MATH 381 Section 1.1

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10 January 2024

1 Get the title of this section

1.1 Introduction

Definition A **proposition** is a sentence that declares a fact that is either true or false but not both.

Definition If p is a proposition, then $\bar{p}$ or $\neg p$ is the **negation** of proposition p, i.e. "it is not the case that p [holds]".

Definition Assume that p and q are propositions. Then the **conjucntion** of p and q, $p \wedge q$, is the proposition p **and** q.

Remark $p \wedge q$ is true when both p **and** q are true. It is false otherwise.

Definition Let p and q be two propositions. We define the **disjunction** of p and q, $p \vee q$ to be proposition p **or** q.

Remark $p \vee q$ is false when both p and q are false. It is true otherwise.

Remark If we have n propositions, its associated table contains 2^n rows.

Definition p and q are two propositions.
The **exclusive** of p and q, $p \oplus q$, is to be a proposition that is true exactly when either p or q is true but not both.

$$p \oplus q = p \vee q - p \wedge q$$

Example
p: "A student can have a salad with dinner."
q: "A student can have soup with dinner."
Express

- $p \wedge q$ "A student can have both a salad and soup with dinner."
- $p \vee q$ "A studet can have a salad or soup or both."
- $p \oplus q$ "A student can have dinner with either salid or soup."

Definition Let p and q be two propositions.
The **conditional statement** $p \to q$ is a proposition: "if p then q".

Remark $p \to q$ is false if $p =$ true and $q =$ false and true otherwise.

Remark Terminology:

- if p then q
- if p, q
- p is sufficient for q
- q if p
- q when p
- a necessary condition for q is p
- q unless $\neg p$
- q provided that p

Proof Prove that the exclusive $p \to q$ is logicaly equivalent to the contrapositive $\neg q \to \neg p$.
(i.e. the two exlusive propositions have the same T or F values)

Definition The **biconditional statement** $p \leftrightarrow q$ is the proposition "p if and only if q". It is true whenever p and q have the same value.

Proof Is the biconditional statement $p \leftrightarrow q$ logically equivalent to $(p \to q) \wedge (q \to p)$? Yes $\because$ the implication between p and q goes both ways $\therefore p \leftrightarrow q \iff (p \to q) \wedge (q \to p)$.

Definition

1. The **converse** of the conditional statement $p \to q$ is $q \to p$.
2. The **contrapositive** of the conditional statement $p \to q$ is $\neg q \to \neg p$.

3. The **inverse** of the conditional statement $p \to q$ is $\neg p \to \neg q$.

Definition

1. A **bit** is a symbol $\{0, 1\}$.
2. A **boolean variable** is a variable that can either be true or false.
3. The **length** of a bitstring is the number of bits in the string.

Example bitwise
$x := 0110110110$
$y := 1100011101$
$x \vee y := 1110111111$
$x \wedge y := 0100010100$
$x \oplus y := 1010101011$