## Package 'strategize'

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Title Optimal Stochastic Interventions with High-dimensional Data
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<b>Description</b> Software for performing optimal stochastic intervention analysis with high-dimensional data.
<b>Depends</b> R (>= $3.3.3$ )
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## Description

Performs cross-validation to select the regularization parameter  $\lambda$  (and, if desired, other hyperparameters) for the <code>OptiConjoint</code> function. This function splits the data by respondent (or user-specified units), trains candidate models under a grid of  $\lambda$  values, and evaluates out-of-sample performance, returning the model that maximizes a chosen criterion (e.g., out-of-sample expected utility or log-likelihood).

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#### Usage

```
cv.OptiConjoint(
 Υ,
 W,
 X = NULL
  lambda_seq = NULL,
  lambda = NULL,
  folds = 2L,
  varcov_cluster_variable = NULL,
  competing_group_variable_respondent = NULL,
  competing_group_variable_candidate = NULL,
 competing_group_competition_variable_candidate = NULL,
 pair_id = NULL,
  respondent_id = NULL,
 respondent_task_id = NULL,
 profile_order = NULL,
 p_list = NULL,
  slate_list = NULL,
 UseOptax = F,
 K = 1,
 nSGD = 100,
 diff = F,
 MaxMin = F,
 UseRegularization = F,
 OpenBrowser = F,
 ForceGaussianFamily = F,
 A_{INIT\_SD} = 0.001,
  TypePen = "KL",
 ComputeSEs = T,
  conda_env = NULL,
  conda_env_required = F,
 confLevel = 0.9,
 nFolds_glm = 3L,
 nMonte_MaxMin = 5L,
 nMonte_Qglm = 100L,
  jax_seed = as.integer(Sys.time()),
 OptimType = "default"
```

## Arguments

W

Χ

Υ	A numeric or binary response vector. If binary (e.g., 0–1), it should correspond
	to forced-choice outcomes (1 if candidate A is chosen; 0 if candidate B is chosen).
	If numeric, please see details in OptiConjoint for how outcomes are handled.

A data frame or matrix representing the randomized conjoint attributes. Each column is a factor or character vector indicating attribute levels for a particular dimension. Multiple columns can be used if the conjoint has multiple attributes.

Optional covariate matrix or data frame for modeling systematic heterogeneity. If K > 1, this is typically required for multi-class or cluster-based models. Otherwise, set X = NULL.

lambda\_seq A numeric vector of candidate  $\lambda$  values for cross-validation. If NULL and lambda

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is also NULL, a sequence of values is automatically generated (e.g., via  $10^seq(-4, 0, length.out = 5) * sd(Y)$ ).

lambda A single user-specified  $\lambda$  value. If provided, cross-validation is effectively dis-

abled unless lambda\_seq is also supplied.

folds An integer or user-specified partitioning indicating the number of cross-validation

folds. Defaults to 2. See Details for how data splitting is done.

varcov\_cluster\_variable

An optional clustering variable for robust standard errors. For instance, if the data is from multiple respondents, specify respondent IDs here for cluster-robust inference (via sandwich estimation). If NULL, no cluster-based variance correction is used.

competing\_group\_variable\_respondent

Optional vector for multi-round or multi-group setups, indicating which respondent belongs to which group. Used for advanced or adversarial designs (e.g., dual-party contexts). If NULL, standard usage is assumed.

 ${\tt competing\_group\_variable\_candidate}$ 

Similar to competing\_group\_variable\_respondent, but for candidate-level grouping. If NULL, standard usage is assumed.

competing\_group\_competition\_variable\_candidate

An optional variable for specifying which candidate is in competition with which group. Relevant if multi-step adversarial frameworks are used.

pair\_id An optional vector (same length as Y) identifying which rows (candidate pairs) belong to the same forced choice. For example, if each respondent evaluates multiple pairs, this ID ensures correct grouping. Required only in certain ad-

vanced difference-in-differences or paired analyses.

respondent\_id A user-specified ID to denote respondent-level grouping, typically used to cluster standard errors or to perform out-of-sample validation by respondent. If NULL, a simple row index is used for splitting.

respondent\_task\_id

Another optional ID for tasks (e.g., each respondent might see multiple tasks). Helps in advanced designs. If NULL, ignored.

profile\_order An optional vector capturing the ordering of candidate profiles within tasks, if multiple profiles are being shown. Used in difference or extended hierarchical

modeling.

p\_list A list of assignment probabilities for each attribute, if known or desired as a

baseline. If NULL, each level is assumed to have uniform probability or derived

from empirical frequencies in W.

slate\_list An optional list specifying alternative or restricted sets of attribute levels. Used

when a subset of attributes is feasible or when bounding certain strategies in an

adversarial design.

