Propositional Logic - Syntactic Sugar

$\varphi \Leftrightarrow \psi := (\neg\varphi \vee \psi) \wedge (\neg\psi \vee \varphi)$ $\quad \varphi \rightarrow \psi := \neg\varphi \vee \psi$

$\varphi \oplus \psi := (\varphi \wedge \neg\psi) \vee (\psi \wedge \neg\varphi)$ $\quad \varphi \bar{\wedge} \psi := \neg(\varphi \wedge \psi)$

$(\alpha \Rightarrow \beta|\gamma) := (\neg\alpha \vee \beta) \wedge (\alpha \vee \gamma)$ $\quad \varphi \bar{\vee} \psi := \neg(\varphi \vee \psi)$

Satisfiability, Validity and Equivalence

$\mathrm{SAT}(\varphi) := \neg\mathrm{VALID}(\neg\varphi)$ $\quad \varphi \Leftrightarrow \psi := \mathrm{VALID}(\varphi \leftrightarrow \psi)$

$\mathrm{VALID}(\varphi) := (\varphi \Leftrightarrow 1)$ $\quad \mathrm{SAT}(\varphi) := \neg(\varphi \Leftrightarrow 0)$.

Conjunctive Normal Form: from truth table, take minterms that are 0. Each minterm is built as an OR of the negated variables. E.g., $(0, 0, 1) \rightarrow (x \vee y \vee \neg z)$.

Distributivity: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Sequent Calculus:

1. Prove validity of ϕ: start with $\{\} \vdash \phi$; ϕ is valid iff $\Gamma \cap \Delta \neq \{\}$ for all leaves; else, counterexample: var is true, if $x \in \Gamma$; false otherwise; "don't care", if variable doesn't appear.

2. Prove satisfiability of ϕ: start with $\{\phi\} \vdash \{\}$; ϕ is satisfiable iff $\Gamma \cap \Delta = \{\}$ for at least one leaf. Satisfying interpretation: same as counterexample.

OPER.	LEFT	RIGHT
NOT	$\frac{\neg\phi,\Gamma\vdash\Delta}{\Gamma\vdash\phi,\Delta}$	$\frac{\Gamma\vdash\neg\phi,\Delta}{\phi,\Gamma\vdash\Delta}$
AND	$\frac{\phi\wedge\psi,\Gamma\vdash\Delta}{\phi,\psi,\Gamma\vdash\Delta}$	$\frac{\Gamma\vdash\phi\wedge\psi,\Delta}{\Gamma\vdash\phi,\Delta \quad \Gamma\vdash\psi,\Delta}$
OR	$\frac{\phi\vee\psi,\Gamma\vdash\Delta}{\phi,\Gamma\vdash\Delta \quad \psi,\Gamma\vdash\Delta}$	$\frac{\Gamma\vdash\phi\vee\psi,\Delta}{\Gamma\vdash\phi,\psi,\Delta}$

Resolution Calculus $\frac{\{\neg x\}\cup C_1 \qquad \{x\}\cup C_2}{C_1 \cup C_2}$

To prove unsatisfiability of given clauses in CNF: If we reach $\{\}$, the formula is unsatisfiable. E.g., $\{\{a\}, \{\neg a, b\}, \{\neg b\}\}$, we get: $\{a\} + \{\neg a, b\} \rightarrow \{b\}; \{b\} + \{\neg b\} \rightarrow \{\}$ (unsatisfiable). To prove validity, prove UNSAT of negated formula.

Linear Clause Forms (Computes CNF) - Bottom up in the syntax tree: convert "operators and variables" into new variable. E.g., $\neg a \vee b$ becomes $x_1 \leftrightarrow \neg a; x_2 \leftrightarrow x_1 \vee b$. Use rules below to find CNF.

$$x \leftrightarrow \neg y \Leftrightarrow (\neg x \vee \neg y) \wedge (x \vee y)$$

$$x \leftrightarrow y_1 \wedge y_2 \Leftrightarrow (\neg x \vee y_1) \wedge (\neg x \vee y_2) \wedge (x \vee \neg y_1 \vee \neg y_2)$$

$$x \leftrightarrow y_1 \vee y_2 \Leftrightarrow (\neg x \vee y_1 \vee y_2) \wedge (x \vee \neg y_1) \wedge (x \vee \neg y_2)$$

$$x \leftrightarrow y_1 \rightarrow y_2 \Leftrightarrow (x \vee y_1) \wedge (x \vee \neg y_1 \vee \neg y_2) \wedge (\neg x \vee \neg y_1 \vee y_2)$$

$$x \leftrightarrow (y_1 \leftrightarrow y_2) \Leftrightarrow (x \vee y_1 \vee y_2) \wedge (x \vee \neg y_1 \vee \neg y_2) \wedge (\neg x \vee y_1 \vee \neg y_2) \wedge (\neg x \vee \neg y_1 \vee y_2)$$

$$x \leftrightarrow y_1 \oplus y_2 \Leftrightarrow (x \vee \neg y_1 \vee y_2) \wedge (x \vee y_1 \vee \neg y_2) \wedge (\neg x \vee y_1 \vee y_2) \wedge (\neg x \vee \neg y_1 \vee \neg y_2)$$

Davis Putnam Procedure - proves SAT; To prove validity: prove unsatisfiability of negated formula. **(1)** Compute Linear Clause Form *(Don't forget to create the last clause $\{x_n\}$)* **(2)**Last variable has to be $\underline{1}$ (true) $\rightarrow$ find implied variables. **(3)**For remaining variables: assume values and compute newly implied variables. **(4)**If contradiction reached: backtrack.

```
Apply(⊙, Bddnode a, b)
 int m; BddNode h, l;
 if isLeaf(a)&isLeaf(b)
       then
  return Eval(⊙,label(a),
        label(b));
 else
  m=max{label(a),label(b)}
  (a0,a1):=Ops(a,m);
  (b0,b1):=Ops(b,m);
  h:=Apply(⊙,a1,b1);
  l:=Apply(⊙,a0,b0);
  return CreateNode(m,h,l)
 end;
end
```

