

## Questions

This test exercise uses data that are available in the data file TestExer6. The question of interest is to model monthly inflation in the Euro area and to investigate whether inflation in the United States of America has predictive power for inflation in the Euro area. Monthly data on the consumer price index (CPI) for the Euro area and the USA are available from January 2000 until December 2011. The data for January 2000 until December 2010 are used for specification and estimation of models, and the data for 2011 are left out for forecast evaluation purposes.

(a) Make time series plots of the CPI of the Euro area and the USA, and also of their logarithm  $\log(\text{CPI})$  and of the two monthly inflation series  $\text{DP} = \Delta \log(\text{CPI})$ . What conclusions do you draw from these plots?

**ans:**

1. cointegration may happens between CPI of the Euro and USA area also of their logarithm  $\log(\text{CPI})$
2. LOG CPI and CPI might not be stationary
3. DP might be stationary

In [1]:

```

%matplotlib inline
import sys
sys.path.append('/Users/CJ/Documents/bitbucket/xforex_v1/xforex_v3')
import pandas as pd
import matplotlib.pyplot as plt
from datetime import datetime
from xforex.BackTesting.econometrics_tools import Econometrics_Tool
import numpy as np
import pprint as pp
import statsmodels.tsa.stattools as ts
from statsmodels.tsa.ar_model import AR
import pandas
from dateutil.relativedelta import relativedelta

dat = pd.read_csv(
    '/Users/CJ/Documents/bitbucket/xforex_v1/xforex_v3/training/econometric
s/week6-time-series/Test6-CPI-round1.txt', sep = '\t')

def get_date_index():
    start = datetime.strptime("01-2000", "%m-%Y")
    end = datetime.strptime("12-2011", "%m-%Y")
    date_generated = []
    date = start
    date_generated.append(date)
    while date < end:
        date = date + relativedelta(months=1)
        date_generated.append(date)
    return date_generated

dat.index = pd.Index(get_date_index())
dat.head()

```

Out[1]:

	YEAR	TREND	CPI_EUR	CPI_USA	LOGPEUR	LOGPUSA	DPEUR	DPUSA
2000-01-01	2000M01	1	105.1	107.6	4.654912	4.678421	NaN	NaN
2000-02-01	2000M02	2	105.4	108.3	4.657763	4.684905	0.002850	0.0064
2000-03-01	2000M03	3	105.8	109.1	4.661551	4.692265	0.003788	0.0073
2000-04-01	2000M04	4	105.9	109.2	4.662495	4.693181	0.000945	0.0009
2000-05-01	2000M05	5	106.0	109.3	4.663439	4.694096	0.000944	0.0009

In [2]:

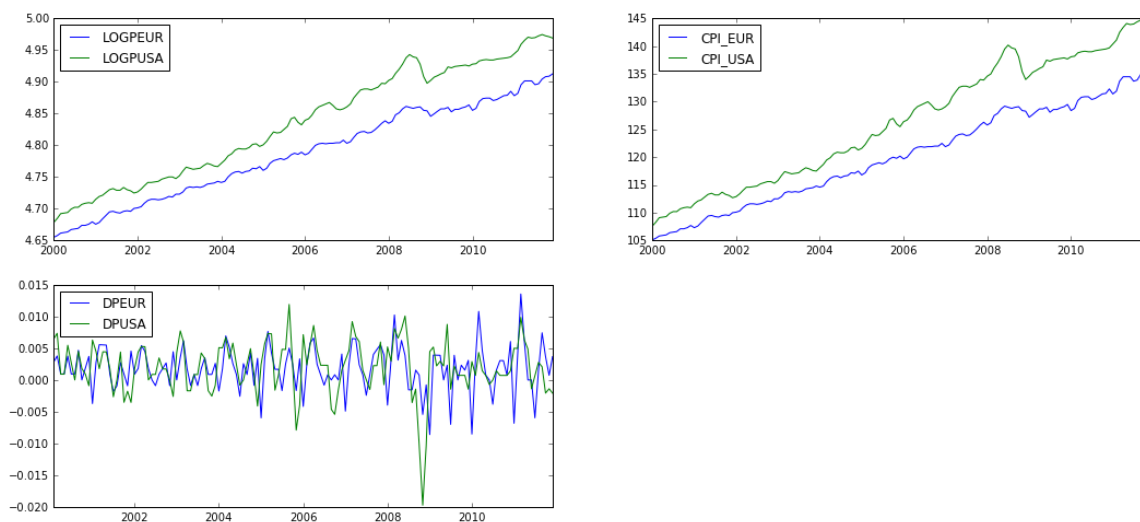
```
plt.figure(1, figsize=(18, 8))
plt.subplot(2,2,1)
plt.plot(dat.index,dat['LOGPEUR'],label = 'LOGPEUR')
plt.plot(dat.index,dat['LOGPUSA'],label = 'LOGPUSA')
plt.legend(loc='best')

plt.figure(1, figsize=(18, 8))
plt.subplot(2,2,2)
plt.plot(dat.index,dat['CPI_EUR'],label = 'CPI_EUR')
plt.plot(dat.index,dat['CPI_USA'],label = 'CPI_USA')
plt.legend(loc='best')

plt.figure(1, figsize=(18, 8))
plt.subplot(2,2,3)
plt.plot(dat.index,dat['DPEUR'],label = 'DPEUR')
plt.plot(dat.index,dat['DPUSA'],label = 'DPUSA')
plt.legend(loc='best')
```

Out[2]:

&lt;matplotlib.legend.Legend at 0x116f21350&gt;



(b) Perform the Augmented Dickey-Fuller (ADF) test for the two  $\log(\text{CPI})$  series. In the ADF test equation, include a constant ( $\alpha$ ), a deterministic trend term ( $\beta t$ ), three lags of  $\text{DP} = \Delta \log(\text{CPI})$  and, of course, the variable of interest  $\log(\text{CPI}_{t-1})$ . Report the coefficient of  $\log(\text{CPI}_{t-1})$  and its standard error and t-value, and draw your conclusion.

**ans::**

p-value for LOGPEUR: 0.19, p-value for LOGPUSA: 0.22, which indicate coefficient of  $\log(\text{CPI}_{t-1})$  is not significant at 5% level. Therefore, the null hypothesis is rejected. Both LOGPUSA and LOGPEUR are not stationary. Check below for details

In [3]:

```
result = {}

# EUROP
result['LOGPEUR'] = ts.adfuller(dat['LOGPEUR'], regression = 'ct', maxlag = 3, a
utolag = None) # maxlag is now set to 3
result['LOGPUSA'] = ts.adfuller(dat['LOGPUSA'], regression = 'ct', maxlag = 3, a
utolag = None) # maxlag is now set to 3
pp.pprint(result)
```

```
{'LOGPEUR': (-2.8263432452941988,
             0.18743718231598833,
             3,
             140,
             {'1%': -4.0249342984693879,
              '10%': -3.1457269825072887,
              '5%': -3.4423275561224487}),
 'LOGPUSA': (-2.7344718644272716,
             0.222050078484473,
             3,
             140,
             {'1%': -4.0249342984693879,
              '10%': -3.1457269825072887,
              '5%': -3.4423275561224487})}
```

(c) As the two series of log(CPI) are not cointegrated (you need not check this), we continue by modelling the monthly inflation series  $DPEUR = \Delta \log(CPI_{EUR})$  for the Euro area. Determine the sample autocorrelations and the sample partial autocorrelations of this series to motivate the use of the following AR model:

$$DPEUR_t = \alpha + \beta_1 DPEUR_{t-6} + \beta_2 DPEUR_{t-12} + \varepsilon_t$$

Estimate the parameters of this model (sample Jan 2000 - Dec 2010)

**ans::**

As shown in the figures below, both autocorrelation and partial autocorrelations are significant for lag 6 and lag 12. Therefore we could use the AR model with lag 6 and lag 12.

