Heat Distribution Simulation with Parallel Algorithms

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1. PROBLEM DESCRPTION AND SYMBOL DEFINITION

1.1 Problem Descrption

There is a room of 50 ft in height and width, at the temperature of $ROOM_TEMP$. A fire place is in the middle of the top wall of the room. It has length of 20 ft and temperature of $FIREPLACE_TEMP$. The wall is at $ROOM_TEMP$ constantly and will not be heated by the fireplace. Simulate the heat distribution when the temperature is balanced. Draw it in 5°C temperature contours.

1.2 Symbol List

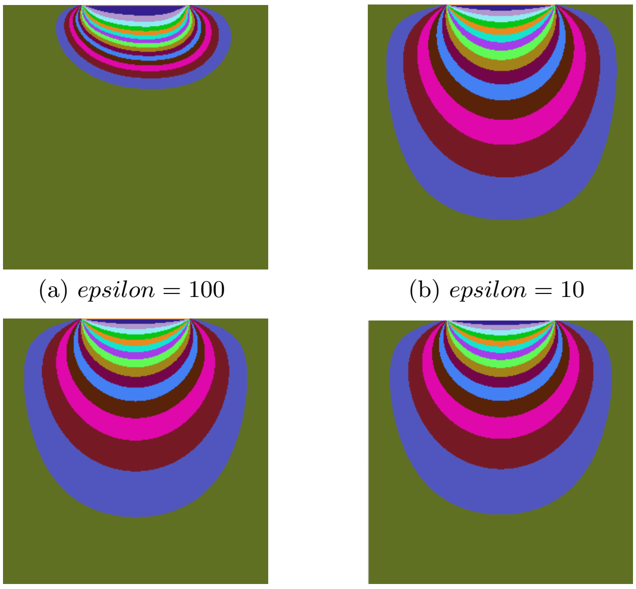
Table 1: Symbol List

Item	Description
x	Cells in height of the room.
y	Cells in width of the room.
$iter$	Max numbers of iterations in each datum.
INC_TIME	Number of different scale of data processed by the increment algorithm/
$INCREMENT$	Constant used by the increment algorithm.
P	Number of processes.
$EPSILON$	Terminating error.
F_INTVAL	Length of each frame (in ms) while displaying the result.
$T_{k,i,j}$	The temperature of cell (i,j) in k th iteration.

2. ALGORITHM DESIGN

2.1 Laplace Approach

For each iteration, each cell's temperature is set to the average of its four neighbours.



(a) $epsilon = 100$

(b) $epsilon = 10$

(c) $epsilon = 1$

(d) $epsilon = 0.1$

Figure 1: Effect of $epsilon$**,** $x = y = 300$

Terminating Condition: Keep doing the iteration until $TotalError < EPSILON$, where

$$TotalError = \sum_{i,j} |T_{k,i,j} - T_{k-1,i,j}| \tag{1}$$

The effect of $epsilon$ is shown as figure 1.

2.2 The Baseline Algorithm

The baseline algorithm is to keep doing the iteration until terminating condition meets. The time complexity is $O(xy \times iter)$, space complexity is $O(xy)$.

2.3 Increment Algorithm

We observed that in initial iterations, the temperature of cells that stay far from the fireplace does not change. It is waste of time calculating this. To speedup the algorithm, we can use better initial values to make the heat field converges faster. To get the initial value, we can use a smaller scale data with $x/INCREMENT$, $y/INCREMENT$ size. Since the data is small, it converges much faster than the original one. Thus, the solution of the smaller data can be used as the initial value of the original data, with scratching the temperature field of $x/INCREMENT$, $y/INCREMENT$ to x, y. This process can be applied multiple times, we set $INC\_TIME = 8$ in this problem.

Assuming the smaller scale problem converges as slow as the original one, the time complexity has an upper bound of $xy \times iter + \frac{xy \times iter}{INCREMENT^2} + \frac{xy \times iter}{INCREMENT^4} + ... = O(xy \times iter)$. This implies the increment algorithm is not slow than the baseline algorithm in asymptotic time complexity. The space complexity is $O(xy)$, the same as the original one.

