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CS 135 Homework 9

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I pledge my honor that I have abided by the Stevens Honor System.

Section 4.1

13.

- a. 10
- c. 0
- e. 6

26. 16, 28, 40, 52, & 64

Section 4.2

22.

- a. Sum = 1022
Product = 101220
- b. Sum = 21210
Product = 111020122

26.

$$(644)_{10} = (10\ 1000\ 0100)_2$$

$$a = 1010000100$$

$$i = 0 - 9$$

$$x = 1$$

$$i = 0. a_0 = 0, x = 1 \text{ and power} = 11^2 \bmod 645 = 121$$

$$i = 1. a_1 = 0, x = 1 \text{ and power} = 121^2 \bmod 645 = 451$$

$$i = 2. a_2 = 1, x = (1 * 451) \bmod 645 = 451 \text{ and power} = 451^2 \bmod 645 = 226$$

$$i = 3. a_3 = 0, x = 451 \text{ and power} = 226^2 \bmod 645 = 121$$

$$i = 4. a_4 = 0, x = 451 \text{ and power} = 121^2 \bmod 645 = 451$$

$$i = 5. a_5 = 0, x = 451 \text{ and power} = 451^2 \bmod 645 = 226$$

$$i = 6. a_6 = 0, x = 451 \text{ and power} = 226^2 \bmod 645 = 121$$

$$i = 7. a_7 = 1, x = (451 * 121) \bmod 645 = 391 \text{ and power} = 121^2 \bmod 645 = 451$$

$$i = 8. a_8 = 0, x = 391 \text{ and power} = 451^2 \bmod 645 = 226$$

$$i = 9. a_9 = 1, x = (391 * 226) \bmod 645 = 1, x = 1$$

Section 4.4

1. $15 * 7 = 105 \equiv 1 \pmod{26}$

6.

$$\text{b. } 89 = 2 * 34 + 21$$

$$34 = 1 * 21 + 13$$

$$21 = 1 * 13 + 8$$

$$13 = 1 * 8 + 5$$

$$8 = 1 * 5 + 3$$

$$5 = 1 * 3 + 2$$

$$3 = 1 * 2 + 1$$

$$2 = 2 * 1$$

Retrace:

$$1 = 1*3 + (-1) * 2 = (-1) * 5 + 2 * 3 = 2 * 8 + (-3) * 5 = (-3) * 13 + 5 * 8$$

$$= 5 * 21 + (-8) * 12 = (-8) * 34 + 13*21 = 13 * 89 + (-34) * 34$$

$$\text{Answer: } -34 \equiv 55 \pmod{89}$$

12.

$$\text{a. } x = 52 \pmod{89}$$

20. All values of k that fit the following: $k = 53 \pmod{60}$, $x = 60*k + 53$

$$34. \gcd(23, 41) = 1$$

$$23^{40} = 1 \pmod{41}$$

$$23^{1002} = 23^{40*25} * 23^2 = 1^{250} * 529 = 37 \pmod{41}$$

Section 4.6 (Assume A = 0, B = 1, ... Z = 25)

1.

a. GR QRW SDVV JR

5.

a. SURRENDER NOW

$$24. 19^{13} \pmod{43 * 59} = 2299$$

$$1900^{13} \pmod{43 * 59} = 1317$$

$$210^{13} \pmod{43 * 59} = 2117$$

$$2299 \ 1317 \ 2117$$

26. SQUIRREL

30. First, Alice and Bob have to agree to use a prime number $p(101)$ and a primitive root $a(2)$ of $p(101)$. Then, Alice has to choose a secret integer $k_1(7)$ and sends Bob $a^{k_1} = 2^7 = 27 \pmod{101}$. Bob then has to choose his secret integer $k_2(9)$ and sends Alice $a^{k_2} = 2^9 = 7 \pmod{101}$. Alice uses the 7 to compute $7^7 = 90 \pmod{101}$. Bob uses the 27 to compute $27^9 = 90 \pmod{101}$. Each key equals 90 so it's a success.