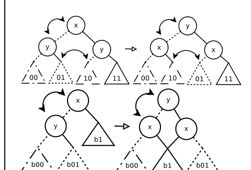
```
Compose(int x, BddNode ψ, α)
 int m; BddNode h, l;
 if x>label(ψ) then
  return ψ;
 elseif x=label(ψ) then
  return ITE(α,high(ψ),
        low(ψ));
 else
  m=max{label(ψ), label(α)}
  (α0,α1):=Ops(α, m);
  (ψ0,ψ1):=Ops(ψ, m);
  h:=Compose(x,ψ1,α1);
  l:=Compose(x,ψ0,α0);
  return CreateNode(m,h,l)
 endif;end
```

```
ITE(BddNode i, j, k)
 int m; BddNode h, l;
 if i = 0 then return k
 elseif i=1 then
  return j
 elseif j=k then
  return k
 else
  m = max{label(i),
      label(j),label(k)}
  (i0,i1):=Ops(i,m);
  (j0,j1):=Ops(j,m);
  (k0,k1):=Ops(k,m);
  l:=ITE(i0,j0,k0);
  h:=ITE(i1, j1, k1);
  return CreateNode(m,h,l)
 end;end
```

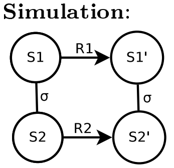
```
Constrain(Φ, β)
 if β=0 then
  ret 0
 elseif Φ ∈ {0,1}(β = 1)
     ret Φ
 else
  m=max{label(β),label(Φ)}
    (Φ0, Φ1):=Ops(Φ,m);
    (β0, β1):=Ops(β,m);
    if β0=0
  ret Constrain(Φ1, β1)
 elseif β1=0 then
  ret Constrain(Φ0, β0)
 else
  l:=Constrain(Φ0, β0);
  h:=Constrain(Φ1, β1);
  ret CreateNode(m,h,l)
endif;endif;end
```

```
Restrict(Φ, β)
 if β=0
  return 0
 elseif
        Φ ∈ {0,1} ∨ (β = 1)

  return Φ
 else
  m=max{label(β),label(Φ)}
  (Φ0, Φ1):=Ops(Φ,m);
  (β0, β1):=Ops(β,m)
 if β0=0
  return Restrict(Φ1, β1)
 elseif β1=0
  return Restrict(Φ0, β0)
 elseif m=label(Φ)
  return CreateNode(m,
   Restrict (Φ1, β1),
   Restrict (Φ0, β0))
 else
  return Restrict(Φ,
  Apply(∨, β0, β1))
 endif;endif;end
```

```
Ops(v,m)
 x:=label(v);
 if m=degree(x)
  return (low(v),high(v))
 else return(v, v)
end;end
----------------------
Other Diagrams:
TODO ZDD FDD

-----
```



Simulation: given $\mathcal{K}_1 = (\mathcal{I}_1, \mathcal{S}_1, \mathcal{R}_1, \mathcal{L}_1)$ and $\mathcal{K}_2 = (\mathcal{I}_2, \mathcal{S}_2, \mathcal{R}_2, \mathcal{L}_2)$; $\sigma \subseteq \mathcal{S}_1 \times \mathcal{S}_2$ is a sim. relation between $\mathcal{K}_1$ and $\mathcal{K}_2$ ($\mathcal{K}_1 \preccurlyeq \mathcal{K}_2$) if: **SIM1-** $(s_1, s_2) \in \sigma$ implies $\mathcal{L}_1(s_1) = \mathcal{L}_2(s_2)$; **SIM2-** for $s_1, s_1' \in \mathcal{S}_1, s_2 \in \mathcal{S}_2$ with $(s_1, s_2) \in \sigma$ and $(s_1, s_1') \in \mathcal{R}_1$, there must be $s_2' \in \mathcal{R}_2$ with $(s_1', s_2') \in \sigma$ $(s_2, s_2') \in \mathcal{R}_2$; **SIM3-** for all $s_1 \in \mathcal{I}_1$, there is a $s_2 \in \mathcal{I}_2$ with $(s_1, s_2) \in \sigma$.



Greatest Simulation Relation

$(s_1, s_2) \in \mathcal{H}_0 \Leftrightarrow \mathcal{L}_1(s_1) = \mathcal{L}_2(s_2)$

$(s_1, s_2) \in \mathcal{H}_{i+1} \Leftrightarrow$

$$\left(\begin{array}{l} (s_1, s_2) \in \mathcal{H}_i \wedge \\ \forall s_1' \in \mathcal{S}_1 . \exists s_2' \in \mathcal{S}_2 . \\ (s_1, s_1') \in \mathcal{R}_1 \rightarrow \quad (s_2, s_2') \in \mathcal{R}_2 \wedge (s_1', s_2') \in \mathcal{H}_i \end{array}\right)$$

$\mathcal{H}_*$ is the greatest simulation relation if **SIM3**: $\mathcal{I}_1 \subseteq \{s_1 \in \mathcal{S}_1 | \exists s_2 \in \mathcal{I}_2.(s_1, s_2) \in \mathcal{H}_*\}$

