)$, we can now attempt to make certain inferences without any specific ontological rules.

The aspect of systems which is supported by Semantic Spacetime is the frequently neglected dual importance of quantitative dynamics with qualitative semantics [33]. Process representations that encode knowledge may have several interpretations. The most common understanding of computation is based on the arithmetic conventions of rings and fields for addition and multiplication of real numbers. Our universal adoption of standard conventions means that we rarely question to consistency or meaning of these rules, yet they are fraught with many ambiguities, which normally only surface when faced with the ‘loose end of’ division by zero.

Our semantic model is relevant here too, as representations of computations may involve either physical interlinking of agents (e.g. agents collaborating on a chip die to perform a computation) or purely informational maps recording conceptual models of the processes (e.g. symbolic algebras).

The fact that we can represent computations as process graphs should be no surprise. It is a special case of expressing reasoning as a graph, from ad hoc stories ‘once upon a time’ to highly constrained algebraic logics. The lesson, however, is that this is not as simple as chaining together arbitrary triplets of named storage locations as in an RDF graph. The usual formalized notion of an ontology is of little use too, since it is too specialized and once always needs to extend or replace it. An ontology can never stabilize except by limiting data or constraining the allowed phenomena.

The types C , E do not ‘propagate’ indefinitely in the sense that L does. They are representative of the snapshot one obtains by freezing temporal evolution in a fixed configuration. We thus need to investigate the idea of scaling for inference. For instance, if we take an example and generalize or specialize it (going ‘up’ in the CONTAINS or EXPRESS direction), does the generalization have the same properties as the example? For example:

- Mark is human.

- Mark is tired.
- All humans are tired.

This attempted syllogism is clearly wrong, but the opposite direction may be true.

- All humans are annoying.
- Mark is human.
- Mark is annoying.

As pointed out by Couch [34, 35], the approximate notion of “might be true” or possibility links is sometimes the best one can do when reasoning, There is no precise logical one hopes with formal ontology. For example:

- If A contains B and A contains C , then B and C might be near one another on the scale of A .
- If an event E involves A and B , then A must have been near B on the occasion of that event.
- If A contains a collection nodes B_i should they inherit properties expressed by A ?
- If a collection of nodes B_i are embedded in $A \rightarrow B_i \rightarrow C$, then all the B_i are symmetrical or equivalent with respect to this particular process and may be considered part of a single supernode with redundant elements.

Interpretations like these are involved in the processes of reasoning by deduction, induction, and abduction, etc. The application of these ideas to unlabelled probabilistic transitions of Markov chains is the way current diffusion models of machine learning attempt to ‘reason’ [18].

3 Graph-algebraic Semantic Spacetime

Based on the foregoing definition of nodes and links, we can say more about the properties of these graphs. A graph may be represented structurally by a number of matrices, in particular the adjacency and incidence matrices, which represent maps of locations and flow gradients for the process concerned.

3.1 Proper names as semantic coordinates

In knowledge systems, such as taxonomies, one attempts to give a unique name to concepts in a contextually appropriate way. The name, although intended as a true representation of its unique meaning is often not unique in practice. In a coordinate system covering a region of space, in which one attempts to label distinct locations, a proper name is a kind of semantic coordinate. Unlike numerical coordinate systems, proper names are often multivalued, which leads both to opportunities and problems when reasoning. For a simple graph representation in which the characteristics of the node are the name itself, name associations may be considered tautologies, since our ability to make distinctions relies on there being observable differences.

The scaling of names to groups and regions is not altogether trivial, however. In statistical subjects, the importance of node or entity *distinguishability* has long been known and was shown by Boltzmann to be associated with the degree to which systems are able to exert a causal influence (free energy and entropy concepts) [25]. This issue of unique identity affects the way we model concepts and things in a graph, particularly when using spacetime as a model.

The concept of proper name interacts with the $\gamma(3, 4)$ types. A graph node is associated with a distinct identity. The numerical identifier of the node or semantic identifier may be associated with its ‘proper

name’, i.e. the collection of symbolic or numerical attributes that are expressed within or outside the node, i.e. the union of interior attributes S and the set of links $\{E(n_i)\}$ of type ‘EXPRESS PROPERTY’. In semantic spacetime, scaling allows us to take an entire book of text as a node in a graph. The proper name of the node is the entire text of the book. The node can be decomposed into smaller parts in a variety of ways to express the meaning of the entity.

- A book, considered as an entity, has its entire text as its unique identifier or proper name. We can give it several aliases, such as a title, a cryptohash or an ISBN number.
- The concept of the book with its unique text is realized in many physical copies, which express the concept of the book by reproduction. The physical books exists and can participate in different events (see figure 6).
- Compressed descriptions of events, things, and concepts can be unified under the umbrella of a name: ‘The Battle Of Britain’, ‘Tractor’, ‘hunger’. As patterns, we define names to be concepts or virtual attributes, rather than physical realizations. Names are usually recognizable patterns, with a variety of manifestations: in speech or writing, etc. It’s the content of the realizations that imbues the name, not the mode of implementation.

3.2 Graph structures: appointments and loops

When several nodes point to a single node as their successor, we call that an appointed node [5]. This leads to a local amplification of flow into the node. Nodes that are pointed to are also called ‘hubs’, while nodes that point to many are also called ‘authorities’ in social graphs [36,37]. An appointing node is the equivalent predecessor or source for several successor nodes. This is a division of the flow through the node into weighted distribution, according to the link weights. Appointing and appointed nodes correlate their appointees implicitly, and thus form a common dependency in reasoning. Such nodes are important for several reasons. Pragmatically, they are absorbing nodes and therefore lead to division by zero issues.

Graphs may contain structures that have both semantic and dynamical consequences.

- *Sources and Sinks*: these are nodes that start and end a path through the graph. They exchange places if one changes the sign of the link type.
- *Appointed nodes*: when several nodes point to a single hub that appointee is called an appointed agent in Promise Theory. The cluster of nodes all pointing / electing a single individual are thus correlated by the appointee (they have it in common). Such structures help us to see processes and process histories.
- For “leads to” arrows, these structures are confluences of arrows or explosions from a point.
- For “contains” arrows, these structures are the containers or shared members
- For “property expression” arrows, these structures are compositions of attributes or shared attributes common to several compositions
- For “near” arrows, these structures are synonym / alias / or density clusters

Appointed nodes that are themselves appointed recursively form nodes that are called ‘central’.

- A central node for ‘leads to’ has a high level of involvement, implying a high mass for flows.
- A central node for ‘contained by’ arrows is the container.

$$\{n_i\} \xrightarrow{-\text{CONTAINS}} n \quad (9)$$

- A central node for ‘contains’ is a member of several containers (a member of many categories).
- A central node for expressing a property is a widely shared or common property.
- A central node for originating properties has a rich spectrum of attributes.
- A central node for being similar or near others implies a high density or a high level of redundancy in interpretation, perhaps a popular concept.

If we don’t recognize these distinctions etc , the four link classes or types run into ambiguities when trying to classify the geometry of arbitrary semantics. For instance, EXPRESS versus CONTAINS are superficially similar, they both represent interior states, however one refers to the physical materialized makeup of a thing while the other refers to a concept.

3.3 Propagation and terminating absorption of the $\gamma(3, 4)$ types

A node that propagates may form chains. Nodes that ‘absorb’ arrows are natural endpoints of graph flows. Events are naturally propagating, without any necessary end. Both concepts and things are absorbing types of node because there is a most primitive attribute in practice.

The mapping between directed graphs and sequences of events (as ST-1 ‘leads to’ arrows) creates an obvious geometry for processes in a graph. These arrows become associated with proper evolution of states, e.g. Hamiltonian evolution in symplectic systems.

Consider examples for how each of the 4 types propagates.

- A terminating L chain sequence is one in which there is no natural followup in the narrative:

$$\text{egg} \xrightarrow{\text{(gestates into)}} \text{caterpillar} \xrightarrow{\text{(becomes)}} \text{a butterfly} \xrightarrow{\text{(flies to)}} \text{tree} \quad (10)$$

Notice that ‘a butterfly’ is really the event of changing into the state of being a butterfly, which is an event, since it wasn’t a butterfly before. Similarly ‘tree’ is really a shorthand for ‘the visitation of the butterfly to the tree’, which depends on the material thing ‘tree’, but is not the tree. These language subtleties trip modellers up frequently. The event may refer to a thing, but it is not the same as the thing. If we neglect to make these distinctions, and believe too literally the meaning of our abbreviations, we fall into the inconsistent the semantics of many knowledge graphs. Leads to chains have no obvious end, unless we choose to restrict them, e.g. to focus on a particular butterfly that eventually dies or transforms back into its constituents. Ideally, we would write the events more clearly (see figure 2),

- A C chain:

$$\text{Mark} \xrightarrow{\text{(owns)}} \text{car} \xrightarrow{\text{(made of)}} \text{atoms} \xrightarrow{\text{(contains)}} \text{quarks} \quad (11)$$

- An E (or P) chain:

$$\text{diagram} \xrightarrow{\text{(has prop)}} \text{visual} \xrightarrow{\text{(has prop)}} \text{colour} \xrightarrow{\text{(has prop)}} \text{blue} \xrightarrow{\text{(has frequency)}} f \xrightarrow{\text{(has units)}} \text{Hz} \quad (12)$$

- An N_t chain:

$$\text{Horsefly} \xrightarrow{\text{(looks like)}} \text{a butterfly} \xrightarrow{\text{(sometimes confused with)}} \text{a moth} \xrightarrow{\text{(resembles)}} \text{angel.} \quad (13)$$

The types of things now enter into these chains of reasoning.

The metaphorical text of the song ‘Love is Like a Butterfly’ can clearly be written as a simple triplet: Love (is like) Butterfly. However, this doesn’t quite work in this more stringent representation:

$$\text{Love} \xrightarrow{\text{(is like)}} \text{a butterfly}, \quad (14)$$

because love is a concept and butterfly is a thing, and these cannot be alike while retaining this type distinction. Clearly natural language works by metaphor much of the time, rather than by logic, and our brains are quite good at seeing through these inferences. This, on the other hand, makes the challenge of more careful logical explanation from natural language a non-trivial challenge. Arguments may be presented for keeping or eliminating metaphor in graphical knowledge representations. As long as we want a more formal calculative framework for reasoning (which is typically what we ask of machines), the shortcuts of natural language are probably best avoided unless we can find a method for replacing simplistic arrows with subgraphs that may be substituted in their place. In a sense, this is what Artificial Neural Network representations are doing on a probabilistic level.

