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I pledge my honor that I have abided by the Stevens Honor System.

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3.

a. It will be proved that for the positive integer n ≥ 1, 1^2 = 1((1)+1)(2(1)+1)/6
1^2 = 1 * 2 * 3 / 6

b. Basis step, P(1):
1^2 = 1*((1)+1)*(2(1)+1)/6
1 = 2*3/6
1 = 1

c. Inductive Hypothesis: $1^2 + 2^2 + \ldots + k^2 = k (k +1) (2k + 1)/6$

d. P(k + 1) = $1^2 + 2^2 + \ldots + k^2 + (k + 1)^2 = (k + 1) (k + 2) (2(k + 1) + 1)/6$
$1^2 + 2^2 + \ldots + k^2 + (k + 1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

e. $(1^2 + 2^2 + \ldots + k^2) + (k + 1)^2 = \frac{k+1}{6}\ [k(2k+1) + 6(k+1)]$
$= \frac{k+1}{6}[2k^2 + k +6k + 6] = \frac{k+1}{6}(2k^2 + 7k +6)$
$= \frac{k+1}{6}(2k+3)(k+2) = \frac{(k+1)(k+2)(2k+3)}{6}$

f. Since the basis step and inductive step are done, the principle of mathematical induction states that the statement is true for every positive integer n.

31. Prove that 2 divides n^2 + n whenever n is a positive integer
Basis step: $1^2 + 1 = 2$
Inductive hypothesis: $k^2 + k$ is divisible by 2
Inductive step:
$(k + 1)^2 + (k + 1) = k^2 + 2k +1 + k + 1 = (k^2 + 3k + 2) = (k^2 + k) + 2(k + 1)$
This step proves that the sum of a multiple of 2 and a multiple of 2 is divisible by 2

Section 5.2

6.

a. 3, 6, 9, 10, 12, 13, 15, 16, and n ≥ 18

b. P(n) means n is the amount of postage you can create with 3 and 10-cent stamps. P(n) is true for all n ≥ 18
Basis step: Let Q(6) = P(18) ^ P(19) ^ P(20):
18 = 3*6, 19=10*1 + 3*3, and 20 = 10*2.
Inductive Hypothesis: Suppose that Q(n – 1) is true, then there must be such that:
$3a_1 + 10b_1 = 3n - 3$
$3a_2 + 10b_2 = 3n - 2$
$3a_3 + 10b_3 = 3n - 1$
Then we find:

$3(a_1 + 1) + 10b_1 = 3n$
$3(a_2 + 1) + 10b_2 = 3n + 1$
$3(a_3 + 1) + 10b_3 = 3n + 2$

c. P(n) means n is the amount of postage you can create with 3 and 10-cent stamps.
Basis step: Let S(n) mean that P(k) is true for all P(6)= S(18) ^ S(19) ^ S(20):
18 = 3*6, 19=10*1 + 3*3, and 20 = 10*2.
Inductive Hypothesis: For n > 20, if S(n) is true, then P(n-2) is also true. Since P(n-2) is true, there exists a, $b \in Z \geq 0$ that satisfies 3a + 10b = n -2. If we add 3 to both sides we get 3(a+1) + 10b = n + . Therefore P(n+1) is true.

Section 5.3

3.

a. f(2) = 2 + 3(-1) = 2 – 3 = -1
f(3) = -1 + 3(2) = -1 + 6 = 5
f(4) = 5 + 3(-1) = 5 – 3 = 2
f(5) = 2 + 3(5) = 2 + 15 = 17

13. $P(n) = f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$
Basis step: $f_1 = 1 = f_2$ -> P(1) is true.
Inductive hypothesis: P(k) is true.
Inductive step, where n = k +1: $f_1 + f_3 + \ldots + f_{2k-1} + f_{2k+1} = f_{2k} + f_{2k+1} = f_{2k+2} = f_{2(k+1)}$

```
(define (increasing? numList)
  (if (or (null? numList) (null? (cdr numList)))
    #t
    (if (>= (car numList) (cadr numList))
       #f
       (increasing? (cdr numList)))))
```

Prove the function returns true for the increasing list (1, 2)

Let R(n) be $f_1 < f_2 < \ldots < f_{n.}$

Basis step: P(1) is true because $f_1 = 1 < f_2$.

Inductive step: Assume R(k) is true. Then $f_1 < f_2 < \ldots < f_k$

y=k+x
Prove that (k, y) is increasing.
k ≥ y? If it isn't, recursive call.
cdr of y null? #t
For any integer x>0, list starting with k, where following integers are k+x, the function will return #t.