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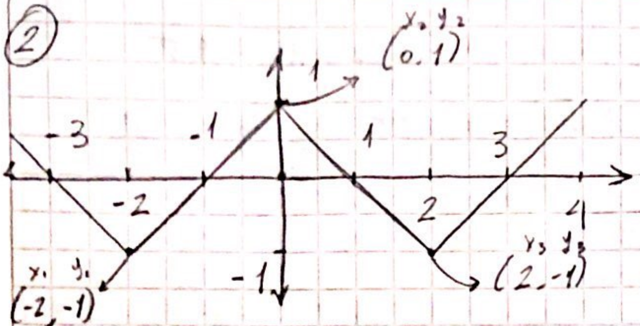
Parcial 2 - Señales y Sistemas

(2)



$$m_1 = \frac{1-(-1)}{0-(-2)} \Rightarrow \frac{2}{2} = 1$$

$$m_2 = \frac{-1-1}{2-0} = \frac{-2}{2} = -1$$

$$y_1 = m(x-x_0)+y_0 \Rightarrow 1(x-(-2))+(-1)$$

$$y_1 = x+2-1 \Rightarrow (x+1) = (t+1)$$

$$y_2 = m_2(x-x_2)+y_2 \Rightarrow -1(x-0)+(1) \Rightarrow (-x+1) = (-t+1)$$

$$a_0 = \frac{2}{4}\left(\int_{-2}^{0} x+1\,dt + \int_{0}^{2} -x+1\,dt\right) \Rightarrow \frac{1}{2}\left(\frac{x^2}{2}+x\Big|_{-2}^{0} + \left(-\frac{x^2}{2}+x\Big|_{0}^{2}\right)\right)$$

$$a_0 = \frac{1}{2}\left[-(2-2)+(-2+2)\right] = 0$$

$$a_k = \frac{1}{2}\left[\int_{-2}^{0} t\cos(k\omega t)\,dt + \int_{-2}^{0}\cos(k\omega t)\,dt - \int_{0}^{2} t\cos(k\omega t)\,dt + \int_{0}^{2}\cos(k\omega t)\right]$$

$u = t \qquad dv = \cos(k\omega t)dt$

$du = dt \qquad v = \frac{\sin(k\omega t)}{k\omega}$

$$a_k = \frac{1}{2}\left[\left(\frac{t\sin(k\omega t)}{k\omega}\Big|_{-2}^{0} - \frac{1}{k\omega}\int_{-2}^{0}\sin(k\omega t)\,dt\right) + \left(\frac{\sin(k\omega t)}{k\omega}\Big|_{-2}^{0}\right) \dots\right.$$

$$\left.\dots - \left(\frac{t\sin(k\omega t)}{k\omega}\Big|_{0}^{2} - \frac{1}{k\omega}\int_{0}^{2}\sin(k\omega t)\,dt\right) + \left(\frac{\sin(k\omega t)}{k\omega}\Big|_{0}^{2}\right)\right]$$

$$a_k = \frac{1}{2}\left[-\frac{1}{(k\omega)^2}\left(-\frac{\cos(k\omega t)}{k\omega}\Big|_{-2}^{0}\right) - \left(-\frac{1}{(k\omega)^2}\left(\frac{-\cos(k\omega t)}{k\omega}\Big|_{0}^{2}\right)\right)\right]$$

$$a_k = \frac{1}{2}\left[\frac{1}{(k\omega)^2}\left(-\left(-\cos(k\omega t)\Big|_{-2}^{0}\right) + \left(-\cos(k\omega t)\Big|_{0}^{2}\right)\right)\right]$$

$$a_k = \frac{1}{2}\left[\frac{1}{(k\omega)^2}\left(\overset{1}{\cos(\cancel{k\omega 0})} - \cos(-k\omega 2) - \left(\cos(k\omega 2) - \overset{1}{\cos(\cancel{k\omega 0})}\right)\right)\right]$$

$$a_k = \frac{1}{2}\left[\frac{1}{(k\omega)^2}\left(1 - \cos(k\omega 2) - \cos(k\omega 2) + 1\right)\right]$$

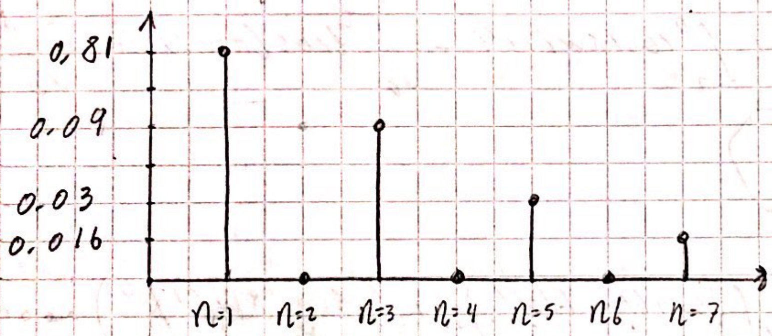
$$a_K = \frac{1}{2}\left[\frac{1}{(Kw)^2}\left(2 - 2\cos(Kw2)\right)\right] \quad ; \; Kw = 2\pi\frac{1}{4}\cdot 2 \cdot K = K\pi$$

$$K^2 w^2 = K^2 \cdot 4\pi^2 \cdot \frac{1}{16} \Rightarrow K^2\frac{\pi^2}{4}$$

$$a_K = \frac{1}{2}\left[\frac{2}{(Kw)^2}\left(1 - \cos(K\pi)\right)\right] = \frac{1}{K^2\frac{\pi^2}{4}}\left[1-(-1)^n\right]$$

$$a_K = \frac{4}{K^2\pi^2}\left(1-(-1)^n\right) \quad ; \quad a_n = \begin{cases} 0, & \text{Pares} \\ \frac{8}{K^2\pi^2}, & \text{impares} \end{cases}$$

