

LIGN 165

Problem Set 2

Please put your answers into a file called `lastname-firstname.clj`. Be careful to follow the instructions exactly and be sure that all of your function definitions use the precise names, number of inputs and input types, and output types as requested in each question. **If you do not follow these instructions, we cannot give you credit for your answer.**

For running your code, you have two options. You can run your code on the course website. Alternatively, you can install Clojure on your local machine and write and debug the answers to each problem in a local copy of `lastname-firstname.clj`. You can find information about installing and using Clojure here <https://clojure.org/>.

You may collaborate with up to three other students on this problem set. You must submit your work individually. If you do not submit a copy of the problem set under your own name, you will not get credit. When you submit your work, you must indicate who you worked with, and what each of your individual contributions were.

Problem 1: Write a procedure, called `sequence-to-power`, that takes a sequence (represented as a list) `x`, and a positive integer `n`, and returns the sequence `xn`. For example, given the sequence (list 'a 'b) and the number 3, the procedure should return (list 'a 'b 'a 'b 'a 'b 'a 'b 'a 'b 'a 'b).

Problem 2: Define L as a language containing a single sequence, $L = \{[a]\}$. (Note: the notation $[a]$ means the same thing as (list 'a), which is what we have been using in class). Write a procedure `in-L?` that takes a sequence (represented as a list), and decides if it is a member of the language L^* . That is, given a sequence x , the procedure should return `true` if and only if x is a member of L^* , and `false` otherwise.

Problem 3: Define the language $a^n b^n$ as follows: it is the set of strings consisting of k a 's followed by k b 's, for all $k > 0$, i.e. $a^n b^n = \{[ab], [aabb], [aaabbb], \dots\}$. Write a procedure `generate-an-bn`, which takes a single argument k (which you can assume to be an integer greater than 0). It should return the k 'th element of the language $a^n b^n$. For example, given the number 3, it should return (list 'a 'a 'a 'b 'b 'b).

Hint: Use the function `add-to-end` that you wrote in Problem Set 1.

Problem 4: Write a procedure `remove-last-element`, which takes a single list l as its argument. The procedure should return the list l with its last element removed. For example, given the input (list 'a 'b 'b), the procedure should return (list 'a 'b).

Hint: Use the procedure `reverse` that you wrote in Problem Set 1.

Problem 5: Write a procedure `recognize-an-bn`, that takes an input string `str` as its argument, and returns `true` if `str` is a member of $a^n b^n$ and `false` otherwise. For example, the procedure should return `false` on the input (list 'a 'a 'a 'b 'b 'a).

Hint: Use the procedures `reverse` and `remove-last-element` from the last problem.

Problem 6: Define L as a language containing a single string. For example, we may have $L = \{[a]\}$ or $L = \{[b]\}$. You can assume that L is presented to you as a single string, for example, one possible value for L is (list 'a'). (Note: In general, you should assume that languages are presented to you as a list of lists, i.e. a list of strings. This problem is a special case, as L is assumed to contain only a single string.)

Define A to be any language consisting of a finite set of sequences. You can assume A is represented as a list of lists. For example, one possible value for A is (list (list 'a 'b) (list 'b 'b)), which would represent the language $\{[ab], [bb]\}$.

Write a procedure **concat-L-A** that takes two arguments, L and A , as defined above. It should return $\text{CONCAT}(L, A)$, the concatenation of L and A . The return value should be represented as a list of lists (i.e., a list of strings).

Problem 7: Let A and B be languages. Find an example of languages A and B such that $\text{CONCAT}(A, B) = \text{CONCAT}(B, A)$.

Problem 8: Let A and B be languages. Find an example of languages A and B such that $\text{CONCAT}(A, B)$ does not equal $\text{CONCAT}(B, A)$

Problem 9: Find an example of a language L such that $L = L^2$, i.e. $L = \text{CONCAT}(L, L)$.