

Propositional Logic Syntactic Sugar

$\varphi \Leftrightarrow \psi := (\neg \varphi \vee \psi) \wedge (\neg \psi \vee \varphi) \quad \varphi \rightarrow \psi := \neg \varphi \vee \psi$
 $\varphi \oplus \psi := (\varphi \wedge \neg \psi) \vee (\psi \wedge \neg \varphi) \quad \varphi \bar{\wedge} \psi := \neg(\varphi \wedge \psi)$
 $(\alpha \Rightarrow \beta | \gamma) := (\neg \alpha \vee \beta) \wedge (\alpha \vee \gamma) \quad \varphi \bar{\vee} \psi := \neg(\varphi \vee \psi)$

Distributivity: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
 $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
D Morgan: $\neg(a \vee b) \equiv (\neg a \wedge \neg b)$
 $\neg(a \wedge b) \equiv (\neg a \vee \neg b)$
CNF: from truth table, take minterms that are 0.
Each minterm is built as an OR of the negated variables. E.g., $(0, 0, 1) \rightarrow (x \vee y \vee \neg z)$.

SAT SOLVERS
Satisfiability, Validity and Equivalence

$\text{SAT}(\varphi) := \neg \text{VALID}(\neg \varphi) \quad \varphi \Leftrightarrow \psi := \text{VALID}(\varphi \leftrightarrow \psi)$
 $\text{VALID}(\varphi) := (\varphi \Leftrightarrow 1) \quad \text{SAT}(\varphi) := \neg(\varphi \Leftrightarrow 0).$

Sequent Calculus:
- *Validity:* start with $\{\} \vdash \phi$; valid iff $\Gamma \cap \Delta \neq \{\}$
FOR ALL leaves.
- *Satisfiability:* start with $\{\phi\} \vdash \{\}$; satisfiable iff $\Gamma \cap \Delta = \{\}$ for AT LEAST ONE leaf.
- Counterexample/sat variable assignment: var is true, if $x \in \Gamma$; false otherwise; "don't care", if variable doesn't appear.

OPER.	LEFT	RIGHT
NOT	$\frac{\neg \phi, \Gamma \vdash \Delta}{\Gamma \vdash \phi, \Delta}$	$\frac{\Gamma \vdash \neg \phi, \Delta}{\Gamma \vdash \phi, \Delta}$
AND	$\frac{\phi \wedge \psi, \Gamma \vdash \Delta}{\phi, \psi, \Gamma \vdash \Delta}$	$\frac{\Gamma \vdash \phi \wedge \psi, \Delta}{\Gamma \vdash \phi, \Delta \quad \Gamma \vdash \psi, \Delta}$
OR	$\frac{\phi \vee \psi, \Gamma \vdash \Delta}{\phi, \Gamma \vdash \Delta \quad \psi, \Gamma \vdash \Delta}$	$\frac{\Gamma \vdash \phi \vee \psi, \Delta}{\Gamma \vdash \phi, \psi, \Delta}$

Resolution Calculus $\frac{\{ \neg x \} UC_1 \quad \{ x \} UC_2}{C_1 UC_2}$

To prove unsatisfiability of given clauses in CNF: If we reach $\{\}$, the formula is unsatisfiable. E.g., $\{\{a\}, \{\neg a, b\}, \{\neg b\}\}$, we get: $\{a\} + \{\neg a, b\} \rightarrow \{b\}$; $\{b\} + \{\neg b\} \rightarrow \{\}$ (unsatisfiable).
To prove validity, prove UNSAT of negated formula.

Davis Putnam Procedure - proves SAT; To prove validity: prove unsatisfiability of negated formula.

- (1) Compute Linear Clause Form
(Don't forget to create the last clause $\{x_n\}$)
- (2) Last variable has to be 1 (true) \rightarrow find implied variables.
- (3) For remaining variables: assume values and compute newly implied variables.
- (4) If contradiction reached: backtrack.

Linear Clause Forms (Computes CNF) - Bottom up in the syntax tree: convert "operators and variables" into new variable. E.g., $\neg a \vee b$ becomes $x_1 \leftrightarrow \neg a$; $x_2 \leftrightarrow x_1 \vee b$. Use rules below to find CNF.

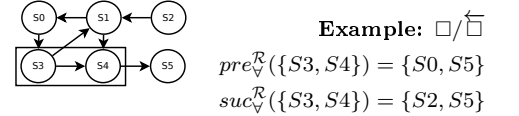
$x \leftrightarrow \neg y \Leftrightarrow (\neg x \vee \neg y) \wedge (x \vee y)$
 $x \leftrightarrow y_1 \wedge y_2 \Leftrightarrow (\neg x \vee y_1) \wedge (\neg x \vee y_2) \wedge (x \vee \neg y_1 \vee \neg y_2)$
 $x \leftrightarrow y_1 \vee y_2 \Leftrightarrow (\neg x \vee y_1 \vee y_2) \wedge (x \vee \neg y_1) \wedge (x \vee \neg y_2)$
 $x \leftrightarrow y_1 \rightarrow y_2 \Leftrightarrow (x \vee y_1) \wedge (x \vee \neg y_1 \vee \neg y_2) \wedge (\neg x \vee \neg y_1 \vee y_2)$
 $x \leftrightarrow (y_1 \leftrightarrow y_2) \Leftrightarrow (x \vee y_1 \vee y_2) \wedge (x \vee \neg y_1 \vee \neg y_2) \wedge (\neg x \vee y_1 \vee \neg y_2) \wedge (\neg x \vee \neg y_1 \vee y_2)$
 $x \leftrightarrow y_1 \oplus y_2 \Leftrightarrow (x \vee \neg y_1 \vee y_2) \wedge (x \vee y_1 \vee \neg y_2) \wedge (\neg x \vee y_1 \vee y_2) \wedge (\neg x \vee \neg y_1 \vee \neg y_2)$

