Solving linear systems

a.)

```
>> A =[14 9 14 6 -10;-11 -11 5 8 6;15 -2 -14 8 -15;14 13 11 -3 -7;0 9 13 5 -14]

A =

    14     9    14     6   -10
   -11   -11     5     8     6
    15    -2   -14     8   -15
    14    13    11    -3    -7
     0     9    13     5   -14
```

```
>> M= horzcat(A,b)

M =

    14     9    14     6   -10    -4
   -11   -11     5     8     6     8
    15    -2   -14     8   -15     6
    14    13    11    -3    -7     0
     0     9    13     5   -14    10
```

```
>> x=rref(M)

x =

    1.0000         0         0         0         0    0.6404
         0    1.0000         0         0         0   -7.2826
         0         0    1.0000         0         0    2.9525
         0         0         0    1.0000         0   -6.3001
         0         0         0         0    1.0000   -4.9043
```

```
>> b= [-4;8;6;0;10]

b =

    -4
     8
     6
     0
    10
```

2a Answer / Explanation –

There is exactly one solution for every equation, meaning that the system is unique (Rank [A]=n).

b1.)

```
>> M= horzcat(A,b)

M =

    14     9    14     6   -10    -4
   -11   -11     5     8     6     8
    15    -2   -14     8   -15     6
    14    13    11    -3    -7     0
     0     9    13     5   -14    10
```

```
>> x=rref(M)

x =

    1.0000         0         0         0         0    0.6404
         0    1.0000         0         0         0   -7.2826
         0         0    1.0000         0         0    2.9525
         0         0         0    1.0000         0   -6.3001
         0         0         0         0    1.0000   -4.9043
```

b2.)

```
>> x1=x(:,end)

x1 =

    0.6404
   -7.2826
    2.9525
   -6.3001
   -4.9043
```

```
>> x2=linsolve(A,b)

x2 =

    0.6404
   -7.2826
    2.9525
   -6.3001
   -4.9044
```

b3.)

```
>> x3=A^(-1)*b

x3 =

    0.6404
   -7.2826
    2.9525
   -6.3001
   -4.9044
```

b4.)

```
>> rref(horzcat(A, eye(5)))

ans =

    1.0000         0         0         0         0   -0.0530    0.0771    0.0427    0.1393   -0.0445
         0    1.0000         0         0         0    0.5436   -0.4570   -0.1933   -0.6956   -0.0293
         0         0    1.0000         0         0   -0.2042    0.1991    0.0539    0.3029    0.0220
         0         0         0    1.0000         0    0.5381   -0.3471   -0.1460   -0.6544   -0.0495
         0         0         0         0    1.0000    0.3521   -0.2329   -0.1264   -0.3997   -0.0875

>> R=rref(horzcat(A, eye(5)))

R =

    1.0000         0         0         0         0   -0.0530    0.0771    0.0427    0.1393   -0.0445
         0    1.0000         0         0         0    0.5436   -0.4570   -0.1933   -0.6956   -0.0293
         0         0    1.0000         0         0   -0.2042    0.1991    0.0539    0.3029    0.0220
         0         0         0    1.0000         0    0.5381   -0.3471   -0.1460   -0.6544   -0.0495
         0         0         0         0    1.0000    0.3521   -0.2329   -0.1264   -0.3997   -0.0875

>> B = R(:,6:10)

B =

   -0.0530    0.0771    0.0427    0.1393   -0.0445
    0.5436   -0.4570   -0.1933   -0.6956   -0.0293
   -0.2042    0.1991    0.0539    0.3029    0.0220
    0.5381   -0.3471   -0.1460   -0.6544   -0.0495
    0.3521   -0.2329   -0.1264   -0.3997   -0.0875

>> x4=B*b

x4 =

    0.6404
   -7.2826
    2.9525
   -6.3001
   -4.9044
```

c.) Comparisons Below

According to the comparisons of x1,x2,x3, and x4 , none of them exactly equal each other. This contradicts what was shown in “a)”, because it shows that there is more than one solution to the linear system, while “a)” showed that it was unique.

```
>> x1-x3

ans =

   1.0e-04 *

   -0.1291
   -0.0872
    0.0119
    0.0032
    0.2559

>> x1-x4

ans =

   1.0e-04 *

   -0.2528
    0.2109
    0.2097
    0.1654
    0.7264

>> x2-x3

ans =

   1.0e-15 *

    0.1110
         0
         0
   -0.8882
         0

>> x2-x4

ans =

   1.0e-04 *

   -0.1238
    0.2981
    0.1978
    0.1622
    0.4705
```

```
>> x3-x4

ans =

   1.0e-04 *

   -0.1238
    0.2981
    0.1978
    0.1622
    0.4705
```