UseOptax Logical. If TRUE, uses the optax Python library (via reticulate) for gradient-

based optimization. If FALSE, uses a default gradient-based approach from  $\mathbf{jax}.$ 

K An integer specifying the number of mixture components or clusters if X is used

(e.g., for multi-class analysis). Defaults to 1 (no mixture).

nSGD An integer number of iterations for gradient-based training. Defaults to 100 but

can be increased if convergence has not been reached.

diff Logical indicating whether a difference-based model (e.g., for forced-choice

or difference-in-outcomes) is used. Defaults to FALSE, but set TRUE in certain

difference-of-utility designs.

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MaxMin Logical indicating whether to use a two-party or multi-agent adversarial ap-

proach in the optimization. If TRUE, a min-max (zero-sum) formulation is employed. Defaults to FALSE (single-agent or average-case optimization).

UseRegularization

Logical; if TRUE, penalty-based regularization is used for the outcome model. Usually set to TRUE for large designs. Defaults to FALSE.

OpenBrowser Logical used in debugging (e.g., opening a browser for interactive inspection

within **jax** or **reticulate**). Defaults to FALSE.

ForceGaussianFamily

Logical indicating whether a Gaussian family (1m-style) is forced for the outcome model, even if Y is binary. Defaults to FALSE.

A\_INIT\_SD A numeric controlling the random initialization scale for unconstrained param-

eters in gradient-based optimization. Defaults to 0.001. Larger values can help

avoid local minima in complex outcome landscapes.

TypePen A character string specifying the type of penalty for the *optimal stochastic in-*

tervention, e.g., "KL", "L2", or "LogMaxProb". The default is "KL".

ComputeSEs Logical; if TRUE, attempts to compute standard errors using M-estimation or the

Delta method. Defaults to TRUE.

conda\_env A character specifying the name of a Conda environment for **reticulate**. If NULL,

the default environment is used.

conda\_env\_required

 $Logical. \ \ If \ \mathsf{TRUE}, \ errors \ if \ the \ specified \ Conda \ environment \ conda\_env \ cannot$ 

be found. Otherwise tries to fall back gracefully.

confLevel The confidence level (between 0 and 1) for interval estimation, default 0.90.

nFolds\_glm An integer specifying the number of folds in internal regression-based cross-

validation (if used) for outcome model selection. Defaults to 3.

nMonte\_MaxMin A positive integer specifying the number of Monte Carlo draws for the min-max

(adversarial) stage, if MaxMin = TRUE. Defaults to 5.

nMonte\_Qglm An integer specifying the number of Monte Carlo draws for evaluating certain

integrals in glm-based approximations, default 100.

jax\_seed An integer seed for the JAX random number generator. Defaults to as.integer(Sys.time()).

OptimType A character describing the optimization routine. Typically "default" uses a

standard gradient-based approach; set "tryboth" or "SecondOrder" for testing

or advanced routines.

#### **Details**

cv.OptiConjoint implements a cross-validation routine for OptiConjoint. First, the data is split into folds parts. For each fold, we train candidate outcome models and compute out-of-sample performance. The best-performing  $\lambda$  is selected. Finally, a refit on the full data is done using the chosen hyperparameters, returning the results of the final OptiConjoint call with  $\lambda$  set to the best value.

The function supports a wide range of conjoints, including forced-choice (where diff = TRUE), multi-cluster outcome modeling (where K>1), and adversarial designs (where MaxMin = TRUE). Regularization for the outcome model or for the candidate distribution can be enabled via UseRegularization and TypePen. Cross-validation is particularly helpful when the data is limited or highly dimensional.

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#### Value

A named list with components:

**PiStar\_point** The estimated optimal probability distribution(s) over candidate profiles  $(\hat{\pi}^*)$ .

**Q\_point\_mEst** The estimated expected outcome (e.g., vote share) under the selected optimal distribution.

**lambda** The chosen  $\lambda$  value from cross-validation (and any other relevant hyperparameters).

**CVInfo** A data frame or matrix summarizing cross-validation results, e.g., in-sample and out-of-sample estimates for each candidate  $\lambda$ .