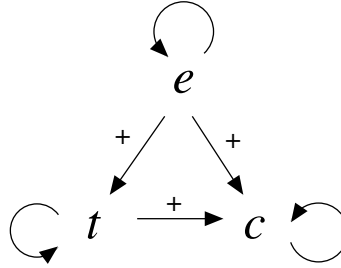


Figure 4: Allowed semantic transitions between node types, by kinds of arrow. There is a separation between virtual or conceptual states and physical or material characteristics. Events are the encapsulating class for both of those.

3.4 Allowed type transitions of $\gamma(3, 4)$

We can now summarize the algebra of rules for valid graphs in the $\gamma(3, 4)$ representation. See figure 4 and table 4.

Due to linguistic inference, once again, it seems that an event ought to be able to express or refer to another event as an attribute, e.g.

$$\text{Party celebrating the Olympics (refers to) the Olympics} \quad (15)$$

The Olympics was an event and the party is an event. However, if we seek a clean distinction we need a more pedantic eye. The use of Olympics in this case refers to the collective memory of the event, not the actual happening event itself—which is a concept without physical manifestation. We might prefer to mention Olympics only once in a knowledge graph, but then we would render all uses of the word equivalent, which is semantic nonsense. The same type of reasoning can be applied to explain why things cannot be expressed as properties. For concepts that seem to express things, e.g.

$$\text{fast food hatred (is about) fast food}, \quad (16)$$

we observe that the reference to ‘fast food’ is not a reference to an instance of fast food, but rather a reference to the whole class of things we call fast food, which is a concept. Our minds quickly create short cuts through these matters, yet we are also intuitively aware of the distinctions. This underlines

TRANSITION	EXPLANATION (EXAMPLE)
$e (\pm L) e$	An event can be followed by or lead to another event
$e (\pm C) e$	An event can contain or be part of another event
$e (N_e) e$	An event can be similar to another event by any criterion
$e (+C) t$	An event as a region of spacetime can contain a thing for its duration
$e (+E) c$	An event can express a property or concept (timestamp)
$e (+E) c$	can event can express a property or another event (celebration of Christmas of 74)
$t (-C) e$	A thing can be part of an event, but an event cannot be part of a thing.
$t (\pm C) t$	A thing can contain or be part of another thing
$t (+E) c$	A thing can express a concept as an attribute (blue car)
$t (N_t) t$	A thing can be close to or like another thing
$c (+E) e$	A concept can refer to an event as an attribute (that one time at band camp)
$c (-E) e$	A concept can be an attribute of an event (a time of happiness)
$c (-E) t$	Concepts can only be attributes expressed by things (blue car)
$c (E) c$	A concept can have properties or be a property of something else e.g. (blue is a colour)
$c (N_c) c$	A concept can be similar to another concept, (aquamarine,turquoise)

Table 4: Explicit transitions allowed for events, things, and concepts through the four link meta-types.

(as is well appreciated) that, without quite sophisticated automated analysis capabilities, natural language could not be understood literally. Our penchant for metaphor is busily at work in natural language [15].

The consistency of these relations can be show using the matrix algebra in section 4. In short, we can use this decomposition to define what are events, things, concepts. One could add more detail, but we are trying to compress description into simple elements rather than exfoliating.

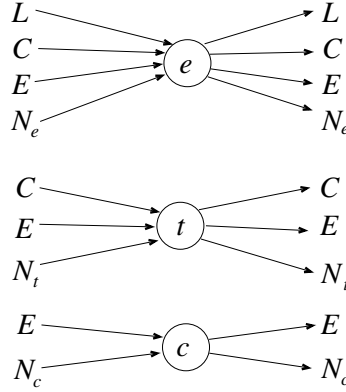


Figure 5: Allowed semantic transitions through node types, by kinds of arrow. Not all of the links are freely joinable, however, so there are restrictions on allowed transitions.

3.5 Inference rules and symmetries

In an unconstrained graph there can be no rules for inference, because rules require some regularity and functional predictability. In Semantic Spacetime, however, one has four broad kinds of relation and three kinds of node or entity.

If we seek a strongly constrained deterministic logic, as in ontology approaches to graphs, the result will be either simplistic or intractable. Inference about common properties are straightforward, though remain speculative.

- If a collection of nodes n_i are contained by a node n_C which expresses property P , then one might infer that the contained nodes *might also have this property*.
- If a node has a property P and is declared to be similar/near another node, then the similar node *might also have this property*.

This possible inferential reasoning goes back to the discovery by Alva Couch in joint work [34, 35], which was subsequently deepened in the development of semantic spacetime. Unlike logical ontological schemas, this kind of reasoning is simpler but inexact. With ontological first order logic, results are either precise or non-existent. In practice, most relations on data that are not carefully designed will fail to yield any result due to the over-constrained nature of first order logic.

A second kind of symmetry concerns patterns of inference in the linkage of nodes. Duplicate nodes may arise in a graph either by accident or by a deliberate encoding of redundancy (degeneracy). If a collection of nodes each possesses the same incoming and outgoing links of a given ST-type, then we can infer that they are functionally equivalent with respect to that process. This implies that they can be treated as a single node (which we refer to as a supernode) for those intents and purposes, though perhaps not all. While the nodes might have the same links for, say, causal trajectory ('leads to'), they might have different properties. This suggests another possible set of inferences or warnings to flag: why is there a partial but not complete equivalence? Is it intentional or accidental. Is it, in fact, an error?

3.6 Absorption by blind alley and by statistical aggregation: scales and degrees of freedom

How do we draw a ring around a region of interest or influence in a graph? The role of an agent boundary in determining what is a value or operation in a given context is important to many of the concepts as we scale up or down a hierarchy of meaning. Concepts emerge by recombination of atomic or genetic concepts and attributes, suggesting that C, E spatial types have semantic limits. L may have frequently have practical limits due to the ephemeral nature of interactions and processes in general, but there is no obvious upper limit to the extent of time.

The structure implied by the SST $\gamma(3, 4)$ model is of a graph composed of snaking causal sequences of events or ST-type 1 ('leads-to') connected nodes, where each node has 'contains' and 'express property' nodes in orbit around them, expressing their interior attributes. These may themselves be in orbit around nodes that contain or express them. A few 'nearness' links provide shortcuts (wormholes) between nodes that are marked as being similar for reasons outside the scope of explicit causal process knowledge.

One can say that events effectively act as bipartite bridging nodes between invariant things and concepts, leading to a temporary connection in the sense that an event has a finite contextual validity or lifetime even if the historical node remains in the graph.

Absorbing regions arise whenever there are source nodes or dead end sinks in a directed graph. Once we descend into the details of a node, by CONTAINS or EXPRESS, we find their invariant attributes. They live in an interior subspace orthogonal to the timeline of events. The timelike ST-type 1 ('leads-to') are analogous to the Hamiltonian evolution in symplectic systems of physics. This orthogonal subspace is analogous to the hidden dimensions of a Kaluza-Klein or string theory, a field space of forces or 'interaction semantics'. The entire directional links for containment and expression point to ultimately absorbing regions, as they are spacelike.

There is an erasure of information in two ways here:

- Flows that end up pooling at sink nodes are aggregated with a loss of distinguishable history or identity.
- Processes that drive several nodes towards an appointed successor merge separate dynamical flows rendering them indistinguishable. Conversely, a node that splits a flow to several successors with a weighted distribution is either sharing (dividing) or amplifying (multiplying) the flow along the links.

Clearly driving independent flows through a common node erases information and semantics, coarse graining the result. If one cares to measure the incoming distribution against the outgoing, with an entropy function, there is a change of entropy—potentially an increase or a reduction depending on how one chooses to define the semantics of the result. One does not avoid process semantic issues merely by adopting a conventional narrative.

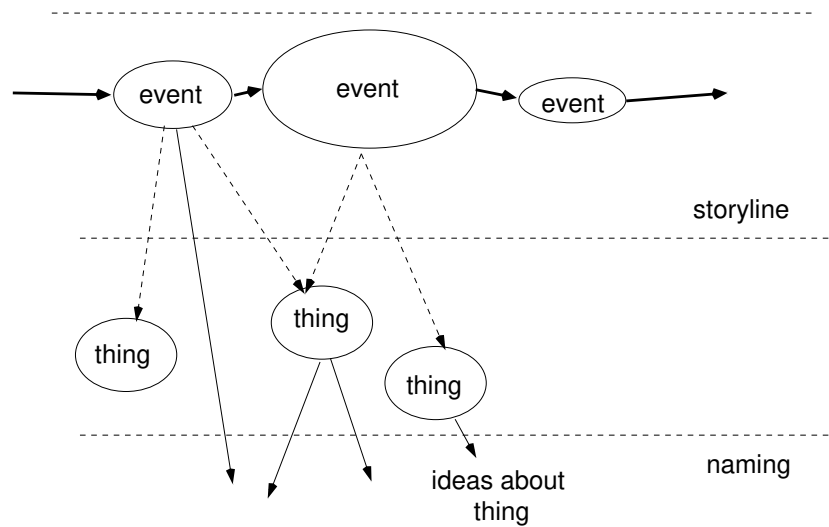


Figure 6: The logical structure of events, things, and concept is subtle. Events happen in real exterior space. Things exist in real exterior space, but ideas about things are interior to the agents that express them. They can become shared by interaction (like entanglement of quantum agents) but they are *a priori* private on the interior of agents.

The significance of absorbing regions, sources and sinks, for a finite system is clear semantically. Dynamically, however, there are different ways to interpret the processes.

There are certain elements that are, in a sense, atomic or irreducible by virtue of being a logical end to a process. States with both incoming and outgoing arrows are transitory but reducible ultimately to the sets of nodes either leading to it or emerging from it. In differential calculus one refers to these processes as retarded and advanced solutions.

Equilibrium solutions are also possible by selecting from both sets of arrow, or removing the arrow direction altogether. These are known as Feynman boundary conditions in physics. The latter is interesting because it introduces a second type of absorption for a process: a statistical absorption, which is associated with *entropy*. Any bulk state of a system, which converges statistically by some separation of scales into an average *probability distribution* of values, may be called statistically stable [17]. For example, an ideal gas at finite temperature has a statistically stable (or maximum entropy) distribution of particle velocities, which it makes no sense to count as distinguishable phenomena. This, at the semantic level of the bulk gas, the thermal state is absorbing—and represents another kind of ‘zero’: placing a

small object into a large (fabled infinite) reservoir at temperature T_0 leads to the process:

$$\text{Equilibrium } T_{\text{object}} = T_0. \quad (17)$$

In other words, semantically, equilibration is another kind of zero operation.