$$DPEUR_t = 0.0004 + 0.1887 * DPEUR_{t-6} + 0.5980 * DPEUR_{t-12} + \varepsilon_t$$

In [4]:

```
dat_sample = dat['2000-01':'2010-12']  
dat_sample.tail()
```

Out[4]:

	YEAR	TREND	CPI_EUR	CPI_USA	LOGPEUR	LOGPUSA	DPEUR	DPUSA
2010-08-01	2010M08	128	130.6	139.2	4.872139	4.935912	0.001533	0.0014
2010-09-01	2010M09	129	131.0	139.3	4.875197	4.936630	0.003058	0.0007
2010-10-01	2010M10	130	131.4	139.4	4.878246	4.937347	0.003049	0.0007
2010-11-01	2010M11	131	131.5	139.5	4.879007	4.938065	0.000761	0.0007
2010-12-01	2010M12	132	132.3	139.7	4.885072	4.939497	0.006065	0.0014

In [5]:

```

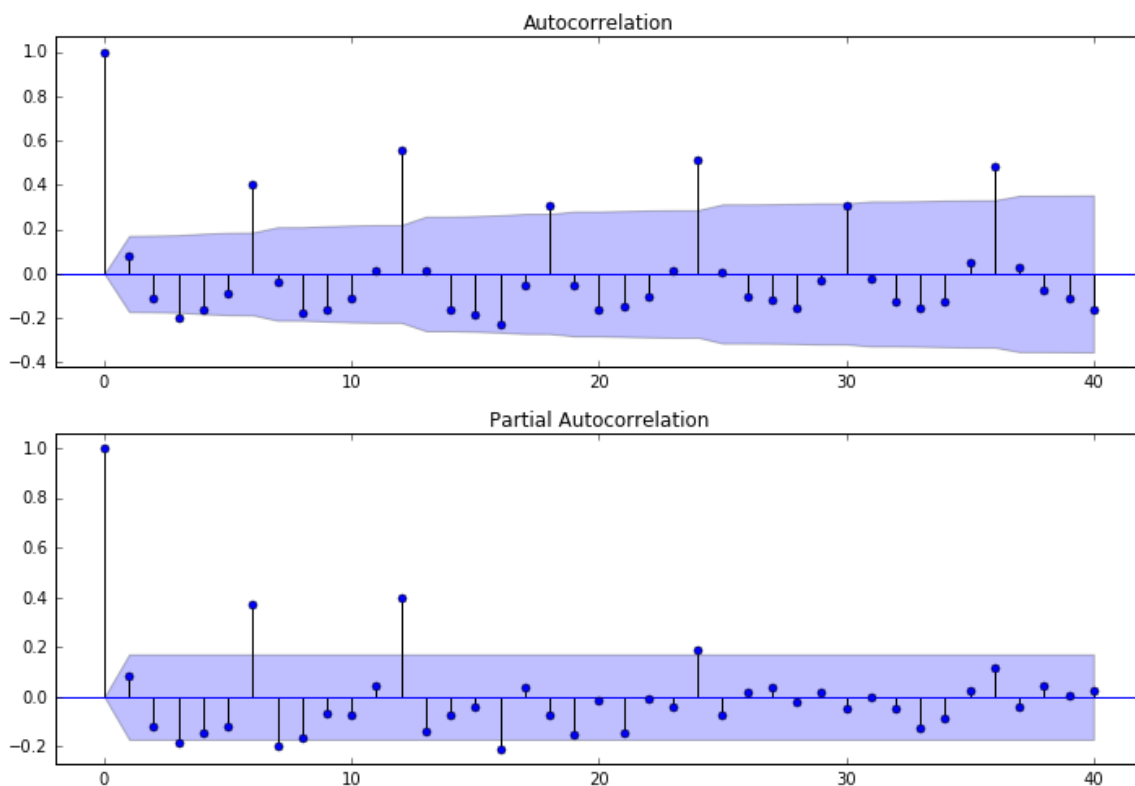
import statsmodels.api as sm

# plot
fig = plt.figure(figsize=(12,8))
ax1 = fig.add_subplot(211)
dat_sample_eur= dat_sample['DPEUR']
fig = sm.graphics.tsa.plot_acf(dat_sample_eur[1:].values.squeeze(), lags=40, ax=ax1)

ax2 = fig.add_subplot(212)
fig = sm.graphics.tsa.plot_pacf(dat_sample_eur[1:], lags=40, ax=ax2)

# result table
# r,q,p = sm.tsa.acf(dat_sample_eur[1:], qstat=True)
# data = np.c_[range(1,41), r[1:], q, p]
# table = pd.DataFrame(data, columns=['lag', "AC", "Q", "Prob(>Q)"])
# print(table.set_index('lag'))