**Other components** Various additional objects useful for inference and debugging (e.g., final model fits, standard error estimates, weighting details).

#### See Also

OptiConjoint for direct optimization of stochastic interventions in conjoint analysis, including average and adversarial settings.

## **Examples**

```
# A minimal example using hypothetical data
set.seed(123)
# Suppose Y is a binary forced choice outcome, W has several attributes (factors)
Y < - rbinom(200, size = 1, prob = 0.5)
W <- data.frame(</pre>
  Gender = sample(c("Male", "Female"), 200, TRUE),
  Age = sample(c("35", "50", "65"), 200, TRUE),
 Party = sample(c("Dem","Rep"),
                                       200, TRUE)
# Cross-validate over a range of lambda
lam_seq <- c(0, 0.001, 0.01, 0.1)
cv_fit <- cv.OptiConjoint(</pre>
  Y = Y,
  W = W
  lambda_seq = lam_seq,
  folds = 2
# Extract optimal lambda and final fit
print(cv_fit$lambda)
print(cv_fit$CVInfo)
print(names(cv_fit$PiStar_point))
```

#### **Description**

OneStep.OptiConjoint implements a single-step ("one-step") approach to estimating a target quantity of interest in high-dimensional conjoint (or factorial) experiments, such as finding an *optimal stochastic intervention* over factor levels. This method can incorporate adversarial or non-adversarial settings, regularization, multi-stage structures (e.g., primaries followed by a general election), and a variety of user-specified outcome models. It returns estimated distributions of factor levels (e.g., candidate attributes) that maximize or minimize an outcome (e.g., vote share), including optional standard errors via M-estimation or the delta method.

## Usage

```
OneStep.OptiConjoint(
  W,
  Υ,
  X = NULL
  K = 1,
  warmStart = FALSE,
  automatic_scaling = TRUE,
  p_list = NULL,
  hypotheticalProbList = NULL,
  pi_init_vec = NULL,
  constrain_ub = NULL,
  nLambda = 10,
  penaltyType = "LogMaxProb",
  testFraction = 0.5,
  log_PrW = NULL,
  LEARNING_RATE_BASE = 0.01,
  cycle_width = 50,
  cycle_number = 4,
  nSGD = 500,
  nEpoch = NULL,
  X_factorized = NULL,
  momentum = 0.99,
  nFullCycles_noRestarts = 1,
  optim_method = "tf",
  sg_method = NULL,
  forceSEs = FALSE,
  clipAT_factor = 100000.
  adaptiveMomentum = FALSE,
  PenaltyType = "L2",
  knownNormalizationFactor = NULL,
  split1_indices = NULL,
  split2_indices = NULL,
  openBrowser = FALSE,
  useHajekInOptimization = TRUE,
  findMax = TRUE,
  quiet = TRUE,
  lambda_seq = NULL,
  lambda\_coef = 0.0001,
  nFolds = 3,
  batch\_size = 50,
  confLevel = 0.90,
```

