MATH 381 Section 2.1

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Sets

Definition A set is an unordered collection of disjoint objects called elements or members. A set is said to contain its elements.
We write $a \in A$ to denote that a is an element of the set A.
We write $a \notin A$ to denote that a is not an element of the set A.

Example

1. Set of vowels
$$V = \{a, e, i, o, u\}$$
2. odd positive integers less than 10
$$\{1, 3, 5, 7, 9\}$$

Definition

1. Two sets are equal if and only if they have the same elements.
2. A set A is a subset of a set B
$$A \subseteq B (\forall x \in A \to x \in B)$$

Remark To prove $A = B$ is to prove that $A \subseteq B$ and $B \subseteq A$.

Theorem 0.1 *For any set S*

1. $\emptyset \subset S$

2. $S \subseteq S$

Definition If there are exactly n distinct elements in set S we call n to be cardinality of S

$$|S| = n$$

E.g. $|\emptyset| = 0$
A set is infinite if it has no finite cardinality.

Definition Given a set S the power of S is the set of all subsets of S

$$\mathcal{P}(S) = \{A | A \subseteq S\}$$

Theorem 0.2 *If S is a finite set*

$$|\mathcal{P}(S)| = 2^{|S|}$$

$\binom{n}{i}$: number of distinct subsets of cardinality i of S (with cardinality n)

Theorem 0.3

$$\binom{n}{k} = \binom{n}{n-k}$$
$$n \in \mathbb{N} \textit{ and } 0 \leq k \leq n$$

Theorem 0.4 *Binomial Theorem*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Theorem 0.5 *If S is finite, then* $|\mathcal{P}(S)| = 2^{|S|} = 2^n$

Proof Plug in $x = y = 1$ into the binomial theorem.

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n}$$

Cartesian Products of Sets

Definition A, B are sets.

1. $A \times B = \{(a, b) \mid a \in A, b \in B\}$
2. consider $A_1, A_2, \ldots, A_n$

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i\}$$

Remark If A and B are finite sets,

1. $|A \times B| = |A| \cdot |B|$

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot \cdots \cdot |A_n|$$

   A_i is a finite set

2. $|P(A_1 \times \cdots \times A_n)| = 2^{|A_1 \times \cdots \times A_n|} = 2^{|A_1| \cdot |A_2| \cdot \cdots \cdot |A_n|}$