Parallel Implementations of Computational Models of Multiple Scattering of Light

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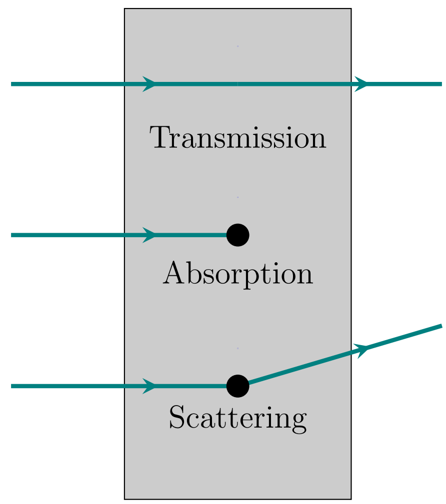
Summary of the Project

Capturing the effects of absorption and scattering on light passing through a medium has various applications in areas such as biomedical optics, atmospheric sciences, and several areas of physics. In the figure below, we show three effects our models will capture as light passes through a medium.



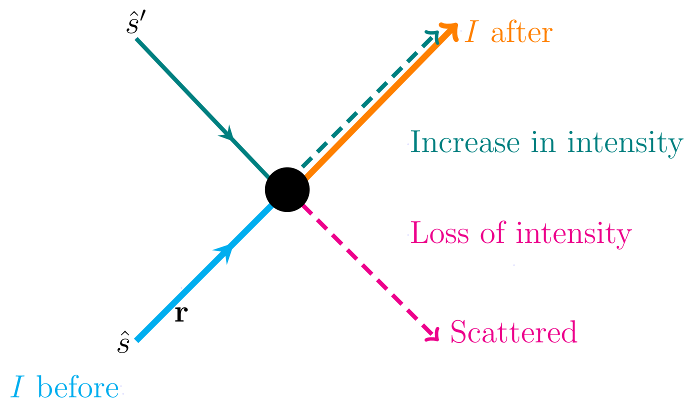
For this project, we plan to numerically approximate the radiative transfer equation in different dimensions. To approximate solutions to this equation, we solve the associated generalized eigenvalue problems by taking advantage of several inherent symmetries of the model. We apply the discrete ordinate method and spacial Fourier Transforms to find approximations to the radiative transfer equation. Parallelization will be used in the construction of necessary matrices as well as in computing one dimensional solutions across a discretized, two dimensional grid. Solutions to the radiative transfer equation are needed to solve related inverse problems in various scientific applications. The model has several opportunities to be parallelized which will greatly decrease computation time.

Aims and Background

Colton was exposed to this problem during an REU at UC Merced with professors Arnold Kim and Boaz Ilan. During the experience, Colton implemented a numerical method to solve the radiative transfer equation in a one-spacial, one-angular dimension case in Matlab. Building upon this, we hope to implement similar methods to solve higher dimensional cases of the radiative transfer equation. This project will benefit the research of professors Kim and Ilan in their large scale simulations which are required to solve related inverse scattering problems.

Approach and Methodology

The discrete ordinate method we plan on implementing is based on using a Gaussian-Legendre quadrature rule to approximate the integral in the right hand side of the radiative transfer equation given below.



$$\underbrace{\hat{s}\cdot\nabla I}_{\text{Change in intensity}} = -\underbrace{(\underbrace{\mu_a I}_{\text{Absorption}} + \underbrace{\mu_s I}_{\text{Scattering}})}_{\text{Loss of intensity}} + \overbrace{\mu_s \int_{S^2} p(\hat{s},\hat{s}')I(\mathbf{r},\hat{s}')\mathrm{d}\hat{s}'}^{\text{Increase in intensity}}$$

For the one spacial, one angular dimensional case we can non-dimensionalize the radiative transfer equation and obtain

$$\mu\frac{\partial I}{\partial \tau} + I = \varpi_0 \int_{-1}^{1} h(\mu,\mu')I(\mu',\tau)\mathrm{d}\mu'$$

where

$$\mu = \cos(\theta)$$

$$\tau = (\mu_a + \mu_s)z$$

with boundary conditions

$$I(\mu,0) = \alpha(\mu) \text{ for } 0 < \mu \leq 1$$

$$I(\mu,\tau_f) = \beta(\mu) \text{ for } -1 \leq \mu < 0$$

In this case, we will use Gaussian Quadrature to approximate the right hand side of the equation.

$$\mu\frac{\partial I}{\partial\tau} + I = \varpi_0 \underbrace{\int_{-1}^{1} h(\mu,\mu')I(\mu',\tau)\mathrm{d}\mu'}_{\text{Approximate using Gaussian Quadrature}}$$

Gaussian Quadrature:

$$\int_{-1}^{1} f(x)\mathrm{d}x \approx \sum_{j=1}^{N} w_j f(x_j)$$

Using this method to approximate the integral is advantageous for several reasons. First, using this quadrature rule allows us to derive several inherent symmetries in the eigenvalues and eigenvectors of the resulting system. Taking advantage of these symmetries will allow us to reduce memory usage by half when setting up and solving the generalized eigenvalue problems associated to each approximation. Furthermore the inherent symmetries will allow us to parallelize when we go to apply boundary conditions to our solutions.

Expected Outcome

At the conclusion of the REU, the team was able to write a Matlab code to solve the radiative transfer equation in one spacial and one angular dimension. This code takes a significant amount of time to run (a few minutes for a relatively small grid). Professor Kim's research simulations to compute solutions to the radiative transfer equation in three spacial and two angular dimensions take days to run. We aim to greatly reduce these computation times and gain reasonable speedup through a parallel implementation.

Bibliography

[1] Kim, Arnold D., and Miguel Moscoso. "Radiative transfer computations for optical beams." Journal of Computational Physics (2003): 50-60. Print.

[2] Stamnes, Knut, et al. "Numerically stable algorithm for discrete-ordinate-method radiative transfer in multiple scattering and emitting layered media." Applied Optics 27.12 (1988): 2502-09. Print.