Equations for the Mathematical model of the gyroscope:

Air Pressure:

To first approximation, we will assume that the pressure drops exponentially, like so:

$$P_0 = Initial, 1\ atm, \qquad P_{(t)} = Air\ Pressure\ over\ time, \qquad c_1 = a\ constant\ to\ measure\ experimentally$$

$$\frac{dP_{(t)}}{dt} = -c_1 * P_{(t)}, \qquad => \qquad P_{(t)} = P_0 * e^{-c_1 t}$$

The value for c_1 will be measured experimentally using the pressure gauge on the vacuum chamber.

Physical Description of Gyroscope:

The gyroscope actually has a total of 12 moving parts, that will be rotating at different speeds, however they are not all equivalently important. The parts, in order of importance are:

- Main Gyroscope:
  - The heaviest part, and the part that's rotation is measured using the tacheometer.
  - Mass: 270.55 g
  - Approximate dimensions:
    - Toroid square ring, radii: 3.25cm, 1.8cm, height 1.1cm (Main part)
    - Flat disk, radius 2.6cm, height 0.2cm (connects main part to axis)
    - Flat disk, radius 0.6cm, height 0.9cm (inner bearing)
    - Flat disk, radius 0.7cm, height 1.4cm (the spindle upon which it can rotate like a top)
  - Notation: ω_M (m for main)
- Shell Piece:
  - This piece initially starts with a rotational velocity of zero, since it is what is held while the gyroscope is spun up with the angle grinder.
  - Mass: 158.55 g
  - Approximate dimensions:
    - Toroid square ring, radii: 1.25cm, 0.85cm, height 1.2cm (enclosing bearing)
    - Toroid triangle ring, radii: 3.6cm, cross sectional area: 0.21 cm^2 (Outer edge where one grasps the object)
    - Flat disk, radius 3.4cm, height 0.2cm (connects outer edge to axis)
    - Flat disk, radius 0.6cm, height 3cm (top handle)
  - Notation: ω_S (s for shell)
- 7 small ball bearings between Main and Shell:
  - These spin much faster, however they are so small that we might not need to consider them.
  - Mass: approximately 0.12 g for each ball, about .84 g total.
  - Approximate dimensions:
    - Rotate at radius of 0.7 cm
    - Sphere of radius 0.15 cm
- 3 large ball bearings in the stationary stand that the gyroscope's handle is placed upon:
  - We shall assume they rotate with the Shell piece for now
  - Mass: 2.1 g per ball, or 6.3 g together
  - Approximate dimensions:

    - Rotate at radius of 0.4 cm
    - Sphere of radius 0.4 cm

Approximation to Accommodate the Ball Bearings:

- Main Gyroscope:
  - Add the 7 small ball bearings by extending the radius of this part:
    - From: Flat disk, radius 0.6cm, height 0.9cm
    - To: Flat disk, radius 0.9cm, height 0.9cm
  - Increasing the mass of the Main part by 2 g and decreasing the Shell part by 2 grams
  - Mass: 272.55 g
- Shell Piece:
  - Add the 3 large ball bearings by extending the radius of this part:
    - From: Flat disk, radius 0.6cm, height 3cm (top handle)
    - To: Flat disk, radius 0.6cm, height 4cm (top handle)
  - Add 6.3 g from these ball bearings to the Shell part.
  - Mass: 162.85 g

Mathematical Model for Angular Velocities of Main and Shell Parts:

Next, we are able to define some differential equations to model the change in the angular velocities of the Main part and the Shell part:

- Define this as the difference between the angular velocities:
  - $\Delta\omega = \omega_M - \omega_S$
- Main Gyroscope:
  - $\omega_{M(0)} = Initial, recorded\ by\ measurement, \quad \omega_{M(t)} = Angular\ velocity\ of\ Main\ over\ time$
  - Drag from surrounding gas:
    - $\left[P_{(t)}\left(c_2 * \omega_{M(t)} + c_3 * \omega_{M(t)}^2\right)\right]$
    - This term is proportional to the air density, and includes terms up to 2nd order.
    - This term quickly disappears due to the vacuum being generated.
  - Drag from friction with Shell piece through ball bearings:
    - $\left[c_4 * \Delta\omega_{(t)} + c_5 * \Delta\omega_{(t)}^2\right]$
    - This term is proportional to the difference in angular velocity of the Main and the Shell, and includes terms up to 2nd order.
  - Drag from any unknown source:
    - $\left[c_6 * \omega_{M(t)}\right]$
    - Assumed to be relatively small, but still present, but only to first order.
  - Differential equation for the Main part's angular velocity:
    - $\frac{d\omega_{M(t)}}{dt} = -\left[P_{(t)}\left(c_2 * \omega_{M(t)} + c_3 * \omega_{M(t)}^2\right)\right] - \left[c_4 * \Delta\omega_{(t)} + c_5 * \Delta\omega_{(t)}^2\right] - \left[c_6 * \omega_{M(t)}\right]$
    - All terms contribute toward decreasing the angular velocity of the main part
    - $\omega_{m(t)}$ Will start out at a very high value, and gradually decrease as the rotational energy is transferred through friction.
- Shell Piece:
  - $\omega_{S(0)} = Initial\ zero\ value = 0,\ \omega_{S(t)} = Angular\ velocity\ of\ Shell\ over\ time$
  - Drag from surrounding gas:
    - $\left[P_{(t)}\left(c_7 * \omega_{S(t)} + c_8 * \omega_{S(t)}^2\right)\right]$
    - This term is proportional to the air density, and includes terms up to 2nd order.

        - This term quickly disappears due to the vacuum being generated.
    - Drag from friction with Main piece through ball bearings:
        - $\left[c_9 * \Delta\omega_{(t)} + c_{10} * \Delta\omega_{(t)}^2\right]$
        - This term is proportional to the difference in angular velocity of the Main and the Shell, and includes terms up to 2nd order.
        - Unlike the Main part, this term acts to increase the angular velocity of the Shell piece
    - Drag from friction with Main piece through gas:
        - $\left[P_{(t)} * c_{11} * \Delta\omega_{(t)}\right]$
        - This term is proportional to the air density.
        - This term is proportional to the difference in angular velocity of the Main and the Shell.
        - This term acts to increase the angular velocity of the Shell piece.
    - Drag from friction with stationary stand through large ball bearings and any other unknown friction:
        - $\left[c_{12} * \omega_{S(t)} + c_{13} * \omega_{S(t)}^2\right]$
        - Friction is definitely present at least through large ball bearings, including to 2nd order.
    - Differential equation for the Shell part's angular velocity:
        - $\frac{d\omega_{S(t)}}{dt} = -\left[P_{(t)}\left(c_7 * \omega_{S(t)} + c_8 * \omega_{S(t)}^2\right)\right] + \left[c_9 * \Delta\omega_{(t)} + c_{10} * \Delta\omega_{(t)}^2\right] + \left[P_{(t)} * c_{11} * \Delta\omega_{(t)}\right] - \left[c_{12} * \omega_{S(t)} + c_{13} * \omega_{S(t)}^2\right]$
        - The 2nd and 3rd terms serve to increases the angular velocity of the Shell, while other terms dampen it.
        - $\omega_{S(t)}$ Will start out at zero, but quickly increase but never exceed the angular velocity of the Main part. At some point it will also start to decrease as the 4th term dominates.

Mathematical Model for Contributions to Measured Change in Measured Mass:

Here we shall list all possible contributions to the measured change in effective weight (measured as mass, but it is a weight) of the gyroscope as it is spinning on the scale. First of all, the objective of this experiment is to measure a centrifugal force that should be due to the rapidly rotating gyroscope parts relative to the Earth's center of mass. We will carefully explain that effect last, however first is this list of effects that must be filtered out of the data:

- Fluid dynamics exert different effective pressures on surfaces that are moving quickly. To account for this we can look at Bernoulli's principle for inspiration. It can also be shown experimentally that if the orientation of the top is reversed (so that the fast spinning surface is pointed downward) then there is an obvious shift in the force when there is gas present. We can model the effect through a term like this:
    - $\left[P_{(t)}\left(c_{14} * \omega_{M(t)}^2 - c_{15} * \omega_{S(t)}^2\right)\right]$
    - In this term we assign a positive influence to the rotational velocity of the Main part because that provides a lifting force by decreasing the pressure on the top surface. Conversely, there is a negative force pushing the top down due to the rotation of the Shell, since it is on the bottom.
    - The term is also proportional to the air pressure since it requires gas and should vanish with the vacuum.
- There is a small buoyancy force that is removed when we remove the gas. However after calculating the order of magnitude for this force at about 0.0002 grams, and after measuring it using our experiment to be 0.000 g, we can safely ignore this effect entirely. It is present, but too small to be detectable with our current equipment.
    - Therefore we shall ignore this effect.
- There is a possibility of the calibration of our scale to be changed when it is placed in a vacuum. There are some other things that can affect it's calibration, such as if the gyroscope's relatively heavy weight is dropped upon it. For this reason, we have recorded a control experiment where we measured any change in calibration of the scale over the course of a normal amount of taking data.

