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Basic 1

1.1 .vimrc

```
syn on
    se ai nu ru cul mouse=a
1 | se cin et ts=2 sw=2 sts=2
   so $VIMRUNTIME/mswin.vim
    colo desert
    se gfn=Monospace\ 14
   noremap <buffer><F9> :! g++ -std=c++14 -02 -Wall - Wshadow '%' -o '%<'<CR>
   noremap <buffer><F5> :! './%<'<CR>
noremap <buffer><F6> :! './%<' < './%<.in'<CR>
noremap <buffer><F7> :! './%<' < './%<.in' > './%<.out'
```

$\mathbf{2}$ Math

5

6 6

Euclidean's Algorithm

```
// a must be greater than b
pair< int, int > gcd( int a, int b ) {
  if ( b == 0 ) return { 1, 0 };
  pair< int, int > q = gcd( b, b % a );
  return { q.second, q.first - q.second * ( a / b ) };
}
```

2.2Big Integer

```
const int base = 10000000000;
const int base_digits = 9;
class Bigint {
 public:
  vector< int > a;
  int sign;
  Bigint() : sign( 1 ) {}
Bigint( long long v ) { *this = v; }
Bigint( const string &s ) { read( s ); }
  void operator=( const Bigint &v ) {
    sign = v.sign;
    a = v.a;
  void operator=( long long v ) {
    sign = 1;
if ( v < 0 ) sign = -1, v = -v;
    for (; v > 0; v = v / base) a.push_back( v % base
  Bigint operator+( const Bigint &v ) const {
    if ( sign == v.sign ) {
      0);
res.a[i] += carry + (i < (int)a.size() ? a[
             i]:0);
         carry = res.a[ i ] >= base;
         if ( carry ) res.a[ i ] -= base;
      return res;
    return *this - ( -v );
  Bigint operator-( const Bigint &v ) const {
    if ( sign == v.sign ) {
      if ( abs() >= v.abs() ) {
        Bigint res = *this;
         for ( int i = 0, carry = 0; i < (int)v.a.size()</pre>
          || carry; ++i ) {
res.a[ i ] -= carry + ( i < (int)v.a.size() ?
    v.a[ i ] : 0 );</pre>
```

```
carry = res.a\lceil i \rceil < 0;
         if ( carry ) res.a[ i ] += base;
       res.trim();
      return res;
    return -( v - *this );
  }
  return *this + ( -v );
void operator*=( int v ) {
  if ( v < 0 ) sign = -sign, v = -v;
  for ( int i = 0, carry = 0; i < (int)a.size() ||
       carry; ++i ) {
    if ( i == (int)a.size() ) a.push_back( 0 );
long long cur = a[ i ] * (long long)v + carry;
carry = (int)( cur / base );
a[ i ] = (int)( cur % base );
  trim();
Bigint operator*( int v ) const {
  Bigint res = *this;
  res *= v;
  return res;
friend pair< Bigint, Bigint > divmod( const Bigint &
     a1, const Bigint &b1 ) {
  int norm = base / ( b1.a.back() + 1 );
Bigint a = a1.abs() * norm;
  Bigint b = b1.abs() * norm;
  Bigint q, r;
  q.a.resize( a.a.size() );
  for ( int i = a.a.size() - 1; i >= 0; i-- ) {
    r *= base;
    r += a.a[ i ];
    int s1 = r.a.size() <= b.a.size() ? 0 : r.a[ b.a.</pre>
         size() ];
    int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[
         b.a.size() - 1 ];
    int d = ((long long)base * s1 + s2) / b.a.back
    ();
r -= b * d;
    while ( r < 0 ) r += b, --d;
q.a[ i ] = d;
  q.sign = a1.sign * b1.sign;
  r.sign = a1.sign;
  q.trim();
  r.trim();
  return make_pair( q, r / norm );
Bigint operator/( const Bigint &v ) const { return
     divmod( *this, v ).first; }
Bigint operator%( const Bigint &v ) const { return
     divmod( *this, v ).second; }
void operator/=( int v ) {
  if (v < 0) sign = -sign, v = -v;
  for ( int i = (int)a.size() - 1, rem = 0; i >= 0;
    long long cur = a[ i ] + rem * (long long)base;
a[ i ] = (int)( cur / v );
    rem = (int)( cur % v );
  trim();
Bigint operator/( int v ) const {
  Bigint res = *this;
  res /= v;
  return res;
int operator%( int v ) const {
  if ( v < 0 ) v = -v;
  int m = 0;
  for ( int i = a.size() - 1; i >= 0; --i ) m = (a[
       i ] + m * (long long)base) % v;
```

```
return m * sign;
void operator+=( const Bigint &v ) { *this = *this +
void operator-=( const Bigint &v ) { *this = *this -
    v; }
void operator*=( const Bigint &v ) { *this = *this *
    v; }
void operator/=( const Bigint &v ) { *this = *this /
bool operator<( const Bigint &v ) const {</pre>
  if ( sign != v.sign ) return sign < v.sign;</pre>