Bisimulation:$\sigma \subseteq \mathcal{S}_1 \times \mathcal{S}_2$ is a bisim. relation between $\mathcal{K}_1$ and $\mathcal{K}_2$ ($\mathcal{K}_1 \approx \mathcal{K}_2$) if: **BISIM1-** $(s_1, s_2) \in \sigma$ implies $\mathcal{L}_1(s_1) = \mathcal{L}_2(s_2)$; **BISIM2a-** $s_1, s_1' \in \mathcal{S}_1, s_2 \in \mathcal{S}_2$, $(s_1, s_2) \in \sigma$, $(s_1, s_1') \in \mathcal{R}_1$, imply that there is $s_2' \in \mathcal{S}_2$ with $(s_1', s_2') \in \sigma$ and $(s_2, s_2') \in \mathcal{R}_2$;**BISIM2b-** $s_2, s_2' \in \mathcal{S}_2, s_1 \in \mathcal{S}_1$, $(s_1, s_2) \in \sigma$, $(s_2, s_2') \in \mathcal{R}_2$, imply that there is $s_1' \in \mathcal{S}_1$ with $(s_1', s_2') \in \sigma$ and $(s_1, s_1') \in \mathcal{R}_1$;**BISIM3a-** for all $s_1 \in \mathcal{I}_1$, there is a $s_2 \in \mathcal{I}_2$ with $(s_1, s_2) \in \sigma$;**BISIM3b-** for all $s_1 \in \mathcal{I}_2$, there is a $s_2 \in \mathcal{I}_2$ with $(s_1, s_2) \in \sigma$.

Greatest Bisimulation Relation (Equivalence)

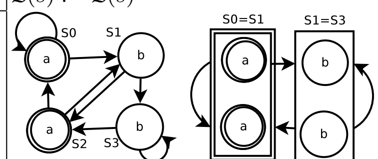
$(s_1, s_2) \in \mathcal{B}_0 \Leftrightarrow \mathcal{L}_1(s_1) = \mathcal{L}_2(s_2)$

$(s_1, s_2) \in \mathcal{B}_{i+1} \Leftrightarrow$

$$\left(\begin{array}{c} (s_1, s_2) \in \mathcal{B}_i \wedge \\ \left[\begin{array}{c} \forall s_1' \in \mathcal{S}_1 . \exists s_2' \in \mathcal{S}_2 . \\ (s_1, s_1') \in \mathcal{R}_1 \rightarrow (s_2, s_2') \in \mathcal{R}_2 \wedge (s_1', s_2') \in \mathcal{B}_i \\ \forall s_2' \in \mathcal{S}_2 . \exists s_1' \in \mathcal{S}_1 . \\ (s_2, s_2') \in \mathcal{R}_2 \rightarrow (s_1, s_1') \in \mathcal{R}_1 \wedge (s_1', s_2') \in \mathcal{B}_i \end{array}\right] \end{array}\right)$$

$\mathcal{B}_*$is the greatest simulation relation if
$\mathcal{I}_1 \subseteq \{s_1 \in \mathcal{S}_1 | \exists s_2 \in \mathcal{I}_2.(s_1, s_2) \in \mathcal{B}_*\}$
$\mathcal{I}_2 \subseteq \{s_2 \in \mathcal{S}_2 | \exists s_1 \in \mathcal{I}_1.(s_1, s_2) \in \mathcal{B}_*\}$

Quotient: given $\mathcal{K} = (\mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{L})$ and the equivalence relation $\sigma \subseteq \mathcal{S} \times \mathcal{S}$; Quotient structure $\mathcal{K}_{/\sigma} = (\widetilde{\mathcal{I}}, \widetilde{\mathcal{S}}, \widetilde{\mathcal{R}}, \widetilde{\mathcal{L}})$: $\widetilde{\mathcal{I}} := \{\{s' \in \mathcal{S} | (s, s') \in \sigma\} | s \in \mathcal{I}\}$
$\widetilde{\mathcal{S}} := \{\{s' \in \mathcal{S} | (s, s') \in \sigma\} | s \in \mathcal{S}\}$
$(\widetilde{s}_1, \widetilde{s}_2) \in \mathcal{R} :\Leftrightarrow \exists s_1' \in \widetilde{s}_1 . \exists s_2' \in \widetilde{s}_2 . (s_1', s_2') \in \mathcal{R}$
$\widetilde{\mathcal{L}}(\widetilde{s}) := \mathcal{L}(s)$



Symbolic Product Computation - given $\mathcal{K}_1 = (\mathcal{V}_1, \varphi_{\mathcal{I}}, \varphi_{\mathcal{R}})$ and $\mathcal{K}_2 = (\mathcal{V}_2, \psi_{\mathcal{I}}, \psi_{\mathcal{R}})$, the product is: $\mathcal{K}_1 \times \mathcal{K}_2 = (\mathcal{V}_1 \cup \mathcal{V}_2, \varphi_{\mathcal{I}} \wedge \psi_{\mathcal{I}}, \varphi_{\mathcal{R}} \wedge \psi_{\mathcal{R}})$

Quantif. $\exists x.\varphi := [\varphi]_x^1 \vee [\varphi]_x^0$ ♣ $\forall x.\varphi := [\varphi]_x^1 \wedge [\varphi]_x^0$

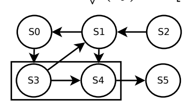
Predecessor and Successor

$\Diamond := pre_{\exists}^{\mathcal{R}}(Q) := \exists x_1', ..., x_n'.\varphi_{\mathcal{R}} \wedge [\varphi_Q]_{x_1,...,x_n}^{x_1',...,x_n'}$

$\overleftarrow{\Diamond} := suc_{\exists}^{\mathcal{R}}(Q) := [\exists x_1, ..., x_n.\varphi_{\mathcal{R}} \wedge \varphi_Q]_{x_1',...,x_n'}^{x_1,...,x_n}$