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```
conda_env = NULL,
conda_env_required = FALSE,
hypotheticalNInVarObj = NULL
)
```

## **Arguments**

W	A matrix or data frame of assigned factor levels in the conjoint. For forced- choice designs, each row generally represents a single profile or a single respondent- task-profile combination. Must have integer, factor, or character columns repre- senting factor levels.
Υ	A numeric or binary outcome vector. Typically Y is 1 if a profile is chosen over its competitor, and 0 otherwise. If findMax = FALSE, the sign is flipped, effectively minimizing Y.
Υ	Ontional matrix or data frame of covariates (or respondent level features). Can

Optional matrix or data frame of covariates (or respondent-level features). Can be used for multi-cluster modeling with K>1.

An integer for the number of mixture components or latent clusters if multicluster modeling is desired. Defaults to 1 (no clusters).

WarmStart Logical. If TRUE, attempts to re-initialize from previous solutions each time lambda or other hyperparameters change. Defaults to FALSE.

automatic\_scaling

p\_list

Logical indicating whether to center or scale  $\boldsymbol{X}$  and  $\boldsymbol{Y}$  automatically. Defaults to TRUE.

A list of baseline factor-level probabilities in the design or assignment mechanism (e.g., the original random assignment distribution). If NULL, the function may assume uniform or empirical distributions.

hypothetical ProbList

An optional list specifying a counterfactual distribution over factor levels. If provided, OneStep.OptiConjoint directly computes and returns the performance or value under that distribution instead of estimating a new optimal distribution.

pi\_init\_vec A numeric vector for initializing the simplex-based representation of factor-level probabilities to be optimized. If NULL, a random initialization is used internally.

constrain\_ub Optional numeric or vector of upper bounds on factor probabilities. If not NULL, can help to enforce constraints in optimization.

nLambda Integer specifying the number of penalty values considered if cross-validation is

performed. Defaults to 10.

penaltyType A character specifying the type of penalty for shifting probabilities (e.g., "LogMaxProb",

"L2", or "KL"). This is an additional penalization on top of PenaltyType for

the outcome model. Defaults to "LogMaxProb".

testFraction Fraction of samples used for holdout in cross-validation. Defaults to 0.5 for a

basic split. If NULL, no split is performed.

log\_PrW Optional numeric vector of log probabilities for each row in W. If omitted, the

function will compute log\_PrW from p\_list given the assumption of indepen-

dent factor-level assignments.

LEARNING RATE BASE

Base learning rate for gradient-based optimizers. Defaults to 0.01.

cycle\_width 
Numeric controlling the frequency of restarts or adaptive learning-rate sched-

ules.

nSGD Number of gradient-descent updates. If nEpoch is provided, that takes prece-

dence.

nEpoch Number of epochs, each pass including length(availableTrainIndices) /

batch\_size mini-batches. If provided, overrides nSGD.

X\_factorized An optional matrix or data frame representing factorized (dummy-coded) ver-

sions of X for advanced modeling. If NULL, the function may factorize internally.

momentum Numeric specifying momentum for stochastic gradient descent. Defaults to

0.99.

nFullCycles\_noRestarts

If >1, repeats training cycles without restarts for a total number of gradient steps.

Useful for stability checks.

optim\_method A character specifying the optimization backend (e.g., "tf" for TensorFlow-

based, or "jax" for JAX-based). Defaults to "tf" if available.

sg\_method A character controlling the type of gradient updates (e.g., "adanorm", "wngrad").

If NULL, a default method is chosen.

forceSEs Logical. If TRUE, attempts to compute standard errors by M-estimation or the

delta method, even if no cross-validation is done.

clipAT\_factor A large numeric to clip gradient norms if they exceed clipAT\_factor.

adaptiveMomentum

Logical. If TRUE, momentum is adapted automatically as the optimization pro-

ceeds. Defaults to FALSE.

PenaltyType A character specifying the type of penalty (e.g., "L2") for the outcome model.

Used only if a penalized approach to outcome model fitting is internally per-

formed.

knownNormalizationFactor

An optional numeric to normalize reweighting for Hajek-based estimators. If

NULL, it is inferred from the sum of weights.

split1\_indices, split2\_indices

Optional vectors of indices partitioning the data for cross-validation or holdout.

If NULL, a random partition is done internally.

openBrowser Logical for debugging. If TRUE, may open an interactive browser for advanced

inspection.

useHajekInOptimization

Logical. If TRUE, uses a Hajek-based reweighting in objective functions for com-

puting the expected outcome under counterfactual probability shifts. Defaults to

TRUE.

findMax Logical. If TRUE, maximizes Y; if FALSE, treats Y as negative of interest (e.g.,

adversity minimization).

quiet Logical controlling the verbosity of printed messages.

lambda\_seq Optional numeric vector of penalty values for cross-validation. If NULL, the

function attempts a default sequence or single value.

lambda\_coef Numeric constant controlling the magnitude of the penalty for the outcome

model. Defaults to 0.0001.

nFolds Number of folds for cross-validation. Defaults to 3.

batch\_size Positive integer specifying the size of mini-batches in each gradient iteration.

Defaults to 50.

confLevel Numeric in  $\backslash((0,1)\backslash)$ , specifying the confidence level for intervals around esti-

mated probabilities or performance measures. Defaults to 0.90.

conda\_env Optional name of a Conda environment with **jax**, **optax**, etc. If NULL, attempts

a default environment.

conda\_env\_required

Logical. If TRUE, errors if the environment conda\_env cannot be found. Otherwise attempts to proceed gracefully.

hypotheticalNInVarObj

Optional numeric specifying an alternative n for certain variance calculations (e.g., hypothesized population size). If NULL, uses the observed sample size.

#### **Details**

This function implements a *one-step M-estimation* approach for directly estimating the "optimal" probability distributions over high-dimensional factors in conjoint or factorial experiments. Rather than a multi-step procedure of (1) outcome modeling followed by (2) re-optimizing factor distributions, the one-step approach can iteratively re-estimate distribution parameters while simultaneously adjusting the outcome model. This allows regularization or advanced modeling to be integrated into the *same* optimization objective, potentially improving finite-sample performance. Support for adversarial or multiple clusters is also available.

By default, OneStep.OptiConjoint attempts to find the distribution(s)  $\pi^*$  that maximizes the average outcome if findMax = TRUE (e.g., maximizing candidate choice share). In adversarial contexts, each cluster or "player" can simultaneously learn a best response. The function is flexible enough to incorporate sub-populations or multiple stages (e.g., primaries plus general elections).

If a user-supplied hypothetical ProbList is given, the function directly computes  $Q(\pi)$  for that distribution instead of estimating. This is useful for evaluating the performance of a known or hypothesized distribution (e.g., "status quo").

Most users do not need to call OneStep.OptiConjoint directly, as this is a lower-level routine. The OptiConjoint or cv.OptiConjoint functions may suffice in many typical workflows.

#### Value

A named list with components, often including:

- PiStar\_point: The estimated optimal or learned distribution(s) over factor levels. If K>1 or if adversarial competition is considered, may return multiple distributions (k1, k2, etc.).
- Q\_point: The estimated performance measure under the learned distribution(s). For example, the average or adversarially optimized outcome.
- Q\_se\_mEst: If available, standard errors via M-estimation or the delta method.
- PiStar\_lb, PiStar\_ub: Lower and upper confidence intervals for factor-level probabilities, if standard errors are computed.
- CVInfo: A data frame or list summarizing cross-validation performance for each candidate lambda.
- ClassProbsXobs, VarCov\_ProbClust, pi\_init\_next, optim\_max\_hajek\_list: Additional objects storing advanced details of the optimization or M-estimation procedure.
- Output.Description: Additional messages describing the run.

#### Note

Advanced arguments like X\_factorized, conda\_env, optim\_method, or specifying adaptiveMomentum are only needed for specialized or larger-scale (GPU-based) computations.

#### References

- Goplerud, M. & Titiunik, R. (2022). Analysis of High-Dimensional Factorial Experiments: Estimation of Interactive and Non-Interactive Effects. ArXiv preprint. - Egami, N. & Imai, K. (2019). Causal Interaction in Factorial Experiments: Application to Conjoint Analysis. Journal of the American Statistical Association, 114(526), 529–540. - Hainmueller, J., Hopkins, D. J., & Yamamoto, T. (2014). Causal Inference in Conjoint Analysis: Understanding Multidimensional Choices via Stated Preference Experiments. Political Analysis, 22(1), 1–30. - (Paper Reference) A forthcoming or accompanying manuscript describing in detail the methods for optimal or adversarial stochastic interventions in conjoint settings.

#### See Also

OptiConjoint for an approach that first fits an outcome model and then re-optimizes factor-level probabilities. \cv.OptiConjoint for cross-validation across candidate values of lambda.

## **Examples**