  if ( a.size() != v.a.size() ) return a.size() *
      sign < v.a.size() * v.sign;</pre>
  for ( int i = a.size() - 1; i >= 0; i-- )
    if ( a[ i ] != v.a[ i ] ) return a[ i ] * sign <
        v.a[ i ] * sign;
  return false;
bool operator>( const Bigint &v ) const { return v <</pre>
    *this; }
bool operator<=( const Bigint &v ) const { return !(</pre>
v < *this ); }
bool operator>=( const Bigint &v ) const { return !(
    *this < v ); }
bool operator==( const Bigint &v ) const { return !(
*this < v ) && !( v < *this ); }
bool operator!=( const Bigint &v ) const { return *
    this < v | | v < *this; }
void trim() {
  while ( !a.empty() && !a.back() ) a.pop_back();
  if (a.empty()) sign = 1;
Bigint operator-() const {
  Bigint res = *this;
  res.sign = -sign;
  return res;
Bigint abs() const {
  Bigint res = *this;
  res.sign *= res.sign;
  return res;
long longValue() const {
  long long res = 0;
  for ( int i = a.size() - 1; i >= 0; i-- ) res = res
  * base + a[ i ];
return res * sign;
friend Bigint gcd( const Bigint &a, const Bigint &b )
     { return b.isZero() ? a : gcd(b, a % b ); }
void read( const string &s ) {
  sign = 1;
  a.clear();
  int pos = 0;
  while ( pos < (int)s.size() && ( s[ pos ] == '-' ||
    s[ pos ] == '+' ) ) {</pre>
    if (\bar{s}[po\bar{s}] == '-') sign = -sign;
    ++pos;
  for ( int i = s.size() - 1; i >= pos; i -=
      base_digits ) {
    a.push_back( x );
  trim();
friend istream &operator>>( istream &stream, Bigint &
    v ) {
  string s;
  stream >> s;
  v.read( s );
```

```
return stream:
friend ostream &operator<<( ostream &stream, const
    Bigint &v ) {
  if ( v.sign == -1 ) stream << '-';</pre>
  stream << ( v.a.empty() ? 0 : v.a.back() );
for ( int i = (int)v.a.size() - 2; i >= 0;
    stream << setw( base_digits ) << setfill( '0' )</pre>
         << v.a[ i ];
  return stream;
static vector< int > convert_base( const vector< int</pre>
    > &a, int old_digits, int new_digits ) {
  vector< long long > p( max( old_digits, new_digits
       ) + 1);
  p[0] = 1;
  for ( int i = 1; i < (int)p.size(); i++ ) p[ i ] =
   p[ i - 1 ] * 10;</pre>
  vector< int > res;
  long long cur = 0;
  int cur_digits = 0;
  for ( int i = 0; i < (int)a.size(); i++ ) {
  cur += a[ i ] * p[ cur_digits ];</pre>
    cur_digits += old_digits;
    while ( cur_digits >= new_digits ) {
       res.push_back( int( cur % p[ new_digits ] ) );
       cur /= p[ new_digits ];
       cur_digits -= new_digits;
    }
  res.push_back( (int)cur );
  while ( !res.empty() && !res.back() ) res.pop_back
      ();
  return res;
typedef vector< long long > vll;
static vll karatsubaMultiply( const vll &a, const vll
     &b ) {
  int n = a.size();
  vll res(n + n);
  if ( n <= 32 ) {
    for ( int i = 0; i < n; i++ )
      for ( int j = 0; j < n; j++ ) res[ i + j ] += a
   [ i ] * b[ j ];</pre>
    return res;
  int k = n \gg 1;
  vll a1( a.begin(), a.begin() + k);
  vll a2( a.begin() + k, a.end() );
vll b1( b.begin(), b.begin() + k );
  vll b2( b.begin() + k, b.end() );
  vll a1b1 = karatsubaMultiply( a1, b1 )
  vll a2b2 = karatsubaMultiply( a2, b2 );
  for ( int i = 0; i < k; i++ ) a2[ i ] += a1[ i ];
  for ( int i = 0; i < k; i++ ) b2[i] += b1[i];
  vll r = karatsubaMultiply(a2, b2)
  for ( int i = 0; i < (int)a1b1.size(); i++ ) r[ i ]</pre>
        -= a1b1[ i ];
  for ( int i = 0; i < (int)a2b2.size(); i++ ) r[ i ]
-= a2b2[ i ];
  for ( int i = 0; i < (int)r.size(); i++ ) res[ i +</pre>
       k ] += r[ i ];
  for ( int i = 0; i < (int)a1b1.size(); i++ ) res[ i
  ] += a1b1['i ];
for ( int i = 0; i < (int)a2b2.size(); i++ ) res[ i
        + n ] += a2b2[i];
  return res;
Bigint operator*( const Bigint &v ) const {
  vector< int > a6 = convert_base( this->a,
       base_digits, 6 );
  vector< int > b6 = convert_base( v.a, base_digits,
  6 );
vll a( a6.begin(), a6.end() );
vll b( b6.begin(), b6.end() );
  while ( a.size() < b.size() ) a.push_back( 0 )</pre>
  while ( b.size() < a.size() ) b.push_back( 0 );</pre>
```