$\Box = pre_{\forall}^{\mathcal{R}}(Q) := \forall x_1', ..., x_n'.\varphi_{\mathcal{R}} \rightarrow [\varphi_Q]_{x_1,...,x_n}^{x_1',...,x_n'}$

$\overleftarrow{\Box} := suc_{\forall}^{\mathcal{R}}(Q) := [\forall x_1, ..., x_n.\varphi_{\mathcal{R}} \rightarrow \varphi_Q]_{x_1',...,x_n'}^{x_1,...,x_n}$



Example: $\Box / \overleftarrow{\Box}$

$pre_{\forall}^{\mathcal{R}}(\{S3, S4\}) = \{S0, S5\}$

$suc_{\forall}^{\mathcal{R}}(\{S3, S4\}) = \{S2, S5\}$

```
pre∀R(Q = {S1, ..., Sn})
for each node n in K:
 if(n points to a node
       that is not in Q)
  n ∉ pre∀R(Q)
 else
  n ∈ pre∀R(Q)
```

```
suc∀R(Q = {S1, ..., Sn})
for each node n in K:
 if(n is pointed by a node
       that is not in Q)
  n ∉ suc∀R(Q)
 else
  n ∈ suc∀R(Q)
```

Tarski-Knaster Theorem: $\mu :=$ starts $\perp \rightarrow$ least fixpoint ♠ $\nu :=$ starts $\top \rightarrow$ greatest fixpoint * ***Rabin-Scott Subset Construction* 1.** Initial state is a set of states containing all the initial states. **2.** For all transitions of a set of states, compute the successors and create a set of states containing all the possible reachable states when performing that transition. **3.** Acceptance condition are set of states containing acceptance states.

Local Model Checking

(1) $\frac{s\vdash_\Phi \varphi\wedge\psi}{\{s\vdash_\Phi\varphi\} \quad \{s\vdash_\Phi\psi\}} \wedge$	(2) $\frac{s\vdash_\Phi \varphi\vee\psi}{\{s\vdash_\Phi\varphi\} \quad \{s\vdash_\Phi\psi\}} \vee$
(3) $\frac{s\vdash_\Phi \Box\varphi}{\{s_1\vdash_\Phi\varphi\}...\{s_n\vdash_\Phi\varphi\}} \wedge$	(4) $\frac{s\vdash_\Phi \Diamond\varphi}{\{s_1\vdash_\Phi\varphi\}...\{s_n\vdash_\Phi\varphi\}} \vee$
(5) $\frac{s\vdash_\Phi \overleftarrow{\Box}\varphi}{\{s_1'\vdash_\Phi\varphi\}...\{s_n'\vdash_\Phi\varphi\}} \wedge$	(6) $\frac{s\vdash_\Phi \overleftarrow{\Diamond}\varphi}{\{s_1'\vdash_\Phi\varphi\}...\{s_n'\vdash_\Phi\varphi\}} \vee$
$\frac{s\vdash_\Phi \mu x.\varphi}{s\vdash_\Phi\varphi}$ $\quad$ $\frac{s\vdash_\Phi \nu x.\varphi}{s\vdash_\Phi\varphi}$	$\frac{s\vdash_\Phi x}{s\vdash_\Phi \mathfrak{D}_\Phi(x)}$ $\quad$ $\frac{\mathfrak{D}_\Phi\text{(replace w.}}{\text{initial form.)}}$
$\{s_1 \ldots s_n\} = suc_{\exists}^{\mathcal{R}}(s)$ and $\{s_1' \ldots s_n'\} = pre_{\exists}^{\mathcal{R}}(s)$	

Approximations and Ranks

If (s,$\mu x.\varphi$) repeats$\rightarrow$return 1	$apx_0(\mu x.\varphi) := 0$
If (s,$\nu x.\varphi$) repeats$\rightarrow$return 0	$apx_0(\nu x.\varphi) := 1$
$apx_{n+1}(\mu x.\varphi) := [\varphi]_x^{apxn(\mu x.\varphi)}$	
$apx_{n+1}(\nu x.\varphi) := [\varphi]_x^{apxn(\nu x.\varphi)}$	

Automata types: G$\rightarrow$Safety; F$\rightarrow$Liveness; FG$\rightarrow$Persistence/Co-Buchi; GF$\rightarrow$Fairness/Buchi.

Automaton Determinization

$\mathbf{NDet_G \rightarrow Det_G}$: 1.Remove all states/edges that do not satisfy acceptance condition; 2.Use Subset construction (Rabin-Scott); 3.Acceptance condition will be the states where {} is never reached.

$\mathbf{\{NDet_F(partial)\ or\ NDet_{prefix}\} \rightarrow Det_{FG}}$: Breakpoint Construction.

$\mathbf{NDet_F\ (total) \rightarrow Det_F}$: Subset Construction.

$\mathbf{NDet_{FG} \rightarrow Det_{FG}}$: Breakpoint Construction.

$\mathbf{NDet_{GF} \rightarrow \{Det_{Rabin}\ or\ Det_{Streett}\}}$: Safra Algorithm.

