Quantum Schur-type dualities of finite and affine type B

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May 7, 2014

(joint work with L. Luo and W. Wang)

Outline

(old) Schur-type dualities of finite type A/B and affine type A

- Classical
- q-Schur algebra level
- Quantum group level

(new) Schur-type dualities of affine type B [L-Luo-Wang2014]

- q-Schur algebra level
- Quantum group level

Outline

Classical Schur duality

- $\mathfrak{gl}_n$: general linear Lie algebra / $\mathbb{C} \Rightarrow$ left action

$$\mathfrak{gl}_n \curvearrowright V^{\otimes d}$$

  Here $V := \mathbb{C}^n$ natural representation of $\mathfrak{gl}_n$

- S_d symmetric group $\Rightarrow$ right action

$$\mathfrak{gl}_n \curvearrowright V^{\otimes d} \curvearrowleft S_d$$

Classical Schur duality

$$\begin{array}{ccccc} \mathfrak{gl}_n & \overset{\psi}{\curvearrowright} & V^{\otimes d} & \overset{\varphi}{\curvearrowleft} & S_d \\ \text{general linear Lie algebra} & & \text{tensor space} & & \text{symmetric group} \end{array}$$

Schur duality (1927)

1. The actions of $\mathfrak{gl}_n$ and S_d on the tensor space $V^{\otimes d}$ commute.
2. The algebras of operators on $V^{\otimes d}$ generated by the actions of $\mathfrak{gl}_n$ and S_d are centralizing algebras of each other. That is,

$$\begin{aligned} \mathrm{End}_{\psi(U(\mathfrak{gl}_n))}(V^{\otimes d}) &\simeq \varphi(\mathbb{C}S_d) \\ \psi(U(\mathfrak{gl}_n)) &\simeq \mathrm{End}_{\varphi(\mathbb{C}S_d)}(V^{\otimes d}) \end{aligned}$$

Deformed objects (Quantum group level)

$$\begin{array}{ccccc}
\underset{\text{universal enveloping algebra}}{U(\mathfrak{gl}_n)} & \curvearrowright & V^{\otimes d} & \curvearrowleft & \underset{\text{group algebra of symmetric group}}{\mathbb{C}[S_d]} \\
\text{q-deformation} \Big\downarrow & & & & \Big\downarrow \text{q-deformation} \\
\underset{\text{quantum group}}{U_q(\mathfrak{gl}_n)} & \curvearrowright & V_q^{\otimes d} & \curvearrowleft & \underset{\text{Hecke algebra}}{\mathcal{H}_d^A}
\end{array}$$

Here $V_q = \mathbb{Q}(q)^n$.

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1. Quantum group level
   - Classical Schur duality
   - finite type A
2. q-Schur algebra level
   - finite type A
   - finite type B
   - affine type A
   - affine type B (new)
3. Quantum group level (cont.)
   - finite type B
4. new affine type B duality
   - The algebra $\widetilde{U}^{\imath}$
   - new Schur duality

Hecke algebra (of finite type A)

The Hecke algebra $\mathcal{H}_d^A$ is a $\mathbb{Q}(q)$-algebra generated by

$$\{T_i \mid i = 1, \ldots, d-1\}$$

subject to

- Braid relations (among T_i's)
- Hecke relations $(T_i - q^{-1})(T_i + q) = 0$.

Hecke algebra action (of finite type A)

- $V_q = \sum_i \mathbb{Q}(q) v_i$
- e.g. when $d = 2$,
  $V_q^{\otimes 2} \curvearrowleft \mathcal{H}_2^A$ by

$$(v_a \otimes v_b)T_1 = \begin{cases} v_b \otimes v_a & \text{if } b > a \\ q^{-1} v_a \otimes v_b & \text{if } b = a \\ v_b \otimes v_a + (q^{-1} - q) v_a \otimes v_b & \text{if } b < a \end{cases}$$

  Specializing $q = 1 \Rightarrow (v_a \otimes v_b)T_1 = v_b \otimes v_a = (v_a \otimes v_b)(12)$

- $V_q^{\otimes d} \curvearrowleft \mathcal{H}_d^A$ similarly

Quantum group (of finite type A)

