ODE and PDE models for cell differentiation

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Biographie – After a Master degree of Mathematics of Modelling in 2020, my keen interest in the biological applications of mathematics led me to pursue in this field as a PhD student at Laboratoire Jacques-Louis Lions. My research focuses on the modelling and the analysis of cell differentiation and its applications in oncology, and is supervised by Pierre-Alexandre Bliman, Nastassia Pouradier Duteil and Camille Pouchol.

Resumé :

Cell differentiation is a biological process in which a cell changes its phenotype (or biological trait) and thus acquires new characteristics.

Cell differentiation phenomena can be modelled by ordinary differential equations which describe the behavior of several molecules in interaction, and for which each stable equilibrium corresponds to a possible phenotype. Such models are widely used in biology in order to describe cell-fate phenomena such as epithelial-mesenchymal transition, hematopoietic stem cells or embryonic stem cells. Nevertheless, these systems have been little studied from a mathematical point of view.

It is well-known that such models may lead to monostability or multistability, depending on the selected parameters. However, extensive numerical simulations have led systems biologists to conjecture that in the vast majority of cases, there cannot be more than two stable points. We proved this result in [1], which brings a deeper understanding of the phenomena necessary for the existence of three phenotypes or more.

After presenting our contribution to the study and the understanding of these ODE models, which describe the behaviour of a single cell, we develop a PDE model describing the behaviour of a cell colony undergoing differentiation. The model we propose includes a local advection term corresponding to the cell differentiation phenomenon, and a non-local reaction term which models the growth of the population. This new advection-reaction model in in line with previous well-known biological models, which take into account advection only or growth only [2]. We are especially interested in the asymptotic behaviour of this model, and we show that it leads to interesting concentration behaviors, determined both by growth and advection.

We will compare this result with those of the two other existing models, illustrate it with numerical schemes and discuss their biological relevance.

Références

[1] Jules Guilberteau, Camille Pouchol, and Nastassia Pouradier Duteil. Monostability and bistability of biological switches. *arXiv preprint arXiv:2104.04227*, 2021.

[2] Benoît Perthame. *Transport equations in biology.* Springer Science & Business Media, 2006.