d.)

```
>> A*x1-b

ans =

   1.0e-03 *

   -0.4964
    0.3999
   -0.5740
   -0.4610
   -0.4196

>> A*x2-b

ans =

   1.0e-13 *

   -0.0355
   -0.1066
   -0.0533
    0.0888
   -0.0355

>> A*x3-b

ans =

   1.0e-14 *

    0.3553
   -0.3553
   -0.5329
    0.1776
    0.3553
```

Ranks - In terms of smallest difference

1. x3 (smallest)
2. x2
3. x1
4. x4 (biggest)

```
>> A*x4-b

ans =

    0.0000
   -0.0003
    0.0011
   -0.0001
    0.0001
```

Problem 3 - LU decomposition

```
>> A

A =

    -3     2     5     1
    12    -4   -20    -2
    -6     0    15     1
    -9     6    35     4

>> %E21(4) %E31(-2) %E41(-3)
>> A(2,:) = A(2,:) + 4*A(1,:);
>> A(3,:) = A(3,:) + -2*A(1,:);
>> A(4,:) = A(4,:) + -3*A(1,:);
>> A

A =

    -3     2     5     1
     0     4     0     2
     0    -4     5    -1
     0     0    20     1

>> %E32(1) %E43(-4)
>> A(3,:) = A(3,:) + 1*A(2,:);
>> A(4,:) = A(4,:) + -4*A(3,:);
>> A

A =

    -3     2     5     1
     0     4     0     2
     0     0     5     1
     0     0     0    -3

>> U=A

U =

    -3     2     5     1
     0     4     0     2
     0     0     5     1
     0     0     0    -3

>> L=eye(4);
>> L

L =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
>> %E21(4) %E31(-2) %E41(-3)
>> %E32(1) %E43(-4)
>> L(2,1) = -4;
>> L(3,1)=2;
>> L(4,1)=3;
>> L(3,2)=-1

L =

     1     0     0     0
    -4     1     0     0
     2    -1     1     0
     3     0     0     1
```

```
>> L(4,3)=4

L =

     1     0     0     0
    -4     1     0     0
     2    -1     1     0
     3     0     4     1
```

```
>> L

L =

     1     0     0     0
    -4     1     0     0
     2    -1     1     0
     3     0     4     1

>> U

U =

    -3     2     5     1
     0     4     0     2
     0     0     5     1
     0     0     0    -3

>> L*U

ans =

    -3     2     5     1
    12    -4   -20    -2
    -6     0    15     1
    -9     6    35     4
```

```
>> A = [-3 2 5 1; 12 -4 -20 -2; -6 0 15 1; -9 6 35 4]

A =

    -3     2     5     1
    12    -4   -20    -2
    -6     0    15     1
    -9     6    35     4

>> L*U

ans =

    -3     2     5     1
    12    -4   -20    -2
    -6     0    15     1
    -9     6    35     4
```

A=L*U

Applying LU and LUP decompositions

a.)

```
>> [L1 U1 P1]=lu(A)

L1 =

    1.0000         0         0         0         0
   -0.7857    1.0000         0         0         0
    0.0714   -0.4713    1.0000         0         0
    0.8571   -0.9349   -0.4300    1.0000         0
   -0.7857    0.0345   -0.0338   -0.2335    1.0000


U1 =

   14.0000   11.0000   -5.0000   -5.0000   -7.0000
         0   18.6429  -11.9286   -0.9286    2.5000
         0         0  -14.2644    9.9195    9.6782
         0         0         0    6.6833   25.4991
         0         0         0         0   -3.3036


P1 =

     0     1     0     0     0
     0     0     0     1     0
     0     0     1     0     0
     1     0     0     0     0
     0     0     0     0     1
```

```
>> P1*A

ans =

    14    11    -5    -5    -7
   -11    10    -8     3     8
     1    -8    -9    10     8
    12    -8    13    -1    13
   -11    -8     4     2    -4

>> L1*U1

ans =

   14.0000   11.0000   -5.0000   -5.0000   -7.0000
  -11.0000   10.0000   -8.0000    3.0000    8.0000
    1.0000   -8.0000   -9.0000   10.0000    8.0000
   12.0000   -8.0000   13.0000   -1.0000   13.0000
  -11.0000   -8.0000    4.0000    2.0000   -4.0000
```