- For this reason, we shall be including a calibration constant, where we can normalize our data using a part of the data that should correspond to absolutely minimal amount of change in calibration.
- Further, the control data gives assurance that this calibration constant is indeed constant over the timeframe where we are taking data.

Measurement Objective: Detect a Centrifugal Force Opposing Gravity

Lastly, the main objective of this experiment is to measure a repulsive force opposing gravity, which should be generated through a centrifugal term. Our gyroscope effectively has two main parts that are both rotating at different angular velocities. Their contribution to this centrifugal term is proportional to the square of their angular velocities and also depends upon their geometries and mass distributions. The general equation for calculating the repulsive centrifugal force is given by:

$$F_C = 2\pi \int\int \left[\frac{\omega^2 r^2}{z}\right] \rho(r,z)\, r\, dr\, dz,$$

$$where\ z\ is\ measured\ from\ the\ center\ of\ the\ Earth, and\ r\ is\ measured\ from\ the\ axis\ of\ rotation$$

We can simplify this since our gyroscope is tiny compared to the Earth, and also by assuming that it has a constant density:

$$Centrifugal\ Force = F = \frac{2\pi\rho}{R_{earth}} \int\int \omega^2 r^3\, dr\, dz$$

$$F_{(t)} = \frac{2\pi}{R_{earth}} \left[\rho_M \omega_{M(t)}^2 \left(\int r^3\, dr\, dz_{\ Main}\right) + \rho_S \omega_{S(t)}^2 \left(\int r^3\, dr\, dz_{\ Shell}\right)\right]$$

We must simply do these cumbersome integrals over the volume of the Main and the Shell parts, and we will have all the mathematical models we need to make a prediction for how large this effect should be for our experiment.

- Main Gyroscope:
  - Mass: 272.55 g
  - Approximate dimensions:
    - Toroid square ring, radii: 3.25cm, 1.8cm, height 1.1cm (Main part)
    - Flat disk, radius 2.6cm, height 0.2cm (connects main part to axis)
    - Flat disk, radius 0.6cm, height 0.9cm (inner bearing)
    - Flat disk, radius 0.7cm, height 1.4cm (the spindle upon which it can rotate like a top)

$$Main = \int r^3\, dr\, dz$$

$$= cm^5 \left[1.1 * \left[\int_{1.8}^{3.25} r^3\, dr\right] + 0.2 * \left[\int_{0}^{2.6} r^3\, dr\right] + 0.9 * \left[\int_{0}^{0.6} r^3\, dr\right] + 1.4 * \left[\int_{0}^{0.7} r^3\, dr\right]\right]$$

$$Main = \frac{cm^5}{4}[1.1 * 101.07 + 0.2 * 45.70 + 0.9 * 0.13 + 1.4 * 0.24] = 30.19\ cm^5$$

$$Volume\ Main = 2\pi \int \mathrm{r}\, dr\, dz$$

$$= 2\pi\ cm^3 \left[1.1 * \left[\int_{1.8}^{3.25} \mathrm{r}\, dr \right] + 0.2 * \left[\int_{0}^{2.6} \mathrm{r}\, dr \right] + 0.9 * \left[\int_{0}^{0.6} \mathrm{r}\, dr \right] + 1.4 * \left[\int_{0}^{0.7} \mathrm{r}\, dr \right] \right]$$

$$V_M = \pi\ [1.1 * 7.32 + 1.35 + 0.32 + 0.69] cm^3 = 32.7\ cm^3$$

$$\rho_M = \frac{272.55\ \mathrm{g}}{32.7\ \ cm^3} = 8.3 \frac{g}{cm^3}$$

- Shell Piece:
  - Mass: 162.85 g
  - Approximate dimensions:
    - Toroid square ring, radii: 1.25cm, 0.85cm, height 1.2cm (enclosing bearing)
    - Toroid square ring, radii: 3.4cm, 3.7cm, height 0.7cm (cross sectional area: 0.21 cm^2) (Outer edge where one grasps the object)
    - Flat disk, radius 3.4cm, height 0.2cm (connects outer edge to axis)
    - Flat disk, radius 0.6cm, height 3cm (top handle)

$$Shell = \int r^3\, dr\, dz$$

$$= cm^5 \left[1.2 * \left[\int_{0.85}^{1.25} r^3\, dr \right] + 0.7 * \left[\int_{3.4}^{3.7} r^3\, dr \right] + 0.2 * \left[\int_{0}^{3.4} r^3\, dr \right] + 3 * \left[\int_{0}^{0.6} r^3\, dr \right] \right]$$

