MATH 381 Section 6.1

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Section 6.1 The Basics of Counting

Counting Functions

How many functions from a set of m elements to a set of n elements?

$$n^m$$

One-to-One Functions

How many 1-to-1 functions from a set with m elements to a set with n elements?

$$n(n-1)\dots(n-m+1)$$

Example Use the product rule to show that the no. of different subsets of a finite set S is $2^{|S|}$. $S = \{x_1, \dots, x_n\}$.

Let $S =$ finite set. List the elements of S in arbitrary order. Recall that there is a 1-to-1 correspondence between subsets of S and bitstrings of length $2^{|S|}$.

$$\text{e.g. } \{x_2, x_3, x_5\} \to \{0, 1, 1, 0, 1, 0, \dots, 0\}$$

By the product rule, there are $2^{|S|}$ bitstrings of length $|S|$, i.e. $|\mathcal{P}(S)| = 2^{|S|}$

Example How many bitstrings of length 8 start with a 1 bit or end with the two bits 00?

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| = 2^7 + 2^6 - 2^5 = 2^5(2^2 + 2 - 1) = 32\dot{5} = 160$

There are $n \mid d$ ways to do a task if it can be done using a procedure that can be carried out in n ways and for each way w, exactly d of the n ways correspond to way w.

Example How many ways are there to seat 4 people at a circular table? (2 sittings are the same if each person has the same left and right neighbors i.e. equal under rotation.)

$$4! = 24 \text{ (to order)}$$
$$\frac{4!}{4} = \frac{24}{4} = 6 \text{ (4 ways to choose a person in seat 1)}$$

Example How many different bitstrings of length 4 do not have consecutive 1s?

$$2^3 = 8 = \# \text{ of bitstrings that have 2 consecutive 1s}$$
$$2^4 = 16 = \# \text{ of total bitstrings}$$
$$2^4 - 2^3 = 8 = \# \text{ of bitstrings with no consecutive 1s}$$