```
## Not run:
# Suppose we have a forced-choice conjoint dataset (W, Y) and baseline probabilities p_list.
# We want to estimate an optimal distribution that maximizes average Y.
set.seed(123)
# X could be respondent covariates, if any
X <- matrix(rnorm(nrow(W)*2), nrow(W), 2)</pre>
result_one_step <- OneStep.OptiConjoint(</pre>
  W = W
  Y = Y,
  X = X
  p_list = p_list,
  nSGD = 400,
  useHajekInOptimization = TRUE,
  penaltyType = "LogMaxProb",
  PenaltyType = "L2",
  lambda_seq = c(0.01, 0.1),
  testFraction = 0.3
# Inspect the estimated distribution over factor levels
str(result_one_step$PiStar_point)
# Evaluate estimated performance
print( result_one_step$Q_point )
## End(Not run)
```

OptiConjoint

Estimate Optimal (or Adversarial) Stochastic Interventions for Conjoint Experiments

#### **Description**

OptiConjoint implements the core methods described in the accompanying paper for learning an optimal or adversarial probability distribution over conjoint factor levels. It is specifically designed for forced-choice conjoint settings (e.g., candidate-choice experiments) and can accommodate scenarios in which a single agent optimizes its strategy in isolation, or in which two (potentially adversarial) agents simultaneously optimize against each other.

This function can be used to find the *optimal stochastic intervention* for maximizing an outcome of interest (e.g., vote choice, rating, or utility), possibly subject to a penalty that keeps the learned distribution close to the original design distribution. It can also incorporate institutional rules (e.g., primaries, multiple stages of choice) by specifying additional arguments. Estimation can be done under standard generalized linear modeling assumptions or more advanced approaches. The function returns estimates of the learned distribution and the associated performance quantity  $(Q(\pi^*))$  along with optional inference based on the (asymptotic) delta method.

## Usage