The quantum group $U_q(\mathfrak{gl}_n)$ is a $\mathbb{Q}(q)$-algebra generated by

$$\{E_i, F_i, D_j, D_j^{-1}\}$$

subject to

- q-Chevalley relations
- q-Serre relations

Remark: The quantum group $U_q(\mathfrak{sl}_n) = \{E_i, F_i, K_i, K_i^{-1}\}$ is related via the embedding

$$\begin{array}{ccc} U_q(\mathfrak{sl}_n) & \hookrightarrow & U_q(\mathfrak{gl}_n) \\ K_i & \mapsto & D_i D_{i+1}^{-1} \end{array}$$

Example: commutivity

Here's an example showing $(F_2(v_3 \otimes v_2))T_1 = F_2((v_3 \otimes v_2)T_1)$

$$\begin{tikzcd}
V_q \otimes V_q \arrow[rrr, "T_1"] \arrow[ddd, "F_2"'] & & & V_q \otimes V_q \arrow[ddd, "F_2"] \\
& v_3 \otimes v_2 \arrow[r, mapsto] \arrow[d, mapsto] \arrow[dr, phantom, "\circlearrowright"] & \begin{array}{c}(q^{-1} - q)v_3 \otimes v_2 \\ +v_2 \otimes v_3\end{array} \arrow[d, mapsto] & \\
& q^{-1}v_3 \otimes v_3 \arrow[r, mapsto] & q^{-2}v_3 \otimes v_3 & \\
V_q \otimes V_q \arrow[rrr, "T_1"'] & & & V_q \otimes V_q
\end{tikzcd}$$

Schur-Jimbo duality

$$\underset{\text{quantum group}}{U_q(\mathfrak{gl}_n)} \quad \overset{\psi}{\curvearrowright} \quad V_q^{\otimes d} \quad \overset{\varphi}{\curvearrowleft} \quad \underset{\text{Hecke algebra}}{\mathcal{H}_d^A}$$

Schur-Jimbo duality (Jimbo1986)

The algebras $U_q(\mathfrak{gl}_n)$ and $\mathcal{H}_d^A$ satisfy double centralizer property. That is,

$$\begin{aligned} \mathrm{End}_{\psi(U_q(\mathfrak{gl}_n))}(V_q^{\otimes d}) &= \varphi(\mathcal{H}_d^A) \\ \psi(U_q(\mathfrak{gl}_n)) &= \mathrm{End}_{\varphi(\mathcal{H}_d^A)}(V_q^{\otimes d}) \end{aligned}$$

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   - new Schur duality

Deformed objects (q-Schur algebra level)

$$\begin{array}{ccc} U_q(\mathfrak{gl}_n) & & \\ \text{quantum group} & & \\ \downarrow & & \\ S_q^A(n,d) \curvearrowright & (\mathbb{Q}(q)^n)^{\otimes d} & \curvearrowleft \mathcal{H}_d^A \\ q\text{-Schur algebra} & & \text{Hecke algebra} \end{array}$$

q-Schur algebra (of finite type A)

- Set of Weights $\Lambda(n,d) = \{(\lambda_1, \dots, \lambda_n) \in \mathbb{N}^n | \sum \lambda_i = d\}$
  $\Rightarrow$ "q-permutation module" $x_\lambda \mathcal{H}_d^A$, $\lambda \in \Lambda(n,d)$
- $\displaystyle\bigoplus_{\lambda \in \Lambda(n,d)} x_\lambda \mathcal{H}_d^A \simeq V_q^{\otimes d}$
- q-Schur algebra $S_q^A(n,d) := \mathrm{End}_{\mathcal{H}_d^A} \left(\displaystyle\bigoplus_{\lambda \in \Lambda(n,d)} x_\lambda \mathcal{H}_d^A \right) \simeq \mathrm{End}_{\mathcal{H}_d^A} \left(V_q^{\otimes d} \right)$

Quantum Schur duality of finite type A

$$\begin{array}{ccccc} U_q(\mathfrak{gl}_n) & & & & \\ \text{quantum group} & & & & \\ \downarrow & & & & \\ S_q^A(n,d) & \curvearrowright & V_q^{\otimes d} & \curvearrowleft & \mathcal{H}_d^A \\ q\text{-Schur algebra} & & & & \text{Hecke algebra} \end{array}$$

