MATH 381 Section 1.3

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Propositional Equivalences

Definition

- A compound proposition that is always true regardless of the truth values of the propositional variables that occur within it is called a **tautology**.
- A compound proposition that is always false regardless of the truth values of the propositional variables that occur within it is called a **contradiction**.
- A compound proposition that is neither true nor false neither always true nor always false is called a **contingency**.

Example

- Tautology: $P \vee \neg P$ is always true. ($\equiv T$)
- $P \wedge \neg P$ is a contradiction. ($\equiv F$)

Remark 2 compound propositions are **logically equivalent** if they have the same truth values.
p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

DeMorgan's Laws

$$\neg(p \wedge q) = \neg p \vee \neg q$$

$$\neg(p \vee q) = \neg p \wedge \neg q$$

Generalization of DeMorgan's Laws

$$\neg(\bigvee_{i=1}^{n} P_i) = \bigwedge_{i=1}^{n} \neg P_i$$

$$\neg(\bigwedge_{i=1}^{n} P_i) = \bigvee_{i=1}^{n} \neg P_i$$

Example Show $p \to q$ and $\neg p \vee q$ are logically equivalent.

Example Show that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
The **distributive** law of disjunction over conjunction

Equivalences

$$P \wedge T \equiv P$$

identity law

$$P \vee F \equiv P$$

identity law [fill with a big brace to caputre both]

$$P \vee T \equiv T$$

dominstaion laws

$$P \wedge F \equiv F$$

domination laws

$$P \vee P \equiv P$$

idempotent law

$$P \wedge P \equiv P$$

$$\neg(\neg p) = p$$

double negation law

$$p \vee q \equiv q \vee p$$

commutative laws

$$p \wedge q \equiv q \wedge p$$

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

associativity laws

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

distributive laws

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

DeMorgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$p \vee \neg p \equiv T$$

Negation law

$$p \wedge \neg p \equiv F$$

$$p \vee (p \wedge q) \equiv p$$

absorption laws

$$p \wedge (p \vee q) \equiv p$$

$$p \vee (p \wedge q) \equiv p \implies (p \vee p) \wedge (p \vee q) = p \wedge (p \vee q)$$

1. $p \to q \equiv \neg p \vee q$
2. $p \to q \equiv \neg q \to \neg p$
3. $p \vee q \equiv \neg p \to q$
4. $p \wedge q \equiv \neg(p \to \neg q)$
5. $\neg(p \to q) \equiv p \wedge \neg q$
6. $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
7. $(p \to r) \wedge (q \to r) \equiv p \vee q \to r$
8. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
9. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
10. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
11. $\neg(p \leftrightarrow \neg q) \equiv p \leftrightarrow \neg q$

1.3.5

Example

$$\neg(p \to q) \equiv \neg(p \to \neg q)$$

Example

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$$

Example Show $p \wedge q \to p \vee q$ is a tautology.

Definition

1. A compound proosition is **satisfiable** if there is an assignment of the truth variables that makes it true.
2. When we have a particular assignment of truth values that make a compound proposition true is a **solution**.
3. To show that a compound proposition is **not satisfiable** you have to show that for any assignment of truth values it is false.

Example Determine whether each compound proposition is satisfiable $\iff$ p, q and r have the same truth value.

1. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$
2. $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$
3. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r)$