* ***Breakpoint Construction* 1.** Each state is composed by two components **2.** Initial state first component is a set of all initial states, and second component is the empty set. Ex.: $(\mathcal{I}, \{\})$. **3.** a successor for a state (Q,Q$_f$) is generated as follows:

$$\begin{cases} \text{If } Q_f = \{\} & (suc_{\exists}^{\mathcal{R}_a}(Q), (suc_{\exists}^{\mathcal{R}_a}(Q) \cap \mathcal{F}) \\ \text{Otherwise} & (suc_{\exists}^{\mathcal{R}_a}(Q), (suc_{\exists}^{\mathcal{R}_a}(Q_f) \cap \mathcal{F}) \end{cases}$$

4. Acceptance states are states where $Q_f \neq \{\}$.

Boolean Operations on ω-Automata

Complement

$$\neg\mathcal{A}_\forall(Q, \mathcal{I}, \mathcal{R}, \mathcal{F}) = \mathcal{A}_\exists(Q, \mathcal{I}, \mathcal{R}, \neg\mathcal{F})$$
$$\neg\mathcal{A}_\exists(Q, \mathcal{I}, \mathcal{R}, \mathcal{F}) = \mathcal{A}_\forall(Q, \mathcal{I}, \mathcal{R}, \neg\mathcal{F})$$

Conjunction

$$(\mathcal{A}_\exists(Q_1, \mathcal{I}_1, \mathcal{R}_1, \mathcal{F}_1) \wedge \mathcal{A}_\exists(Q_2, \mathcal{I}_2, \mathcal{R}_2, \mathcal{F}_2)) = \mathcal{A}_\exists(Q_1 \cup Q_2, \mathcal{I}_1 \wedge \mathcal{I}_2, \mathcal{R}_1 \wedge \mathcal{R}_2, \mathcal{F}_1 \wedge \mathcal{F}_2)$$

Disjunction

$$(\mathcal{A}_\exists(Q_1, \mathcal{I}_1, \mathcal{R}_1, \mathcal{F}_1) \vee \mathcal{A}_\exists(Q_2, \mathcal{I}_2, \mathcal{R}_2, \mathcal{F}_2)) =$$
$$\mathcal{A}_\exists \left(\begin{array}{c} Q_1 \cup Q_2 \cup \{q\}, \\ (\neg q \wedge \mathcal{I}_1) \vee (q \wedge \mathcal{I}_2), \\ (\neg q \wedge \mathcal{R}_1 \wedge \neg q') \vee (q \wedge \mathcal{R}_2 \wedge q'), \\ (\neg q \wedge \mathcal{F}_1) \vee (q \wedge \mathcal{F}_2) \end{array}\right)$$

If both automata are totally defined,

$$(\mathcal{A}_\exists(Q_1, \mathcal{I}_1, \mathcal{R}_1, \mathcal{F}_1) \vee \mathcal{A}_\exists(Q_2, \mathcal{I}_2, \mathcal{R}_2, \mathcal{F}_2)) = \mathcal{A}_\exists(Q_1 \cup Q_2, \mathcal{I}_1 \wedge \mathcal{I}_2, \mathcal{R}_1 \wedge \mathcal{R}_2, \mathcal{F}_1 \vee \mathcal{F}_2)$$

Eliminate Nesting - Acceptance condition **must** be an automata of the same type

$$\mathcal{A}_\exists(Q^1, \mathcal{I}_1^1, \mathcal{R}_1^1, \mathcal{A}_\exists(Q^2, \mathcal{I}_1^2, \mathcal{R}_1^2, \mathcal{F}_1))$$
$$= \mathcal{A}_\exists(Q^1 \cup Q^2, \mathcal{I}_1^1 \wedge \mathcal{I}_1^2, \mathcal{R}_1^1 \wedge \mathcal{R}_1^2, \mathcal{F}_1))$$

Boolean Operations of G

(1)$\neg G\varphi = F\neg\varphi$ $\quad$ (2)$G\varphi \wedge G\psi = G[\varphi \wedge \psi]$

(3)$G\varphi \vee G\psi = \mathcal{A}_\exists(\{p, q\}, p \wedge q, [p' \leftrightarrow p \wedge \varphi] \wedge [q' \leftrightarrow q \wedge \psi], G[p \vee q])$

Boolean Operations of F

(1)$\neg F\varphi = G\neg\varphi$ (2)$F\varphi \vee F\psi = F[\varphi \vee \psi]$

(3)$F\varphi \wedge F\psi = \mathcal{A}_\exists(\{p,q\}, \neg p \wedge \neg q, [p' \leftrightarrow p \vee \varphi] \wedge [q' \leftrightarrow q \vee \psi], F[p \wedge q])$

Boolean Operations of FG

(1)$\neg FG\varphi = GF\neg\varphi$ (2)$FG\varphi \wedge FG\psi = FG[\varphi \wedge \psi]$

(3)$FG\varphi \vee FG\psi = \mathcal{A}_\exists(\{q\}, \neg q, q' \leftrightarrow (q \Rightarrow \psi | \neg\varphi), FG[\neg q \vee \psi])$

Boolean Operations of GF

(1)$\neg GF\varphi = FG\neg\varphi$ (2)$GF\varphi \vee GF\psi = GF[\varphi \vee \psi]$

(3)$GF\varphi \wedge GF\psi = \mathcal{A}_\exists(\{q\}, \neg q, q' \leftrightarrow (q \Rightarrow \neg\psi | \varphi), GF[q \wedge \psi])$

Transformation of Acceptance Conditions

Reduction of G

$G\varphi = \mathcal{A}_\exists(\{q\}, q, \varphi \wedge q \wedge q', Fq))$
$G\varphi = \mathcal{A}_\exists(\{q\}, q, q' \leftrightarrow q \wedge \varphi, FGq)$
$G\varphi = \mathcal{A}_\exists(\{q\}, q, q' \leftrightarrow q \wedge \varphi, GFq)$

Reduction of F

$F\varphi$ can **not** be expressed by $NDet_G$
$F\varphi = \mathcal{A}_\exists(\{q\}, \neg q, q' \leftrightarrow q \vee \varphi, FGq)$
$F\varphi = \mathcal{A}_\exists(\{q\}, \neg q, q' \leftrightarrow q \vee \varphi, GFq)$