Proposition (Dipper-James1991)

The algebras $S_q^A(n,d)$ and $\mathcal{H}_d^A$ satisfy double centralizer property. That is,

$$\begin{aligned} \mathrm{End}_{S_q^A(n,d)}(V_q^{\otimes d}) &= \mathcal{H}_d^A \\ S_q^A(n,d) &= \mathrm{End}_{\mathcal{H}_d^A}(V_q^{\otimes d}) \end{aligned}$$

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   - new Schur duality

Hecke algebra (of finite type B)

$$\underset{0}{\circ} \Longleftarrow \underset{1}{\circ} \text{ --- } \cdots \text{ --- } \underset{d-1}{\circ}$$

The Hecke algebra $\mathcal{H}_d^B$ is a $\mathbb{Q}(q)$-algebra generated by

$$\{T_i \mid i = 0, 1 \ldots, d-1\}$$

subject to

- Hecke relations $(T_i - q^{-1})(T_i + q) = 0$
- Braid relations (among T_i's)

Remark: $\mathcal{H}_d^A \hookrightarrow \mathcal{H}_d^B$

Hecke algebra action (of finite type B)

- $V_q^{\otimes d} \curvearrowleft \mathcal{H}_d^B$ is described by $V_q^{\otimes d} \curvearrowleft \mathcal{H}_d^A$ and $V_q^{\otimes d} \curvearrowleft T_0$
- e.g. $d = 2$,

$$(v_a \otimes v_b)T_0 = \begin{cases} v_{-a} \otimes v_b & \text{if } a > 0 \\ v_{-a} \otimes v_b + (q^{-1} - q)v_a \otimes v_b & \text{if } a < 0 \end{cases}$$

Quantum Schur duality of finite type B

We define the q-Schur algebra $S_q^\imath(n,d)$ of finite type B similarly.

$$\underset{q\text{-Schur algebra}}{S_q^\imath(n,d)} \curvearrowright \quad V_q^{\otimes d} \quad \curvearrowleft \underset{\text{Hecke algebra}}{\mathcal{H}_d^B}$$

Proposition (R. M. Green1997)

The algebras $S_q^\imath(n,d)$ and $\mathcal{H}_d^B$ satisfy double centralizer property.

Remark: There are other algebras having the right to be called the q-Schur algebra of type B: the q-Schur^2 algebra [Du-Scott2000] and the (Q,q)-Schur algebra [Dipper-James-Mathas1998].

Outline

Affine Hecke algebra (of type A)

The affine Hecke algebra $\widehat{\mathcal{H}}_d^A$ is a $\mathbb{Q}(q)$-algebra generated by

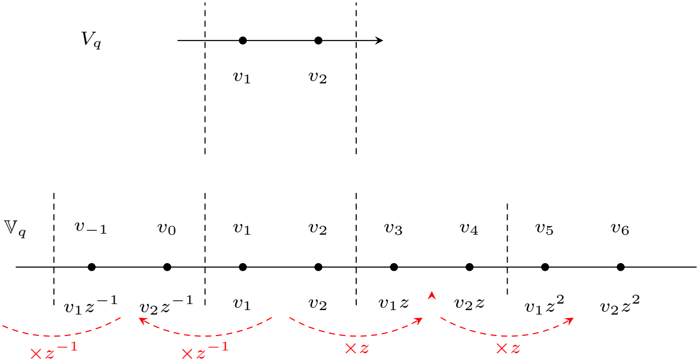
$$\{T_i | i = 1, \ldots, d-1\} \cup \{X_i, X_i^{-1} | i = 1, \ldots, d\}$$

such that

- $\langle T_i \rangle \simeq \mathcal{H}_d^A$
- $\langle X_i, X_i^{-1} \rangle$ is a Laurent polynomial ring
- T_i, X_j satisfy some mixed relations

Affine Hecke algebra action (of type A)

- We extend V_q periodically by setting $\mathbb{V}_q = V_q \otimes \mathbb{Q}(q)[z, z^{-1}]$ e.g.



