MATH 381 Section 5.2

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Strong Induction and Well-Ordering

Strong Induction

1. $P(1)$ holds
2. $\forall 1 \leq j \leq k \quad P(j) \to P(k+1)$

Then we conclude $P(n)$ holds for $n \geq 1$.

Definition A number is called **prime** if it is non-unital and the only divisors of the number are 1 and itself.

$$1 \mid p \qquad p \mid p$$

($p = 1$ is not prime by definition)

Example Show that if $n \in \mathbb{Z}$, then either n is prime or n can be written as a product of primes. (In fact, this decomposition is unique.)

Proof Strong Induction

$$\begin{array}{rl} P(n) & n \text{ can be written as a product of primes.} \\ n \geq 2 & \end{array}$$

1. $P(2), P(3), P(4), P(5), P(6), P(8)$
2. Assume $P(j)$ holds for any $1 \leq j \leq n$.
   We claim $P(n+1)$ holds.

Let $n+1 \in \mathbb{Z}, n \geq 2$

(a) either $n+1$ is prime

(b) or $n+1$ is not prime i.e. $\exists d \in \mathbb{N}, d \neq 1 \wedge d = n+1$

$$\begin{aligned} d \mid n+1 &\implies n+1 = a \cdot d \\ \begin{matrix} d \neq 1, n+1 \\ a \neq 1, n+1 \end{matrix} &\implies a, d < n+1 \qquad\qquad 2 \leq a, d \leq n \end{aligned}$$

Theorem 0.1 *A simple polygon with n sides, where $n \in \{n \in \mathbb{N} \mid n \geq 3\}$, can be triangulated into $n-2$ triangles.*

Definition

1. A **polygon** is a closed geometric figure consisting of a sequence of line segments with $s_1, \ldots, s_n$ as sides.

2. A polygon is **simple** if no consecutive sides intersect.

3. A polygon is **convex** if any line segment connecting 2 points in the interior of the polygon lies inside the polygon.

4. A **diagonal** is a line connecting 2 vertices of a polygon.