3 Data Structure

3.1 Disjoint Set

```
class DisjointSet {
  public:
    static const int N = 1e5 + 10;
    int p[ N ];
    void Init( int x ) {
       for ( int i = 1; i <= x; ++i ) p[ i ] = i;
    }
    int Find( int x ) { return x == p[ x ] ? x : p[ x ] =
            Find( p[ x ] ); }
    void Union( int x, int y ) { p[ Find( x ) ] = Find( y
            ); }
};</pre>
```

3.2 Segement Tree with Lazy Tag

```
#define L( X ) ( X << 1 )
#define R( X ) ( ( X << 1 ) + 1 ) #define mid ( ( l + r ) >> 1 )
class SegmentTree {
 public:
   static const int N = 1e5 + 10;
   int arr[ N ], st[ N << 2 ], lazy[ N << 2 ];</pre>
  inline void Pull( int now ) { st[ now ] = max( st[ L(
    now ) ], st[ R( now ) ] ); }
inline void Push( int now, int l, int r ) {
     if ( lazy[ now ] != 0 ) {
        if ( l != r ) {
           st[ L( now ) ] += lazy[ now ];
st[ R( now ) ] += lazy[ now ];
lazy[ L( now ) ] += lazy[ now ];
lazy[ R( now ) ] += lazy[ now ];
        lazy[now] = 0;
   void Build( int now, int l, int r ) {
      if ( l == r ) {
        st[ now ] = arr[ l ];
        return;
      Build( L( now ), l, mid );
     Build( R(\text{now}), mid + 1, r);
     Pull( now );
   void Update( int ql, int qr, int value, int now, int
      l, int r ) {
if ( ql > qr || l > qr || r < ql ) return;</pre>
      Push( now, 1, r );
     if (`l == ql'&& qr == r ) {
  st[ now ] += value;
        lazy[ now ] += value;
        return;
```

```
if ( qr <= mid )</pre>
     Update(ql, qr, value, L( now ), l, mid ); else if ( mid < ql )
       Update( ql, qr, value, R( now ), mid + 1, r );
       Update( ql, mid, value, L( now ), l, mid );
       Update( mid + 1, qr, value, R(now), mid + 1, r
     Pull( now );
   int Query( int ql, int qr, int now, int l, int r ) {
  if ( ql > qr || l > qr || r < ql ) return 0;</pre>
     Push( now, 1, r );
if ( l == ql && qr == r ) return st[ now ];
     if ( qr <= mid )
     return Query( ql, qr, L( now ), l, mid );
else if ( mid < ql )</pre>
        return Query( ql, qr, R( now ), mid + 1, r );
     else {
        int left = Query( ql, mid, L( now ), l, mid );
        int right = Query( mid + 1, qr, R( now ), mid +
             1, r);
        int ans = max( left, right );
        return ans;
};
```