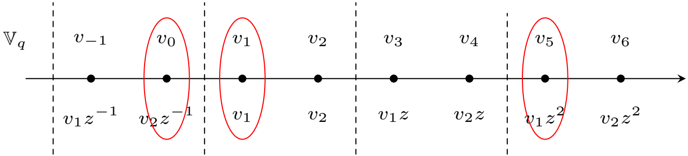
Each $v_f \in \mathbb{V}_q$ has a unique expression $v_f = \underbrace{v_i}_{\in V_q} z^a$

Affine Hecke algebra action (of type A)

- Each element $M_f = v_{f(1)} \otimes \ldots \otimes v_{f(d)} \in \mathbb{V}_q^{\otimes d}$ has a unique expression

$$\underbrace{M_{\overline{f}}}_{\in V_q^{\otimes d}} z_f = (v_{\overline{f}(1)} \otimes v_{\overline{f}(d)}) z_1^{a_1} \ldots z_d^{a_d}$$

- e.g.



$$\underbrace{v_1 \otimes v_5 \otimes v_0}_{\in \mathbb{V}_q^{\otimes 3}} = (\underbrace{v_1 \otimes v_1 \otimes v_2}_{\in V_q^{\otimes 3}}) z_2^2 z_3^{-1}$$

Affine Hecke algebra action (of type A)

- $\mathbb{V}_q^{\otimes d} \curvearrowleft \widehat{\mathcal{H}}_d^A$ by

$$M_f X_j = q^{2(j-1)}(\ldots \otimes v_{f(j)-n} \otimes \ldots)$$
$$M_f T_i = \begin{cases} M_{f\cdot s_i} & +(q^{-1}-q)M_{\overline{f}}P_0(f,i) & \text{if } \overline{f}(i+1) > \overline{f}(i) \\ q^{-1}M_{\overline{f}}z_{f\cdot s_i} & +(q^{-1}-q)M_{\overline{f}}P_0(f,i) & \text{if } \overline{f}(i+1) = \overline{f}(i) \\ M_{f\cdot s_i} & +(q^{-1}-q)M_{\overline{f}}P_1(f,i) & \text{if } \overline{f}(i+1) < \overline{f}(i) \end{cases}$$

Here $P_1, P_0 \in \mathbb{Z}[z_1, z_1^{-1} \ldots, z_d, z_d^{-1}]$

- In the central region ($z_f = 1$), $P_1 \equiv 1$ and $P_0 \equiv 0$ and hence the action is the same as in finite case.

Quantum Schur duality of affine type A

We define the q-Schur algebra $\widehat{S}_q^A(n,d)$ of affine type A similarly.

$$\underset{\text{affine } q\text{-Schur algebra}}{\widehat{S}_q^A(n,d)} \quad \curvearrowright \quad \mathbb{V}_q^{\otimes d} \quad \curvearrowleft \quad \underset{\text{affine Hecke algebra}}{\widehat{\mathcal{H}}_d^A}$$

Proposition (R. M. Green1999)

The algebras $\widehat{S}_q^A(n,d)$ and $\widehat{\mathcal{H}}_d^A$ satisfy double centralizer property.

Outline

Affine Hecke algebra (of type B)

The affine Hecke algebra $\widehat{\mathcal{H}}_d^B$ is a $\mathbb{Q}(q)$-algebra generated by

$$\{T_i \mid i = 0, 1 \ldots, d-1\} \cup \{X_j, X_j^{-1} \mid j = 1, \ldots, d\}$$

such that

- $\langle T_i \rangle \simeq \mathcal{H}_d^B$
- $\langle X_i, X_i^{-1} \rangle$ is a Laurent polynomial ring
- T_i, X_j satisfy some mixed relations

Affine Hecke algebra action (of type B)

- We want $\mathbb{V}_q^{\otimes d} \curvearrowleft \widehat{\mathcal{H}}_d^B$ by the same formulation

$$M_f T_i = \begin{cases} M_{f\cdot s_i} & +(q^{-1}-q)M_{\overline{f}}P_0(f,i) \quad \text{if } \overline{f}(i+1) > \overline{f}(i) \\ q^{-1}M_{\overline{f}}z_{f\cdot s_i} & +(q^{-1}-q)M_{\overline{f}}P_0(f,i) \quad \text{if } \overline{f}(i+1) = \overline{f}(i) \\ M_{f\cdot s_i} & +(q^{-1}-q)M_{\overline{f}}P_1(f,i) \quad \text{if } \overline{f}(i+1) < \overline{f}(i) \end{cases}$$

  where $P_1, P_0 \in \mathbb{Z}[z_0, z_0^{-1}, z_1, z_1^{-1}, \ldots, z_d, z_d^{-1}]$.
- The extra generator T_0 causes trouble since z_0 doesn't make sense in the affine A setting. We overcome it by defining $z_0 = z_1^{-1}$ and hence $P_1, P_0 \in \mathbb{Z}[z_1, z_1^{-1}, \ldots, z_d]$ fit perfectly in this picture.