```
OptiConjoint(
 Υ,
 W,
 X = NULL
 lambda,
  varcov_cluster_variable = NULL,
  competing_group_variable_respondent = NULL,
  competing_group_variable_candidate = NULL,
  competing_group_competition_variable_candidate = NULL,
  pair_id = NULL,
  respondent_id = NULL,
  respondent_task_id = NULL,
 profile_order = NULL,
 p_list = NULL,
  slate_list = NULL,
 K = 1,
 nSGD = 100,
 diff = FALSE.
 MaxMin = FALSE,
 UseRegularization = FALSE,
 OpenBrowser = FALSE,
 ForceGaussianFamily = FALSE,
 A_INIT_SD = 0,
  TypePen = "KL"
 ComputeSEs = TRUE,
  conda_env = NULL,
  conda_env_required = FALSE,
  confLevel = 0.90,
  nFolds_glm = 3L,
  folds = NULL,
 nMonte_MaxMin = 5L,
 nMonte_Qglm = 100L,
 UseOptax = FALSE,
  jax_seed = as.integer(Sys.time()),
 OptimType = "tryboth"
```

#### **Arguments**

Υ

A numeric or binary vector of observed outcomes, typically in  $\{0,1\}$  for forced-choice conjoint tasks, indicating whether the profile was selected. For instance, Y = 1 if candidate A was chosen over candidate B, and Y = 0 otherwise. The length must match the number of rows in W.

W

A matrix or data frame representing the assigned levels of each factor in a conjoint design (one column per factor). Each row corresponds to a single profile. For forced-choice tasks, a given respondent may have contributed multiple rows if you reshape pairwise choices into long format. If the experiment used multiple factors D, with each factor having  $L_d$  levels, W should capture all factor assignments accordingly.

Χ

An optional matrix or data frame of additional covariates, often respondent-level features (e.g., respondent demographics). If K > 1, X may be used internally to fit multi-cluster or multi-component outcome models, or to allow cluster-specific effect estimation for more granular insights. Defaults to NULL.

lambda

A numeric scalar or vector giving the regularization penalty (e.g., in Kullback-Leibler or L2 sense) used to shrink the learned probability distribution(s) of factor levels toward a baseline distribution p\_list. Typically set via either domain knowledge or cross-validation.

varcov\_cluster\_variable

An optional vector of cluster identifiers (e.g., respondent IDs) used to form a robust variance-covariance estimate of the outcome model. If NULL, the usual IID assumption is made. Defaults to NULL.

competing\_group\_variable\_respondent, competing\_group\_variable\_candidate

competing\_group\_competition\_variable\_candidate Optional variables that mark competition group membership of respondents or candidate profiles. Particularly relevant in adversarial settings (MaxMin = TRUE) or multi-stage electoral settings, e.g., capturing the party of each respondent or candidate. Defaults to

pair\_id

A factor or numeric vector identifying the forced-choice pair. If each row of W is a single profile, pair\_id groups the rows belonging to the same choice set. Defaults to NULL.

respondent\_id, respondent\_task\_id

Another set of optional identifiers. respondent\_id marks each respondent across tasks, while respondent\_task\_id can define unique IDs for repeated measurements from the same respondent across multiple tasks. Useful for advanced clustering or robust SEs. Defaults to NULL.

profile\_order

If each forced-choice is shown with different ordering (e.g., Candidate A vs. Candidate B), profile\_order can label each row accordingly. Helpful for ensuring consistent labeling of reference vs. opposing profiles. Defaults to NULL.

p\_list

An optional list describing the baseline probability distribution over factor levels in W. Typically derived from the initial design distribution or uniform assignment distribution. If NULL, the function may assume uniform or attempt to estimate the distribution from W.

slate\_list

An optional list (or lists) providing custom "slates" of candidate features (and their associated probabilities). Used in more advanced or adversarial setups where certain combinations must be included or excluded. If NULL, no special constraints beyond the usual factor-level distributions are applied.

Κ

Integer specifying the number of latent clusters for multi-component outcome models. If K = 1, no latent clustering is done. Defaults to 1.

nSGD Integer specifying the number of stochastic gradient descent (or gradient-based) iterations to use when learning the optimal distributions. Defaults to 100.

diff Logical indicating whether the outcome Y represents a first-difference or difference-

based metric. In forced-choice contexts, typically diff = FALSE. Defaults to

FALSE.

MaxMin Logical controlling whether to enable the max-min adversarial scenario. When

TRUE, the function searches for a pair of distributions (one for each competing party or group) such that each party's distribution is optimal given the other

party's distribution. Defaults to FALSE.

UseRegularization

Logical indicating whether to regularize the outcome model (in addition to any penalty lambda on the distribution shift). This can help avoid overfitting in high-

dimensional designs. Defaults to FALSE.

OpenBrowser Logical for debugging; if TRUE, the function may open an interactive browser

window upon encountering certain conditions. Typically FALSE.

ForceGaussianFamily