3.3 Copy on Write Segement Tree

```
// tested with ASC 29 B
#define mid ( (l + r) \gg 1 )
class Node {
public:
  int value, l, r, who;
  Node() {}
  Node( int_v ) : value( v ) { l = r = who = 0; }
class SegmentTree {
public:
  static const int N = 1e9;
  vector< Node > st;
  inline void Pull( int now ) {
    int lchild = st[ now ].l;
    int rchild = st[ now ].r;
if ( lchild != 0 ) {
       st[ now ].value = st[ lchild ].value;
       st[ now ].who = st[ lchild ].who;
    if ( rchild != 0 and st[ rchild ].value > st[ now
       ].value ) {
st[ now ].value = st[ rchild ].value;
       st[ now ].who = st[ rchild ].who;
    }
  void Build() {
    st.push_back( Node() ); // Null Node
st.push_back( Node( 0 ) );
  void Update( int ql, int qr, int value, int who, int
   now = 1, int l = 1, int r = N ) {
   if ( ql > qr or qr < l or ql > r ) return;
    if ( l == ql and qr == r ) {
      st[ now ].value = value;
st[ now ].who = who;
       return;
    if ( qr <= mid ) {</pre>
       if ( st[ now ].l == 0 ) {
         st[ now ].l = st.size();
         st.push_back( Node( 0 ) );
       Update( ql, qr, value, who, st[ now ].l, l, mid )
    else if ( mid < ql ) {</pre>
       if ( st[now].r == 0 )
         st[ now ].r = st.size();
```

```
st.push_back( Node( 0 ) );
       Update( ql, qr, value, who, st[ now ].r, mid + 1,
     else {
       if ( st[ now ].l == 0 ) {
  st[ now ].l = st.size();
          st.push_back( Node( 0 ) );
       if (st[now].r == 0) {
          st[now ].r = st.size();
          st.push_back( Node( 0 ) );
       Update( ql, mid, value, who, st[ now ].l, l, mid
       Update( mid + 1, qr, value, who, st[ now ].r, mid
             + 1, r );
     Pull( now );
   pair< int, int > Query( int ql, int qr, int now = 1,
     int l = 1, int r = N) {
if ( ql > qr or qr < l or ql > r ) return { 0, 0 };
     if ( l == ql and qr == r ) {
       return { st[ now ].value, st[ now ].who };
     if ( qr <= mid ) {</pre>
       if ( st[ now ].l == 0 ) return { 0, 0 };
       return Query( ql, qr, st[ now ].l, l, mid );
     else if ( mid < ql ) {</pre>
       if ( st[ now ].r == 0 ) return { 0, 0 };
return Query( ql, qr, st[ now ].r, mid + 1, r );
     else {
       pair< int, int > lchild = { 0, 0 };
       if ( st[ now ].l != 0 ) lchild = Query( ql, mid,
    st[ now ].l, l, mid );
pair< int, int > rchild = { 0, 0 };
       if ( st[ now ].r != 0 ) rchild = Query( mid + 1,
       qr, st[ now ].r, mid + 1, r );
pair< int, int > ans = { 0, 0 };
        if ( lchild.first > ans.first ) {
          ans.first = lchild.first;
          ans.second = lchild.second;
       if ( rchild.first > ans.first ) {
          ans.first = rchild.first;
          ans.second = rchild.second;
       return ans;
  }
};
```

3.4 Persistent Segement Tree

```
\ensuremath{//} tested with spoj MKTHNUM - K-th Number
#define mid ( (l+r) >> 1)
class Node {
 public:
  int value, l, r;
  Node() { value = l = r = 0; }
class SegmentTree {
 public:
  static const int N = 1e5 + 10;
  int ver_size, st_size;
  vector< int > ver;
  vector< Node > st;
  SegmentTree() {
    ver_size = st_size = 0;
    ver.resize( N );
st.resize( 70 * N );
    ver[ ver_size++ ] = 1;
    st[0] = st[1] = Node();
    st_size = 2;
```

```
void AddVersion() {
  ver[ ver_size++ ] = st_size++;
     st[ver[ver_size - 1]] = st[ver[ver_size - 2]
   inline void Pull( int now ) {
     int lchild = st[ now ].l, rchild = st[ now ].r;
st[ now ].value = st[ lchild ].value + st[ rchild
          ].value;
   void Build( int now = 1, int l = 1, int r = N ) {
     if ( l == r ) return;
     st[ now ].l = st_size++;
     st[now].r = st_size++;
     Build( st[ now ].l, l, mid );
Build( st[ now ].r, mid + 1, r );
     Pull( now );
   void Update( int prv_now, int now, int pos, int l =
     1, int r = N ) {
if ( l == r ) {
       st[ now ].value += 1;
       return;
     if ( pos <= mid ) {</pre>
       st[ now ].l = st_size++;
st[ st[ now ].l ] = st[ st[ prv_now ].l ];
       Update( st[ prv_now ].l, st[ now ].l, pos, l, mid
     else {
       st[ now ].r = st_size++;
       st[ st[ now ].r ] = st[ st[ prv_now ].r ];
Update( st[ prv_now ].r, st[ now ].r, pos, mid +
            1, r);
     Pull( now );
  pair< int, bool > Query( int prv_now, int now, int k,
         int l = 1, int r = N) {
     int prv_value = st[ prv_now ].value, now_value = st
          [ now ].value;
     if ( l == r and now_value - prv_value == k )
       return make_pair( l, true );
     else if ( now_value - prv_value < k )</pre>
       return make_pair( now_value - prv_value, false );
     pair< int, bool > child = Query( st[ prv_now ].l,
          st[ now ].l, k, l, mid );
     if ( child.second == false ) {
       k -= st[ st[ now ].l ].value - st[ st[ prv_now ].
l ].value;
       child = Query( st[ prv_now ].r, st[ now ].r, k,
            mid + 1, r);
     return child;
};
```