Quantum Schur duality of affine type B

We define the q-Schur algebra $\widehat{S}^{\imath}_q(n,d)$ of affine type B similarly.

$$\underset{\text{affine } q\text{-Schur algebra}}{\widehat{S}^{\imath}_q(n,d)} \curvearrowright \mathbb{V}_q^{\otimes d} \curvearrowleft \underset{\text{affine Hecke algebra}}{\widehat{\mathcal{H}}^{B}_d}$$

Proposition (L-Luo-Wang, 2014)

The algebras $\widehat{S}^{\imath}_q(n,d)$ and $\widehat{\mathcal{H}}^{B}_d$ satisfy double centralizer property.

Quantum Schur-type dualities

finite type A	affine type A
$U_q(\mathfrak{gl}_n)$ $\downarrow$ $S_q^A(n,d) \quad \curvearrowright V_q^{\otimes d} \curvearrowleft \quad \mathcal{H}_d^A$	$U_q(\widehat{\mathfrak{gl}}_n)$ $\downarrow$ $\widehat{S}_q^A(n,d) \quad \curvearrowright \mathbb{V}_q^{\otimes d} \curvearrowleft \quad \widehat{\mathcal{H}}_d^A$
finite type B	**affine type B**
? $\downarrow$ $S_q^\imath(n,d) \quad \curvearrowright V_q^{\otimes d} \curvearrowleft \quad \mathcal{H}_d^B$	?? $\downarrow$ $\widehat{S}_q^\imath(n,d) \quad \curvearrowright \mathbb{V}_q^{\otimes d} \curvearrowleft \quad \widehat{\mathcal{H}}_d^B$

Remark: There is other type B duality replacing $U_q(\mathfrak{gl}_n)$ by $U_q(\mathfrak{so}_{2n+1})$ and $\mathcal{H}_d^B$ by q-Brauer algebra.

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   - The algebra $\widetilde{U}^\imath$
   - new Schur duality

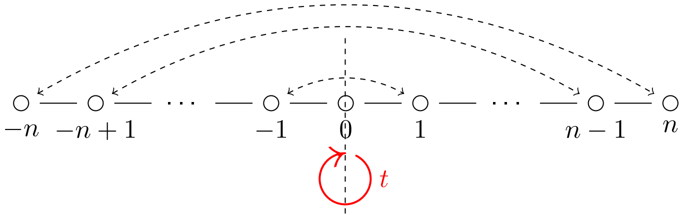
Coideal subalgebra (of finite type B)

- The algebra $U^\imath$ is generated by

$$\{e_i, f_i, k_i, k_i^{-1}\} \cup \{t\}$$

subject to some Serre-type relations

- t arises from the Dynkin diagram involution

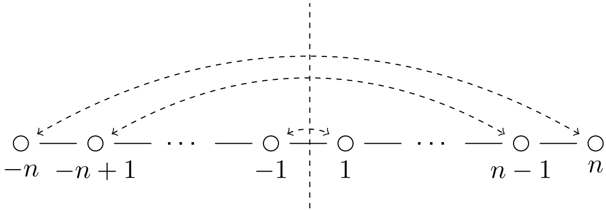


Coideal subalgebra (of finite type B)

- It has a counterpart, the algebra $U^\jmath$ generated by

$$\{e_i, f_i, k_i, k_i^{-1}\}$$

- From the Dynkin diagram involution one sees there is no t.