A pseudocode is shown as algorithm 1.

Algorithm 1 The increment algorithm of heat distribution problem

```
function HeatDistribution($x$, $y$, $INC\_TIME$)
    if $INC\_TIME > 0$ then
        $T' \Leftarrow$ HeatDistribution($x/INCREMENT$, $y/INCREMENT$, $INC\_TIME - 1$)
    else
        $T' \Leftarrow$ the initial heat field
    end if
    for $i = 0 \to x - 1$ do
        for $j = 0 \to y - 1$ do
            $T_{i,j} \Leftarrow T'_{i/INCREMENT, j/INCREMENT}$
        end for
    end for
    while $T$ not converges do
        Laplace-iterate($T$)
    end whilereturn $T$
end function
```

2.4 OpenMP Parallel Algorithm

We make the problem parallel by processing the rows of rooms parallel. The time complexity reduces to $O(xy \times iter/P)$. We use static scheduling, thus the number of create and join operation will be $O(P \times iter)$.

2.5 PThread Parallel Algorithm

The pthread algorithm has nearly same implmentation as OpenMP. Excepting mutex has been used to wake up and wait for threads. The number of create and join operation has been reduced to $O(P)$, while the time and space complexity remains the same as OpenMP, i.e., $O(xy \times iter/P)$ and $O(xy)$.

2.6 MPI Parallel Algorithm

We divide the room into blocks for the subprocesses to compute. Blocks scheme is better because it have lower communication overhead comparing with strip scheme.

$$StripOverhead = 2max\{x, y\}P \quad (2)$$

$$BlockOverhead = 4max\{x, y\}P^{0.5} \quad (3)$$

Apparently, $StripOverhead > BlockOverhead$ when $P > 4$. A MPI pseudocode is as algorithm 2.

The time complexity is $O(xy \times iter/P)$, space complexity is

Algorithm 2 The MPI increment algorithm of heat distribution problem

```
function HeatDistribution($x$, $y$, $INC\_TIME$)
    if $INCREMENT\_TIME > 0$ then
        $T' \Leftarrow$ HeatDistribution($x/INCREMENT$, $y/INCREMENT$, $INCREMENT\_TIME - 1$)
    else
        $T' \Leftarrow$ the initial heat field
    end if
    for $i = 0 \to x - 1$ do
        for $j = 0 \to y - 1$ do
            $T_{i,j} \Leftarrow T'_{i/INCREMENT, j/INCREMENT}$
        end for
    end for
    scatter $T$ to subprocesses
    while $T$ not converges do
        Laplace-iterate($T$)
    end while
    gather $T$ from subprocesses return $T$
end function
function Laplace-iterate($T$)
    send topmost, bottommost row and leftmost, rightmost columns to neighbours
    receive topmost, bottommost row and leftmost, rightmost columns from neighbours
    $T'' \Leftarrow T$
    for $i = 0 \to x/P^{0.5} - 1$ do
        for $j = 0 \to y/P^{0.5} - 1$ do
            $T_{i,j} \Leftarrow (T''_{i-1,j} + T''i+1, j + T''i, j-1 + T''i, j+1)/4$
        end for
    end forreturn $T$
end function
```

$O(xy)$, communication overhead is $O(max\{x, y\}P^{0.5} \times iter + xy \times INC\_TIME)$, approximately $O(max\{x, y\}P^{0.5} \times iter)$.

2.7 Summary

The complexity of mentioned algorithms is summarized as table 2.