3.5 Rope

```
#include<ext/rope>
using namespace __gnu_cxx;
// inserts c before p.
iterator insert(const iterator& p, charT c) :
// inserts n copies of c before p.
iterator insert(const iterator& p, size_t n, charT c) :
// inserts the character c before the ith element.
void insert(size_t i, charT c) :
// erases the element pointed to by p.
void erase(const iterator& p) :
// erases the range [f, 1)
void erase(const iterator& f, const iterator& l) :
// Appends a C string.
void append(const charT* s) :
void replace(const iterator& f, const iterator& l,
    const rope& x)
void replace(const iterator& f, const iterator& l,
    const charT* s)
void replace(const iterator& f1, const iterator& l1,
    const charT* f2, const charT* 12)
```

```
void replace(const iterator& f1, const iterator& l1,
    const iterator& f2, const iterator& 12)
void replace(const iterator& p, const rope& x)
void replace(size_t i, size_t n, const rope& x)
void replace(size_t i, size_t n, charT c)
void replace(size_t i, size_t n, const charT* f, const
    charT* 1)
void replace(size_t i, size_t n, const iterator& f,
    const iterator& 1)
rope substr(iterator f, iterator l) const
rope substr(const_iterator f, const_iterator l) const
rope substr(size_t i, size_t n = 1) const
3.6 pb_ds
/***********PB_DS priority_queue***********/
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
typedef priority_queue<T,less<T>,pairing_heap_tag> PQ;
typedef PQ::point_iterator PQit;
point_iterator push(const_reference key)
void modify(point_iterator it, const_reference key)
void erase(point_iterator it)
T top()
void pop()
point_iterator begin()
point_iterator end()
void join(priority_queue &other)
template<class Pred> void split(Pred prd,
    priority_queue &other) //Other will contain only
    values v for which prd(v) is true. When calling
    this method, other's policies must be equivalent to this object's policies.
template<class Pred> size_type erase_if(Pred prd) //
    Erases any value satisfying prd; returns the number
     of value erased.
//1. push will return a point_iterator, which can be
    saved in a vector and modify or erase afterward.
//2. using begin() and end() can traverse all elements
    in the priority_queue.
```

//3. after join, other will be cleared.

and rc_binomial_heap_tag

binary_heap_tag

//similar to std::map

size(), return end()

錄子樹的mapped_value的和。

class _Alloc> struct my_nd_upd {

TREE;

 $//when T = _$

//4. for optimizing Dijkstra, use pairing_heap

//7. when using only push, pop and join, use

#include <ext/pb_ds/assoc_container.hpp> #include <ext/pb_ds/tree_policy.hpp>

//8. when using modify, use pairing_heap_tag or

//5. binary_heap_tag is better that std::priority_queue

//6. pairing_heap_tag is better than binomial_heap_tag

thin_heap_tag /*********************************/

typedef tree<K, T, less<K>, rb_tree_tag, Node_Update>

//1. iterator find_by_order(size_type order) return the smallest order-th element(e.x. when order = 0

return the smallest element), when order > TREE.

//2. size_type order_of_key(const_reference key) return

other, 然後把*this當中所有大於key的元素移到other

//自定義Node_Update : 查詢子段和的map<int, int>,需要紀

template<class Node_CItr, class Node_Itr, class Cmp_Fn,</pre>

typedef int metadata_type ; //額外信息,這邊用int inline void operator()(Node_Itr it,Node_CItr end_it){

Node_Itr l=it.get_l_child(), r=it.get_r_child();

void join(tree &other) //other和*this的值域不能相交

void split(const_reference key, tree &other) // 清空

number of elements smaller than key

virtual Node_CItr node_begin () const = 0;

int left = 0 , right = 0;
if(l != end_it) left = l.get_metadata();

virtual Node_CItr node_end () const = 0;