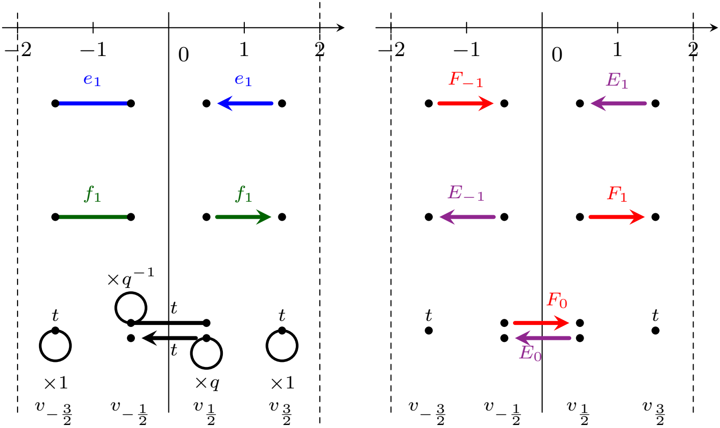
- In this talk we will focus on the more interesting algebra $U^\imath$.

Coideal subalgebra (of finite type B)

- $U^\imath$ is a subalgebra of $U = U_q(\mathfrak{sl}_\bullet) = \langle E_i, F_i, K_i, K_i^{-1} \rangle / \sim$
- The coproduct $\Delta : U \to U \otimes U$ restricts to $\Delta : U^\imath \to U^\imath \otimes U$.
  $\Rightarrow U^\imath$ is a (right) coideal subalgebra of U and $(U, U^\imath)$ form a quantum symmetric pair.

Coideal subalgebra action (of finite type B)

- $U^\imath$-action v.s. U-action



$\imath$-Schur duality of finite type B,

$$
\begin{array}{ccccccc}
U \supseteq & \underset{\text{coideal subalgebra}}{U^\imath} & \overset{\psi}{\curvearrowright} & V_q^{\otimes d} & \overset{\varphi}{\curvearrowleft} & \underset{\text{Hecke algebra}}{\mathcal{H}_d^B} \\
& \downarrow & & & & \\
& \underset{q\text{-Schur algebra}}{S_q^B(n,d)} & \curvearrowright & V_q^{\otimes d} & \curvearrowleft & \underset{\text{Hecke algebra}}{\mathcal{H}_d^B}
\end{array}
$$

Proposition (Bao-Wang, 2013)

1. The algebras $U^\imath$ and $\mathcal{H}_d^B$ satisfy double centralizer property.
2. $(U, U^\imath)$ form a quantum symmetric pair.

Example: commutivity

- This commutivity is highly nontrivial!

$$
\begin{tikzcd}
V_q \otimes V_q \arrow[r, "T_0"] \arrow[d, "t"'] & V_q \otimes V_q \arrow[d, "t"] \\
V_q \otimes V_q \arrow[r, "T_0"'] & V_q \otimes V_q
\end{tikzcd}
$$

$$
\begin{tikzcd}
v_{-\frac{1}{2}} \otimes v_{\frac{1}{2}} \arrow[r, mapsto] \arrow[d, mapsto] & \begin{array}{r} (q^{-1}-q)\, v_{-\frac{1}{2}} \otimes v_{\frac{1}{2}} \\ +\ v_{\frac{1}{2}} \otimes v_{\frac{1}{2}} \end{array} \arrow[d, mapsto] \\
\begin{array}{r} q^{-2}\, v_{-\frac{1}{2}} \otimes v_{\frac{1}{2}} \\ +q^{-1}\, v_{\frac{1}{2}} \otimes v_{\frac{1}{2}} \\ +\ v_{-\frac{1}{2}} \otimes v_{-\frac{1}{2}} \end{array} \arrow[r, mapsto] & \begin{array}{r} (q^{-1}-q)\, v_{-\frac{1}{2}} \otimes v_{-\frac{1}{2}} \\ +q^{-2}\, v_{\frac{1}{2}} \otimes v_{\frac{1}{2}} \\ +q^{-3}\, v_{-\frac{1}{2}} \otimes v_{\frac{1}{2}} \\ +\ v_{\frac{1}{2}} \otimes v_{-\frac{1}{2}} \end{array}
\end{tikzcd}
$$