```



In [6]:

```

from statsmodels.tsa.tsatools import lagmat
DPEUR12 = lagmat(dat['DPEUR'], maxlag=12, trim='Both')
DPEUR_lag12 = DPEUR12[:,11]
DPEUR_lag6 = DPEUR12[:,5]
X = pd.DataFrame({'lag6': DPEUR_lag6, 'lag12': DPEUR_lag12})[1:]
X = sm.add_constant(X)
y = dat['DPEUR'][13:]

X.index = y.index
X = X['2000-01':'2010-12']
y = y['2000-01':'2010-12']
ar_model = sm.OLS(y, X).fit()
ar_model.summary()

```

Out[6]:

## OLS Regression Results

<b>Dep. Variable:</b>	DPEUR	<b>R-squared:</b>	0.423
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.413
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	42.55
<b>Date:</b>	Thu, 22 Sep 2016	<b>Prob (F-statistic):</b>	1.38e-14
<b>Time:</b>	16:46:11	<b>Log-Likelihood:</b>	542.43
<b>No. Observations:</b>	119	<b>AIC:</b>	-1079.
<b>Df Residuals:</b>	116	<b>BIC:</b>	-1071.
<b>Df Model:</b>	2		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[95.0% Conf. Int.]
<b>const</b>	0.0004	0.000	1.365	0.175	-0.000 0.001
<b>lag12</b>	0.5980	0.084	7.157	0.000	0.432 0.763
<b>lag6</b>	0.1887	0.077	2.442	0.016	0.036 0.342

<b>Omnibus:</b>	10.599	<b>Durbin-Watson:</b>	1.626
<b>Prob(Omnibus):</b>	0.005	<b>Jarque-Bera (JB):</b>	19.700
<b>Skew:</b>	-0.321	<b>Prob(JB):</b>	5.27e-05
<b>Kurtosis:</b>	4.887	<b>Cond. No.</b>	406.

(d) Extend the AR model of part (c) by adding lagged values of monthly inflation in the USA at lags 1, 6, and 12. Check that the coefficient at lag 6 is not significant, and estimate the ADL model

$$DPEUR_t = \alpha + \beta_1 DPEUR_{t-6} + \beta_2 DPEUR_{t-12} + \gamma_1 DPUSA_{t-1} + \gamma_2 DPUSA_{t-12} + \varepsilon_t$$

(sample Jan 2000 - Dec 2010).

**ans:**

p-value for lag6 is 0.3, not significant at 5% level.

The adl model is

$$DPEUR_t = \alpha + 0.1687 * DPEUR_{t-6} + 0.6552 * DPEUR_{t-12} + 0.2326 * DPUSA_{t-1} - 0.2265 * DPU$$