4 Matrix representations of graphs

A graph has an associated matrix representation for mapping the topology onto a linear map acting on rings or fields. The matrix representation is a crucial map between arithmetic algebraic rules and the realization of processes as graphical structures.

The *adjacency matrix* A and its transpose A^T are square matrices, whose rows and columns are the node labels, and whose non-negative elements represent quantitative link weights. The numerical values are typically set to 1 in elementary texts on graph theory, but they can have relative weights as well as semantic labels. These may be chosen as non-negative real numbers. An *undirected* graph (with arrows in both directions) is symmetrical about the leading diagonal, i.e. $A = A^T$. A directed graph is asymmetrical and its source and sink nodes, which are the starts and ends of paths, lead to zero rows and columns, which consequently lead to zero eigenvalues of the matrix. This proves to be important in a number of ways, as it implies the matrix is non-invertible, preventing predictive process-path reversals. A complementary *line* or ‘join’ graph matrix is the complement of the adjacency matrix, where rows and columns are represented by the links joining nodes. Both of these may be found from the incidence matrix.

The *incidence matrix* and its complement describe the emission and absorption of link lines from and to nodes. Since they are local to a single node, they may be associated with the promised intent of the nodes concerned. The standard conventions in most texts are for undirected graphs and are unhelpful here. Directed and labelled graphs with self-referential loops require a separation of the incident matrix into two parts. These are analogous to the offer and acceptance promises in Promise Theory, and the matrix elements factor from a Hadamard product form into two complementary matrices that are effectively the square roots of the adjacency matrix: $I^{(+)}$ and $I^{(-)}$.

$$\hat{I}^{(+)} \hat{I}^{(-)} = \hat{A} + \hat{C} \quad (18)$$

for some diagonal matrix C belonging to the Cartan subalgebra of the flow. When there are no auto-referential (pumping) self-loops, C is proportional to the identity matrix. The incidence matrices $I^{(+)}$ and $I^{(-)}$ correspond to the Promise Theoretic *offer* and *acceptance* promise rates [5]:

$$\hat{I}^{(+)} = \begin{matrix} & L & C & E & N_e & N_t & N_c \\ \begin{matrix} e \\ t \\ c \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad (19)$$

$$\hat{I}^{(-)} = \begin{matrix} & e & t & c \\ \begin{matrix} L \\ C \\ E \\ N_e \\ N_t \\ N_c \end{matrix} & \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad (20)$$

Notice that these are not simple transverses of one another, as they would be in a simple undirected graph, since a thing cannot be expressed (a forbidden state transition). We distinguish nearness links for each of the three types e, t, c for convenience.

Rates of change within the processes can be represented as derivative. The ‘dynamical’ graph derivative for node values is defined $\nabla_i v_j = v_i - v_j$. This corresponds to the usual Newtonian derivative $\partial_x v(x)$ for a function which is distributed over graph nodes $\vec{v}(N)$. There is a second notion of rate of change for a graph: because links define both direction and value between each pair of nodes, they also behave as a vector field, which has a gradient role of its own. The matrix of links mapping to links forms a dual ‘line graph’ representation in which there are rows and columns for every independent link, no matter the nodes they connect.

4.1 Matrices for $\gamma(3, 4)$ skeleton

The $\gamma(3, 4)$ skeleton graph can be represented, without explicit nodes only type names, as a set of matrices characterizing the graph. Certain rules about process semantics mean that transitions between certain node types are limited to specific kinds of arrow:

$$\hat{A} = A(n_i \mapsto n_j) = \begin{matrix} & e & t & c \\ \begin{matrix} e \\ t \\ c \end{matrix} & \begin{pmatrix} \pm L, \pm C, \pm E, N_e & +C & +E \\ -C & \pm C, N_t & +E \\ -E & -E & \pm E, N_c \end{pmatrix} \end{matrix} \quad (21)$$

This can be decomposed into a number of generators with antisymmetric (or anti-Hermitian) signatures:

$$\hat{A}_L = \begin{matrix} & e & t & c \\ \begin{matrix} e \\ t \\ c \end{matrix} & \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{matrix} \quad (22)$$

$$\hat{A}_C = \begin{matrix} & e & t & c \\ \begin{matrix} e \\ t \\ c \end{matrix} & \begin{pmatrix} \pm 1 & 1 & 0 \\ -1 & \pm 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{matrix} \quad (23)$$

$$\hat{A}_E = \begin{matrix} & e & t & c \\ \begin{matrix} e \\ t \\ c \end{matrix} & \begin{pmatrix} \pm 1 & 0 & -1 \\ 0 & 0 & +1 \\ -1 & -1 & \pm 1 \end{pmatrix} \end{matrix} \quad (24)$$

$$\hat{A}_N = \begin{matrix} & e & t & c \\ \begin{matrix} e \\ t \\ c \end{matrix} & \begin{pmatrix} 1_e & 0 & 0 \\ 0 & 1_t & 0 \\ 0 & 0 & 1_c \end{pmatrix} \end{matrix} \quad (25)$$

Conversely, arrows in a trajectory can only be joined by certain types of node as a path join matrix J :

$$J_{\gamma(3,4)} \left(\overset{a}{\rightarrow} n_i \overset{a'}{\rightarrow} \right) = \begin{matrix} & L & C & E & N \\ \begin{matrix} L \\ C \\ E \\ N \end{matrix} & \begin{pmatrix} e & e & e & e \\ e & e, t & e, t & e, t \\ e & e, t & e, t, c & e, t, c \\ e & e, t & e, t, c & e, t, c \end{pmatrix} \end{matrix} \quad (26)$$

- A process L must terminate on a final event or never.
- A property attribute process E must terminate on an atomic concept (property).

- A containment process C must terminate on an atomic thing (component).
- A metric similarity process N need not terminate within the scope of connected nodes.

The key point for directed graphs, representing finite processes of different types, is the existence of isolated states that absorb and emit transitions of the graph. In a ring or field this is not the usual case. There one has ‘translational invariance’ or group transformations that are essentially unlimited, for both addition and multiplication, with the important exception of the zero element, which is the one absorbing state under multiplication.

4.2 Stable regions of a graph, information, emission and absorption

Particularly for directed graphs, but also for some undirected ones, an important feature within a graph is the number of nodes at which the link flow converges (when absorbed by a node) or diverges (when emitted by a node), i.e. for in- and out-degrees greater than 1. These are confluence and branching points in the arrow vector field, which are literal and metaphorical singularities. As remarked in [38], emission and absorption by nodes is associated with zero operations.

There are certain operators which transmute interior state to exterior movement in the graph. These are related to ladder operators known in the context of differential equations, and they are further connected to absorbing states of the graph. Convergent semantics with idempotence of an operation one some end state comes about from ending up in a cycle that has the identity element as its final state. This is used to good effect in enforcing policy choices [38]:

$$\begin{aligned}\hat{O}\vec{v} &= \vec{v}_0 \\ \hat{O}\vec{v}_0 &= \vec{v}_0\end{aligned}\tag{27}$$

Consider figure 7: We can write this algebraically as:

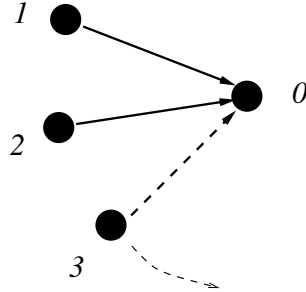


Figure 7: A convergence of flows at a point is an absorbing region of the graph, associated with a singularity.

$$\begin{aligned}\text{Arrow } n_1 &= n_0 \\ \text{Arrow } n_2 &= n_0 \\ \text{Arrow } n_3 &= n_0,\end{aligned}\tag{28}$$

which is clearly isomorphic to

$$\begin{aligned}0 \cdot 1 &= 0 \\ 0 \cdot 2 &= 0 \\ 0 \cdot 3 &= 0.\end{aligned}\tag{29}$$

in other words, by virtue of ending up at the same state ‘0’, the arrows lead the process into a location which does not remember the route by which it arrived there. If there are no arrows flowing away from node 0 then the node is completely absorbing (a kind of black hole for the process), and it is called a *sink* node. The adjoint, in which arrows are reversed would be a node that emits arrows from nowhere, called a *source*. Even with arrows both incoming and outgoing, any such multi-line convergence is singular, and indeed this is represented in the adjacency matrix of the graph by the existence of zero eigenvalues—and, indeed, the implications for invertability of the zero operations. Inverse of zero is a topic that several authors have discussed.

A hub operation of this kind turns a distribution into a single value and vice versa. Clearly, one can turn n values into one by averaging or some selection process, but turning one value into n requires more information. One can duplicate, triplicate, etc values to send out identically to multiple redundant destinations. Then the distribution of values is unchanged. Alternatively, one can use a new source of information in different directions to determine the result in different directions. One possibility is to use a differentiated distribution of link weights. However one chooses to inject information, it has to come from outside the starting node.

With multiple states distinguished semantically, rather than representing different locations on a neutral number line, we now have more responsibility to define the degrees of freedom carefully. In physics, one uses entropy concepts to describe and measure the extent and homogeneity of a distribution over states. The Shannon entropy [39,40] is defined over some distribution of states partitioned into N choices, and measure each with an alphabet of states $1 \dots C$.

$$p_i = x_i / \sum_i x_i \quad (30)$$

$$S = - \sum_{i=1}^N p_i \log_C p_i. \quad (31)$$

This is maximal $S = \log_C N$, when $p_i = 1/C$, $\forall i$, and minimal $S = 0$ when $p_i = 1$ for some choice. If we increase the resolution or alphabet of the distribution $N \rightarrow \infty$, then the entropy gets larger. If we coarse grain by making $C \rightarrow \infty$ then the entropy approaches zero and then ceases to be defined for normal arithmetic rules [41]. The ambiguity lies in how we count the implicit dimension of the state space (see figure 9 and the discussion below). The issue here is that a complicated process has more attributes or ‘degrees of freedom’ than a simplistic view of the number line.