_gnu_pbds::null_type, become std::set //when Node_Update = tree_order_statistics_node_update, TREE become a ordered TREE with two new functions:

```
if(r != end_it) right = r.get_metadata();
const_cast<metadata_type&>(it.get_metadata())=
      left+right+(*it)->second;
  //operator()功能是將節點it的信息更新, end_it表空節點
  //it是Node_Itr, *之後變成iterator, 再取->second變節點
       的mapped_value
  inline int prefix_sum (int x) {
    int ans = 0;
    Node_CItr it = node_begin();
    while(it!=node_end()){
      Node_CItr l = it.get_l\_child() , r = it.
      get_r_child();
if(Cmp_Fn()(x , (*it)->first)) it = 1;
      else {
         ans += (*it)->second;
         if(l != node_end ()) ans += l.get_metadata();
         it = r;
      }
    return ans;
  inline int interval_sum(int l ,int r)
  {return prefix_sum(r)-prefix_sum(l-1);}
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/hash_policy.hpp>
__gnu_pbds::cc_hash_table<Key, Mapped>
__gnu_pbds::gp_hash_table<Key, Mapped>
//支援find和operator[]
/*********************************/
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/trie_policy.hpp>
typedef trie<string, null_type,
    trie_string_access_traits<>, pat_trie_tag,
              trie_prefix_search_node_update> pref_trie;
pref_trie.insert(const string &str);
auto range = pref_trie.prefix_range(const string &str);
for(auto it = range.first; it != range.second; ++it)
cout << *it << '\n';</pre>
                       push
                                     modify
                                               erase
                                                        join
                              pop
   std::priority_queue
                       \lg(n)
                              \lg(n)
                                     n \lg(n)
                                               n \lg(n)
                                                       n \lg(n)
   pairing_heap_tag
                              \lg(n)
                                      \lg(n)
                                               \lg(n)
    binary\_heap\_tag
                       \lg(n)
                              \lg(n)
                                                         n
 binomial_heap_tag
rc_binomial_heap_tag
                              \lg(n)
                                      \lg(n)
                                               \lg(n)
                                                       \lg(n)
                              lg(n)
                                      lg(n)
                                               lg(n)
                                                       \lg(n)
    thin_heap_tag
                                    \lg(n)[ps]
                                               \lg(n)
                              \lg(n)
                                                         n
ps: 1 if increased_key only else \lg(n)
```

4 graph

4.1 Dijkstra's Algorithm

```
vector< pair< int, int > > v[ N ];
vector< int > Dijkstra( int s ) {
  // n: number of nodes
  vector< int > d(n + 1, 1e9);
  vector< bool > visit( n + 1, false );
  d[s] = 0;
  priority_queue< pair< int, int >, vector< pair< int,
   int > >, greater< pair< int, int > > > pq;
pa.push( make_pair( d[ s ], s ) );
  while (1) {
     int now = -1;
     while ( !pq.empty() and visit[ now = pq.top().
           second ] ) pq.pop()
     if ( now == -1 or visit[ now ] ) break;
     visit[ now ] = true;
     for ( int i = 0; i < v[ now ].size(); ++i ) {
  int child = v[ now ][ i ].first;
  int w = v[ now ][ i ].second;</pre>
       if ( !visit[ child ] and ( d[ now ] + w ) < d[</pre>
          child ] ) {
d[ child ] = d[ now ] + w;
```

```
pq.push( make_pair( d[ child ], child ) );
}
}
return d;
```

4.2 Tarjan's Algorithm

4.3 Jump Pointer Algorithm

```
// Build: O( VlogV ), Query: O( logV )
int tin[ N ], tout[ N ], ancestor[ N ][ 20 ];
vector< int > v[ N ];
void dfs( int now, int pnow ) {
   tin[ now ] = ++now_time;
   ancestor[ now ][ 0 ] = pnow;
   for ( int i = 1; i < 20; ++i )
     ancestor[ now ][ i ] = ancestor[ ancestor[ now ][ i - 1 ] ][ i - 1 ];
   for ( auto child : v[ now ] )
  if ( child != pnow ) dfs( child, now );
   tout[ now ] = ++now_time;
bool check_ancestor( int x, int y ) { return ( tin[ x ]
<= tin[ y ] and tout[ x ] >= tout[ y ] ); }
int find_lca( int x, int y ) {
  if ( check_ancestor( x, y ) ) return x;
if ( check_ancestor( y, x ) ) return y;
   for ( int i = 19; i >= 0; --i )
      if ( !check_ancestor( ancestor[ x ][ i ], y ) ) x =
   ancestor[ x ][ i ];
return ancestor[ x ][ 0 ];
}
```