In [7]:

```
# europ
DPEUR12 = lagmat(dat['DPEUR'], maxlag=12, trim='Both')
DPEUR_lag12 = DPEUR12[:,11]
DPEUR_lag6 = DPEUR12[:,5]

# usa
DPUSA12 = lagmat(dat['DPUSA'], maxlag=12, trim='Both')
usa_lag1 = DPUSA12[:,0]
usa_lag12 = DPUSA12[:,11]
usa_lag6 = DPUSA12[:,5]

X = pd.DataFrame({'eur_lag6': DPEUR_lag6, \
                  'eur_lag12': DPEUR_lag12, \
                  'usa_lag1': usa_lag1, \
                  'usa_lag12': usa_lag12, \
                  'usa_lag6': usa_lag6})[1:]

X = sm.add_constant(X)
y = dat['DPEUR'][13:]
X.index = y.index
X = X['2000-01':'2010-12']
y = y['2000-01':'2010-12']
adl_model = sm.OLS(y, X).fit()
adl_model.summary()
```

Out[7]:

## OLS Regression Results

<b>Dep. Variable:</b>	DPEUR	<b>R-squared:</b>	0.560
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.541
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	28.79
<b>Date:</b>	Thu, 22 Sep 2016	<b>Prob (F-statistic):</b>	9.84e-19
<b>Time:</b>	16:46:11	<b>Log-Likelihood:</b>	558.57
<b>No. Observations:</b>	119	<b>AIC:</b>	-1105.
<b>Df Residuals:</b>	113	<b>BIC:</b>	-1088.
<b>Df Model:</b>	5		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[95.0% Conf. Int.]
<b>const</b>	0.0004	0.000	1.545	0.125	-0.000 0.001
<b>eur_lag12</b>	0.6367	0.087	7.279	0.000	0.463 0.810
<b>eur_lag6</b>	0.2030	0.079	2.584	0.011	0.047 0.359
<b>usa_lag1</b>	0.2264	0.051	4.429	0.000	0.125 0.328
<b>usa_lag12</b>	-0.2300	0.054	-4.247	0.000	-0.337 -0.123
<b>usa_lag6</b>	-0.0561	0.055	-1.024	0.308	-0.165 0.052

<b>Omnibus:</b>	10.601	<b>Durbin-Watson:</b>	2.011
<b>Prob(Omnibus):</b>	0.005	<b>Jarque-Bera (JB):</b>	15.289
<b>Skew:</b>	0.443	<b>Prob(JB):</b>	0.000479
<b>Kurtosis:</b>	4.517	<b>Cond. No.</b>	512.

In [8]:

```
# europ
DPEUR12 = lagmat(dat['DPEUR'], maxlag=12, trim='Both')
DPEUR_lag12 = DPEUR12[:,11]
DPEUR_lag6 = DPEUR12[:,5]

# usa
DPUSA12 = lagmat(dat['DPUSA'], maxlag=12, trim='Both')
usa_lag1 = DPUSA12[:,0]
usa_lag12 = DPUSA12[:,11]
usa_lag6 = DPUSA12[:,5]

X = pd.DataFrame({'eur_lag6': DPEUR_lag6, \
                  'eur_lag12': DPEUR_lag12, \
                  'usa_lag1': usa_lag1, \
                  'usa_lag12': usa_lag12})[1:]

X = sm.add_constant(X)
y = dat['DPEUR'][13:]
X.index = y.index
X = X['2000-01':'2010-12']
y = y['2000-01':'2010-12']
adl_model = sm.OLS(y, X).fit()
adl_model.summary()
```

Out[8]:

## OLS Regression Results

<b>Dep. Variable:</b>	DPEUR	<b>R-squared:</b>	0.556
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.541
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	35.71
<b>Date:</b>	Thu, 22 Sep 2016	<b>Prob (F-statistic):</b>	2.55e-19
<b>Time:</b>	16:46:11	<b>Log-Likelihood:</b>	558.02
<b>No. Observations:</b>	119	<b>AIC:</b>	-1106.
<b>Df Residuals:</b>	114	<b>BIC:</b>	-1092.
<b>Df Model:</b>	4		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[95.0% Conf. Int.]
<b>const</b>	0.0003	0.000	1.267	0.208	-0.000 0.001
<b>eur_lag12</b>	0.6552	0.086	7.651	0.000	0.486 0.825
<b>eur_lag6</b>	0.1687	0.071	2.374	0.019	0.028 0.310
<b>usa_lag1</b>	0.2326	0.051	4.582	0.000	0.132 0.333
<b>usa_lag12</b>	-0.2265	0.054	-4.189	0.000	-0.334 -0.119