In thermodynamic statistical physics, the Boltzmann entropy mimics the Clausius entropy as a measure of the energy in a system, which by virtue of being distributed indistinguishably around the system, has lost its capacity to do work. This is because statistical absorption is just as powerful as absorption by a single state or node of a process. In either case, the ability to make distinctions over a prescribed scale is the relevant issue. No measure of information can be conserved by processes that converges or diverges into a change of degrees of freedom, on any scale, and hence cannot be restored without an injection new boundary information for the inverse operation at or above the scale of aggregation.

4.3 Sources and sinks

A state is absorbing in a semantic graph because this reflects the interpretation we *intend* for it. In other words, it’s no accident that we end up with absorption. The same is true in arithmetic if one is careful, but there are cases where one is led to seek answers in ways that bump into problems concerning the incompleteness of definition.

If we think of the values in the field as nodes in a graph, with arbitrary many nodes, then operations that take us from one value to another may be represented as links with particular arrows that represent the operational semantics.

Graph transformations are used widely in machine learning diffusion models for image reconstruction and enhancement. They are also of interest here in a process knowledge representation for tracing the contextual relevance of the map, which I'll return to below.

Consider nodes 0,1,2 in the confluent junction in figure 7. The partial adjacency matrix is:

$$\hat{A} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad (32)$$

From this, the transposed adjacency acts as a forward stepping operator over the vector landscape of internal node values, and the untransposed matrix is a backwards stepping operator when acting on internal graph node values, represented as a vector $\vec{v}^T = (v_1, v_2, \dots)$:

$$\hat{F} = \begin{pmatrix} 0 & f_1 & f_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \hat{B} = \begin{pmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ b_1 & 0 & 0 \end{pmatrix}, \quad (33)$$

so that

$$\hat{F}\vec{v} = \begin{pmatrix} 0 & f_1 & f_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} (f_1x_1 + f_2x_2) \\ 0 \\ 0 \end{pmatrix}. \quad (34)$$

We note that \hat{B} and \hat{F} are not inverses of one another, since they both contain zero eigenvalues, i.e. have determinants of zero. Operating on the graph's internal state with this stepping operator, we see that the values from nodes 1 and 2 are shunted onto node 0, with the weighting determined by the adjacency matrix link weights. The original value at node 0 falls off the end of the absorbing node into the void and is unrecoverable. If we now try to reverse this in order to restore the original information, we see that the absorbing node wipes out the memory of the system rendering an inverse impossible without an input of new information:

$$\hat{B}\hat{F}\vec{v} = \begin{pmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ b_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1x_1 + f_2x_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ b_1(f_1x_1 + f_2x_2) \\ b_2(f_1x_1 + f_2x_2) \end{pmatrix}. \quad (35)$$

If we select the values for \vec{f} and \vec{b} appropriately, we can partially restore the original state, but not without specific knowledge of the original configuration and the ability to inject appropriate values into other nodes. The absorbing node x_3 's value is lost forever unless we insert a value by hand (as a matter of policy). and the initially unused source value x_1 has been injected. The values of b would typically involve division by the dimension or node degree $k_i n$ of the absorbing junction node 0.

$$(f_1x_1 + f_2x_2) \rightarrow 1, b/f \rightarrow \frac{1}{2}. \quad (36)$$

Why is the reverse operation not equal to the inverse matrix? This can be traced to the zero eigenvalues (and zero columns in the \hat{F} operator). The computation of a direct inverse would require a division by zero, which could yield any value, from eqn (29).

For a 3×3 matrix we can write the inverse explicitly for arbitrary real numbers $a, b, c, d, e, f, g, h, i, j$:

$$M = \begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix} \quad (37)$$

The transposed cofactor matrix (adjugate) forms the linear combinations which can be reverse engineered to yield cancellations or determinants, giving a formula:

$$M^{-1} = \frac{1}{\det(M)} \begin{pmatrix} (ej - if) & -(bj - ic) & (bf - ec) \\ -(dj - hf) & (aj - hc) & (af - dc) \\ (di - he) & -(ai - hb) & (ae - db) \end{pmatrix} \quad (38)$$

where $\det(M) = a(ej - if) - b(dj - hf) + c(di - he)$. The difficulty arises in the scaling of the values to renormalize the diagonal to $\vec{1}$:

$$M M^T = M^T M = I = \vec{1}. \quad (39)$$

For a sparse graph, most of the values a, b, \dots are zero. The ability to evaluate the expression is then unclear, since the rules for rings and fields do not admit division by zero. Here there are so many zeroes in multiplication that one has to see them as strings of operations rather than mere numbers.

The renormalization of the inverse is related to an overall factor of the determinant of the matrix. Since the determinant is the product of the eigenvalues, it is zero if there is a zero eigenvalue, which corresponds to a zero row or column. For a directed graph, this corresponds to a node which goes nowhere i.e. an absorbing state.

The effect of $\hat{B}\hat{F}$ is to eliminate the degree of freedom at x_0 and to coarse grain the values of the others so that they become equal. The only way to restore the original distribution is to encode the original values by a memory operation $\hat{M}(\vec{x}, \hat{F}) \rightarrow \hat{B}$. Indeed, this is schematically how diffusion models of machine learning restore images: by training an inverse process to capture the process memory of destroying the image through insertion of noise. The presence of 0^{-1} in the determinant or inverse indicates the need to remember past history, a snapshot of the past to ‘roll back’ to, as pointed out in [38].

We can now associate the semantics of stepping and state exposure with the semantic spacetime properties.

The same inverse notions apply to the semantic graphs. If we try to shift context up or down a branching graph, in the information hierarchy, crucial context may be lost. The relevance of the path is reduced by the dimension of the possible alternative pathways. This loss is happening inhomogeneously all over the graph where a process is ongoing. While this might initially seem harmless, making sense of the result requires a continuous input of new information to keep the inverse on course—but, on course to where? Deciding this selection requires an intentional act, i.e. an intentional insertion of policy information at each stage. In diffusion models, this is provided by prompt information from a user.

As with logics, if one attempts to remember complete and precise information, then one would only be able to generate results that were explicitly input. No recombinative mixing or ‘lateral thinking’ could enter the process to create something new (however derivative).

In earlier work, I proposed a T-rank algorithm for maintaining a more stable entropy distribution over a \vec{v} by pumping graph emission self-referentially to counter absorption. While this works, it also emphasizes that an ad hoc input is needed to obtain a stable answer, and that this ad hoc prescription basically determines the outcome with possibly only a shadow of the original constraints intact.

4.4 Frobenius-Perron eigenvector theorem

The eigenvector equation

$$M\vec{v} = \lambda\vec{v}, \quad (40)$$

for some matrix M and vector \vec{v} has solutions called eigenvectors, which are intrinsic (eigen) properties of the matrix in some sense. The equation can be applied to graph matrices by taking the adjacency matrix

$M \mapsto A$, which is non-negative, over the nodes of a graph $\Gamma(N, V) \mapsto (\vec{v}, A)$. This technique is widely used in social network analysis to perform so-called importance ranking of nodes in the graph [37].

The Frobenius-Perron theorem for non-negative matrices states that the largest or principal eigenvector of any such graph will be entirely positive. More significantly, the semantics of this vector attach to the eigenvalue equation: Iterating equation (40) represents a recursive propagation of node values over links, weighted by the link values, which implies that multiplying any non-zero vector repeatedly by A will converge \vec{v} towards the principal eigenvector.

The purely positive addition of purely positive values must yield the highest eigenvalue. An undirected graph is in flow equilibrium, so there is no net direction to the movement of values over the links. The equilibrium distribution of \vec{v} in the principal eigenvector thus represents the reservoir water level at each of the nodes (flow capacitance) at equilibrium. In social networks, this is equated with social capital or ‘importance ranking’ (an important person is someone with many important friends).

For an undirected graph, where node connections propagate without opposition, all the value flows along directed paths to sink nodes, where it pools. The normal algorithm for computing the eigenvector by operating many times with A onto a vector of ones $\vec{1}$ fails in this case to give the right answer, essentially due to the ambiguity of division by zero. When there are zero rows (absorbing nodes) the effect of this computation is simply zero. A more careful analysis shows that the vector components for absorbing nodes should be non-zero as this is where all the flow piles up, but the multiplication by zero overwhelms the normalization of the vector by λ , which may be zero itself for the absorbing nodes yielding $0/0 \mapsto 1$.

5 Graph semantics of operational arithmetic computation

We can illustrate some of the semantic choices in basic arithmetic operations using graphs as the domain ‘space’ to model what we mean by the operations. The standard meanings are so ingrained in us from an early age that it might seem strange to question them, yet doing so is quite instructive. The purpose of doing so is not to necessarily change conventions or solve some inconsistencies, but to point out how a few common themes trace back to interpretational ambiguities that we take for granted in mundane arithmetic. These become crucial to our understanding when we adopt semantically rich knowledge representations, such as in artificial reasoning.

Without adopting the standard jargon of rings and fields, arithmetic concerns the rules for counting and measuring amounts of ‘stuff’. It uses formal quantities called x to model ‘amounts’ and handles operations of augmenting and combining (+), depleting (-), duplicating (\cdot) and sharing (/) operations. These operations should work in both quantitative and qualitative interpretations. The abstractions either expose or conceal information however.

In the algebra of rings and fields, the status of numbers as positions or as shifts is blurred into a single set theoretic domain along the number line. Apart from this conventional interpretation, the question of semantics remains ambiguous for graphical representations of operations. Should numbers be thought of as value locations or as part of transformational operations? Should we treat numbers as events, things, or concepts? And perhaps more pertinently, do we muddle these interpretations carelessly in dealing with numbers? This question is particularly interesting in the service of Quantum Theory, where operator algebras and numbers co-mingle extensively.

As an illustration of this, we can begin by looking at graph representations of basic arithmetic reasoning and ask what are natural interpretations for addition, subtraction, multiplication, and division where the domain of mapping is no longer simply a single copy of the number line. Although this feels slightly self-indulgent, it’s illustrative of the fundamental issues in attributing semantics to formal processes and therefore underpins everything else in a form which is familiar to all readers. Let’s consider some examples, which might not be exhaustive.