<pre>Compose(int x, BddNode ψ, α) int m; BddNode h, l; if m := 0 then return k else if x>label(ψ) then return ψ; else if x=label(ψ) then return ITE(α, high(ψ), low(ψ)); else m:=max{label(ψ), label(α)}; (α_0, α_1) := Ops(α, m); (ψ_0, ψ_1) := Ops(ψ, m); h := Compose(x, ψ_1, α_1); l := Compose(x, ψ_0, α_0); return CreateNode(m, h, l) endif; end</pre>	<pre>ITE(BddNode i, j, k) int m; BddNode h, l; if i = 0 then return k else if i=1 then return j else if j=k then return k else m = max{label(i), label(j), label(k)}; (i_0, i_1) := Ops(i, m); (j_0, j_1) := Ops(j, m); (k_0, k_1) := Ops(k, m); l := ITE(i_0, j_0, k_0); h := ITE(i_1, j_1, k_1); return CreateNode(m, h, l) end; end</pre>
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Quantif. $\exists x. \varphi := [\varphi]_x^1 \vee [\varphi]_x^0 \quad \forall x. \varphi := [\varphi]_x^1 \wedge [\varphi]_x^0$

Predecessor and Successor
 $\diamond := pre_{\nabla}^R(Q) := \exists x'_1, \dots, x'_n. \varphi_R \wedge [\varphi_Q]_{x'_1, \dots, x'_n}^{x_1, \dots, x_n}$
 $\diamond := suc_{\nabla}^R(Q) := [\exists x_1, \dots, x_n. \varphi_R \wedge \varphi_Q]_{x'_1, \dots, x'_n}^{x_1, \dots, x_n}$

$\square = pre_{\nabla}^R(Q) := \forall x'_1, \dots, x'_n. \varphi_R \rightarrow [\varphi_Q]_{x'_1, \dots, x'_n}^{x_1, \dots, x_n}$
 $\sqsupset = suc_{\nabla}^R(Q) := [\forall x_1, \dots, x_n. \varphi_R \rightarrow \varphi_Q]_{x'_1, \dots, x'_n}^{x_1, \dots, x_n}$



$pre_{\nabla}^R(Q = \{S_1, \dots, S_n\})$ for each node n in \mathcal{K} : if (n points to a node that is not in Q) $n \notin pre_{\nabla}^R(Q)$ else $n \in pre_{\nabla}^R(Q)$	$suc_{\nabla}^R(Q = \{S_1, \dots, S_n\})$ for each node n in \mathcal{K} : if (n is pointed by a node that is not in Q) $n \notin suc_{\nabla}^R(Q)$ else $n \in suc_{\nabla}^R(Q)$
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Tarski-Knaster Theorem: $\mu :=$ starts $\perp \rightarrow$ least fixpoint $\blacktriangledown \nu :=$ starts $\top \rightarrow$ greatest fixpoint

Local Model Checking	
$\frac{s \vdash \varphi \wedge \psi}{\{s \vdash \varphi\} \quad \{s \vdash \psi\} \wedge}$	$\frac{s \vdash \varphi \vee \psi}{\{s \vdash \varphi\} \quad \{s \vdash \psi\} \vee}$
$\frac{s \vdash \varphi \sqcup \varphi}{\{s_1 \vdash \varphi\} \dots \{s_n \vdash \varphi\} \wedge}$	$\frac{s \vdash \varphi \diamond \varphi}{\{s_1 \vdash \varphi\} \dots \{s_n \vdash \varphi\} \vee}$
$\frac{s \vdash \varphi \square \varphi}{\{s'_1 \vdash \varphi\} \dots \{s'_n \vdash \varphi\} \wedge}$	$\frac{s \vdash \varphi \diamond \varphi}{\{s'_1 \vdash \varphi\} \dots \{s'_n \vdash \varphi\} \vee}$
$\frac{s \vdash \varphi \mu x. \varphi \quad s \vdash \varphi \nu x. \varphi}{s \vdash \varphi} \quad \frac{s \vdash \varphi \quad \mathfrak{D}\varphi(\text{replace w. initial form.})}{s \vdash \varphi \mathfrak{D}\varphi(x)}$	
$\{s_1 \dots s_n\} = suc_{\nabla}^R(s)$ and $\{s'_1 \dots s'_n\} = pre_{\nabla}^R(s)$	

Approximations and Ranks	
If $(s, \mu x. \varphi)$ repeats \rightarrow return 1	$apx_0(\mu x. \varphi) := 0$
If $(s, \nu x. \varphi)$ repeats \rightarrow return 0	$apx_0(\nu x. \varphi) := 1$
$apx_{n+1}(\mu x. \varphi) := [\varphi]_x^{apx_n(\mu x. \varphi)}$	
$apx_{n+1}(\nu x. \varphi) := [\varphi]_x^{apx_n(\nu x. \varphi)}$	

AUTOMATA
Automata types: G \rightarrow Safety; F \rightarrow Liveness;
FG \rightarrow Persistence/Co-Buchi; GF \rightarrow Fairness/Buchi.

