Questions

Consider again the application in lecture 5.5, where we have analyzed response to a direct mailing using the following logit specification

$$Pr[resp_i = 1] = \frac{exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

for i = 1, . . . , 925.

The maximum likelihood estimates of the parameters are given by Variable Coefficient Std. Error t-value p-value Intercept -2.488 0.890 -2.796 0.005 Male 0.954 0.158 6.029 0.000 Active 0.914 0.185 4.945 0.000 Age 0.070 0.036 1.964 0.050 (Age/10)2 -0.069 0.034 -2.015 0.044

(a) Show that

$$\frac{\partial Pr[respi = 1]}{\partial agei} + \frac{\partial Pr[respi = 0]}{\partial agei} = 0$$

ans:

$$Pr[respi = 0] = 1 - Pr[respi = 0]$$
$$\frac{\partial Pr[respi = 1]}{\partial agei} = -\frac{\partial Pr[respi = 0]}{\partial agei}$$

then

$$\frac{\partial Pr[respi = 1]}{\partial agei} + \frac{\partial Pr[respi = 0]}{\partial agei} = 0$$

(b) Assume that you recode the dependent variable as follows:

$$respnew_i = -resp_i + 1$$

. Hence, positive response is now defined to be equal to zero and negative response to be equal to 1. Use the odds ratio to show that this transformation implies that the sign of all parameters change.

ans:

$$odds = \frac{Pr[resp_i^{new} = 1]}{Pr[resp_i^{new} = 0]} = \frac{Pr[resp_i = 0]}{Pr[resp_i = 1]}$$

$$e^{(\beta_0^{new} + \beta_1^{new} male_i + \beta_2^{new} active_i + \beta_3^{new} age_i + \beta_4^{new} (age_i/10)^2)} = e^{-(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

then

$$\beta_0^{new} = \beta_0, \beta_1^{new} = \beta_1, \beta_2^{new} = \beta_2, \beta_3^{new} = \beta_3, \beta_4^{new} = \beta_4$$

(c) Consider again the odds ratio positive response versus negative response:

$$\frac{Pr[resp_i = 1]}{Pr[resp_i = 0]} = exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)$$

During lecture 5.5 you have seen that this odds ratio obtains its maximum value for age equal to 50 years for males as well as females. Suppose now that you want to extend the logit model and allow that this age value is possibly different for males than for females. Discuss how you can extend the logit specification.

ans: create a dummy variables for female to count on the possible different for males and females.

$$\frac{Pr[resp_i = 1]}{Pr[resp_i = 0]} = exp(\beta_0 + r_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2 + r_2 female_i)$$