The first question for semantics is to ask how we actually mean to represent a number to be added,

subtracted, multiplied or divided. There is no unique answer to this question, but we can naively imagine people counting pebbles or abacus beads. The standard rules for calculating are guided by a principle of closure around a set of numbers: when we combine numbers, the result should be a number of the same ‘type’.

$$+ : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}. \quad (41)$$

This is clear enough in the limited realm of mathematics, because the semantics of the ordered number line \mathbb{R} have already been defined (see figure 10). The number line is a key visualization of the process that works well for a geometrical interpretation of addition, but less well for a bulk interpretation. In our standard definition of division, for example, which is based on the primacy of the number line, there are ‘design issues’ to consider: if we wish our answer to a division operation to remain (mapped back into) the scope of the number line, then we can no longer both map an agent of size x back to an agent of size x *and* split the original inventory amount into a parts for all values without expanding the concept of numbers to include fractional amounts. Once one introduces new numbers, negative numbers and so forth, the consistency of the whole is jeopardized unless one can close the operations convincingly. This had resulted in the invention or discovery of symbols representing $i = \sqrt{-1}$, and more recently \perp or Φ for concepts including $1/0$ [42, 43].

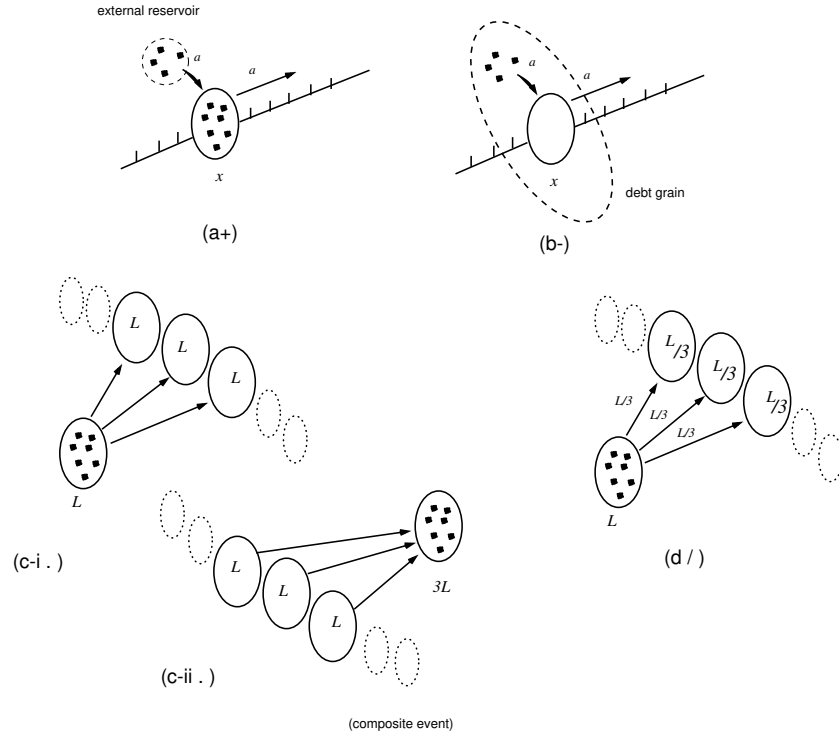


Figure 8: Some semantic interpretations of addition, subtraction, multiplication, and division in graphical form. A graph has more degrees of freedom than the number line automorphisms of rings and fields. Notice that multiplication has at least two possible interpretations: as an outgoing amplification of nodes (with an expansive dimensional meaning) or as an additive aggregation from nodes (mapping onto a single value more like the conventional ring/field interpretation).

5.1 Arithmetic

Let's consider the arithmetic operations on the number line as an example. These operations define operations on rational numbers in graphical terms (see figure 8), but run into difficulties with irrational numbers. In order to cover irrational numbers (which some mathematicians deny the existence of) one has to introduce infinities and limits on the interior of agents. These are fascinating issues that can only be mentioned in passing here (some discussions can be found in [43–48]).

Is there such a thing as a pure number? Arithmetic algebra defines unary $1 \mapsto -1$ and binary $1+1 \mapsto 2$ operators, in which the values are often pictured as locations on the real number line \mathbb{R}^1 . Euclidean geometry builds on this idea to coordinatize \mathbb{R}^n and make the explicit connection between value and location in a spatial construct. A graph is an analogous structure to a Euclidean vector space

The algebraic structures of geometry and arithmetic are so ingrained in daily norms that we seldom stop to confront the details of why these structures work. Representing these processes as graphs is an interesting exercise in self-consistent representation, as algebraic graph treatments involve matrix arithmetic, which is a generalization of ordinary arithmetic for partially coherent parallel processes. The stories or explanations we tell about these operations sometimes deviate from the actual rules and results provided by rings and fields. We can try to use graphical representations to elucidate the intended meanings.

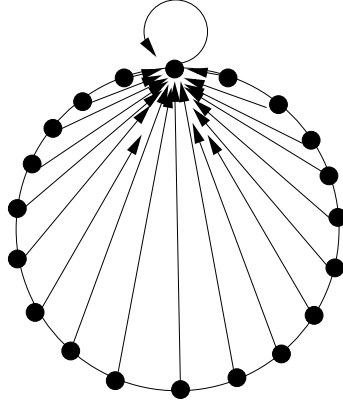


Figure 9: A compact representation of a zero multiplication interpreted graphically on a circular topology with L rather than ∞ nodes. When inverting this, the number of possible shares is represented by the number of arrows, like the vector results in eqns (53) and (54). The common interpretation of $n/0 \rightarrow \infty$ could be associated with these arrows, so should the answer be $n/0 \rightarrow 1/L$ in a finite system?

5.2 Special arithmetic states 1 and 0

Key to establishing the axioms and theorems of rings, fields, and groups are the special values 1 and 0. These binary values have attained legendary status in the digital age, but their significances are more important than binary arithmetic. They are the two stable fixed points of arithmetic:

$$\begin{aligned} 1 \cdot 1 &= 1 \\ n \cdot 1 &= n \\ n \cdot 0 &= 0 \end{aligned} \tag{42}$$

These are graphical processes (see figure 8) Here there is a special value 0 which is an absorbing state, i.e. arrows that enter do not leave (figure 9)).

$$0 \times 0 \mapsto 0 \quad (43)$$

$$1 \times 0 \mapsto 0 \quad (44)$$

$$2 \times 0 \mapsto 0 \quad (45)$$

$$3 \times 0 \mapsto 0 \quad (46)$$

$$4 \times 0 \mapsto 0 \quad (47)$$

$$\dots L \times 0 \mapsto 0. \quad (48)$$

In this interpretation, the proposal that $x/0 \mapsto \infty$ is a statement of the dimension or degeneracy of the inverse map. The inverse map is not single values and thus a prescription for interpreting it is needed. In machine learning diffusion models, for example, this inverse is interpreted as a Markov process and a policy decision is used to invert the map to get ‘something from nothing’.

More generally one can introduce a new value (something like the square root of minus 1) such that $x/0 \equiv \perp$. But what kind of object is \perp ? In the finite graphical interpretation, one might imagine that the dimension of the result would be L .

5.3 Addition as a graphical process

Translation is one dominant interpretation for additive arithmetic. Consider the expression:

$$x' = x + a \quad (49)$$

where x, a are what we intend to mean as ‘numbers’ (counts or measures). This operation of addition has two common interpretations that are easily distinguished in terms of agents. In order to use agents, we only need to assume that the set of agents is countable. In this model A number may refer to:

1. A location of an agent on the number line \mathbb{R} , which is a externalized total ordering of agents according to their number proper identities. The meaning of addition is then to translate or redefine the labels from coordinate position x to position $x+a$. Relativity has implications for the interpretation of these operations too.
2. The amount of interior holdings of the agent of some counter (e.g. money or energy etc). The meaning of addition is then an augmentation of the holdings from x to $x+a$. We say this is interior to the agent, because its position hasn’t changed in our new interpretation. However, one could also argue that this is the same as introducing additional exterior dimensions that are private to our invariant meaning for location (as one does in Kaluza-Klein or string theories of physics, for instance). The question of where a new things come from, or how the semantics of a (an increment) differ from the semantics of x (a state) is typically brushed aside.

The difference between interior and exterior ‘locations’, allowing us to distinguish exterior location from interior holdings clearly has ‘boundary value’ implications.

Indulging the forbearance of the reader for a moment, let’s examine this in more detail, since the issues are central to semantics of space and time and all graph based knowledge representations. We take the first of these additive cases whimsically to mean a translation along the number line, as in figure 10: In most cases, we think of x and a as being representative of numbers belonging to the same set, not to different copies of a set that looks like \mathbb{R} , yet this is misleading if not inaccurate. In both cases, the closure of the rational or real numbers under addition is a formal identification of outcomes back onto the same space, so while we write (41), we actually intend:

$$+ : \mathbb{R} \mapsto \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}. \quad (50)$$

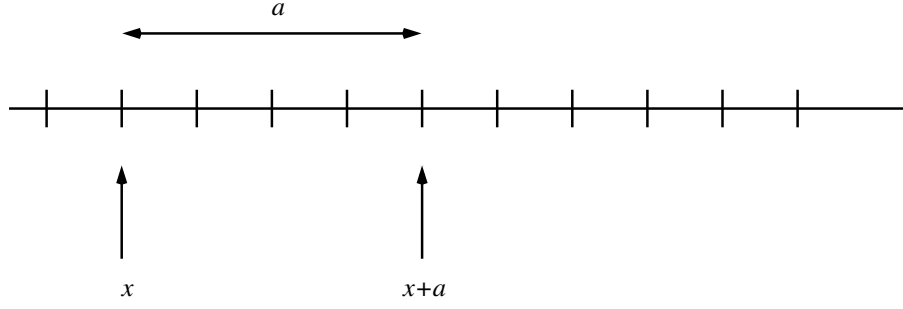


Figure 10: Addition pictured as a geometric translation. This is a common image in physics, where groups under addition are used to represent motion.

The final map involves a loss of information. If we introduce an entropy of representation For other operations one gets away with the same final mapping.

From a geometrical perspective we can think of x as a position, and a as being a argument to an operation that shifts the state of our marker from x to $x + a$. Implicit in this idea is, of course, the idea that the successive values along \mathbb{R}^1 are totally ordered so that the coordinatization is in one-to-one correspondence with elements of some spacetime. If we pedantically represent (49) as an operation on x , then we could write:

$$\hat{O}_a x \mapsto x' : |x'| = |x + a|, \quad (51)$$

i.e. an operation of type argument or magnitude a on the initial position x leads to a new location, in which the location x' has a label whose value corresponds to $x + a$. Without clear semantics, this is only a tautology. Indeed, it is only convention that would lead us to believe the result of operating on x with \hat{O}_a would be a position and not some new thing. This is not clear from (41).