Automaton Determinization
NDet_G \rightarrow Det_G: 1.Remove all states/edges that do not satisfy acceptance condition; 2.Use Subset construction (Rabin-Scott); 3.Acceptance condition will be the states where $\{\}$ is never reached.
{NDet_F(partial) or NDet_{prefix}} \rightarrow Det_{FG}: Breakpoint Construction.
NDet_F (total) \rightarrow Det_F: Subset Construction.
NDet_{FG} \rightarrow Det_{FG}: Breakpoint Construction.
NDet_{GF} \rightarrow {Det_{Rabin} or Det_{Streett}}: Safra Algorithm.
Boolean Operations on ω -Automata

Complement

$\neg A_{\forall}(Q, \mathcal{I}, \mathcal{R}, \mathcal{F}) = A_{\exists}(Q, \mathcal{I}, \mathcal{R}, \neg \mathcal{F})$
 $\neg A_{\exists}(Q, \mathcal{I}, \mathcal{R}, \mathcal{F}) = A_{\forall}(Q, \mathcal{I}, \mathcal{R}, \neg \mathcal{F})$

Conjunction
 $(A_{\exists}(Q_1, \mathcal{I}_1, \mathcal{R}_1, \mathcal{F}_1) \wedge A_{\exists}(Q_2, \mathcal{I}_2, \mathcal{R}_2, \mathcal{F}_2)) = A_{\exists}(Q_1 \cup Q_2, \mathcal{I}_1 \wedge \mathcal{I}_2, \mathcal{R}_1 \wedge \mathcal{R}_2, \mathcal{F}_1 \wedge \mathcal{F}_2)$

Disjunction
 $(A_{\exists}(Q_1, \mathcal{I}_1, \mathcal{R}_1, \mathcal{F}_1) \vee A_{\exists}(Q_2, \mathcal{I}_2, \mathcal{R}_2, \mathcal{F}_2)) = A_{\exists}\left(\begin{matrix} Q_1 \cup Q_2 \cup \{q\}, \\ (\neg q \wedge \mathcal{I}_1) \vee (q \wedge \mathcal{I}_2), \\ (\neg q \wedge \mathcal{R}_1 \wedge \neg q') \vee (q \wedge \mathcal{R}_2 \wedge q'), \\ (\neg q \wedge \mathcal{F}_1) \vee (q \wedge \mathcal{F}_2) \end{matrix}\right)$

If both automata are totally defined,
 $(A_{\exists}(Q_1, \mathcal{I}_1, \mathcal{R}_1, \mathcal{F}_1) \vee A_{\exists}(Q_2, \mathcal{I}_2, \mathcal{R}_2, \mathcal{F}_2)) = A_{\exists}(Q_1 \cup Q_2, \mathcal{I}_1 \wedge \mathcal{I}_2, \mathcal{R}_1 \wedge \mathcal{R}_2, \mathcal{F}_1 \vee \mathcal{F}_2)$

Eliminate Nesting - Acceptance condition **must** be an automata of the same type
 $A_{\exists}(Q^1, \mathcal{I}_1^1, \mathcal{R}_1^1, A_{\exists}(Q^2, \mathcal{I}_1^2, \mathcal{R}_1^2, \mathcal{F}_1)) = A_{\exists}(Q^1 \cup Q^2, \mathcal{I}_1^1 \wedge \mathcal{I}_1^2, \mathcal{R}_1^1 \wedge \mathcal{R}_1^2, \mathcal{F}_1)$

Boolean Operations of G
(1) $\neg G\varphi = F\neg\varphi$ (2) $G\varphi \wedge G\psi = G[\varphi \wedge \psi]$
(3) $G\varphi \vee G\psi = A_{\exists}(\{p, q\}, p \wedge q, [p' \leftrightarrow p \wedge \varphi] \wedge [q' \leftrightarrow q \wedge \psi], G[p \vee q])$

Boolean Operations of F
(1) $\neg F\varphi = G\neg\varphi$ (2) $F\varphi \vee F\psi = F[\varphi \vee \psi]$
(3) $F\varphi \wedge F\psi = A_{\exists}(\{p, q\}, \neg p \wedge \neg q, [p' \leftrightarrow p \vee \varphi] \wedge [q' \leftrightarrow q \vee \psi], F[p \wedge q])$

Boolean Operations of FG
(1) $\neg FG\varphi = GF\neg\varphi$ (2) $FG\varphi \wedge FG\psi = FG[\varphi \wedge \psi]$
(3) $FG\varphi \vee FG\psi = A_{\exists}(\{q\}, \neg q, q' \leftrightarrow (q \Rightarrow \psi | \neg \varphi), FG[\neg q \vee \psi])$

Boolean Operations of GF
(1) $\neg GF\varphi = FG\neg\varphi$ (2) $GF\varphi \vee GF\psi = GF[\varphi \vee \psi]$
(3) $GF\varphi \wedge GF\psi = A_{\exists}(\{q\}, \neg q, q' \leftrightarrow (q \Rightarrow \neg \psi | \varphi), GF[q \wedge \psi])$

Transformation of Acceptance Conditions
Reduction of G
 $G\varphi = A_{\exists}(\{q\}, q, \varphi \wedge q \wedge q', Fq)$
 $G\varphi = A_{\exists}(\{q\}, q, q' \leftrightarrow q \wedge \varphi, FGq)$
 $G\varphi = A_{\exists}(\{q\}, q, q' \leftrightarrow q \wedge \varphi, GFq)$

Reduction of F
 $F\varphi$ can **not** be expressed by $NDet_G$
 $F\varphi = A_{\exists}(\{q\}, \neg q, q' \leftrightarrow q \vee \varphi, FGq)$
 $F\varphi = A_{\exists}(\{q\}, \neg q, q' \leftrightarrow q \vee \varphi, GFq)$

Reduction of FG
 $FG\varphi$ can **not** be expressed by $NDet_G$
 $FG\varphi = A_{\exists}(\{q\}, \neg q, q \rightarrow \varphi \wedge q', Fq)$