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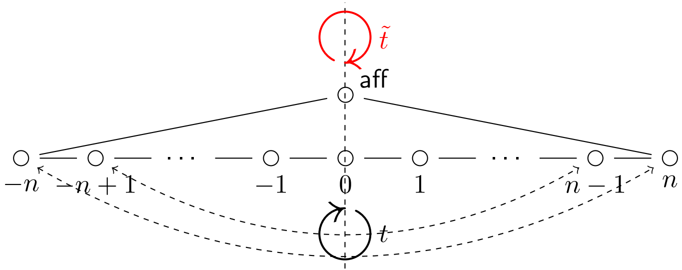
Constructing $\widetilde{U}^\imath$

- We want to construct an analogue in affine type B.
- The algebra $\widetilde{U}^\imath$ is a $\mathbb{Q}(q)$-algebra generated by

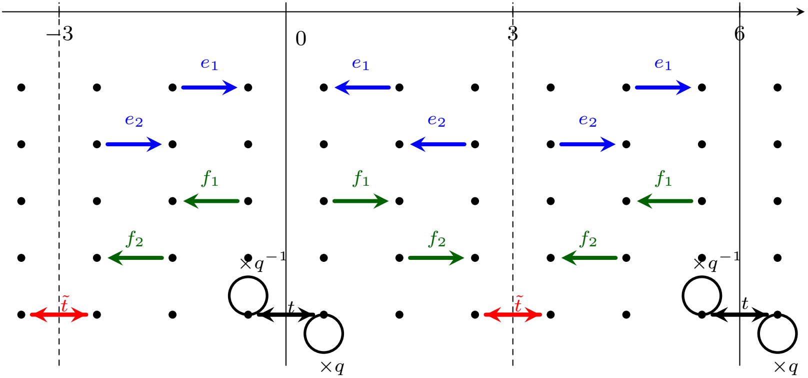
$$\{e_i, f_i, k_i, k_i^{-1}\} \cup \{t, \widetilde{t}\}$$

  subject to similar Serre-type relations
- $\tilde{t}$ arises from the Dynkin diagram involution



action of $\widetilde{U}^\imath$ (of affine type B)

- $\widetilde{U}^\imath$-action $\approx$ periodic $U^\imath$-action, while $\tilde{t}$ is slightly different.



$\imath$-Schur duality of affine type B

$$U_q(\widehat{\mathfrak{sl}}_\bullet) \supseteq \widetilde{U}^\imath \quad \curvearrowright \quad \mathbb{V}_q^{\otimes d} \quad \curvearrowleft \quad \underset{\text{affine Hecke algebra}}{\widehat{\mathcal{H}}_d^B}$$

Theorem (L-Luo-Wang2014)

The actions of $\widetilde{U}^\imath$ and $\widehat{\mathcal{H}}_d^B$ on $\mathbb{V}_q^{\otimes d}$ commute

Remark: $(U_q(\widehat{\mathfrak{sl}}_\bullet), \widetilde{U}^\imath)$ form a quantum symmetric pair

Question: How do we achieve double centralizer property?

$\imath$-Schur duality of affine type B

We plan to "extend" $\widetilde{U}^\imath$ to an affine coideal subalgebra $\widehat{U}^\imath \subseteq U_q(\widehat{\mathfrak{gl}}_\bullet)$ by adding some central elements

$$
\begin{array}{ccccccc}
U_q(\widehat{\mathfrak{sl}}_\bullet) & \supseteq & \widetilde{U}^\imath & & & & \\
\downarrow & & \downarrow & & & & \\
U_q(\widehat{\mathfrak{gl}}_\bullet) & \supseteq & \underset{\text{affine coideal subalgebra}}{\widehat{U}^\imath} & \curvearrowright & \mathbb{V}_q^{\otimes d} & \curvearrowleft & \underset{\text{affine Hecke algebra}}{\widehat{\mathcal{H}}_d^B} \\
 & & \downarrow & & & & \\
 & & \underset{\text{affine } q\text{-Schur algebra}}{\widehat{S}_q^\imath(n,d)} & & & &
\end{array}
$$

We expect (work in progress)

1. $\widehat{U}^\imath$ and $\widehat{\mathcal{H}}_d^B$ have double centralizer property.
2. $\widetilde{U}^\imath$ and $\widehat{\mathcal{H}}_d^B$ have double centralizer property if $n > d$.

Remark: Similar result holds for affine type A [Ginzburg-Vasserot, Lusztig, Green].

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