Next, consider the interpretation as an interior amount. Aggregation is one kind of addition. We might call it ‘semantic addition’ as opposed to addition as a translation along the number line. Now location and translation are exchanged for inventory count and input amount, which combine back to a new inventory amount. The question of where the input comes from (a different place than the agent’s own storage, thus violating conservation) is sidestepped in the same way one sidesteps the semantics of changes in thermodynamics: by inventing fictitious infinite reservoirs that are approximately conserved and so on. This is a familiar trick in handling arithmetical anomalous cases too, such as with division.

To further illustrate the points here, consider an alternative matrix representation of addition, which acts as a bridge between the interior and exterior forms, since it uses an explicit extra dimension to distinguish and represent the add and a (see also [38]).

$$\begin{pmatrix} 1 \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ x + a \end{pmatrix} \quad (52)$$

In this form, x and a belong to different ‘Euclidean’ dimensions, or rows in the row space of the matrix. Clearly the expression also applies even when $a = x$, so $2x = x + x$, though the name $2x$ now assumes a new aspect of the algebra in linear multiplication. An entire chain of interpretation is implicit in these semantics. In (52), the original meaning is embedded in the new, so this is hardly progress. However, it serves to illustrate that we can make representations as complicated as we like, with as many dimensions as we see fit. Matrices are of interest in this case, because they offer the bridge between graphs and Euclidean vector spaces. We have intuitions about both, but these are easily muddled.

In a higher dimensional vector space \mathbb{R}^n or a graph $\Gamma(N, L)$, we need to select a direction from the possible independent directions available. Directional generators in \mathbb{R}^n can be made easily from suitable matrices or tensors in a given coordinate system, assuming homogeneous and isotropic spaces. For a graph it is generally more complicated, because there is no consistent set of directions from which to choose: each point points to a select set of neighbour destinations, which is typically different at every node. At the same time, the entire graph adjacency matrix can be thought of as a stepping operator for propagating interior field values from node to node [37].

In a graphical representation, the superficial similarities between $\Gamma(N, L)$ and \mathbb{R}^n are clear, and are frequently exploited in embeddings, e.g. for artificial intelligence feature representations for artificial neural networks, in which one plays with interior and exterior dimensional representations freely to enable independent processes.

5.4 Subtraction as a graphical process

Subtraction semantics $x - a$ imply taking away part of a measurable state. Once again, our interpretation will depend on the how we give meaning to x itself. As a translation, subtraction is the same as addition with only a reversal of direction (assuming that the reverse direction exists, which is assumed in vector spaces but is not generally true in graphs or vector fields). This is a convenient group theoretic property, because we need an inverse for every operation to complete a symmetry.

If, on the other hand, x refers to the amount of inventory held by an agent there is no opposite direction for filling states, unless one take on the concept of debt. This occurs in physics too because of the attachment to a principle of conservation. Whether the conservation principles are actual physical truths about the universe or simply self-consistent ways of counting slowly varying bulk quantities is probably still unresolved. The changes in the way money is allocated point to a similar issue. Money is not conserved but we calculate with it as if it were. This is the purpose of rings and fields: to uphold a simple set of semantics that broadly enforce conservation of amounts.

The addition and subtraction thus have similar semantics but with opposite effect on the final states of an agent. A translation forwards is similar to a translation backwards. An addition of inventory is similar to a removal of inventory, which the same question about what become of the balance of the change. Apart from the change of flow direction for a , subtraction is not substantially different from addition until we reach amounts that involve subtracting from zero. This leads back to the question of conservation and into what ‘afterlife’ these amounts a actually go to. The concept of ‘debt’ is based on this conundrum, and the resolution involves playing with coarse grains of time over which balance is restored and ordering can be sacrificed for the greater good of maintaining the conservation mythology. Subtraction then forces us to embrace a wider world of dynamical behaviours, detailed balances, and on going interactions with external reservoirs in order to give these ideas meaning. In doing so, it forces and explicit introduction of averaging over time or space, repackaged by sleight of hand into coarse grained or statistical conservation.

If we return to the translation interpretation of addition, pushing beyond the end of the number line would mean losing the value altogether. Topology offers alternatives here. Either translations fall off the end of a finite number line into a void, or one might identify the ends into a toroidal or spherical topology (a circle in one dimension), in which case a translation enters into the realm of modulo (‘clock’) arithmetic, wrapping around with remainders. All calendar phenomena follow this approach, with finite repetitions. Multi-valuedness of counting functions on the finite space is avoided by introducing more and more dimensions to the clock (minutes, hours, days, years, etc) for the parameters to spread into to answer the question of where the extra information goes. This is equivalent to adding new nodes to a graph to prevent paths from coming to an end.

Finally, if one is unable to map all values back into a closed compact set of dimensions as in a ring, then one may attribute the growth to information loss by introducing a coarse-graining concept for garbage collection to wipe away unrecoverable information as an undesirable. Again, thermodynamics

(which is an explicit agent model of energy phenomena) has confronted the quantitative aspects of these issues before. Our goal here is to extend this to the semantic aspects too.

The semantics of conservation quickly point out an unavoidable link between statistical (aggregate) information and elementary changes, which leads to ideas of entropy.

In both cases, the algebra of rings gives a convenient answer to a question, but by riding roughshod over specific semantics and normalizing a response. The answer provided however is not necessarily an answer to the question we intended to ask. Theoreticians learn to be cautious in the use of these conventions.

5.5 Multiplication as a graphical process

Multiplication addresses the duplication, triplication, and n -ification of an amount. However, it also plays a role in the meaning of repeated addition. One sees both interpretations in play in mathematics: multiple addition has ‘translational’ semantics, while multiplication of directions has Cartesian ‘direct product’ semantics. Again, our understanding stems from integer amounts, and generalizes conceptually to real numbers later. Once defined for integers, ring and field algebras allow one to close multiplication by non-integer values. (figure 8)

Once again there are interior and exterior interpretations of multiplication as an operation. What aspect of x do we wish to multiply? Referring to figure 8c, we could start with an agent whose inventory is x and wish to multiply the inventory amount within the agent, or we could imagine a multiplication of the agent itself, as a container, containing the equal amounts of inventory. No now x is an agent with certain holdings, and a is an operational parameter with no direct connection to the number line. It is not a distance. It’s semantics are new.

As before, with addition, we conventionally choose that x and a and $x \cdot a$ all map to the same space \mathbb{R} , and so (in agent terms) we want to map each starting agent to a copies that are of the same type, and importantly contain the same amount of inventory. This only works in practice for integer multiples, but we can then postulate the concept of partial agents to complete the calculational picture, in the same way that we use debt to delay the realization of the amount in practice.

Once again, these issues are harder to avoid in physics, and have been confronted in thermodynamics and statistical physics by Boltzmann and others. There are deep connections here to the notion of distinguishable and indistinguishable states and ‘particles’.

In ring algebra, we can think of multiplication as successive addition. This is a result of the final automorphism, mapping outcomes back onto the original number line. While an external observer can attest to the consistency of the answer in a counting scheme, several additions are very different from the commonplace everyday semantics of multiplication (which involves manufacturing new copies of a state) to make more things.

Conveniently, the association of inverse multiplication with division helps to cement this. A similar association occurs in the Fundamental Theorem of Calculus, which proves that indefinite (symbolic) integration is the inverse of differentiation and vice versa. The semantics of differentiation as a gradient, and integration as a summation do not obviously make this clear, yet there is a methodological precision (involving the infinitesimal limits of intervals that allows the operations to be co-related).

What single aspect of the operation should be mapped back into \mathbb{R} : is it the amount received by each agent, the number of copies, or the total inventory of all copies?

5.6 Division

Last but certainly not least, the semantics of division drive us more forcibly into the realm of agent interpretations (figure 8). Division has no obvious translational interpretation, but it can be related to the scaling and partitioning of distances—dividing a journey into a number of ‘legs’, or a procedure into a

number of subroutines. This interpretation is connected to the renormalization group in mathematical physics [49].

A more common conception for division is simply to share a bulk amount between a number of recipients, e.g. in a marketplace. In agent terms, there is (or at least could be) a container or binding force to hold the multiple separable parts in different ‘buckets’. In the absence of further information, a sharing out of x into a parts implies that the amounts are a -furcated into equal measures. There is no particular reason why the measures should be equal, except for a sense of simplicity and symmetry. Prime numbers become an immediate source of worry: what if the inventory is already atomic or indivisible, how can we share amounts that do not divide exactly? Already we are faced with the invention of real number semantics or the concept of remainders. What does it mean to share out the holdings of an agent into equal parts of the same size, with or without remainders? A remainder turns a division not into a single number, but a doublet of (share per recipient, remainder):

$$5/3 \mapsto (1, 2) \quad (53)$$

Matters of definition play an underestimable role. Division, like multiplication, does not yield a single unambiguous interpretation for its answer. Are we counting the number of partitions, the total amount of stuff shared out, or the size of an individual share? The answer could, after all, refer to either the number of agents with equal shares or the amount within each agent.

The usual answer gives a value which is the average size of the original amount given to each of the agents sharing it. Since this is equal, by argument, there is a unique value that can be mapped back into the original system of numbers. It’s meaning is lost, however. If we share a proton between three agents, they would all get different quarks with unequal properties. Only by throwing away information can the result be a single value rather than a vector of the partitioning.