$FG\varphi = A_{\exists}\left(\begin{matrix} \{p, q\}, & \neg p \wedge \neg q, \\ \left[\begin{matrix} (p \rightarrow p') \wedge (p' \rightarrow p \vee \neg q) \wedge \\ (q' \leftrightarrow (p \wedge \neg q \vee \neg \varphi) \vee (p \wedge q)) \end{matrix} \right], \\ G\neg q \wedge Fp \end{matrix}\right)$
 $FG\varphi = A_{\exists}\left(\begin{matrix} \{p, q\}, & \neg p \wedge \neg q, \\ \left[\begin{matrix} (p \rightarrow p') \wedge (p' \rightarrow p \vee \neg q) \wedge \\ (q' \leftrightarrow (p \wedge \neg q \vee \neg \varphi) \vee (p \wedge q)) \end{matrix} \right], \\ GF[p \wedge \neg q] \end{matrix}\right)$

TEMPORAL LOGICS
(S1) Pure LTL: AFGa
(s2) LTL + CTL: AFa
(S3) Pure CTL: AGEFa
(S4) CTL*: AFGa \vee AGEFa

Remarks Beware of Finite Paths

E and A quantify over infinite paths.
 $A\varphi$ **holds** on every state that has no infinite path;
 $E\varphi$ is **false** on every state that has no infinite path;
A0 holds on states with only finite paths;
E1 is **false** on state with only finite paths;
 $\square 0$ holds on states with no successor states;
 $\diamond 1$ holds on states with successor states.
 $F\varphi = \varphi \vee XF\varphi$ $G\varphi = \varphi \wedge XG\varphi$
 $[\varphi U \psi] = \psi \vee (\varphi \wedge X[\varphi U \psi])$
 $[\varphi B \psi] = \neg \varphi \wedge (\varphi \vee X[\varphi B \psi])$
 $[\varphi W \psi] = (\psi \wedge \varphi) \vee (\neg \psi \wedge X[\varphi W \psi])$
Weak Equivalences
 $*[\varphi U \psi] := [\varphi \underline{U} \psi] \vee G\varphi$ $*[\varphi B \psi] := [\varphi \underline{B} \psi] \vee G\neg \psi$
*same to past version $[\varphi W \psi] := \neg[(\neg \varphi) \underline{W} \psi]$
 $\tilde{X}\varphi := \neg \tilde{X}\neg \varphi$ at t0: weak true. strong false

Negation Normal Form
 $\neg(\varphi \wedge \psi) = \neg \varphi \vee \neg \psi$ $\neg(\varphi \vee \psi) = \neg \varphi \wedge \neg \psi$
 $\neg \neg \varphi = \varphi$ $\neg X\varphi = X\neg \varphi$
 $\neg G\varphi = F\neg \varphi$ $\neg F\varphi = G\neg \varphi$
 $\neg[\varphi U \psi] = [(\neg \varphi) \underline{B} \psi]$ $\neg[\varphi \underline{U} \psi] = [(\neg \varphi) B \psi]$
 $\neg[\varphi B \psi] = [(\neg \varphi) \underline{U} \psi]$ $\neg[\varphi \underline{B} \psi] = [(\neg \varphi) U \psi]$
 $\neg A\varphi = E\neg \varphi$ $\neg E\varphi = A\neg \varphi$
 $\neg \tilde{X}\varphi = \tilde{X}\neg \varphi$ $\neg \tilde{X}\varphi = \tilde{X}\neg \varphi$
 $\neg \tilde{F}\varphi = \tilde{G}\neg \varphi$ $\neg \tilde{F}\varphi = \tilde{G}\neg \varphi$

$\neg[\varphi \underline{U} \psi] = [(\neg \varphi) \underline{\tilde{B}} \psi]$ $\neg[\varphi \underline{\tilde{U}} \psi] = [(\neg \varphi) \tilde{B} \psi]$
 $\neg[\varphi \tilde{B} \psi] = [(\neg \varphi) \underline{\tilde{U}} \psi]$ $\neg[\varphi \tilde{U} \psi] = [(\neg \varphi) \underline{\tilde{B}} \psi]$

LTL Syntactic Sugar: analog for past operators
 $G\varphi = \neg[1 \underline{U} (\neg \varphi)]$ $F\varphi = [1 \underline{U} \varphi]$

$[\varphi W \psi] = \neg[(\neg \varphi \vee \neg \psi) \underline{U} (\neg \varphi \wedge \psi)]$
 $[\varphi \underline{W} \psi] = [(\neg \psi) \underline{U} (\varphi \wedge \psi)]$ ($\neg \psi$ holds until $\varphi \wedge \psi$)
 $[\varphi B \psi] = \neg[(\neg \varphi) \underline{U} \psi]$
 $[\varphi \underline{B} \psi] = [(\neg \psi) \underline{U} (\varphi \wedge \neg \psi)]$ (ψ can't hold when φ holds)
 $[\varphi U \psi] = \neg[(\neg \psi) \underline{U} (\neg \varphi \wedge \neg \psi)]$
 $[\varphi \underline{U} \psi] = [\varphi \underline{U} \psi] \vee G\varphi$
 $[\varphi \underline{U} \psi] = \neg[(\neg \psi) U (\neg \varphi \wedge \neg \psi)]$
 $[\varphi \underline{U} \psi] = \neg[(\neg \psi) W (\varphi \rightarrow \psi)]$
 $[\varphi \underline{U} \psi] = [\psi \underline{W} (\varphi \rightarrow \psi)]$
 $[\varphi \underline{U} \psi] = \neg[(\neg \varphi) B \psi]$ (φ doesn't matter when ψ holds)
 $[\varphi \underline{U} \psi] = [\psi \underline{B} (\neg \varphi \wedge \neg \psi)]$

