Christian DiPietrantonio
ME M311: Computational Methods to Viscious FLows
Video Assignment 01
February 7, 2026

Problem Statement: Watch all the video lectures, and derive the governing equation for the kinetic energy, defined as k = 1/2(u2 + v2 + w2). The technique involves multiplying the x-momentum equation by the u velocity, y-momentum by the v velocity, and z-momentum equation by the w velocity, and eventually add all three equations up. Show your derivation process.

Solution:

We must start this derivation with the momentum equations as mentioned above. First, we have the x-momentum equation:

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial_y} + \frac{\partial \tau_{zx}}{\partial z} + S_{Mx} \tag{1}$$

Next, we have the y-momentum equation:

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial_x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + S_{My} \tag{2}$$

Finally, we have the z-momentum equation:

$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial_y} + \frac{\partial \tau_{zz}}{\partial z} + S_{Mz} \tag{3}$$

We can now multiply each equation by its respective velocity component, doing so yields the following for x-momentum:

$$\rho u\frac{Du}{Dt} = -u\frac{\partial p}{\partial x} + u\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial_y} + \frac{\partial \tau_{zx}}{\partial z}\right) + uS_{Mx} \tag{4}$$

For y-momentum:

$$\rho u\frac{Dv}{Dt} = -u\frac{\partial p}{\partial y} + u\left(\frac{\partial \tau_{xy}}{\partial_x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + uS_{My} \tag{5}$$

And for z-momentum:

$$\rho u\frac{Dw}{Dt} = -u\frac{\partial p}{\partial z} + u\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial_y} + \frac{\partial \tau_{zz}}{\partial z}\right) + uS_{Mz} \tag{6}$$