MATH 381 Final Review

Prof. Olivia Dumitrescu

29 April 2024

Final Exam Material

Included

1. 2 problems on Chapter 1
2. 3 problems on Divisibility
3. 2 problems on Induction
4. 2-3 problems on Relations 9.5
5. 2-3 problems on Binomial Coefficients 6.3 + 6.4

Not Included

1. All of chapter 2?
2. 2.5
3. 6.1 + 6.2
4. 6.5

Example Are the following equivalence relations?

$$A = \{f : \mathbb{Z} \to \mathbb{Z}\}$$

$$\{(f, g) \mid f(1) = g(1)\}$$

Yes

$$\{(f, g) \mid f(0) = g(0) \vee f(1) = g(1)\}$$

No, transitivity

$$\{(f, g) \mid f(x) - g(x) = 1 \quad \forall x \in \mathbb{Z}\}$$

No, reflexivity, symmetry and transitivity?

Example

1. Count # functions from $\mathbb{Z}_7$ to $\mathbb{Z}_n$

$$n^7$$

2. Count # injective functions $\mathbb{Z}_7$ to $\mathbb{Z}_n$

$$n \geq 7 \to \frac{n!}{(n-7)!} = n(n-1)(n-2)\dots(n-6)$$

3. Count # of bijective functions from $\mathbb{Z}_7$ to $\mathbb{Z}_n$

$$n = 7 \to 7!$$

4. Count # injective functions that do not contain 0 and 1 in the range

$$n \geq 9 \to \frac{(n-2)!}{(n-9)!} = (n-2)(n-3)\dots(n-8)$$