$$Main\ = \frac{cm^5}{4} [1.2 * 1.92 + 0.7 * 53.78 + 0.2 * 133.63 + 3 * 0.13] = 16.77\ cm^5$$

$$Volume\ Shell = 2\pi \int \mathrm{r}\, dr\, dz$$

$$= 2\pi\ cm^3 \left[1.2 * \left[\int_{0.85}^{1.25} \mathrm{r}\, dr \right] + 0.7 * \left[\int_{3.4}^{3.7} \mathrm{r}\, dr \right] + 0.2 * \left[\int_{0}^{3.4} \mathrm{r}\, dr \right] + 3 * \left[\int_{0}^{0.6} \mathrm{r}\, dr \right] \right]$$

$$V_S = \pi\ [1.01 + 1.49 + 2.31 + 1.08] cm^3 = 18.5\ cm^3$$

$$\rho_S = \frac{162.85\ \mathrm{g}}{18.5\ cm^3} = 8.8 \frac{g}{cm^3}$$

Using these calculations, we can finally generate our prediction for the difference in measured mass due to the centrifugal force:

$$\Delta Mass = \frac{F_{(t)}}{g_e} = \frac{s^2}{980\ cm} \frac{2\pi}{637 * 10^6 cm} \left[\omega_{M(t)}^2 (250.6\ g\ cm^2) + \ \omega_{S(t)}^2 (147.6\ g\ cm^2) \right]$$

$$\Delta Mass = \left[2.5 * \omega_{M(t)}^2 + 1.5 * \omega_{S(t)}^2\right] E^{-9} s^2\ g$$

For example, if our angular velocities were:

$$\omega_M = 16000\ RPM, \qquad \omega_S = 8000\ RPM, \qquad then\ we\ convert\ these\ values\ to\ angular\ velocity\ in\ radians:$$

$$\omega_M = 16000 \frac{2\pi\ rad}{60\ s} = 1664\ s^{-1},\ \ and\ \omega_S = 832\ s^{-1}$$

$$\Delta Mass = \ (6.92\ E^6 + 1.04\ E^6)\ E^{-9}\ g = 0.00796\ g\ = 0.008\ g$$

$$\omega_M = 22000\ RPM, \qquad \omega_S = 6000\ RPM, \qquad then\ we\ convert\ these\ values\ to\ angular\ velocity\ in\ radians:$$

$$\omega_M = 2288\ s^{-1},\ \ and\ \omega_S = 624\ s^{-1}$$

$$\Delta Mass = \ (1.309\ E^7 + 0.058\ E^7)\ E^{-9}\ g = 0.01367\ g = \ 0.014\ g$$

If I had a way to drastically increase the RPMs to much higher values, then:

$$\omega_M = 100000\ RPM, \qquad \omega_S = 60000\ RPM,$$
$$then\ we\ convert\ these\ values\ to\ angular\ velocity\ in\ radians:$$

$$\omega_M = 10400\ s^{-1},\ \ and\ \omega_S = 6240\ s^{-1}$$

$$\Delta Mass = \ (2.70\ E^8 + 0.58\ E^8\ )E^{-9}\ g = 0.328\ g$$

These values are significantly lower than the effect that seems to be measured via the experiment.

Calculating the Effect of the Earth's Rotation:

This equation is actually only valid for a rotating object that is not also in a constant translational velocity:

$$F_C = 2\pi \int\int \left[\frac{\omega^2 r^2}{z}\right] \rho(r,z)\ r\ dr\ dz$$

Radius of Earth: $R_E = 6.37\ E^6\ m$

Angular velocity of Earth: $\omega_E = \frac{2\pi}{24\ hour}\frac{1\ hour}{3600\ s} = 7.27\ E^{-5}\ s^{-1}$

Latitude that our experiment has been done at: $43.07^o N$

Therefore, we need to account for the fact that our experiment is also rotating about the Earth at a velocity of:
$V_E = \cos(Latitude^o)\ \omega_E R_E = \ \cos(43.07^o) * 7.27\ E^{-5}\ s^{-1} *\ 6.37\ E^6\ m = 338.3 \frac{m}{s}\ Eastward$