5 Flow

5.1 Bipartite Matching

```
// O( ( V + E ) * sqrt( V ) )
class BipartiteMatching {
public:
    static const int N = 1e5 + 10;  // total number of
        nodes, n + m
    static const int NIL = 0;
    static const int INF = ( 1 << 28 );
    vector< int > G[ N ];
    int n, m, match[ N ], dist[ N ];
```

```
// n: number of nodes on left side, nodes are
       numbered 1 to n
  // m: number of nodes on right side, nodes are
       numbered n+1 to n+m
  // G = NIL[0] \cup G1[G[1---n]] \cup G2[G[n+1---n+m]]
  bool BFS() {
    int i, u, v, len;
queue< int > Q;
    for ( i = 1; i <= n; i++ )
       if ( match[ i ] == NIL ) {
  dist[ i ] = 0;
         Q.push( i );
       else
         dist[ i ] = INF;
    dist[ NIL ] = INF;
    while ( !Q.empty() ) {
       u = Q.front();
       Q.pop();
       if ( u != NIL ) {
         len = G[ u ].size();
         for ( i = 0; i < len; i++ ) {
    v = G[ u ][ i ];
           if ( dist[ match[ v ] ] == INF ) {
  dist[ match[ v ] ] = dist[ u ] + 1;
  Q.push( match[ v ] );
           }
         }
      }
    return ( dist[ NIL ] != INF );
  bool DFS( int u ) {
    int i, v, len;
    if ( u != NIL ) {
       len = G[u].size();
       for ( i = 0; i < len; i++ ) {
    v = G[ u ][ i ];
         if ( dist[ match[ v ] ] == dist[ u ] + 1 ) {
           if ( DFS( match[ v ] ) ) {
              match[ v ] = u;
match[ u ] = v;
              return true;
           }
         }
       dist[ u ] = INF;
       return false;
    return true;
  int HopcroftKarp() {
    int matching = 0, i;
    // match[] is assumed NIL for all vertex in G
    while ( BFS() )
       for (i = 1; i \le n; i++)
         if ( match[ i ] == NIL && DFS( i ) ) matching
    return matching;
  void AddEdge( int u, int v ) { G[ u ].push_back( n +
  int Solve() { return HopcroftKarp(); }
};
```

5.2 MaxFlow (ISAP)

```
vector< Edge > G[ MAXV * 2 ];
   int iter[ MAXV * 2 ], d[ MAXV * 2 ], gap[ MAXV * 2 ],
   void Init( int x ) {
      tot = x + 2;
      s = x + 1, t = x + 2;

for (int i = 0; i \le tot; i + + 1) {
              ].clear();
        iter[ i ] = d[ i ] = gap[ i ] = 0;
      }
   void AddEdge( int u, int v, int c ) {
      G[ u ].push_back( Edge( v, c, SZ( G[ v ] ) ) );
G[ v ].push_back( Edge( u, 0, SZ( G[ u ] ) - 1 ) );
   int DFS( int p, int flow ) {
  if ( p == t ) return flow;
  for ( int &i = iter[ p ]; i < SZ( G[ p ] ); i++ ) {</pre>
        Edge &e = G[ p ][ i ];
        if (e.c > 0 \& \bar{d}[p] == d[e.v] + 1) {
           int f = DFS( e.v, min( flow, e.c ) );
           if (f) {
             e.c -= f
             G[ e.v ][ e.r ].c += f;
             return f;
           }
        }
      if ( --gap[d[p]] ) == 0 )
        d[ s ] = tot;
      else {
        d[ p ]++;
iter[ p ] = 0;
++gap[ d[ p ] ];
      return 0;
   int Solve() {
      int res = 0;
gap[ 0 ] = tot;
      for ( res = 0; d[ s ] < tot; res += DFS( s, INF ) )</pre>
      return res;
   }
};
```