P1 one used pivoting to find the best possible result in the lu computation.

b.)

```
>> A = [12 -8 13 -1 13;14 11 -5 -5 -7;1 -8 -9 10 8;-11 10 -8 3 8;-11 -8 4 2 -4]

A =

    12    -8    13    -1    13
    14    11    -5    -5    -7
     1    -8    -9    10     8
   -11    10    -8     3     8
   -11    -8     4     2    -4

>> [L1 U1 P1]=lu(A)

L1 =

    1.0000         0         0         0         0
   -0.7857    1.0000         0         0         0
    0.0714   -0.4713    1.0000         0         0
    0.8571   -0.9349   -0.4300    1.0000         0
   -0.7857    0.0345   -0.0338   -0.2335    1.0000
U1 =

   14.0000   11.0000   -5.0000   -5.0000   -7.0000
         0   18.6429  -11.9286   -0.9286    2.5000
         0         0  -14.2644    9.9195    9.6782
         0         0         0    6.6833   25.4991
         0         0         0         0   -3.3036


P1 =

     0     1     0     0     0
     0     0     0     1     0
     0     0     1     0     0
     1     0     0     0     0
     0     0     0     0     1

>> y=linsolve(L1,P1*b)

y =

   -4.0000
   -0.1429
   -4.7816
    5.2388
    4.9237
>> x=linsolve(U1,y)

x =

    0.3190
    2.9609
    3.8235
    6.4703
   -1.4904

>> L1*y

ans =

    -4
     3
    -5
     4
     7

>> P1*b

ans =

    -4
     3
    -5
     4
     7
```

L1y=P1b

```
>> U1*x

ans =

   -4.0000
   -0.1429
   -4.7816
    5.2388
    4.9237

>> y

y =

   -4.0000
   -0.1429
   -4.7816
    5.2388
    4.9237
```

Ux=y

c.)

```
>> A*x-b

ans =

   1.0e-14 *

         0
   -0.2665
    0.7105
    0.5329
   -0.0888
```

Produces difference that is close to zero

Continue problem in upper right-hand side

d.)

```
>> [L2 U2 P2]=lu(A)

L2 =

    1.0000         0         0         0         0
   -0.7857    1.0000         0         0         0
    0.0714   -0.4713    1.0000         0         0
    0.8571   -0.9349   -0.4300    1.0000         0
   -0.7857    0.0345   -0.0338   -0.2335    1.0000


U2 =

   14.0000   11.0000   -5.0000   -5.0000   -7.0000
         0   18.6429  -11.9286   -0.9286    2.5000
         0         0  -14.2644    9.9195    9.6782
         0         0         0    6.6833   25.4991
         0         0         0         0   -3.3036


P2 =

     0     1     0     0     0
     0     0     0     1     0
     0     0     1     0     0
     1     0     0     0     0
     0     0     0     0     1
```

```
>> Y=linsolve(L2,P2*B)

Y =

  -15.0000  -14.0000    4.0000   -2.0000   13.0000  -16.0000   15.0000   -3.0000  -15.0000   14.0000
  -12.7857  -19.0000    9.1429  -12.5714   30.2143  -32.5714    7.7857  -15.3571   -8.7857    5.0000
   -1.9540    4.0460   -5.9770  -22.7816   15.3103    4.7931  -14.4023    9.9770   11.9310    6.3563
   18.0639  -11.0226  -11.4515   -9.8351    9.6873  -16.6745    1.2280    4.5050   -5.2256  -19.5923
  -27.1923  -11.7821    7.9509   12.7941   -5.0470   -4.1803    1.3166  -10.4378  -10.2994    7.4671

>> X=linsolve(U2,Y)

X =

   -2.6042   -1.6583    0.2392    1.0764   -0.3639   -0.6675    0.7638   -0.3315   -1.7479    0.9430
  -12.3292   -7.6790    3.7319    5.7682   -0.7745   -5.2055    1.7873   -5.9541   -6.3431    2.1213
  -14.2378   -8.4734    3.9802    8.2215   -3.0822   -4.5698    1.9245   -6.4700   -7.5367    1.9793
  -28.7020  -15.2566    7.4692   13.3046   -4.3794   -7.3228    1.7043  -11.3808  -12.6768    5.6923
    8.2312    3.5665   -2.4068   -3.8728    1.5278    1.2654   -0.3985    3.1596    3.1177   -2.2603
```

```
>> L2*Y

ans =

  -15.0000  -14.0000    4.0000   -2.0000   13.0000  -16.0000   15.0000   -3.0000  -15.0000   14.0000
   -1.0000   -8.0000    6.0000  -11.0000   20.0000  -20.0000   -4.0000  -13.0000    3.0000   -6.0000
    3.0000   12.0000  -10.0000  -17.0000    2.0000   19.0000  -17.0000   17.0000   15.0000    5.0000
   18.0000   -7.0000  -14.0000   10.0000  -14.0000   -2.0000   13.0000   12.0000  -15.0000  -15.0000
  -20.0000    1.0000    8.0000   17.0000  -17.0000   11.0000  -10.0000  -10.0000    2.0000    1.0000

>> P2*B

ans =

   -15   -14     4    -2    13   -16    15    -3   -15    14
    -1    -8     6   -11    20   -20    -4   -13     3    -6
     3    12   -10   -17     2    19   -17    17    15     5
    18    -7   -14    10   -14    -2    13    12   -15   -15
   -20     1     8    17   -17    11   -10   -10     2     1
```