Reduction of FG

$FG\varphi$ can **not** be expressed by $NDet_G$
$FG\varphi = \mathcal{A}_\exists(\{q\}, \neg q, q \rightarrow \varphi \wedge q', Fq)$

$$FG\varphi = \mathcal{A}_\exists\left(\begin{array}{c}\{p,q\}, \qquad \neg p \wedge \neg q,\\ \left[\begin{array}{c}(p \rightarrow p') \wedge (p' \rightarrow p \vee \neg q) \wedge\\ (q' \leftrightarrow (p \wedge \neg q \vee \neg\varphi) \vee (p \wedge q))\end{array}\right],\\ G\neg q \wedge Fp\end{array}\right)$$

$$FG\varphi = \mathcal{A}_\exists\left(\begin{array}{c}\{p,q\}, \qquad \neg p \wedge \neg q,\\ \left[\begin{array}{c}(p \rightarrow p') \wedge (p' \rightarrow p \vee \neg q) \wedge\\ (q' \leftrightarrow (p \wedge \neg q \vee \neg\varphi) \vee (p \wedge q))\end{array}\right],\\ GF[p \wedge \neg q]\end{array}\right)$$

Temporal Logics *Beware of Finite Paths*

E and A quantify over infinite paths.
Aφ holds on every state that has no infinite path;
Eφ is false on every state that has no infinite path;
A0 holds on states with only finite paths;
E1 is false on state with only finite paths;
$\Box$0 holds on states with no successor states;
$\Diamond$1 holds on states with successor states.

$F\varphi = \varphi \vee XF\varphi$ $\quad G\varphi = \varphi \wedge XG\varphi$
$[\varphi\ U\ \psi] = \psi \vee (\varphi \wedge X[\varphi\ U\ \psi])$
$[\varphi\ B\ \psi] = \neg\psi \wedge (\varphi \vee X[\varphi\ B\ \psi])$
$[\varphi\ W\ \psi] = (\psi \wedge \varphi) \vee (\neg\psi \wedge X[\varphi\ W\ \psi])$

Negation Normal Form

$\neg(\varphi \wedge \psi) = \neg\varphi \vee \neg\psi$ $\quad \neg(\varphi \vee \psi) = \neg\varphi \wedge \neg\psi$
$\neg\neg\varphi = \varphi$ $\quad \neg X\varphi = X\neg\varphi$
$\neg G\varphi = F\neg\varphi$ $\quad \neg F\varphi = G\neg\varphi$
$\neg[\varphi\ U\ \psi] = [(\neg\varphi)\ \underline{B}\ \psi]$ $\quad \neg[\varphi\ \underline{U}\ \psi] = [(\neg\varphi)\ B\ \psi]$
$\neg[\varphi\ B\ \psi] = [(\neg\varphi)\ \underline{U}\ \psi]$ $\quad \neg[\varphi\ \underline{B}\ \psi] = [(\neg\varphi)\ U\ \psi]$
$\neg A\varphi = E\neg\varphi$ $\quad \neg E\varphi = A\neg\varphi$
$\neg\overleftarrow{X}\varphi = \underline{\overleftarrow{X}}\neg\varphi$ $\quad \neg\underline{\overleftarrow{X}}\varphi = \overleftarrow{X}\neg\varphi$
$\neg\overleftarrow{G}\varphi = \overleftarrow{F}\neg\varphi$ $\quad \neg\overleftarrow{F}\varphi = \overleftarrow{G}\neg\varphi$
$\neg[\varphi\ \overleftarrow{U}\ \psi] = [(\neg\varphi)\ \underline{\overleftarrow{B}}\ \psi]$ $\quad \neg[\varphi\ \underline{\overleftarrow{U}}\ \psi] = [(\neg\varphi)\ \overleftarrow{B}\ \psi]$
$\neg[\varphi\ \overleftarrow{B}\ \psi] = [(\neg\varphi)\ \underline{\overleftarrow{U}}\ \psi]$ $\quad \neg[\varphi\ \underline{\overleftarrow{B}}\ \psi] = [(\neg\varphi)\ \overleftarrow{U}\ \psi]$

LTL Syntactic Sugar: analog for past operators

$G\varphi = \neg[1\ \underline{U}\ (\neg\varphi)]$ $\quad F\varphi = [1\ \underline{U}\ \varphi]$
$[\varphi\ W\ \psi] = \neg[(\neg\varphi \vee \neg\psi)\ U\ (\neg\varphi \wedge \psi)]$
$[\varphi\ \underline{W}\ \psi] = [(\neg\psi)\ \underline{U}\ (\varphi \wedge \psi)]$ $(\neg\psi$ *holds until* $\varphi \wedge \psi)$
$[\varphi\ B\ \psi] = \neg[(\neg\varphi)\ \underline{U}\ \psi)]$
$[\varphi\ \underline{B}\ \psi] = [(\neg\psi)\ \underline{U}\ (\varphi \wedge \neg\psi)]$ $(\psi$ *can't hold when* φ *holds)*
$[\varphi\ U\ \psi] = \neg[(\neg\psi)\ \underline{U}\ (\neg\varphi \wedge \neg\psi)]$
$[\varphi\ U\ \psi] = [\varphi\ \underline{U}\ \psi] \vee G\varphi$
$[\varphi\ \underline{U}\ \psi] = \neg[(\neg\psi)\ U\ (\neg\varphi \wedge \neg\psi)]$
$[\varphi\ \underline{U}\ \psi] = \neg[(\neg\psi)\ W\ (\varphi \rightarrow \psi)]$
$[\varphi\ \underline{U}\ \psi] = [\psi\ \underline{W}\ (\varphi \rightarrow \psi)]$
$[\varphi\ \underline{U}\ \psi] = \neg[(\neg\varphi)\ B\ \psi]$ $(\varphi$ *doesn't matter when* ψ *holds)*
$[\varphi\ \underline{U}\ \psi] = [\psi\ \underline{B}\ (\neg\varphi \wedge \neg\psi]$