What matters is the square of the total velocity for each differential part of the gyroscope. The total velocity is given by:

$$\vec{V} = \ \overrightarrow{V_E} + \overrightarrow{V_\theta} = (V_E - r\omega \sin\theta)\hat{x} + (r\omega\cos\theta)\hat{y}, \qquad V^2 = V_E^2 - 2V_E r\omega \sin\theta + \ r^2\omega^2$$

$$F_C = \frac{\rho}{R_E} \iiint_0^{2\pi} V^2\ r\ dr\ dz\ d\theta = \ \frac{\rho}{R_E} \iiint_0^{2\pi} [V_E^2 - 2V_E r\omega \sin\theta + \ r^2\omega^2]\ r\ dr\ dz\ d\theta$$

However, from this integral we can see that the rotational velocity of the Earth should not matter. The first term proportional to V_E^2 is already accounted for through the control of the experiment as it is the centrifugal force of the Earth's rotation, which is already accounted for. The third term is what we have already calculated when we originally ignored the rotation of the Earth. The second term evaluates to zero since it includes this integral:

$$-\int_0^{2\pi} \sin\theta \; d\theta = \cos\theta \,|_0^{2\pi} = 0$$

Therefore, there should not be any net effect from the Earth's rotation.

Calculating Effect of Small Irregularity in Gyroscope:

Now, let's challenge the assumption that the gyroscope is perfectly cylindrically symmetric. It is not a perfect object, and will have very small differences in mass distribution, and the effect of this will be difficult to detect. Therefore, let's apply a bit of perturbation theory to assume that there is a small extra spec of mass on the gyroscope that will model the total irregularity of mass distribution. Therefore assume that we add a small spec like this:

$$Mass\ of\ Pertubation = M_p, \qquad where\ M_p \ll M_M = Mass\ of\ main\ gyroscope$$

Let's assume that this extra mass, rotates with the main gyroscope at $r_p = 3cm, and\ at\ angle\ \theta$. We can still ignore the first term since it is handled by the control. Now, there actually will be an effect related to the velocity of the Earth because the 2nd integral no longer disappears.

$$F_p = \frac{M_p}{R_E} V^2 = \frac{M_p}{R_E}\left[\, {r_p}^2 {\omega_{M(t)}}^2 - 2V_E r_p \omega_{M(t)} \sin\left(\omega_{M(t)} t\right)\right], \qquad where\ angle\ \theta = \omega_{M(t)} t$$

For our experiment, the typical values of $V_E, r_p, and\ \omega_{M(t)}$ imply that the 2nd term is dominant, by over a factor of 10, so we can neglect the first term to see that the Centrifugal force caused by the perturbation oscillates at the same angular frequency, and is 7the value of:

$$F_p = \frac{2V_E r_p M_p}{R_E} \omega_{M(t)} \sin\left(\omega_{M(t)} t\right)$$

If we calculate the ratio of the centrifugal force from the perturbation compared to the centrifugal force from the part of the main gyroscope, then we get that:

$$\left|\frac{F_p}{F_C}\right| = \left|\frac{F_p}{g_e\ \Delta Mass}\right| \sim \frac{2V_E r_p M_p \omega_{M(t)}}{g_e\ R_E\left(2.5 * \omega_{M(t)}^2\ E^{-9} s^2\ g\right)} \sim \frac{2 * \ 3.4\ E^4 cm * 3cm}{980\ cm * 637\ E^6 cm * \ 2.5\ E^{-9} s\ g}\left[\frac{M_p}{\omega_{M(t)}}\right] = 130.7\ \frac{M_p\ s}{\omega_{M(t)}\ g}$$

If we assume that $M_p = 1\ g,$
$and\ \omega_{M(t)}\ is\ about\ 10^3 s^{-1}\ then, \left|\frac{F_p}{F_C}\right| \sim \frac{1}{10}\ which\ is\ actually\ quite\ a\ substantial\ effect$. Therefore, under ideal circumstances where $M_p = 0\ g$, the rotation of the Earth does not matter, however if there is a small perturbation, the rotation of the Earth contributes an oscillating difference in the centrifugal term that provides a net zero effect, but the magnitude can be as large as about 10% if we assume the gyroscope has a 1g distribution error at 10000 RPMs. Still, this cannot explain an overall much larger experimental measurement than predicted by the theory.

Measuring any Effect of Earth's Magnetic Field:

Although there has been no measurement of the local magnetic field, it is assumed to be just the Earth's normal magnetic field. We took three rounds of data without a magnetic shield, and we took 3 rounds of data with a magnetic shield installed. The magnetic shield was created using a mu-metal fabric, and should be strong enough to negate the Earth's magnetic field as well as just about all EM-radiation.

No data available yet.