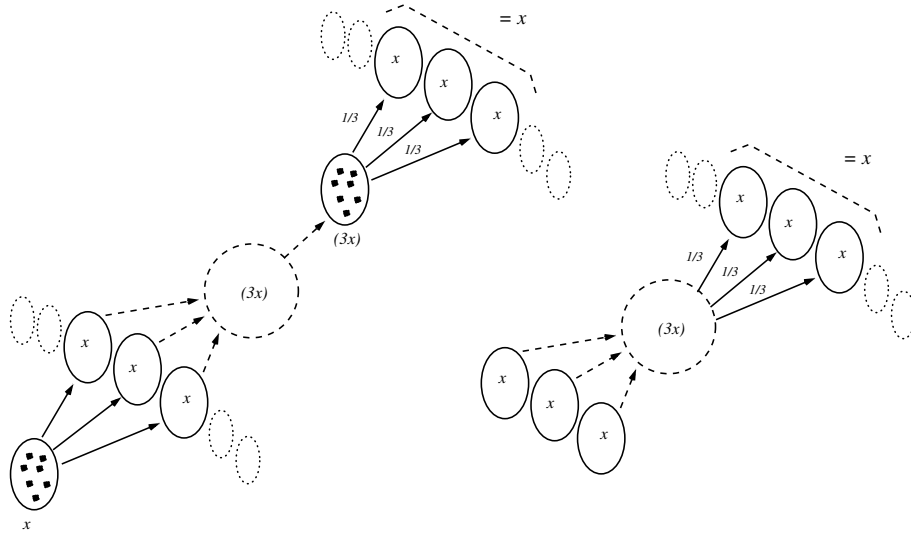


Figure 11: Multiplication and division as inverse processes. Here we represent $3x/3 = x$ as a process graph.

Division by N is a partitioning of the a set into N parts. This makes obvious sense for all numbers except for numbers less than 1. For example, division by 2 leads to two sets that are defined to be equivalent and thus the return value is the result for one of them. We do not write:

$$4/2 \neq (2, 2) \quad (54)$$

This could be interpreted in two ways: the 2 could mean each received a share of 2 or that there are two copies (a ‘redundancy’ or ‘degeneracy’ of 2). What about values smaller than 1? Dividing a set into half agents would mean that there are

$$\frac{x}{\frac{1}{2}} \quad (55)$$

implies that half agents would be twice as many

$$\frac{x}{0.5} = ? \quad (56)$$

The conventional answer of $2x$ here involves a renormalization of amounts. It is not so much an answer as a redefinition of the scale used for the answer. Again the semantic legerdemain leads to the convenience of an inverse operation to multiplication, at the expense of metaphysical caution.

5.7 Division by zero semantics

The more fraught issue of division by zero must also be handled by a more careful convention. It’s possible interpretations are intimately caught up in the processes it might represent—which should issue a warning to those who wish to fully abstract a universal meaning for numbers. Dividing a set into zero parts does not have an obvious mundane meaning. It could mean that the result is nothing as no-one receives any share of the initial state, or it could mean everything since the initial amount remains undivided. Alternatively, one could think of the limit $1/0 \rightarrow \infty$ as the metaphysical effort or force needed to break through the barrier of zero recipients, punching a hole through to the number line. That’s an interesting picture but with semantics that apply only to certain process scenarios.

In another sense, multiplication and division share many similarities, not only as adopted inverses for fields. Both involve nodes that can be considered to absorb or redistribute values, with arrows responsible for moving, aggregating, combining, distributing, and even creating and deleting information based on the topology and the semantics of the graph.

Conservation of some counter (flow continuity) is one of the most common criteria for defining processes, in order to have a consistent normalization of quantities. This is pervasive in physics at least for idealized closed systems. Suppose we argue graphically (figure 8) that division is the sharing of an incoming amount like a flow into a parts; then why do we not follow multiplication and ask for the answer to represent the sum inventory of all the participants? Then one would simply say that $x/a = x$ because the amount of information hasn’t actually changed: the total amount was conserved. Alternatively, we could introduce a partitioning into new dimensions, as in modulo arithmetic, and say that $x/a \mapsto \{b_1, b_2, \dots, b_a\}$ for some $b_i \sim x/a$. Then we sacrifice the property of mapping back to an object of the same or similar type, as an inverse for multiplication within \mathbb{R} .

In mapping the result back into \mathbb{R} , we make a choice about which kind of answer we want to keep. Choosing division to be the inverse of multiplication has many advantages and few anomalies, but the anomalies point to cases where the result makes more sense in other interpretations. Division by zero is the obvious case. As with the debt concept for subtraction (e.g. subtracting a negative amount becoming addition of a positive amount), division by zero involves handing information across the boundary from some ‘interior inventory’ to or from ‘somewhere else’. One cannot actually realize a negative amount in order to subtract it, but we can *post hoc* interpret an addition as having the same effect. For equilibrium processes, however, one can take away a depletion process leading to a net shift in the balance to a positive movement. This is like the concept of ‘electron holes’ in solid state physics. In the case of division by zero, however, the handover to zero recipients is manifestly cavalier as it throws away the meaning of a transfer altogether. There is no recipient. In Promise Theory, however, this does correspond to an physical scenario, where there is an agent offering x but there being no agent accepting the amount:

$$x \cdot (0) : x \xrightarrow{+x} \{\} \quad (57)$$

If an agent insists on propagating (imposing) the value, there is no agent to catch the state, we have to define what happens to it. The question of where the information goes is harder to paper over with a simple concept like a minus sign. There is now a translation into the void, from which no inverse can recover the value. Multiplication involves copying so it does not conserve amounts, unless we postulate its inverse.

If we define the answer to division by the average share amount received by each agent, then a conventional renormalization of scale would send the amount to infinity as a limiting process, but only because we want to imagine integer divisors as discussed above. If we ask instead what is the total inventory for all agents, thinking again about conservation, we multiply the shares by the recipients:

$$x/0 \mapsto b \cdot 0 = 0. \quad (58)$$

Then we admit that the shares have been operated away by shrinking the size of the receiver set to nothing, violating the conservation. Clearly the absorbing fixed point property of zero is problematic not so much semantically as operationally. Once again the information is simply cast into the void with these semantics. Which of the zeroes is supposed to win? The existence of an agent as a container surely wins over its imagined contents. If we have no semantic attachment there is no rational way to resolve the difference between the zero. This might not matter in the context of a neutral calculation, but it might be the difference between an image of a dog or a cat in a process of image reconstruction [18].

What then is an appropriate return value for the function associated with this division? We have several returnable characteristics for the process: the input values, the dimensions of the graph, values associated with the ring etc. What if the return value is the number of agents involved in the sharing? We can imagine the sharing process as being a fair weighted split of the input x into L pieces.

$$x/L \mapsto \frac{x}{\dim(\{L\})} \quad (59)$$

$$x/0 \mapsto \frac{x}{\dim(\{x\} \mapsto 0)} \quad (60)$$

When we interpret with agents like this, we are assuming that they are either blank “stem cell” agents initially or that they can track where stuff comes from. Semantic labels make the latter possible in computing and biology, but elementary agents may not have enough resources to track this information. If we think of the division as part of a process involving agents with memory of state, then the recipient agents might already contain a non-zero amount to begin with (like a bank account balance), so that their final state becomes the answer rather than simply the dividing transaction. If the agent already had a non-zero offset b_0 which could regularize the value $b = b_0 + x/\dim(\{x\})$ it alters the way we handle the outcome. For \mathbb{R} $\dim(x \mapsto 0) \mapsto \infty$, but for a ring of dimension L , $\dim(x \mapsto 0) = 1/L$, which would imply $x/0 \mapsto xL < \infty$.

Sharing nothing between no recipients is no more or less well defined. The only natural answer within the scope of the real numbers is 0. The only value for which this makes invariant sense is $x = 1$, because the absorbing property of 0 interacts with the idempotence of 1 as a stable fixed point under multiplication except for zero. However, in this configuration, the division by zero eliminates the zero issue.

$$k \frac{x}{0} = \frac{x}{k'0} = k \frac{1}{0} \mapsto 0 \quad (61)$$

This in turn suggests that, if we wish to map $0/0$ back into \mathbb{R} then the only plausible value is

$$\frac{0}{0} = 0 \cdot \frac{x}{0} = 0 \quad (62)$$

The problem is not so much the value as the process semantics of the operation (see figure11).

Finally, we may note briefly that there is another example of division of states in which a zero state can be inverted unambiguously into a finite integer number. This is for ladder operators in Fock space algebra [50] or Lie algebra root and weights [51]. The zero state is defined by an absorbing state for the annihilation operator \hat{a} .

$$\begin{aligned}\hat{a}|1\rangle &= |0\rangle \\ \hat{a}|0\rangle &= 0.\end{aligned}\tag{63}$$

Reconstruction, like the \hat{F} and \hat{B} graph operators for directed processes, give:

$$(a^\dagger)^n|0\rangle \mapsto |n\rangle\tag{64}$$

It suggests that one could define

$$0/0 = 1, \quad 0/0^2 = 2, \dots \quad 0/0^n = n\tag{65}$$

This is because the logarithmic property transmutes powers into addition, introducing a new question associated with the definition of logarithms over rings and fields. Clearly the full semantics of divisions are poorly represented by rings and fields, in a similar way that analyticity is incompletely by the reals without a wider scope of complex numbers.

6 Conclusions

Building on the Semantic Spacetime model as a set of guiding principles for graph representations, we can simplify the selection of proper link identification by adopting the $\gamma(3, 4)$ representation to remove unnecessary ambiguities, without adopting ontology or a detailed first order logic. The graphical properties of the algebra are then postulated to be compatible with all expected processes and inferences can now be written down explicitly and be verified by matrix algebra.

It remains for future work to understand how to understand whether knowledge is trustworthy [52,53] by the balance between the different node etc's e, t, c affect the reliability of a knowledge representation. If a knowledge structure contains only concepts, it lacks grounding in truth and can easily disconnect with reality. Could this explain what we are seeing with artificial intelligence knowledge, fake news, cult beliefs and extremism? This is a problem of interest for a more empirical review.

Technically, the presence of absorbing states is inevitable in graphs with non trivial adjacencies. These absorbing states need to be interpreted carefully. If we consider the graphs to be dynamic flows, they lead to loss of information at the edges of the system. The wider problems of consistent transformations within a finite structure have a common theme: the absence of invertability, due to states that are absorbing. In Markov processes, there is no memory to prevent loss of information, but one assumes that information is redistributed in such a way that it isn't actually lost. Absorbing states behave in a similar way, until the moment we want to reverse flows that lead to them. Remedies are analogous connection with division by zero remedies. It would be interesting to study the effects of the various remedies on matrix inversion for directed graphs in more detail.

Axioms and logical primitives already stand alone as 'boundary information' or irreducible knowledge. Such boundary information is thus intimately related to the absorption and emissions of nodes in a directed graph. As one uses the SST structures to scale structures, such features will continue to exist on many scales. Whole regions of a graph can be absorbing. Hierarchical graphs, such as taxonomies and spanning trees, also have nodes that are both the beginning and end of some flow. Finiteness demands this. Only in quasi-continuum models, groups and semi-groups, are we able to define 'translationally invariant' systems that go on and on in different directions. So the infinity of possible outcomes for end

states avoids dealing with beginning and end states. This absence of boundary is used frequently in physical models such as field theories, to argue for smooth continuity. Clearly, the concept of zero and infinity are complementary in this process sense.