$b_K = 0 \rightarrow$ Funcion Par



Para $n=1 = \frac{8}{1^2\pi^2} = 0{,}81$

Para $n=2 \Rightarrow 0$

Para $n=3 = \frac{8}{3^2\pi^2} = 0{,}09$

Para $n=4 \Rightarrow 0$

Para $n=5 \Rightarrow \frac{8}{5^2\pi^2} = 0{,}03$

Para $n=6 = 0$

Para $n=7 = \frac{8}{7^2\pi^2} = 0{,}016$

③

$$x(t) = \frac{1}{2}\,\mathrm{sgn}\left(t+\frac{1}{2}\right) + \frac{1}{2} - u\left(t-\frac{1}{2}\right)$$

$$y(t) = x(t) * h(t)$$

$$h(t) = 2u\left(t+\frac{1}{2}\right) - \mathrm{sgn}\left(t-\frac{1}{2}\right) - 1$$

Encontrar $y(t)$ y $Y(\omega)$:

Para $x(t) \Rightarrow$

① $\mathrm{sgn}(t) \leftrightarrow \frac{2}{j\omega} \rightarrow \mathrm{sgn}\left(t+\frac{1}{2}\right) \leftrightarrow \left(\frac{2}{j\omega}\right) e^{\frac{j\omega}{2}}$

$\rightarrow \frac{1}{2}\,\mathrm{sgn}\left(t+\frac{1}{2}\right) \leftrightarrow \frac{1}{2}\left(\frac{2}{j\omega}\right) e^{\frac{j\omega}{2}} \Rightarrow \frac{e^{\frac{j\omega}{2}}}{j\omega}$

② $\frac{1}{2} \leftrightarrow \frac{2}{2}\pi\delta(\omega) \Rightarrow \pi\delta(\omega)$

③ $u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega} \Rightarrow u\left(t-\frac{1}{2}\right) \leftrightarrow \left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{-\frac{j\omega}{2}}$

$$X(\omega) = \frac{e^{\frac{j\omega}{2}}}{j\omega} + \pi\delta(\omega) - \left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{-\frac{j\omega}{2}}$$

Para $h(t) \Rightarrow$

① $u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega} \rightarrow u\left(t+\frac{1}{2}\right) \leftrightarrow \left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{\frac{j\omega}{2}}$

$2u\left(t+\frac{1}{2}\right) \leftrightarrow 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{\frac{j\omega}{2}}$

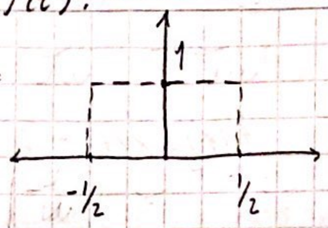
② $\mathrm{sgn}(t) \leftrightarrow \frac{2}{j\omega} \rightarrow \mathrm{sgn}\left(t-\frac{1}{2}\right) \leftrightarrow \left(\frac{2}{j\omega}\right) e^{-\frac{j\omega}{2}}$

③ $1 \leftrightarrow 2\pi\delta(\omega)$

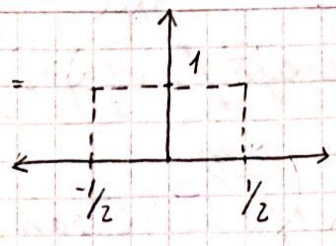
$$H(\omega) = 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{\frac{j\omega}{2}} - \left(\frac{2}{j\omega}\right) e^{-\frac{j\omega}{2}} - 2\pi\delta(\omega)$$

Para $y(t)$:

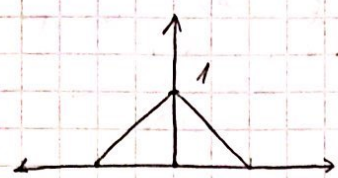
$x(t) =$



$h(t) =$



$y(t) = x(t) * h(t) \Rightarrow$



Para $Y(w)$:

$$X(w) = \frac{e^{\frac{jw}{2}}}{jw} - \frac{e^{-\frac{jw}{2}}}{jw} \quad , \quad h(w) = \frac{2e^{\frac{jw}{2}}}{jw} - \frac{2e^{-\frac{jw}{2}}}{jw}$$

$$Y(w) = \left[\frac{2}{w}\left(\frac{e^{\frac{jw}{2}} - e^{-\frac{jw}{2}}}{2j}\right)\right]\left[\frac{1}{w}\left(\frac{e^{\frac{jw}{2}} - e^{-\frac{jw}{2}}}{2j}\right)\right]$$

$$Y(w) = \frac{8\sin^2\left(\frac{w}{2}\right)}{w^2}$$

$Y(w) = X(w)\,H(w) \Rightarrow$