CTL Syntactic Sugar: analog for past operators
Existential Operators
 $EF\varphi = E[1 \underline{U} \varphi]$
 $EG\varphi = E[\varphi U 0]$
 $E[\varphi U \psi] = E[\varphi \underline{U} \psi] \vee EG\varphi$
 $E[\varphi B \psi] = E[(\neg \psi) \underline{U} (\varphi \wedge \neg \psi)] \vee EG\neg \psi$
 $E[\varphi B \psi] = E[(\neg \psi) U (\varphi \wedge \neg \psi)]$
 $E[\varphi B \psi] = E[(\neg \psi) \underline{U} (\varphi \wedge \neg \psi)]$
 $E[\varphi \underline{B} \psi] = E[(\neg \psi) \underline{U} (\varphi \wedge \neg \psi)]$
 $E[\varphi W \psi] = E[(\neg \psi) \underline{U} (\varphi \wedge \psi)]$
 $E[\varphi \underline{W} \psi] = E[(\neg \psi) \underline{U} (\varphi \wedge \psi)]$
Universal Operators
 $AX\varphi = E\neg EX\neg \varphi$
 $AG\varphi = E\neg[1 \underline{U} \neg \varphi]$
 $AF\varphi = E\neg EG\neg \varphi$
 $AF\varphi = E\neg[(\neg \varphi) U 0]$
 $A[\varphi U \psi] = E\neg[(\neg \psi) \underline{U} (\neg \varphi \wedge \neg \psi)]$
 $A[\varphi \underline{U} \psi] = E\neg[(\neg \psi) \underline{U} (\neg \varphi \wedge \neg \psi)] \wedge \neg EG\neg \psi$
 $A[\varphi \underline{U} \psi] = E\neg[(\neg \psi) U (\neg \varphi \wedge \neg \psi)]$
 $A[\varphi B \psi] = E\neg[(\neg \varphi) \underline{U} \psi]$
 $A[\varphi B \psi] = E\neg[(\neg \varphi) U \psi]$
 $A[\varphi \underline{B} \psi] = E\neg[(\neg \varphi \vee \psi) \underline{U} \psi] \wedge \neg EG(\neg \varphi \vee \psi)$

$A[\varphi \ W \ \psi] = \neg E[(\neg \psi) \ \underline{U} \ (\neg \varphi \wedge \psi)]$
 $A[\varphi \ \underline{W} \ \psi] = \neg E[(\neg \psi) \ \underline{U} \ (\neg \varphi \wedge \psi)] \wedge \neg EG\neg \psi$
 $A[\varphi \ \underline{\underline{W}} \ \psi] = \neg E[(\neg \psi) \ \underline{U} \ (\neg \varphi \wedge \psi)]$
CTL* to CTL - Existential Operators

$EX\varphi = EXE\varphi$
 $EF\varphi = EFE\varphi$ $EFG\varphi \equiv EFEG\varphi$

$E[\varphi \ W \ \psi] = E[(E\varphi) \ W \ \psi]$
 $E[\varphi \ \underline{W} \ \psi] = E[(E\varphi) \ \underline{W} \ \psi]$
 $E[\psi \ \underline{U} \ \varphi] = E[\psi \ \underline{U} \ E(\varphi)]$
 $E[\psi \ \underline{\underline{U}} \ \varphi] = E[\psi \ \underline{\underline{U}} \ E(\varphi)]$
 $E[\varphi \ B \ \psi] = E[(E\varphi) \ B \ \psi]$
 $E[\varphi \ \underline{B} \ \psi] = E[(E\varphi) \ \underline{B} \ \psi]$

obs. $EGF\varphi \neq EGEF\varphi \rightarrow$ can't be converted

CTL* to CTL - Universal Operators

$AX\varphi = AXA\varphi$
 $AG\varphi = AGA\varphi$
 $A[\varphi \ W \ \psi] = A[(A\varphi) \ W \ \psi]$
 $A[\varphi \ \underline{W} \ \psi] = A[(A\varphi) \ \underline{W} \ \psi]$
 $A[\varphi \ \underline{U} \ \psi] = A[A(\varphi) \ \underline{U} \ \psi]$
 $A[\varphi \ \underline{\underline{U}} \ \psi] = A[A(\varphi) \ \underline{\underline{U}} \ \psi]$
 $A[\psi \ B \ \varphi] = A[\psi \ B \ (E\varphi)]$
 $A[\psi \ \underline{B} \ \varphi] = A[\psi \ \underline{B} \ (E\varphi)]$

Eliminate boolean op. after path quantify

$[\varphi_1 \ \underline{U} \ \psi_1] \wedge [\varphi_2 \ \underline{U} \ \psi_2] =$

$$\left[(\varphi_1 \wedge \varphi_2) \ \underline{U} \ \left(\psi_1 \wedge [\varphi_2 \ \underline{U} \ \psi_2]^\vee \right) \right]$$

 $[\varphi_1 \ \underline{U} \ \psi_1] \wedge [\varphi_2 \ U \ \psi_2] =$

$$\left[(\varphi_1 \wedge \varphi_2) \ \underline{U} \ \left(\psi_1 \wedge [\varphi_2 \ U \ \psi_2]^\vee \right) \right]$$

 $[\varphi_1 \ U \ \psi_1] \wedge [\varphi_2 \ U \ \psi_2] =$

$$\left[(\varphi_1 \wedge \varphi_2) \ \underline{U} \ \left(\psi_1 \wedge [\varphi_2 \ U \ \psi_2]^\vee \right) \right]$$

Equivalences and Tips

$[\varphi B \psi] \equiv \psi \text{ can't hold when } \varphi \text{ hold}$
 $[\varphi U \psi] \equiv [\varphi \underline{U} \psi] \vee G\varphi$
 $[a \underline{U} Fb] \equiv Fb$
 $F[a \underline{U} b] \equiv Fb \equiv [Fa \underline{U} Fb]$
 $[\varphi B \psi] \equiv [\varphi \underline{B} \psi] \vee G\neg \psi$
 $F[aBb] \equiv F[a \wedge \neg b]$
 $[\varphi W \psi] \equiv \neg[\neg \varphi \underline{W} \psi]$
 $AEA \equiv A$ $GFX \equiv GFX$
 $FF\varphi \equiv F\varphi$ $GG\varphi \equiv G\varphi$
 $GF\varphi \equiv XGF\varphi \equiv FGF\varphi \equiv GGF\varphi \equiv GFGF\varphi \equiv FGGF\varphi$
 $FG\varphi \equiv XFG\varphi \equiv FFG\varphi \equiv GFG\varphi \equiv GFFG\varphi \equiv FGF\varphi$