<b>Omnibus:</b>	10.148	<b>Durbin-Watson:</b>	2.014
<b>Prob(Omnibus):</b>	0.006	<b>Jarque-Bera (JB):</b>	15.792
<b>Skew:</b>	0.386	<b>Prob(JB):</b>	0.000372
<b>Kurtosis:</b>	4.609	<b>Cond. No.</b>	481.

(e) Use the models of parts (c) and (d) to make two series of 12 monthly inflation forecasts for 2011. At each month, you should use the data that are then available, for example, to forecast inflation for September 2011 you can use the data up to and including August 2011. However, do not re-estimate the model and use the coefficients as obtained in parts (c) and (d). For each of the two forecast series, compute the values of the root mean squared error (RMSE), mean absolute error (MAE), and the sum of the forecast errors (SUM). Finally, give your interpretation of the outcomes. **ans:**

See the cell below for the results.

In [12]:

```
DPEUR12 = lagmat(dat['DPEUR'], maxlag=12, trim='Both')
DPEUR_lag12 = DPEUR12[:,11]
DPEUR_lag6 = DPEUR12[:,5]
X = pd.DataFrame({'lag6': DPEUR_lag6, 'lag12': DPEUR_lag12})[1:]
X = sm.add_constant(X)
y = dat['DPEUR'][13:]
X.index = y.index
ar_y_fitted = ar_model.predict(X['2011-01':'2011-12'])
```

In [13]:

```
adl_model.summary()

# europ
DPEUR12 = lagmat(dat['DPEUR'], maxlag=12, trim='Both')
DPEUR_lag12 = DPEUR12[:,11]
DPEUR_lag6 = DPEUR12[:,5]

# usa
DPUSA12 = lagmat(dat['DPUSA'], maxlag=12, trim='Both')
usa_lag1 = DPUSA12[:,0]
usa_lag12 = DPUSA12[:,11]
usa_lag6 = DPUSA12[:,5]

X = pd.DataFrame({'eur_lag6': DPEUR_lag6, \
                  'eur_lag12': DPEUR_lag12, \
                  'usa_lag1': usa_lag1, \
                  'usa_lag12': usa_lag12})[1:]

X = sm.add_constant(X)
y = dat['DPEUR'][13:]
X.index = y.index
adl_y_fitted = adl_model.predict(X['2011-01':'2011-12'])
```

In [11]:	adl	ar
----------	-----	----

```
import math
from tabulate import tabulate

def rmse(y_fitted, y_true):
    sub = np.array([a-b for a,b in zip(y_fitted, y_true)])
    return math.sqrt(sum((sub)**2)/len(y_fitted))
def mae(y_fitted, y_true):
    sub = np.array([a-b for a,b in zip(y_fitted, y_true)])
    return sum(abs(sub))/len(y_fitted)

def SumOfError(y_fitted, y_true):
    '''
    sum of forcast errors
    '''
    sub = np.array([a-b for a,b in zip(y_fitted, y_true)])
    return sum(sub)

y_true = y['2011-01':'2011-12']

adl_metrics = [rmse(adl_y_fitted, y_true), mae(adl_y_fitted, y_true), SumOfError(
l_y_fitted, y_true)]
ar_metrics = [rmse(ar_y_fitted, y_true), mae(ar_y_fitted, y_true), SumOfError(ar
_y_fitted, y_true)]

metrics = pd.DataFrame({'adl': adl_metrics, 'ar': ar_metrics})
metrics.index = ['RMSE', 'MAE', 'SUM']
metrics
```

Out[11]:

	adl	ar
RMSE	0.002111	0.002324
MAE	0.001404	0.001693
SUM	-0.000478	-0.005065

In [ ]: