MATH 381 Section 1.6

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Rules of Inference

Definition An argument in propositional logic is a sequence of propositions where all but the final proposition are premises and the final one is the conclusion.

Remark An argument is valid if the truth values of all its premises implies the truth of its conclusion.

Example

1. Determine whether the argument is valid
2. Determine if the conclusion is true

If $\sqrt{2} > \frac{3}{2}$, then $(\sqrt{2})^2 > \frac{9}{4}$.

1. true $\because p \to q$
2. false $\because q$ is false

	Rules of Inference	Tautology	Name
1	p $p \to q$ $\therefore q$	$p \wedge (p \to q) \to q$	Modus ponens
2	$\neg q$ $p \to q$ $\therefore \neg p$	$\neg q \wedge (p \to q) \to \neg p$	Modus tollens
3	$p \to q$ $q \to r$ $\therefore p \to r$	$(p \to q) \wedge (q \to r) \to (p \to r)$	Hypothetical syllogism
4	$p \vee q$ $\neg p$ $\therefore q$	$(p \vee q) \wedge \neg p \to q$	Disjunctive Syllogism
5	p $\therefore p \vee q$	$p \to p \vee q$	Addition
6	$p \wedge q$ $\therefore p$	$p \wedge q \to p$	Simplification
7	$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$(p \vee q) \wedge (\neg p \vee r) \to q \vee r$	Resolution

Remark The propostiion $(p \to q) \wedge q \to p$ is not a tautology i.e. it has a false value if $p = F$ and $q = T$. This is because it is not birectional implication. Similarly, the proposition $(p \to q) \wedge \neg p \to \neg q$ is not a tautology. This is because if p is false, then $p \to q$ will always be true no matter q. It is akin to taking the contrapositive and its conclusion to assert the hypothesis.

Example If it rains today, then we will not have a barbecue today.
If we don't have a barbecue today, then we will have a barbecue tomorrow.
Therefore, if it rains today, then we will have a barbecue tomorrow.
p: It rains today
q: We will not have a barbecue today
r: We will have a barbecue tomorrow

$$\begin{array}{c} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array} \qquad \text{Hypothetical Syllogism}$$

Example Premises

1. If you send me an email message, then I will finish writing the program.
   $p \to q$
2. If you do not send me an email message, then I will go to sleep early.
   $\neg p \to r$
3. If I go to sleep early, then I will wake up feeling refreshed.
   $r \to s$

Conclusion
If I do not finish writing the program, then I will wake up feeling refreshed.
$\neg q \to s$

propositions
p: You send me an email message
q: I will finish writing the program
r: I will go to sleep early
s: I will wake up feeling refreshed

$$p \to q \iff \neg q \to \neg p \tag{1}$$
$$\neg p \to r \tag{2}$$
$$\neg q \to r \qquad [Hypothetical Syllogism] \tag{3}$$
$$r \to s \tag{4}$$
$$\neg q \to s \qquad [Hypothetical Syllogism] \tag{5}$$

Example hypothesis

1. Jasmine is skiing or it is not snowing.
   $p \vee \neg q$
2. It is snowing or Bant is playing hockey.
   $q \vee r$
3. Implies Jasmine is skiing or Bant is playing hockey
   $p \vee r$

1. p: Jasmine is skiing.
2. q: It is snowing.
3. r: Bant is playing hockey.

$$\begin{aligned} & p \vee \neg q \\ & q \vee r \\ & p \vee r \qquad [Hypothetical Syllogism] \end{aligned}$$

Example Show that premises imply the conclusion $p \vee s$

1. $$(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$$

2. $$r \rightarrow s = \neg r \vee s$$

3. $$\begin{aligned} (p \vee r) & \\ \neg r \vee s & \\ p \vee s & \qquad \text{Resolution} \end{aligned}$$

4. $$\begin{aligned} p \wedge q & \\ p & \qquad \text{Simplification} \end{aligned}$$

Fallacies

Is the proposition a tautology?

$$(p \to q) \land q \to p$$

Example Is the following argument valid?

1. If you solve every problem in the textbook, then you will learn discrete mathematics.
   $p \to q$
2. You did learn discrete mathematics.
   q

Therefore, you solved every problem in the book.
This is a fallacy, because q does not imply p by $p \to q$

Example Premises

1. Everyone in Discrete Math has taken a course in Computer Science.
2. Manta is a student in this class.
3. **Conclusion**: Manta has taken a course in computer science.

Domain is UNC Students.

- $A(x)$: x is a student in this class.
- $B(x)$: x has taken a Computer Science course.

1. $\forall x(A(x) \to B(x))$
2. Manta $\in A(x)$.
3. **Conclusion**: Manta has taken a course in Computer Science.