CTL Syntactic Sugar: analog for past operators

Existential Operators

$EF\varphi = E[1\ \underline{U}\ \varphi]$
$EG\varphi = E[\varphi\ U\ 0]$
$E[\varphi\ U\ \psi] = E[\varphi\ \underline{U}\ \psi] \vee EG\varphi$
$E[\varphi\ B\ \psi] = E[(\neg\psi)\ \underline{U}\ (\varphi \wedge \neg\psi)] \vee EG\neg\psi$
$E[\varphi\ B\ \psi] = E[(\neg\psi)\ U\ (\varphi \wedge \neg\psi)]$
$E[\varphi\ \underline{B}\ \psi] = E[(\neg\psi)\ \underline{U}\ (\varphi \wedge \neg\psi)]$
$E[\varphi\ \underline{B}\ \psi] = E[(\neg\psi\ \underline{U}\ (\varphi \wedge \neg\psi)]$
$E[\varphi\ W\ \psi] = E[(\neg\psi)\ \underline{U}\ (\varphi \wedge \psi)] \vee EG\neg\psi$
$E[\varphi\ W\ \psi] = E[(\neg\psi)\ U\ (\varphi \wedge \psi)]$
$E[\varphi\ \underline{W}\ \psi] = E[(\neg\psi)\ \underline{U}\ (\varphi \wedge \psi)]$

Universal Operators

$AX\varphi = \neg EX\neg\varphi$
$AG\varphi = \neg E[1\ \underline{U}\ \neg\varphi]$
$AF\varphi = \neg EG\neg\varphi$
$AF\varphi = \neg E[(\neg\varphi)\ U\ 0]$
$A[\varphi\ U\ \psi] = \neg E[(\neg\psi)\ \underline{U}\ (\neg\varphi \wedge \neg\psi)]$
$A[\varphi\ \underline{U}\ \psi] = \neg E[(\neg\psi)\ \underline{U}\ (\neg\varphi \wedge \neg\psi)] \wedge \neg EG\neg\psi$
$A[\varphi\ \underline{U}\ \psi] = \neg E[(\neg\psi)\ U\ (\neg\varphi \wedge \neg\psi)]$
$A[\varphi\ B\ \psi] = \neg E[(\neg\varphi)\ \underline{U}\ \psi]$
$A[\varphi\ \underline{B}\ \psi] = \neg E[(\neg\varphi)\ U\ \psi]$
$A[\varphi\ \underline{B}\ \psi] = \neg E[(\neg\varphi \vee \psi)\ \underline{U}\ \psi] \wedge \neg EG(\neg\varphi \vee \psi)$
$A[\varphi\ W\ \psi] = \neg E[(\neg\psi)\ \underline{U}\ (\neg\varphi \wedge \psi)]$
$A[\varphi\ \underline{W}\ \psi] = \neg E[(\neg\psi)\ \underline{U}\ (\neg\varphi \wedge \psi)] \wedge \neg EG\neg\psi$
$A[\varphi\ \underline{W}\ \psi] = \neg E[(\neg\psi)\ U\ (\neg\varphi \wedge \psi)]$

CTL to $\mu - Calculus(\Phi_{inf} = \nu y.\Diamond y)$

$EX\varphi = \Diamond(\Phi_{inf} \wedge \varphi)$
$EG\varphi = \nu x.\varphi \wedge \Diamond x$
$EF\varphi = \mu x.\Phi_{inf} \wedge \varphi \vee \Diamond x$
$E[\varphi \underline{U} \psi] = \mu x.(\Phi_{inf} \wedge \psi) \vee \varphi \wedge \Diamond x$
$E[\varphi U \psi] = \nu x.(\Phi_{inf} \wedge \psi) \vee \varphi \wedge \Diamond x$
$E[\varphi \underline{B} \psi] = \mu x.\neg\psi \wedge (\Phi_{inf} \wedge \varphi \vee \Diamond x)$
$E[\varphi B \psi] = \nu x.\neg\psi \wedge (\Phi_{inf} \wedge \varphi \vee \Diamond x)$
$AX\varphi = \Box(\Phi_{inf} \rightarrow \varphi)$
$AG\varphi = \nu x.(\Phi_{inf} \rightarrow \varphi) \wedge \Box x$
$AF\varphi = \mu x.\varphi \vee \Box x$
$A[\varphi \underline{U} \psi] = \mu x.\psi \vee (\Phi_{inf} \rightarrow \varphi) \wedge \Box x$
$A[\varphi U \psi] = \nu x.\psi \vee (\Phi_{inf} \rightarrow \varphi) \wedge \Box x$
$A[\varphi \underline{B} \psi] = \mu x.(\Phi_{inf} \rightarrow \neg\psi) \wedge (\varphi \vee \Box x)$
$A[\varphi B \psi] = \nu x.(\Phi_{inf} \rightarrow \neg\psi) \wedge (\varphi \vee \Box x)$

