MATH 381 Section 6.3

Prof. Olivia Dumitrescu

1 April 2024

Section 6.3 Permutations

$n \in \mathbb{N}$
$n! = n(n-1)(n-2)\dots 2 \cdot 1$

Example $S = \{1, 2, 3\}$ The arragement $\{3, 1, 2\}$ is a premutation of S

- 2-permutation of S

$$\{1,2\} \quad \{2,1\} \quad \{1,3\} \quad \{3,1\} \quad \{2,3\} \quad \{3,2\}$$

$n = 3, r = 2$
$P(n, r)$ is the number of r-permutations of a set of order n.

Theorem 0.1 *If n is a positive integer and*

$$1 \leq r \leq n \qquad r \in \mathbb{N}$$

$$\begin{cases} P(n,r) = n(n-1)(n-2)\dots(n-r+1) \\ P(n,0) = 1 \qquad n \in \mathbb{N} \end{cases}$$

Corollary 0.2 $n \in \mathbb{N} \qquad 0 \leq r \leq n \qquad r \in \mathbb{N}$

$$P(n,r) = \frac{n!}{(n-r)!} = \frac{n(n-1)\dots(n-r+1)(n-r)\dots 1}{(n-r)(n-r-1)\dots 1}$$

Example How many permutations of letters $ABCDEFGH$ contain the string ABC?
$ABC = X$
\# of permutations $= XDEFGH$
i.e. # rearrangements $= 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Remark $n! = \#$ of arrangements of a set of order n.

Definition An r-combination of a set of ordern n is an unordered selection of r elements of the set. (Equivalently, it is the number of subsets of order r of a set of order n.)

Example

1. $P(3,2) = 6$
2. $C(3,2) = 3$

Theorem 0.3 $n \in \mathbb{N}, 0 \leq r \leq n$

1.
$$P(n,r) = \frac{n!}{(n-r)!} = n \cdot (n-1) \ldots (n-r+1)$$

2.
$$C(n,r) = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

Corollary 0.4 $n, r \in \mathbb{N} \qquad 0 \leq r \leq n$
Then $C(n,r) = C(n, n-r)$ *or*

$$\binom{n}{r} = \binom{n}{n-r}$$