a) $E(b_R) = E\big((X_1'X_1)^{-1}X_1'(X_1\beta_1 + X_2\beta_2 + \varepsilon)\big)$

$= E((X_1'X_1)^{-1}X_1'X_1\beta_1 + (X_1'X_1)^{-1}X_1'X_2\beta_2 + (X_1'X_1)^{-1}X_1'\varepsilon)$

$Using\ (X_1'X_1)^{-1}X_1'X_1 = I,\ E(\varepsilon) = 0\ and\ X, \beta\ are\ fixed:$

$E(b_R) = \beta_1 + P\beta_2,\ where\ P = (X_1'X_1)^{-1}X_1'X_2$

b) $var(b_R) = E\left(\big(b_R - E(b_R)\big)\big(b_R - E(b_R)\big)'\right)$

$$\begin{aligned}
b_R - E(b_R) &= (X_1'X_1)^{-1}X_1'(X_1\beta_1 + X_2\beta_2 + \varepsilon) - (\beta_1 + P\beta_2) \\
&= \beta_1 + P\beta_2 + (X_1'X_1)^{-1}X_1'\varepsilon - \beta_1 - P\beta_2 \\
&= (X_1'X_1)^{-1}X_1'\varepsilon
\end{aligned}$$

$\big(b_R - E(b_R)\big)' = ((X_1'X_1)^{-1}X_1'\varepsilon)' = (\varepsilon'X_1(X_1'X_1)^{-1})$

$=> var(b_R) = E((X_1'X_1)^{-1}X_1'\varepsilon\varepsilon'X_1(X_1'X_1)^{-1})$

$Using\ E(\varepsilon\varepsilon') = \sigma^2 I,$

$var(b_R) = (X_1'X_1)^{-1}X_1'\sigma^2 I X_1(X_1'X_1)^{-1} = \sigma^2(X_1'X_1)^{-1}$

c) $b_R = (X_1'X_1)^{-1}X_1'y = (X_1'X_1)^{-1}X_1'(X_1b_1 + X_2b_2 + e) = b_1 + Pb_2 + (X_1'X_1)^{-1}X_1'e$

$Due\ to\ orthogonality, X_1'e = 0. Hence,$
$b_R = b_1 + Pb_2$

d) $P = (X_1'X_1)^{-1}X_1'X_2, where:$
$X_1': (2 \times 500)\ matrix$
$X_1: (500 \times 2)\ matrix$
$=> (X_1'X_1)^{-1}: (2 \times 2)\ matrix$
$=> (X_1'X_1)^{-1}X_1': (2 \times 500)\ matrix$
$X_2: (500 \times 3)\ matrix$
$=> (X_1'X_1)^{-1}X_1'X_2: (2 \times 3)\ matrix$
$Hence\ P\ is\ matrix\ with\ the\ following\ 3\ columns:$
$Column\ 1: (X_1'X_1)^{-1}X_1'(Age)$
$Column\ 2: (X_1'X_1)^{-1}X_1'(Parttime)$
$Column\ 3: (X_1'X_1)^{-1}X_1'(Educ)$

e) $In\ lecture\ 2.1:$

$$Age = 40.05 - 0.11Female$$
$$Educ = 2.26 - 0.49Female$$
$$Parttime = 0.2 + 0.25Female$$

$$From\ this: P = \begin{bmatrix} 40.05 & 2.26 & 0.2 \\ -0.11 & -0.49 & 0.25 \end{bmatrix}$$

f) $In\ lecture\ 2.5, the\ following\ results\ were\ found:$
$$Log(Wage)_i = 3.053 - 0.041Female_i + 0.031Age_i + 0.233Educ_i - 0.365Parttime_i + e_i$$

$This\ concludes$:

$$b_1 = \begin{bmatrix} 3.053 \\ -0.041 \end{bmatrix} \qquad b_2 = \begin{bmatrix} 0.031 \\ 0.233 \\ -0.365 \end{bmatrix}$$

Then,
$b_R = b_1 + Pb_2$

Taking P from part e);

$$b_R = \begin{bmatrix} 3.053 \\ -0.041 \end{bmatrix} + \begin{bmatrix} 40.05 & 2.26 & 0.2 \\ -0.11 & -0.49 & 0.25 \end{bmatrix} \begin{bmatrix} 0.031 \\ 0.233 \\ -0.365 \end{bmatrix}$$
$$= \begin{bmatrix} 4.748 \\ -0.2498 \end{bmatrix}$$

In lecture 2.1, the results for b_R were as follows;

$$= \begin{bmatrix} 4.73 \\ -0.25 \end{bmatrix}$$

Showing minor differences in values which can be explained by the rounding off of the coefficients in the lecture slides.