There are no moving bodies in the Semantic Spacetime discussed here, however intrinsic properties of space can be passed along to relocate from node to node or from agent to agent. This is called Motion Of The Third Kind [1, 54]. In a sense it is forced to ‘invent’ the semantic split between matter and spacetime in order to resolve ambiguities about changing states, as natural philosophers found necessary in the formative years of physics. The semantics of material and conceptual constructs are similar. It’s also reminiscent of the arguments about whether one should call interior properties real or not, which continues to rage in the field of Quantum Mechanics.

In modern Artificial Intelligence or reasoning models, where many of the techniques originate from the palette of mathematical physics, there is a tendency to paint path selection and inference with the broad brush of probability theory, without questioning too much what the probabilities mean. The labels that can remember inverses have to be trained explicitly at some expense in shadow representations of knowledge. The relatively recent forays into context and relevance modelling are suprisingly overdue in artificial neural network representaions. Semantic Spacetime belongs to this effort. Semantic spacetime is not a natural language model. Natural language remains a very different way of compressing intentional descriptions with rich semantic content, which undoubtedly relies on the capabilities of an evolved brain for bridging the gulfs between broad and sometimes audacious inferences. The possibility of scanning natural language and compiling a compact SST representation is nevertheless an intriguing possibility of great interest in connection with generative Artificial Intelligence, with or without Large Language Models.

A software implementation of this work is available at [55].

References

- [1] M. Burgess. Spacetimes with semantics (i). *arXiv:1411.5563*, 2014.
- [2] M. Burgess. Spacetimes with semantics (ii). *arXiv.org:1505.01716*, 2015.
- [3] M. Burgess. Spacetimes with semantics (iii). *arXiv:1608.02193*, 2016.
- [4] M. Burgess. A spacetime approach to generalized cognitive reasoning in multi-scale learning. *arXiv:1702.04638*, 2017.
- [5] J.A. Bergstra and M. Burgess. *Promise Theory: Principles and Applications (second edition)*. *χtAxis* Press, 2014,2019.
- [6] W3C. Defining n-ary relations on the semantic web: use with individuals. <http://www.w3.org/TR/2004/WD-swbp-n-aryRelations-20040721/>.
- [7] Mark Burgess. Testing the quantitative spacetime hypothesis using artificial narrative comprehension (i) : Bootstrapping meaning from episodic narrative viewed as a feature landscape, 2020.
- [8] Mark Burgess. Testing the quantitative spacetime hypothesis using artificial narrative comprehension (ii) : Establishing the geometry of invariant concepts, themes, and namespaces, 2020.
- [9] D. Watt. *Programming language syntax and semantics*. Prentice Hall, New York, 1991.
- [10] B.C. Pierce. *Basic Category Theory for Computer Scientists*. MIT Press, 1991.

- [11] B. Coecke and A. Kissinger. *Picturing Quantum Processes*. Cambridge, 2017.
- [12] R.W. Langacker. *Cognitive Grammar, A Basic Introduction*. Oxford, Oxford, 2008.
- [13] I. Heim and A. Kratzer. *Semantics in Generative Grammar*. Blackwell, 1998.
- [14] J. Strassner. *Handbook of Network and System Administration*, chapter Knowledge Engineering Using Ontologies. Elsevier Handbook, 2007.
- [15] G. Deutsche. *The Unfolding of Language*. Academic Press, 2005.
- [16] M.B. Hesse. *Forces and Fields: The Concept of Action at a Distance in the History of Physics*. Dover, 1962.
- [17] G.R. Grimmett and D.R. Stirzaker. *Probability and random processes (3rd edition)*. Oxford scientific publications, Oxford, 2001.
- [18] C.M. Bishop and H. Bishop. *Deep Learning*. Springer, 2023.
- [19] N.M. Cirone, A. Orvieto, B. Walker, C. Salvi, and T. Lyons. Theoretical foundations of deep selective state-space models, 2025.
- [20] J. A. Bergstra. Division by zero, a survey of options. *Transmathematica*, pages 1–20, 2019.
- [21] M. Burgess. *Smart Spacetime*. χ tAxis Press, 2019.
- [22] R.P. Feynman. Space-time approach to quantum electrodynamics. *Physical Review*, 76:769, 1949.
- [23] F.J. Dyson. The radiation theories of tomonaga, schwinger and feynman. *Physical Review*, 75:486, 1949.
- [24] L.F. Abbott. Introduction to the background field method. *Acta Physica Polonica*, B13:33, 1992.
- [25] F. Reif. *Fundamentals of statistical mechanics*. McGraw-Hill, Singapore, 1965.
- [26] J. Myrheim. Statistical geometry. CERN preprint TH.2538, August 1978.
- [27] R.D. Sorkin. Causal sets: Discrete gravity. *arXiv:gr-qc/0309009*, 2003.
- [28] S. Surya. The causal set approach to quantum gravity. *arXiv:1903.11544 [gr-qc]*, 2019.
- [29] E.L. Robertson. Triadic relations: An algebra for the semantic web. *Lecture Notes in Computer Science*, 3372:91–108, 2005.
- [30] N. Fanizzi, C. d’Amato, and F. Esposito. Evolutionary clustering in description logics: Controlling concept formation and drift in ontologies. pages 808–821, 09 2008.
- [31] Michael Uschold. Ontology and database schema: What’s the difference? *Applied Ontology*, 10:243–258, 2015.
- [32] R. Milner. *The space and motion of communicating agents*. Cambridge, 2009.
- [33] Mark Burgess. *In Search of Certainty - The Science of Our Information Infrastructure*. χ tAxis Press, November 2013.
- [34] A. Couch and M. Burgess. Compass and direction in topic maps. (*Oslo University College preprint*), 2009.

- [35] A. Couch and M. Burgess. Human-understandable inference of causal relationships. In *Proc. 1st International Workshop on Knowledge Management for Future Services and Networks*, Osaka, Japan, 2010.
- [36] Jon M. Kleinberg. Authoritative sources in a hyperlinked environment. *Journal of the ACM*, 46(5):604–632, 1999.
- [37] J. Bjelland, M. Burgess, G. Canright, and K. Engø-Monsen. Eigenvectors of directed graphs and importance scores: dominance, t-rank, and sink remedies. *Data Mining and Knowledge Discovery*, 20(1):98–151, 2010.
- [38] M. Burgess and A. Couch. On system rollback and totalized fields: An algebraic approach to system change. *J. Log. Algebr. Program.*, 80(8):427–443, 2011.
- [39] C.E. Shannon and W. Weaver. *The Mathematical Theory of Communication*. University of Illinois Press, Urbana, 1949.
- [40] T.M. Cover and J.A. Thomas. *Elements of Information Theory*. (J.Wiley & Sons., New York), 1991.
- [41] Jan A Bergstra and John V Tucker. Expressing entropy and cross-entropy in expansions of common meadows, 2025.
- [42] Jan A. Bergstra and Alban Ponse. Division by zero in common meadows, 2021.
- [43] J.A.D.W. Anderson. Perspex machine ix: Transreal analysis. In *Electronic Imaging*. San Jos, 2007.
- [44] J.A.D.W. Anderson, N. Völker, and A. A. Adams. Perspex machine viii, axioms of transreal arithmetic. In *Proc. SPIE 6499. Vision Geometry XV*, 2007.
- [45] J.A. Bergstra and J.V. Tucker. The rational numbers as an abstract datatype. *Journal of the ACM*, 54:1–25, 2007.
- [46] J. A. Bergstra and J. V. Tucker. The data type variety of stack algebras. *Annals of Pure and Applied Logic*, 73(1):11–36, 1995.
- [47] J. A. Bergstra and J. V. Tucker. Division safe calculation in totalised fields. *Theory of Computing Systems*, 43:410–424, 2008.
- [48] J. A. Bergstra, Y. Hirshfield, and J. V. Tucker. Meadows and the equational specification of division. *Theoretical Computer Science*, 410:1261–1271, 2009.
- [49] G.I. Barenblatt. *Scaling, self-similarity, and intermediate asymptotics*. Cambridge, 1996.
- [50] S. Schweber. *Relativistic Quantum Field Theory*. Harper & Row, 1961.
- [51] H.F. Jones. *Groups, Representations, and Physics*. Institute of Physics, 1998.
- [52] M. Burgess. Notes on trust as a causal basis for social science. *SSRN Archive*, available at <http://dx.doi.org/10.2139/ssrn.4252501> (DOI: 10.2139/ssrn.4252501), August 2022.
- [53] M. Burgess. Trust and trustability: An idealized operational theory of economic attentiveness. *preprint paper* (DOI: 10.13140/RG.2.2.26862.28480/1), April 2023.
- [54] M. Burgess. Motion of the third kind (i) notes on the causal structure of virtual processes for privileged observers. DOI: 10.13140/RG.2.2.30483.35361 (notes available on Researchgate), 2021.
- [55] M. Burgess. Sstorytime project. <https://github.com/markburgess/SSTorytime/tree/main>, 2025.

A Examples

Only events can lead to events.

1. Hammering leads to noise

Possible: “The activity of hammering” L(leads to) “the event of noise” Wrong: “The activity of hammering” C(contains) “noise”

Possible: “The concept of hammering” E(may have property) “noise”

2. His stubbornness led to indignation

Possible: “The event in which he was stubborn” (led to) “An event in which there was indignation”

Wrong: “The concept of stubbornness” E(has the property) “the concept of indignation”

3. That cake is just like your house!

Unlikely: “The cake (thing)” N(is similar to) “your house (thing)”

Likely: “The appearance of the cake (concept)” E(has property of mapping to) “the appearance of your house”

4. The virus caused his death

Impossible: “The virus concept or thing” L(led to) “the event of his death”

Possible: “the viral infection event” L(led to) “the event of his death”

5. Professor Plumb murders Ms Scarlet in the library

“The event of plumb murders scarlet” (is an example of) “concept of murder”

“The event of plumb murders scarlet” E(has the attribute) “concept of murder”

“The concept of plumb murders scarlet” E(is an example of) “concept of murder”