$GF(x \vee y) \equiv GFx \vee GFy$
 $E(\varphi \wedge \psi) \equiv E\varphi \wedge E\psi \text{ (in general)}$
 $E(\varphi \vee \psi) \equiv E\varphi \vee E\psi$
 $AG(\varphi \wedge \psi) \equiv AG\varphi \wedge AG\psi$

###**MONADIC PREDICATE**

S1S
First order terms are defined as follows:

$-0 \in Term_{\Sigma}^{S1S}$
 $-t \in V_{\Sigma} | typ_{\Sigma}(t) = \mathbb{N} \subseteq Term_{\Sigma}^{S1S}$
 $-SUC(\tau) \in Term_{\Sigma}^{S1S} \text{ if } \tau \in Term_{\Sigma}^{S1S}$

Formulas ζ_{S1S} are defined as:
 $\neg p^{(t)} \in L_{S1S}$ (predicate p at time t)
 $\neg \neg \varphi, \varphi \wedge \psi \in L_{S1S}$
 $\neg \exists t. \varphi \in L_{S1S}$
 $\neg \exists p. \varphi \in L_{S1S}$
 where:

$-\tau \in Term_{\Sigma}^{S1S}$
 $-\varphi, \psi \in \zeta_{S1S}$
 $-t \in V_{\Sigma} \text{ with } typ_{\Sigma}(t) = \mathbb{N}$
 $-p \in V_{\Sigma} \text{ with } typ_{\Sigma}(p) = \mathbb{N} \rightarrow \mathbb{B}$
LO2

first order terms are defined as:
 $-t \in V_{\Sigma} | typ_{\Sigma}(t) = \mathbb{N} \subseteq Term_{\Sigma}^{LO2}$

formulas LO2 are defined as:

$-t1 < t2 \in L_{LO2}$
 $-p^{(t)} \in L_{LO2}$
 $-\neg \varphi, \varphi \wedge \psi \in L_{LO2}$
 $-\exists t. \varphi \in L_{LO2}$
 $-\exists p. \varphi \in L_{LO2}$

where:

$-t, t1, t2, \tau \in V_{\Sigma} \text{ with } typ_{\Sigma}(t) = typ_{\Sigma}(t1) = typ_{\Sigma}(t2) = \mathbb{N}$
 $-\varphi, \psi \in \zeta_{LO2}$
 $-t \in V_{\Sigma} \text{ with } typ_{\Sigma}(t) = \mathbb{N}$
 $-p \in V_{\Sigma} \text{ with } typ_{\Sigma}(p) = \mathbb{N} \rightarrow \mathbb{B}$

LO2' Consider the following set $\zeta_{LO2'}$ of formulas:

$-Subst(p, q), Sing(p), \text{ and } PSUC(p, q) \text{ belong to } \zeta_{LO2'}$
 $-\neg \varphi, \varphi \wedge \psi$
 $-\exists p. \varphi$

where $-\varphi, \psi \in \zeta_{LO2'}$

$-p \in V_{\Sigma} \text{ with } typ_{\Sigma}(p) = \mathbb{N} \rightarrow \mathbb{B}$

$\zeta_{LO2'}$ has nonnumeric variables

numeric variable t is replaced by a singleton set p_t

$\zeta_{LO2'}$ is as expressive as LO2 and S1S

###**TRANSLATIONS**

CTL* Modelchecking to LTL model checking

Let's φ_i be a pure path formula (without path quantifiers), Ψ be a propositional formula, abbreviate subformulas $E\varphi$ and $A\psi$ working bottom-up the syntax tree to obtain the following

normal form: $\phi = \text{let } \begin{bmatrix} x_1 = A\varphi_1 \\ \vdots \\ x_n = A\varphi_n \end{bmatrix} \text{ in } \Psi \text{ end}$

Use LTL model checking to compute

$Q_i := \llbracket A\varphi_i \rrbracket_{\mathcal{K}_{i-1}}$, where $\mathcal{K}_0 := \mathcal{K}$ and \mathcal{K}_{i+1} is obtained from \mathcal{K}_i by labelling the states Q_i with x_i .

Finally compute $\llbracket \Psi \rrbracket_{\mathcal{K}_n}$

function LO2_S1S(Φ)

case Φ of
 $(t1 < t2) : \text{return } \exists p. [\forall t. p^{(t)} \rightarrow p^{(SUC(t))}] \wedge \neg p^{(t1)} \wedge p^{(t2)} ;$
 $p^{(t)} : \text{return } p^{(t)} ;$
 $\neg \varphi : \text{return } \neg LO2_S1S(\varphi) ;$
 $\varphi \wedge \psi : \text{return } LO2_S1S(\varphi) \wedge LO2_S1S(\psi) ;$
 $\exists t. \varphi : \text{return } \exists t. LO2_S1S(\varphi) ;$
 $\exists p. \varphi : \text{return } \exists p. LO2_S1S(\varphi) ;$

end

end

function S1S_LO2(Φ)