Table 2: Complexity of heat simulation algorithm

Algorithm	Time(computation)
Baseline	$O(xy \times iter)$
Increment	$O(xy \times iter)$
OpenMP_Increment	$O(xy \times iter/P)$
PThread_Increment	$O(xy \times iter/P)$
MPI_Increment	$O(xy \times iter)$
Algorithm	**Time(overhead)**
Baseline	$O(xy)$
Increment	$O(xy)$
OpenMP_Increment	$O(xy)$
PThread_Increment	$O(xy)$
MPI_Increment	$O(xy)$
Algorithm	**Space**
Baseline	$O(xy \times iter)$
Increment	$O(xy \times iter)$
OpenMP_Increment	$O(xy \times iter/P)$
PThread_Increment	$O(xy \times iter/P)$
MPI_Increment	$O(xy \times iter)$

3. EXPERIMENTAL METHOLOGY

The experiment is done on a dual chip computer with 12 cores at 2.93GHz and an cluster with 20 nodes respectively.

Table 3: Experiment Configurations

System Configurations1	Intel Xeon X5670x2, 6 Cores 2.93GHz, 12MB Cache
System Configurations2	20 nodes, each nodes is Intel Xeon X5670x2, 6 Cores 2.93GHz, 12MB Cache
Compiler (PThread&OpenMP)	gcc
Compiler (MPI)	Intel C Compiler
MPI	Intel MPI
PThread	PThread
OpenMP	OpenMP

The parameters of base test data is as table **??**, all the experiment parameters are modified from the beast test data.

4. EXPERIMENTAL RESULTS

4.1 Baseline

The performance of the baseline algorithm on base test data is as table 5.

Table 4: Experiment Configurations

Parameter	Value
x	300
y	300
$iter$	1000000(until converges)
$epsilon$	1
INC_TIME	8
$INCREMENT$	1.6

Table 5: Baseline Algorithm Performance

iterations	time/s
67125	120.132760

4.2 Speedup of the Increment Algorithm

4.2.1 Impact of the scale factor $INCREMENT$

We run the base test data and tried different scale factors, from 1.3 to 1.9. The total number of iterations and time consumption is as figure 2. We can observe that both number of iterations and time consumption reached minimum at $INCREMENT = 1.6$.

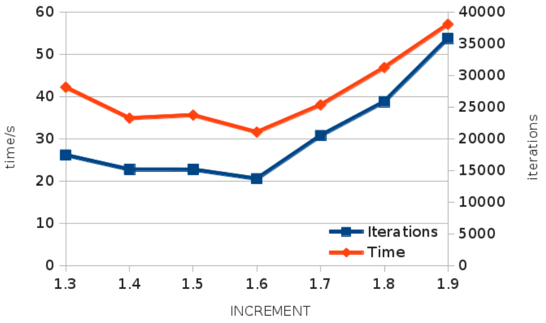


Figure 2: Impact of the scale factor $INCREMENT$

4.2.2 Impact of problem scale

To make the parameter selection more convincing, we calculate $INCREMENT$ at different scale, as table 6.

The time consumption at different scale and $INCREMENT$ is as figure 3. About 6x speedup is achieved at the base test data.

The time consumption and speedup of baseline and increment algorithm at different scale is shown as figure 4, 5. At larger scale, the scaling up will be more precise, but $epsilon$ for each cell is more strict. So the speedup increase with fluctuation.

4.2.3 Impact of $epsilon$

To make the parameter selection more convincing, we calculate $INCREMENT$ at different $epsilon$, as table 7.

The time consumption at different $epsilon$ and $INCREMENT$ is as figure 6.

The time consumption and speedup of baseline and increment algorithm at different $epsilon$ is shown as figure 7,

Table 6: Optimal $INCREMENT$ at different scale

x, y	200	250	300	350
Optimal $INCREMENT$	1.6	1.4	1.6	1.4

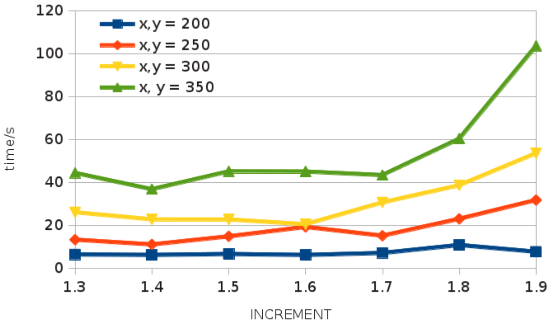


Figure 3: Time consumption of baseline and increment algorithm with different scale and $INCREMENT$