Logical indicating whether to force a Gaussian-based outcome modeling approach, even if Y is binary or forced-choice. If FALSE, the function attempts to

choose a more appropriate link (e.g., "binomial"). Defaults to FALSE.

A\_INIT\_SD Numeric scalar specifying the standard deviation for random initialization of un-

constrained parameters used in the gradient-based search over factor-level prob-

abilities. Defaults to 0.

TypePen A character string specifying the type of penalty (e.g., "KL", "L2", or "LogMaxProb")

used in the objective function for shifting the factor-level probabilities away

from the baseline p\_list. Defaults to "KL".

ComputeSEs Logical indicating whether standard errors should be computed for the final es-

timates (via the delta method or related expansions). Defaults to TRUE.

conda\_env A character string naming a Python conda environment that includes jax, op-

 ${f tax},$  and other dependencies. If not NULL, the function attempts to activate that

environment. Defaults to NULL.

conda\_env\_required

Logical; if TRUE, raises an error if the environment given by conda\_env cannot be activated. Otherwise, the function attempts to proceed with any available

installation. Defaults to FALSE.

confLevel Numeric in (0,1), specifying the confidence level for intervals or credible bounds.

Defaults to 0.90.

nFolds\_glm Integer specifying the number of folds (default 3L) for internal cross-validation

used in certain outcome model or regularization steps. Defaults to 3L.

folds An optional user-supplied partitioning or CV scheme, overriding nFolds\_glm.

Defaults to NULL.

max-min steps, e.g., sampling from the opposing candidate's distribution to ap-

proximate expected payoffs. Defaults to 5L.

nMonte\_Qglm Integer specifying the number of Monte Carlo samples for evaluating or approx-

imating the quantity of interest under certain outcomes or distributions. Defaults

to 100L.

UseOptax Logical indicating whether to use the optax library for gradient-based optimiza-

tion in JAX (TRUE) or a built-in method (FALSE). Defaults to FALSE.

jax\_seed Integer seed for reproducible JAX-based computations. Defaults to as.integer(Sys.time()).

OptimType A character string for choosing which optimizer or approach is used internally (e.g., "default", "SecondOrder", or "tryboth"). Defaults to "tryboth".

#### **Details**

**Modeling the outcome:** Internally, OptiConjoint may fit a generalized linear model or a more flexible approach (such as multi-cluster factorization) to learn the mapping from factor-level assignments W (and optional covariates X) onto outcomes Y. Once these outcome coefficients are estimated, the function uses gradient-based or closed-form solutions to find the *optimal stochastic intervention(s)*, i.e., new factor-level probability distributions that maximize an expected outcome (or solve the max-min adversarial problem).

Adversarial or strategic design: When MaxMin = TRUE, the function attempts to solve a zero-sum game in which one agent (say, "A") chooses its distribution to maximize vote share, while the other ("B") simultaneously chooses its distribution to minimize "A"'s vote share. In many settings, competing\_group\_variable\_respondent and related arguments help define which respondents belong to the "A" or "B" sub-electorate (e.g., a primary). The final solution is a mixed-strategy Nash equilibrium, if it exists, for the forced-choice environment. This can be used to compare or interpret real-world candidate positioning in multi-stage elections.

**Regularization:** The argument lambda penalizes how far the learned distribution strays from the baseline distribution p\_list. This helps avoid overfitting in high-dimensional designs. Different penalty types can be selected via TypePen.

**Implementation details:** Under the hood, this function may rely on **jax** for automatic differentiation. By default, it uses an internal gradient-based approach. If UseOptax = TRUE, the optax library is used for optimization. The function can automatically detect or load a **conda** environment if specified, though advanced users can pass conda\_env\_required = TRUE to enforce that environment activation is mandatory.

#### Value

A named list containing:

PiStar\_point An estimate of the (possibly multi-cluster or adversarial) optimal distribution(s) over the factor levels. If MaxMin = TRUE and K = 1, returns a pair of distributions. If K > 1, returns a list, with each element corresponding to a separate cluster.

\item{\code{PiStar\_se}}{A list of standard errors for the entries of \code{PiStar\_point},
computed if \code{ComputeSEs = TRUE}.}

\item{\code{Q\_point\_mEst}}{Point estimates of the corresponding average or adversarially optimized outcome, e.g., estimated expected utility or vote share.}

\item{\code{Q\_se\_mEst}}{Standard error (if computed) for \code{Q\_point\_mEst}.}

\item{\code{PiStar\_lb}, \code{PiStar\_ub}}{Lower and upper bounds for the factor-level
probabilities in \code{PiStar\_point}, if \code{ComputeSEs = TRUE} and a confidence
level is provided.}

\item{\code{CVInfo}}{A data frame or list summarizing cross-validation performance for
different values of \code{lambda}, if relevant.}

\item{\code{estimationType}}{A string describing whether the solution was found via a