5.3 MinCostMaxFlow

```
// 0( V^2 * F )
class MinCostMaxFlow {
 public:
  static const int MAXV = 2000;
  static const int INF = 1e9;
  struct Edge {
     int v, cap, w, rev;
     Edge() {}
     Edge( int t2, int t3, int t4, int t5 ) : v( t2 ),
         cap( t3 ), w( t4 ), rev( t5 ) {}
  int V, s, t;
  vector< Édge > g[ MAXV ];
  void Init( int n ) {
    V = n + 4;
                               // total number of nodes
// s = source, t = sink
    s = n + 1, t = n + 4;
     for ( int i = 1; i <= V; i++ ) g[ i ].clear();</pre>
  // cap: capacity, w: cost
  void AddEdge( int a, int b, int cap, int w ) {
    g[ a ].push_back( Edge( b, cap, w, (int)g[ b ].size
          () );
     g[ b ].push_back( Edge( a, 0, -w, (int)g[ a ].size
         () - 1 ) );
  int d[ MAXV ], id[ MAXV ], mom[ MAXV ];
bool inqu[ MAXV ];
int qu[ 2000000 ], ql, qr;
// the size of qu should be much large than MAXV
  int MncMxf() {
     int INF = INF;
     int mxf = 0, mnc = 0;
```

```
while (1) {
fill(d+1,d+1+V,INF);
       fill( inqu + 1, inqu + 1 + V, 0 );
      fill( mom + 1, mom + 1 + V, -1);
      mom[s] = s;
      d[\bar{s}] = 0;
       ql = 1, qr = 0;
      qu[ ++qr ] = s;
inqu[ s ] = 1;
      while ( ql <= qr )</pre>
         int u = qu[ ql++ ];
         inqu[ u ] = 0;
         for ( int i = 0; i < (int)g[ u ].size(); i++ )</pre>
           Edge \&e = g[u][i];
           int v = e.v;
           if ( e.cap > 0 && d[ v ] > d[ u ] + e.w ) {
             d[ v ] = d[ u ] + e.w;
mom[ v ] = u;
             id[ v ] = i;
             if (!inqu[v]) qu[++qr] = v, inqu[v]
         }
      if ( mom[ t ] == -1 ) break;
       int df = INF;
       for ( int u = t; u != s; u = mom[ u ] ) df = min(
            df, g[ mom[ u ] ][ id[ u ] ].cap );
       for ( int u = t; u != s; u = mom[ u ] ) {
   Edge &e = g[ mom[ u ] ][ id[ u ] ];
         e.cap -= df;
         g[e.v][e.rev].cap += df;
      mxf += df;
      mnc += df * d[ t ];
    return mnc;
};
```

5.4 BoundedMaxFlow

// node from $0 \sim \text{size} - 1$

```
class Graph {
public:
  Graph( const int &size )
      : size_( size + 2 ),
        source_( size ),
        sink_{size} + 1),
        edges_( size_ ),
        capacity_( size_, vector< int >( size_, 0 ) );
        lower_bound_( size_, vector< int >( size_, 0 )
        lower_bound_sum_( size_, 0 ) {}
 void AddEdge( int from, int to, int lower_bound, int
      capacity ) {
    edges_[ from ].push_back( to );
    edges_[ to ].push_back( from );
    capacity_[ from ][ to ] += capacity - lower_bound;
    lower_bound_[ from ][ to ] += lower_bound;
    lower_bound_sum_[ from ] += lower_bound;
    lower_bound_sum_[ to ] -= lower_bound;
  int MaxFlow() {
    int expected_source = 0, expected_sink = 0;
    for ( int i = 0; i < source_; ++i )
  if ( lower_bound_sum_[ i ] > 0 ) {
        capacity_[ i ][ sink_ ] = lower_bound_sum_[ i
        edges_[ i ].push_back( sink_ );
edges_[ sink_ ].push_back( i );
        expected_sink += lower_bound_sum_[ i ];
      else if ( lower_bound_sum_[ i ] < 0 ) {</pre>
         capacity_[ source_ ][ i ] = -lower_bound_sum_[
             i ];
        edges_[ source_ ].push_back( i );
```

```
expected_source -= lower_bound_sum_[ i ];
   int Flow = 0;
   while ( BFS( source_, sink_ ) )
for ( auto &from : edges_[ sink_ ] ) {
       if ( from_[ from ] == -1 ) continue;
        from_[ sink_ ] = from;
        int current_Flow = numeric_limits< int >::max()
        for ( int i = sink_; i != source_; i = from_[ i
             1)
          current_Flow = min( current_Flow, capacity_[
        from_[ i ] ][ i ] );
if ( not current_Flow ) continue;
        for ( int i = sink_; i != source_; i = from_[ i
             ]){
          capacity_[ from_[ i ] ][ i ] -= current_Flow;
capacity_[ i ][ from_[ i ] ] += current_Flow;
       Flow += current_Flow;
   if ( Flow != expected_source ) return -1;
   return Flow;
 int Flow( int from, int to ) { return lower_bound_[
     from ][ to ] + capacity_[ to ][ from ]; }
private:
 bool BFS( int source, int sink ) {
   queue< int > Q;
   Q.push( source );
   from_ = vector< int >( size_, -1 );
from_[ source ] = source;
   while ( !Q.empty() ) {
     int node = Q.front();
     Q.pop();
     if ( node == sink ) continue;
     for ( auto &neighbour : edges_[ node ] )
       if ( from_[ neighbour ] == -1 and capacity_[
            node ][ neighbour ] > 0 ) {
          from_[ neighbour ] = node;
          Q.push( neighbour);
   return from_[ sink ] != -1;
int size_, source_, sink_;
vector< vector< int > > edges_
 vector< vector< int > > capacity_;
 vector< vector< int > > lower_bound_;
 vector< int > lower_bound_sum_;
 vector< int > from_;
```