Equivalent material layer to an RF sheath

A. Introduction

COMSOL can incorporate a material layer in front of surfaces. Here we derive the electrical properties (complex dielectric or conductivity and permittivity) in order that such a material layer would simulate a sheath.

B. Sheath impedance

We begin by writing down the impedance of a sheath in SI units (SI units are the units of this document unless otherwise explicitly indicated) and relating the sheath impedance to the dimensionless parameter discussed in Refs. 1 and 2. From Eq. (A13) of Ref. 1 we have

$$Z_{sh} = Z_0 \hat{z} \tag{1}$$

where Z_{sh} is the sheath impedance of a surface in Ohms, $\hat{z}$ is the dimensionless sheath impedance parameter and

$$Z_0(\text{Ohms}) = \frac{9\times 10^{11} 4\pi\lambda_{de}(\text{cm})}{\omega_{pi}(\text{s}^{-1}) A_\perp(\text{cm}^2)} \tag{2}$$

Here λ_{de} and ω_{pi} are to be evaluated at the sheath entrance. Using $\mu_0 = 4\pi\times 10^{-7}$ H/m, $c = 3\times 10^8$ m/s and $\varepsilon_0 = 1/(\mu_0 c^2)$ F/m we find that in SI units

$$Z_0 = \frac{\lambda_{de}}{\varepsilon_0 \omega_{pi} A_\perp} \tag{3}$$

(i.e. where λ_{de} and $A_\perp$ are now in m and m^2 respectively).

C. Equivalent layer

The voltage across the sheath is related to the total current flowing through the sheath by

$$V_{sh} = Z_{sh} I_{sh} \tag{4}$$

Using Eq. (1) and $I_{sh} = J_n A_\perp$ where J_n is the sheath current density normal to the surface and $A_\perp$ is the area of the surface, we have

$$J_n = \frac{I_{sh}}{A_\perp} = \frac{V_{sh}}{A_\perp Z_{sh}} = \frac{V_{sh}}{A_\perp Z_0 \hat{z}} \tag{5}$$

For an equivalent material layer, we have

$$J_n = \sigma E = \sigma \frac{V_{sh}}{d} \tag{6}$$

where d is the layer thickness. Equating the two yields

$$\frac{\sigma}{d} = \frac{\hat{y}}{A_\perp Z_0} = \varepsilon_0 \frac{\omega_{pi}}{\lambda_{de}} \hat{y} \tag{7}$$

where $\hat{y} = 1/\hat{z}$ is the dimensionless sheath admittance. Thus, in terms of the complex conductivity we have

$$\sigma = \varepsilon_0 \hat{y} \omega_{pi} \frac{d}{\lambda_{de}} \tag{8}$$

or equivalently in terms of the complex dielectric susceptibility, using $\varepsilon = i\sigma/\omega$,

$$\varepsilon = i\varepsilon_0 \frac{\hat{y}}{\hat{\omega}} \frac{d}{\lambda_{de}} \tag{9}$$

where $\hat{\omega} = \omega / \omega_{pi}$ is the dimensionless RF frequency.

The real-valued equivalent conductivity is clearly

$$\sigma = \varepsilon_0 \omega_{pi} \frac{d}{\lambda_{de}} \mathrm{Re}(\hat{y}) \tag{10}$$

while the real-valued equivalent relative permittivity is

$$\varepsilon_r = -\frac{\mathrm{Im}(\hat{y})}{\hat{\omega}} \frac{d}{\lambda_{de}} \tag{11}$$

Note that $\mathrm{Im}(\hat{y}) < 0$ almost always for RF sheaths, i.e. they behave capacitively.

D. Summary

Eqs. (10) and (11) plus the python (or equivalent) routine for calculating $\hat{y}(\hat{\omega}, \hat{\Omega}, b_n, \xi)$ are all that is needed to complete the equivalent material layer analysis. Here $\hat{\omega} = \omega / \omega_{pi}$, $\hat{\Omega} = \Omega_i / \omega_{pi}$, $b_n = \mathbf{b} \cdot \mathbf{n}$, $\xi = eV_{sh}/T_e$, $\Omega_i = eB/m_i c$, $\mathbf{b} = \mathbf{B}/B$, $\mathbf{n}$ is the unit surface normal and V_{sh} is the 0-peak RF voltage across the sheath (or dielectric layer).

E. Check on the conversions

It is straightforward to check the result in the limit of a capacitive sheath. Up to a constant which is essentially unity, the dimensionless admittance of a capacitive sheath is given by Eq. (42) of Ref. 2 as

$$\hat{y} = -i \frac{\hat{\omega}}{\hat{\Delta}} \tag{12}$$

where $\hat{y} = \hat{\Delta} = D / \lambda_{de}$ is the dimensionless sheath width. Substituting into Eq. (11) yields

$$\varepsilon_r = \frac{d}{\Delta} \tag{13}$$

which is the expected result for obtaining equal capacitances of a vacuum layer of width Δ and a dielectric layer of width d.

F. Sheath voltage - consistency check

For a thin layer, the normal electric field in the layer E_n should be constant across the layer, and layer voltage $V = d\, E_n$ should be a proxy for the sheath voltage.

Let us make a change of notation and use subscript s to denote sheath (simulated by the layer) and subscript p for plasma. Matching across the interface, we have

$$\varepsilon_s E_s = \overleftrightarrow{\varepsilon}_p \cdot \mathbf{E}_p \tag{14}$$

where E_s is now the normal voltage in the layer and ε_s is the complex scalar dielectric in the layer. The sheath voltage is

$$V_s = d_s E_s = d_s \frac{\overleftrightarrow{\varepsilon}_p \cdot \mathbf{E}_p}{\varepsilon_s} \tag{15}$$

where both dielectrics are relative to ε_0. Employing Eq. (9) we have

$$V_s = d_s E_s = -i\hat{\omega}\lambda_{de} \frac{\overleftrightarrow{\varepsilon}_p \cdot \mathbf{E}_p}{\hat{y}} \tag{16}$$

which demonstrates that the sheath voltage, so calculated, is independent of the layer width d_s as it should be. Finally, employing Eqs. (1) and (3)

$$V_s = -i\omega Z_s A_\perp \varepsilon_0 \overleftrightarrow{\varepsilon}_p \cdot \mathbf{E}_p = Z_s A_\perp \overleftrightarrow{\sigma}_p \cdot \mathbf{E}_p = \mathbf{J}_p A_\perp Z_s = I_p Z_s = I_s Z_s \tag{17}$$

where in the final step we have used the continuity of current across the interface. The result is again as expected. The sheath voltage proxy is just the sheath current times the sheath impedance.

References

1. J. R. Myra and D. A. D'Ippolito, "Radio frequency sheaths in an oblique magnetic field," Phys. Plasmas **22**, 062507 (2015).
2. J. R. Myra, "Physics-based parametrization of the surface impedance for radio frequency sheaths," Phys. Plasmas **24**, 072507 (2017).