case Φ of
 $p^{(n)} : \text{return } \exists t0...tn. p^{(tn)} \wedge zero(t0) \wedge \bigwedge_{i=0}^{n-1} succ(ti, ti+1) ;$
 $p^{(t0+n)} : \text{return } \exists t1...tn. p^{(tn)} \wedge \bigwedge_{i=0}^{n-1} succ(ti, ti+1) ;$
 $\neg \varphi : \text{return } \neg S1S_LO2(\varphi) ;$
 $\varphi \wedge \psi : \text{return } S1S_LO2(\varphi) \wedge S1S_LO2(\psi) ;$
 $\exists t. \varphi : \text{return } \exists t. S1S_LO2(\varphi) ;$
 $\exists p. \varphi : \text{return } \exists p. S1S_LO2(\varphi) ;$

end
end
function ElimFO(Φ) (LO2 TO LO2')
case Φ of
 $t1 = t2 : \text{return } Subset(q_{t1}, q_{t2}) \wedge Subset(q_{t2}, q_{t1})$
 $t1 < t2 : \Psi \equiv \forall q1. \forall q2. PSUC(q1, q2) \rightarrow [Subset(q1, p) \rightarrow Subset(q2, p)] ;$
 $\text{return } \exists p. \Psi \wedge \neg Subset(qt1, p) \wedge Subset(qt2, p) ;$
 $p^{(t)} : \text{return } Subset(qt, p)$
 $\neg \varphi : \text{return } \neg ElimFO(\varphi) ;$
 $\varphi \wedge \psi : \text{return } ElimFO(\varphi) \wedge ElimFO(\psi) ;$
 $\varphi \vee \psi : \text{return } ElimFO(\varphi) \vee ElimFO(\psi) ;$
 $\exists t. \varphi : \text{return } \exists qt. Sing(qt) \wedge ElimFO(\varphi) ;$
 $\exists p. \varphi : \text{return } \exists p. ElimFO(\varphi) ;$

end

end

function Tp2Od($t0, \Phi$) *temporal to L01*

case Φ of
 $is_var(\Phi) : \Psi^{(t0)} ;$
 $\neg \varphi : \text{return } \neg Tp2Od(\varphi) ;$
 $\varphi \wedge \psi : \text{return } Tp2Od(\varphi) \wedge Tp2Od(\psi) ;$
 $\varphi \vee \psi : \text{return } Tp2Od(\varphi) \vee Tp2Od(\psi) ;$
 $X\varphi : \Psi := \exists t1. (t0 < t1) \wedge \forall t2. t0 < t2 \rightarrow t1 \leq t2 \wedge Tp2Od(t1, \varphi) ;$
 $[\varphi \underline{U} \psi] : \Psi := \exists t1. t0 \leq t1 \wedge Tp2Od(t1, \psi) \wedge interval((t0, 1, t1, 0), \varphi) ;$
 $[\varphi B \psi] : \Psi := \forall t1. t0 \leq t1 \wedge interval((t0, 1, t1, 0), \neg \varphi) \rightarrow Tp2Od(t1, \neg \psi) ;$
 $\overleftarrow{X}\varphi : \Psi := \forall t1. (t1 < t0) \wedge (\forall t2. t2 < t0 \rightarrow t2 \leq t1) \rightarrow Tp2Od(t1, \varphi) ;$
 $\overleftarrow{X}\varphi : \Psi := \exists t1. (t1 < t0) \wedge (\forall t2. t2 < t0 \rightarrow t2 \leq t1) \wedge Tp2Od(t1, \varphi) ;$
 $[\varphi \underline{\underline{U}} \psi] : \Psi := \exists t1. t1 \leq t0 \wedge Tp2Od(t1, \psi) \wedge interval((t1, 0, t0, 1), \varphi) ;$
 $[\varphi \overleftarrow{B} \psi] : \Psi := \forall t1. t1 \leq t0 \wedge interval((t1, 0, t0, 1), \neg \varphi) \rightarrow Tp2Od(t1, \neg \psi) ;$

end

return Ψ

end

function interval(l, φ)

case Φ of
 $(t0, 0, t1, 0) : \text{return } \forall t2. t0 < t2 \wedge t2 < t1 \rightarrow Tp2Od(t2, \varphi) ;$
 $(t0, 0, t1, 1) : \text{return } \forall t2. t0 < t2 \wedge t2 \leq t1 \rightarrow Tp2Od(t2, \varphi) ;$
 $(t0, 1, t1, 0) : \text{return } \forall t2. t0 \leq t2 \wedge t2 < t1 \rightarrow Tp2Od(t2, \varphi) ;$
 $(t0, 1, t1, 1) : \text{return } \forall t2. t0 \leq t2 \wedge t2 \leq 3t1 \rightarrow Tp2Od(t2, \varphi) ;$

end

end

ω -Automaton to LO2

$A_{\exists}(q1, ..., qn, \psi I, \psi R, \psi F)$ (input automaton)
 $\exists q1...qn. \Theta LO2(0, \psi I) \wedge (\forall t. \Theta LO2(t, \psi R)) \wedge (\forall t1 \exists t2. t1 < t2 \wedge \Theta LO2(t2, \psi F))$

Where $\Theta LO2(t, \Phi)$ is:

$-\Theta LO2(t, p) := p(t)$ for variable p
 $-\Theta LO2(t, X\psi) := \Theta LO2(t+1, \psi)$
 $-\Theta LO2(t, \neg \psi) := \neg \Theta LO2(t, \psi)$
 $-\Theta LO2(t, \varphi \wedge \psi) := \Theta LO2(t, \varphi) \wedge \Theta LO2(t, \psi)$
 $-\Theta LO2(t, \varphi \vee \psi) := \Theta LO2(t, \varphi) \vee \Theta LO2(t, \psi)$

LTL to ω -automata

$\Phi(X\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow X\varphi, \Phi(q)_x)$