LY=PB

UX=Y

```
>> U2*X

ans =

  -15.0000  -14.0000    4.0000   -2.0000   13.0000  -16.0000   15.0000   -3.0000  -15.0000   14.0000
  -12.7857  -19.0000    9.1429  -12.5714   30.2143  -32.5714    7.7857  -15.3571   -8.7857    5.0000
   -1.9540    4.0460   -5.9770  -22.7816   15.3103    4.7931  -14.4023    9.9770   11.9310    6.3563
   18.0639  -11.0226  -11.4515   -9.8351    9.6873  -16.6745    1.2280    4.5050   -5.2256  -19.5923
  -27.1923  -11.7821    7.9509   12.7941   -5.0470   -4.1803    1.3166  -10.4378  -10.2994    7.4671

>> Y

Y =

  -15.0000  -14.0000    4.0000   -2.0000   13.0000  -16.0000   15.0000   -3.0000  -15.0000   14.0000
  -12.7857  -19.0000    9.1429  -12.5714   30.2143  -32.5714    7.7857  -15.3571   -8.7857    5.0000
   -1.9540    4.0460   -5.9770  -22.7816   15.3103    4.7931  -14.4023    9.9770   11.9310    6.3563
   18.0639  -11.0226  -11.4515   -9.8351    9.6873  -16.6745    1.2280    4.5050   -5.2256  -19.5923
  -27.1923  -11.7821    7.9509   12.7941   -5.0470   -4.1803    1.3166  -10.4378  -10.2994    7.4671
```

AX-B is on the next page

d.) continued

```
>> A*X-B

ans =

   1.0e-13 *

   -0.4263    0.2309   -0.0355   -0.0355   -0.0355    0.1288    0.0178    0.0178   -0.1776    0.0355
    0.0178   -0.0888    0.0089    0.0622         0    0.0533         0    0.0311    0.0355    0.0355
   -0.2842    0.0711    0.0888    0.1066         0         0    0.0355    0.0711         0         0
    0.1421   -0.1776    0.0355    0.0711         0   -0.1066    0.0133   -0.1776    0.0355   -0.0355
    0.3553    0.0178   -0.0178   -0.0711         0   -0.0178   -0.0178    0.0888   -0.0355   -0.0711
```

A*X-B is close to the zero matrix as shown in the matrix above.

Bubble Sort

There are 5! Combinations that can be done in example, we are told to arrange the below only using adjacent numbers:

(1 2 3 4 5)→(4 1 5 3 2)

The most efficient way I found to solve by hand was to move the desired number to the front (left) of the order in order of the left to right (so if the number the number that comes first in left to right takes precedent in moving):

Step	Order
Original	(1 2 3 4 5)
1	(1 2 4 3 5)
2	(1 4 2 3 5)
3	(4 1 2 3 5)
4	(4 1 2 5 3)
5	(4 1 5 2 3)
6	(4 1 5 3 2)

You can try moving two adjacent numbers at once, but then you need to solve back to from the switch, and then it ends up creating more work.