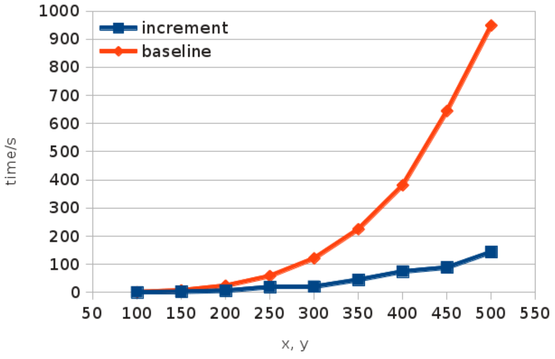


Figure 4: Time consumption of baseline and increment algorithm as scale increase

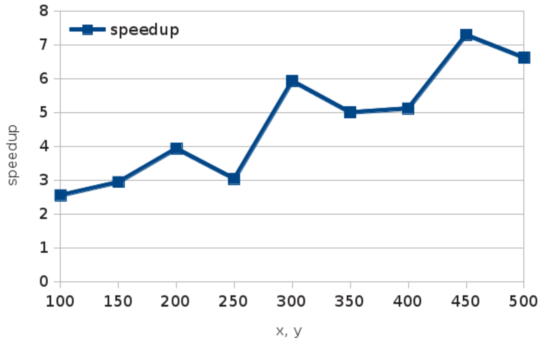


Figure 5: Speedup of baseline and increment algorithm as scale increase

Table 7: Optimal $INCREMENT$ at different $epsilon$

$Epsilon$	1	0.1	0.01
Optimal $INCREMENT$	1.6	1.6	1.6

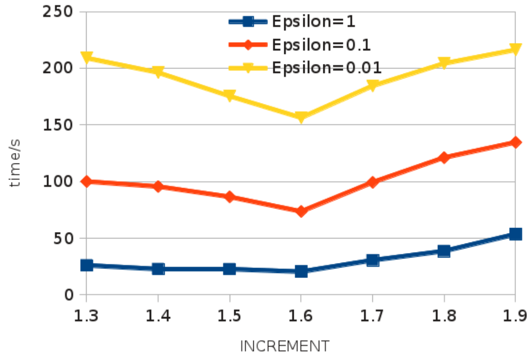


Figure 6: Time consumption of baseline and increment algorithm with different $epsilon$ and $INCREMENT$

8. The scaling up brings intrinsic error, relatively fixed. If $epsilon$ scales down, the speedup converging from the initial state to the intrinsic error state will be outnumber by that computing from the intrinsic error state to $epsilon$ error state. So the speedup is converging to 1. However, since we proved that $epsilon = 1$ is enough, the effect can be neglected.

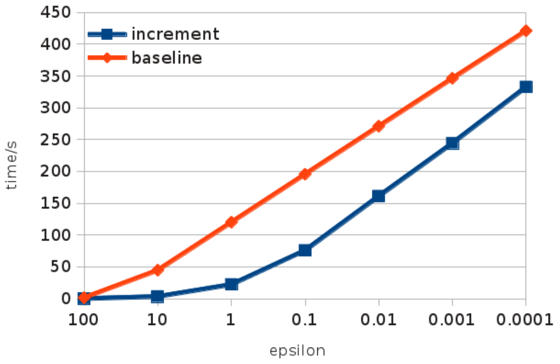


Figure 7: Time consumption of baseline and increment algorithm as $epsilon$ decrease

4.3 Parallel Algorithm Speedup

The parallel algorithm time consumption and speedup on machine 1 is shown as figure 9, 10. About 60x speedup is achieved by OpenMP with 12 cores.

5. CONCLUSION

By appling the increment algorithm and OpenMP, up to 60x of increment is achieved. MPI on clusters can provide further speedup.