CTL* to CTL - Existential Operators

$EX\varphi = EXE\varphi$
$EF\varphi = EFE\varphi$ $\qquad EFG\varphi \equiv EFEG\varphi$
$E[\varphi\ W\ \psi] = E[(E\varphi)\ W\ \psi]$
$E[\varphi\ \underline{W}\ \psi] = E[(E\varphi)\ \underline{W}\ \psi]$
$E[\psi\ U\ \varphi] = E[\psi\ U\ E(\varphi)]$
$E[\psi\ \underline{U}\ \varphi] = E[\psi\ \underline{U}\ E(\varphi)]$
$E[\varphi\ B\ \psi] = E[(E\varphi)\ B\ \psi]$
$E[\varphi\ \underline{B}\ \psi] = E[(E\varphi)\ \underline{B}\ \psi]$

obs. $EGF\varphi \neq EGEF\varphi \rightarrow$ can't be converted

CTL* to CTL - Universal Operators

$AX\varphi = AXA\varphi$
$AG\varphi = AGA\varphi$
$A[\varphi\ W\ \psi] = A[(A\varphi)\ W\ \psi]$
$A[\varphi\ \underline{W}\ \psi] = A[(A\varphi)\ \underline{W}\ \psi]$
$A[\varphi\ U\ \psi] = A[A(\varphi)\ U\ \psi]$
$A[\varphi\ \underline{U}\ \psi] = A[A(\varphi)\ \underline{U}\ \psi]$
$A[\psi\ B\ \varphi] = A[\psi\ B\ (E(\varphi)]$
$A[\psi\ \underline{B}\ \varphi] = A[\psi\ \underline{B}\ (E(\varphi)]$

Eliminate boolean op. after path quantify

$$[\varphi_1\ \underline{U}\ \psi_1] \wedge [\varphi_2\ \underline{U}\ \psi_2] = \left[(\varphi_1 \wedge \varphi_2)\ \underline{U}\ \begin{pmatrix}\psi_1 \wedge [\varphi_2\ \underline{U}\psi_2] \vee\\ \psi_2 \wedge [\varphi_1\ \underline{U}\psi_1]\end{pmatrix}\right]$$

$$[\varphi_1\ \underline{U}\ \psi_1] \wedge [\varphi_2\ U\ \psi_2] = \left[(\varphi_1 \wedge \varphi_2)\ \underline{U}\ \begin{pmatrix}\psi_1 \wedge [\varphi_2\ U\psi_2] \vee\\ \psi_2 \wedge [\varphi_1\ \underline{U}\psi_1]\end{pmatrix}\right]$$

$$[\varphi_1\ U\ \psi_1] \wedge [\varphi_2\ U\ \psi_2] = \left[(\varphi_1 \wedge \varphi_2)\ \underline{U}\ \begin{pmatrix}\psi_1 \wedge [\varphi_2\ U\psi_2] \vee\\ \psi_2 \wedge [\varphi_1\ U\psi_1]\end{pmatrix}\right]$$

CTL* Modelchecking to LTL model checking

Let's φ_i be a pure path formula (without path quantifiers), Ψ be a propositional formula, abbreviate subformulas $E\varphi$ and $A\psi$ working bottom-up the syntax tree to obtain the following normal form: $\phi = \text{let} \begin{bmatrix} x_1 = A\varphi_1\\ \vdots\\ x_n = A\varphi_n \end{bmatrix} \text{in } \Psi \text{ end}$

Use LTL model checking to compute $Q_i := [\![A\varphi_i]\!]_{\mathcal{K}_{i-1}}$, where $\mathcal{K}_0 := \mathcal{K}$ and $\mathcal{K}_{i+1}$ is obtained from $\mathcal{K}_i$ by labelling the states Q_i with x_i. Finally compute $[\![\Psi]\!]_{\mathcal{K}_n}$

LTL to ω-automata

$\phi\langle X\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow X\varphi, \phi\langle q\rangle_x)$
$\phi\langle X\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q_0, q_1\}, 1, (q_0 \leftrightarrow \varphi) \wedge (q_1 \leftrightarrow Xq_0), \phi\langle q_1\rangle_x)$
$\phi\langle G\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \varphi \wedge Xq, \phi\langle q\rangle_x \wedge GF[\varphi \rightarrow q])$
$\phi\langle F\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \varphi \wedge Xq, \phi\langle q\rangle_x \wedge GF[q \rightarrow \varphi])$
$\phi\langle[\varphi\ U\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \psi \vee \varphi \wedge Xq, \phi\langle q\rangle_x \wedge GF[\varphi \rightarrow q])$
$\phi\langle[\varphi\ \underline{U}\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \psi \vee \varphi \wedge Xq, \phi\langle q\rangle_x \wedge GF[q \rightarrow \psi])$
$\phi\langle[\varphi\ B\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \neg\psi \wedge (\varphi \vee Xq), \phi\langle q\rangle_x \wedge GF[q \vee \psi])$
$\phi\langle[\varphi\ \underline{B}\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \neg\psi \wedge (\varphi \vee Xq), \phi\langle q\rangle_x \wedge GF[q \rightarrow \varphi])$
$\phi\langle\overleftarrow{X}\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \varphi, \phi\langle q\rangle_x)$
$\phi\langle\overleftarrow{X}\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \varphi, \phi\langle q\rangle_x)$
$\phi\langle\overleftarrow{G}\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \varphi \wedge q, \phi\langle\varphi \wedge q\rangle_x)$
$\phi\langle\overleftarrow{F}\varphi\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \varphi \vee q, \phi\langle\varphi \vee q\rangle_x)$
$\phi\langle[\varphi\ \overleftarrow{U}\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \psi \vee \varphi \wedge q, \phi\langle\psi \vee \varphi \wedge q\rangle_x)$
$\phi\langle[\varphi\ \underline{\overleftarrow{U}}\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \psi \vee \varphi \wedge q, \phi\langle\psi \vee \varphi \wedge q\rangle_x)$
$\phi\langle[\varphi\ \overleftarrow{B}\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \neg\psi \wedge (\varphi \vee q), \phi\langle\neg\psi \wedge (\varphi \vee q)\rangle_x)$
$\phi\langle[\varphi\ \underline{\overleftarrow{B}}\ \psi]\rangle_x \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \neg\psi \wedge (\varphi \vee q), \phi\langle\neg\psi \wedge (\varphi \vee q)\rangle_x)$

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.