$\Phi(X\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q0, q1\}, 1, (q0 \leftrightarrow \varphi) \wedge (q1 \leftrightarrow Xq0), \Phi(q1)_x)$
 $\Phi(G\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow \varphi \wedge Xq, \Phi(q)_x \wedge GF[\varphi \rightarrow q])$
 $\Phi(F\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow \varphi \wedge Xq, \Phi(q)_x \wedge GF[q \rightarrow \varphi])$
 $\Phi([\varphi \ U \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow \psi \vee \varphi \wedge Xq, \Phi(q)_x \wedge GF[\varphi \rightarrow q])$
 $\Phi([\varphi \ \underline{U} \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow \psi \vee \varphi \wedge Xq, \Phi(q)_x \wedge GF[q \rightarrow \psi])$
 $\Phi([\varphi \ B \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow \neg \psi \wedge (\varphi \vee Xq), \Phi(q)_x \wedge GF[q \vee \psi])$
 $\Phi([\varphi \ \underline{B} \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, 1, q \leftrightarrow \neg \psi \wedge (\varphi \vee Xq), \Phi(q)_x \wedge GF[q \rightarrow \varphi])$
 $\Phi(\overleftarrow{X}\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, q, Xq \leftrightarrow \varphi, \Phi(q)_x)$
 $\Phi(\overleftarrow{X}\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, \neg q, Xq \leftrightarrow \varphi, \Phi(q)_x)$
 $\Phi(\overleftarrow{G}\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, q, Xq \leftrightarrow \varphi \wedge q, \Phi(\varphi \wedge q)_x)$
 $\Phi(\overleftarrow{F}\varphi)_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, \neg q, Xq \leftrightarrow \varphi \vee q, \Phi(\varphi \vee q)_x)$
 $\Phi([\varphi \ \underline{\underline{U}} \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, q, Xq \leftrightarrow \psi \vee \varphi \wedge q, \Phi(\psi \vee \varphi \wedge q)_x)$
 $\Phi([\varphi \ \underline{\underline{B}} \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, \neg q, Xq \leftrightarrow \psi \vee \varphi \wedge q, \Phi(\psi \vee \varphi \wedge q)_x)$
 $\Phi([\varphi \ \overleftarrow{B} \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, q, Xq \leftrightarrow \neg \psi \wedge (\varphi \vee q), \Phi(\neg \psi \wedge (\varphi \vee q))_x)$
 $\Phi([\varphi \ \underline{\underline{B}} \ \psi])_x \Leftrightarrow \mathcal{A}_{\exists}(\{q\}, \neg q, Xq \leftrightarrow \neg \psi \wedge (\varphi \vee q), \Phi(\neg \psi \wedge (\varphi \vee q))_x)$
CTL to μ -Calculus ($\Phi_{inf} = \nu y. \Diamond y$)
 $EX\varphi = \Diamond(\Phi_{inf} \wedge \varphi)$
 $EG\varphi = \nu x. \varphi \wedge \Diamond x$
 $EF\varphi = \mu x. \Phi_{inf} \wedge \varphi \vee \Diamond x$
 $E[\varphi \underline{U} \psi] = \mu x. (\Phi_{inf} \wedge \psi) \vee \varphi \wedge \Diamond x$
 $E[\varphi U \psi] = \nu x. (\Phi_{inf} \wedge \psi) \vee \varphi \wedge \Diamond x$
 $E[\varphi \underline{B} \psi] = \mu x. \neg \psi \wedge (\Phi_{inf} \wedge \varphi \vee \Diamond x)$
 $E[\varphi B \psi] = \nu x. \neg \psi \wedge (\Phi_{inf} \wedge \varphi \vee \Diamond x)$
 $AX\varphi = \Box(\Phi_{inf} \rightarrow \varphi)$
 $AG\varphi = \nu x. (\Phi_{inf} \rightarrow \varphi) \wedge \Box x$
 $AF\varphi = \mu x. \varphi \vee \Box x$
 $A[\varphi \underline{U} \psi] = \mu x. \psi \vee (\Phi_{inf} \rightarrow \varphi) \wedge \Box x$
 $A[\varphi U \psi] = \nu x. \psi \vee (\Phi_{inf} \rightarrow \varphi) \wedge \Box x$
 $A[\varphi \underline{B} \psi] = \mu x. (\Phi_{inf} \rightarrow \neg \psi) \wedge (\varphi \vee \Box x)$
 $A[\varphi B \psi] = \nu x. (\Phi_{inf} \rightarrow \neg \psi) \wedge (\varphi \vee \Box x)$
G and μ -calculus (safety property)
 $-\nu x. \varphi \wedge \Diamond x_K$
 $-\text{Contains states } s \text{ where an infinite path } \pi \text{ starts with } \forall t. \pi^{(t)} \in [\varphi]_K$
 $-\varphi \text{ holds always on } \pi$
F and μ -calculus (liveness property)
 $-\mu x. \varphi \vee \Diamond x_K$
 $-\text{Contains states } s \text{ where a (possibly finite) path } \pi \text{ starts with } \exists t. \pi^{(t)} \in [\varphi]_K$
 $-\varphi \text{ holds at least once on } \pi$
FG and μ -calculus (persistence property)
 $-\mu y. [\nu x. \varphi \wedge \Diamond x] \vee \Diamond y_K$
 $-\text{Contains states } s \text{ where an infinite path } \pi \text{ starts with } \exists t1. \forall t2. \pi^{(t1+t2)} \in [\varphi]_K$
 $-\varphi \text{ holds after some point on } \pi$
GF and μ -calculus (fairness property)
 $-\nu y. [\mu x. (y \wedge \varphi) \vee \Diamond x]_K$
 $-\text{Contains states } s \text{ where an infinite path } \pi \text{ starts with } \forall t1. \exists t2. \pi^{(t1+t2)} \in [\varphi]_K$
 $-\varphi \text{ holds infinitely often on } \pi$