6. EXPERIENCE

1. On memory intensive applicantions, parallel algorithm may not provide speedup.

2. Replacing create and join with mutexs wake up and wait may cause the program faster.

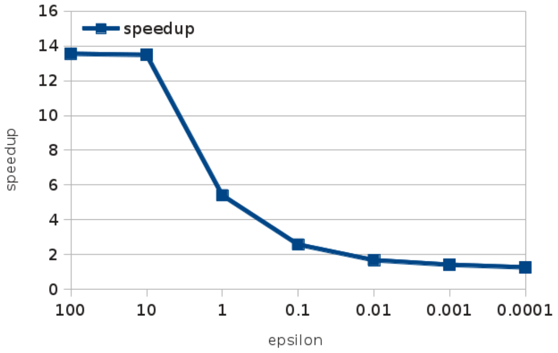


Figure 8: Speedup of baseline and increment algorithm as *epsilon* decrease

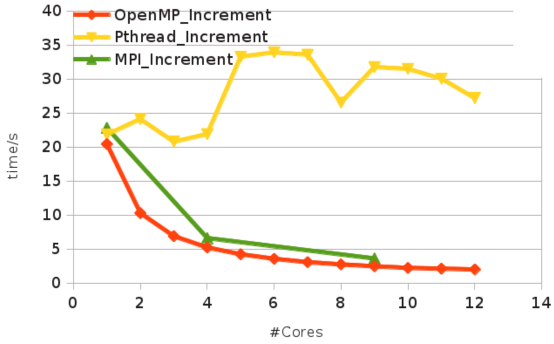


Figure 9: Time consumption of parallel algorithm

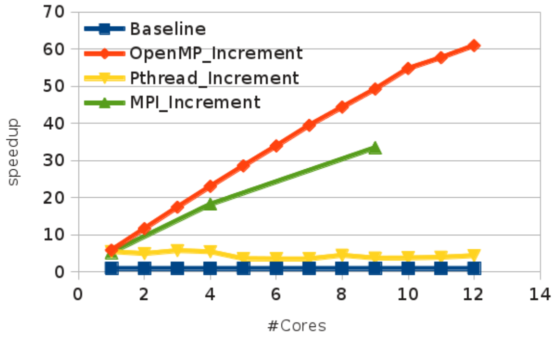


Figure 10: Speedup of parallel algorithm

7. APPENDIX A: INSTRUCTION FOR USING THE EXPERIMENT PROGRAMS

Compiler Options:

DISPLAY: Create output.

Compile:

make all: Make all programs, without display.

make -f Makefile_Display all: Make all programs, with display.

Command Line Parameters:

PThread&OpenMP: ./temperature_openxy/pthread x, y, $iter$, INC_TIME, $INCREMENT$, P, $epsilon$

MPI: mpirun -n P ./temperature_mpi x, y, $iter$, INC_TIME, $INCREMENT$, $epsilon$

8. APPENDIX B: PROGRAMS

Listing 1: const.h

```
#ifndef _CONST
#define _CONST

#define FRAME_INTERVAL 20
#define X_REFRESH_RATE 1000

#define ROOM_TEMP 20
#define FIRE_TEMP 100

#endif
```