```
single-step or multi-step approach (e.g., \code{"TwoStep"}, \code{"OneStep"}).}
\item{\code{...}}{Additional elements storing internal details (e.g., the fitted outcome model, jacobian matrix, or optimization logs).}
```

#### References

- Hainmueller, J., Hopkins, D. J., & Yamamoto, T. (2014). Causal Inference in Conjoint Analysis: Understanding Multidimensional Choices via Stated Preference Experiments. *Political Analysis*, 22(1), 1–30.
- Egami, N., & Imai, K. (2019). Causal Interaction in Factorial Experiments: Application to Conjoint Analysis. *Journal of the American Statistical Association*, 114(526), 529–540.
- Goplerud, M., & Titiunik, R. (2022). Analysis of High-Dimensional Factorial Experiments: Estimation of Interactive and Non-Interactive Effects. *arXiv preprint arXiv:2207.XXXX*.
- (Paper Reference) A forthcoming or accompanying manuscript describing in detail the methods for *optimal* or *adversarial* stochastic interventions in conjoint settings.

#### See Also

cv.OptiConjoint for cross-validation across candidate values of lambda. See also OneStep.OptiConjoint for a function that implements a "one-step" approach to M-estimation of the same target quantity.

## **Examples**

```
## Not run:
  # Suppose we have a forced-choice conjoint dataset with
  # factor matrix W, outcome Y, and baseline probabilities p_list
  # Basic usage: single agent optimizing expected outcome
  opt_result <- OptiConjoint(</pre>
      Y = Y,
      W = W,
      lambda = 0.1,
      p_list = p_list,
     MaxMin = FALSE,
TypePen = "KL",
                            # No adversarial component
# Kullback-Leibler penalty
      nSGD = 200
                               \# # of gradient descent iterations
  )
  # Inspect the learned distribution and performance
  print(opt_result$PiStar_point)
  print(opt_result$Q_point_mEst)
  print(opt_result$CVInfo)
                                        # If cross-validation was used
  # Adversarial scenario with multi-stage structure
  # E.g., define 'competing_group_variable_respondent' for two parties' supporters
  adv_result <- OptiConjoint(</pre>
      Y = Y,
      W = W,
      lambda = 0.2,
      p_list = p_list,
      MaxMin = TRUE,
                               # Solve zero-sum game across two sets of respondents
      competing_group_variable_respondent = partyID,
      nSGD = 300
  )
```

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```
# 'adv_result' now contains distributions for each party's candidate
# that approximate a mixed-strategy Nash equilibrium
print(adv_result$PiStar_point$k1) # Party A distribution
print(adv_result$PiStar_point$k2) # Party B distribution
## End(Not run)
```

strategize.plot

Implements...

## Description

Implements...

## Usage

```
OneStep.OptiConjoint(...)
```

## Arguments

Х

Description

#### **Details**

 ${\tt OneStep.OptiConjoint\ Description}$ 

• Description

## Value

z Description

## **Examples**

```
# Analysis
OptiConjoint_analysis <- OneStep.OptiConjoint()
print( OptiConjoint_analysis )</pre>
```

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