Listing 2: models.h

```
#ifndef _MODELS
#define _MODELS

#include <memory.h>
#include <stdlib.h>
#include "const.h"

#define legal(x, n) ( (x)>=0 && (x)<(n) )

typedef struct TemperatureField
{
        int x, y;
        double **t;
        double *storage;
}TemperatureField;

void deleteField(TemperatureField *field);

void newField(TemperatureField *field, int x, int y
{
        TemperatureField temp = *field;
        field->storage = malloc( sizeof(double) * x
        field->t = malloc( sizeof(double*) * x );
        field->x = x;
        field->y = y;
        int i, j;
        for (i=0; i<x; ++i)
                field->t[i] = &field->storage[i*y];
        if (sourceX)
        {
                double scaleFactorX = (double)sourceX/x;
                double scaleFactorY = (double)sourceY/y;
                for (i=0; i<x; ++i)
                        for (j=0; j<y; ++j)
                                field->t[i][j] = temp.t[(int)(i*scaleFactorX)][(int)(j*scaleFactorY
                deleteField(&temp);
        }
        else memset(field->storage, 0, sizeof(double)*x*y);
}

void initField(TemperatureField *field)
{
        int i, j;
        for (i=0; i<field->x; ++i)
                for (j=0; j<field->y; ++j)
                        field->t[i][j] = 20.0f;
}

void refreshField(TemperatureField *field, int initX, int initY, int thisX, int thisY, int allX, i
{
        int j;
        for (j=allY*3/10; j<allY*7/10; ++j)
            if (legal(-initX, thisX)&&legal(j-initY, thisY))
                field->t[-initX][j-initY] = 100.0f;
}

TemperatureField* myClone(TemperatureField *field, int X, int Y)
{
        int i, j;
        TemperatureField *ret = malloc(sizeof(TemperatureField));
        ret->x = X;
        ret->y = Y;
        ret->storage = malloc(sizeof(double)*ret->x*ret->y);
        ret->t = malloc(sizeof(double*)*ret->x);
        for (i=0; i<ret->x; ++i)
                ret->t[i] = &ret->storage[i*ret->y];
        for (i=0; i<X; ++i)
                for (j=0; j<Y; ++j)
                        ret->t[i][j] = field->t[i][j];
        return ret;
}

void deleteField(TemperatureField *field)
{
        free(field->t);
        free(field->storage);
        //free(field);
}

#endif
```

Listing 3: display.h

```
/* Initial Mandelbrot program */


#include <X11/Xlib.h>
#include <X11/Xutil.h>
#include <X11/Xos.h>
#include <stdio.h>
#include <string.h>
#include <math.h>
#include <stdlib.h>
#include "models.h"
#include "const.h"

Window          win;
/* initialization for a window */
unsigned
int             width, height,
/* window size */
                border_width,
/*border width in pixels */
                idth, display_height,  /* size of s
                screen;
/* which screen */

char            *window_name = "Temperature Simula
GC              gc;
unsigned
long            valuemask = 0;
XGCValues       values;
Display         *display;
XSizeHints      size_hints;
Pixmap          bitmap;
FILE            *fp, *fopen ();
Colormap        default_cmap;
XColor          color[256];

int temperature_to_color_pixel(double t)
{
        return color[(int)(t/5.0f)].pixel;
}

void XWindow_Init(TemperatureField *field)
{
        XSetWindowAttributes attr[1];

        /* connect to Xserver */

        if (  (display = XOpenDisplay (display_name
           fprintf (stderr, "drawon: cannot connect
                                        XDisplayName (displ
        exit (-1);
        }

        /* get screen size */

        screen = DefaultScreen (display);

        /* set window size *///XFlush (display);

        width = field->y;
        height = field->x;

        /* create opaque window */

        border_width = 4;
        win = XCreateSimpleWindow (display, RootWin
                                        width, height, widt
                                        BlackPixel (display
        size_hints.flags = USPosition|USSize;
        size_hints.x = 0;
        size_hints.y = 0;
        size_hints.width = width;
        size_hints.height = height;
        size_hints.min_width = 300;
        size_hints.min_height = 300;

        XSetNormalHints (display, win, &size_hints);
        XStoreName(display, win, window_name);

        /* create graphics context */

        gc = XCreateGC (display, win, valuemask, &values);

        default_cmap = DefaultColormap(display, screen);
        XSetBackground (display, gc, WhitePixel (display, screen));
        XSetForeground (display, gc, BlackPixel (display, screen));
        XSetLineAttributes (display, gc, 1, LineSolid, CapRound, JoinRound);

        attr[0].backing_store = Always;
        attr[0].backing_planes = 1;
        attr[0].backing_pixel = BlackPixel(display, screen);

        XChangeWindowAttributes(display, win, CWBackingStore | CWBackingPlanes | CWBackingPixel, a

        XMapWindow (display, win);
        XSync(display, 0);

        /* create color */
        int i;
        for (i=0; i<20; ++i)
        {
            color[i].green = rand()%65535;
            color[i].red = rand()%65535;
            color[i].blue = rand()%65535;
            color[i].flags = DoRed | DoGreen | DoBlue;
            XAllocColor(display, default_cmap, &color[i]);
        }
}

void XResize(TemperatureField *field)
{
    XResizeWindow(display, win, field->y, field->x);
}

void XRedraw(TemperatureField *field)
{
    int i, j;
    for (i=0; i<field->x; ++i)
        for (j=0; j<field->y; ++j)
        {
                XSetForeground(display, gc, temperature_to_color_pixel(field->t[i][j]));
                XDrawPoint (display, win, gc, j, i);
        }
    XFlush (display);
}
```

Listing 4: main_openmp_increment.h

```
#include "const.h"
#include "models.h"
#include "display.h"
#include <omp.h>

#define legal(x, n) ( (x)>=0 && (x)<(n) )
#define start_time clock_gettime(CLOCK_MONOTONIC, &
#define end_time clock_gettime(CLOCK_MONOTONIC, &fi
#define time_elapsed_ns (long long)(finish.tv_sec-s
#define time_elapsed_s (double)(finish.tv_sec-start
#define NOT_FIRE_PLACE i

int iteration, threads;
int INCREMENT_TIME;
double EPSILON;
double INCREMENT;
TemperatureField *field;
TemperatureField *tempField, *swapField;

int dx[4] = {0, -1, 0, 1};
int dy[4] = {1, 0, -1, 0};

int x, y, iter_cnt;

double temperature_iterate(TemperatureField *field)
{
        ++iter_cnt;
        refreshField(field, 0, 0, field->x, field->y
        int i, j, d;
        double ret = 0;
#pragma omp parallel for schedule(static) private(j
        for (i=0; i<field->x; ++i){
                for (j=0; j<field->y; ++j)
                {
                        tempField->t[i][j] = 0;
                        for (d=0; d<4; ++d)
                                if ( legal(i+dx[d]
                                        tempField->
                                else
                                        tempField->
                        tempField->t[i][j] /= 4;
                        if (NOT_FIRE_PLACE)
                                ret += fabs(tempFi
                }
        }
        return ret;
}

int main(int argc, char **argv)
{
    struct timespec start, finish;
    start_time
    if (argc<8)
    {
            printf("Usage: %s x y iteration INCREM
    }
    sscanf(argv[1], "%d", &x);
    sscanf(argv[2], "%d", &y);
    sscanf(argv[3], "%d", &iteration);
    sscanf(argv[4], "%d", &INCREMENT_TIME);
    sscanf(argv[5], "%lf", &INCREMENT);
    sscanf(argv[6], "%d", &threads);
    sscanf(argv[7], "%lf", &EPSILON);
    omp_set_num_threads(threads);

    field = malloc(sizeof(TemperatureField));
    tempField = malloc(sizeof(TemperatureField));
    field->x = y;
    field->y = x;
#ifdef DISPLAY
    XWindow_Init(field);
#endif

    int iter, inc;
    int *X_Size = malloc(sizeof(int)*INCREMENT_TIME);
    int *Y_Size = malloc(sizeof(int)*INCREMENT_TIME);
    X_Size[INCREMENT_TIME-1] = x;
    Y_Size[INCREMENT_TIME-1] = y;
    for (inc=INCREMENT_TIME-2; inc>=0; --inc)
    {
        X_Size[inc] = X_Size[inc+1] / INCREMENT;
        Y_Size[inc] = Y_Size[inc+1] / INCREMENT;
    }

    for (inc=0; inc<INCREMENT_TIME; ++inc)
    {
        if (!inc)
        {
            newField(field, X_Size[inc], Y_Size[inc], 0, 0);
            newField(tempField, X_Size[inc], Y_Size[inc], 0, 0);
            initField(field);
        }
        else
        {
            newField(field, X_Size[inc], Y_Size[inc], X_Size[inc-1], Y_Size[inc-1]);
            newField(tempField, X_Size[inc], Y_Size[inc], X_Size[inc-1], Y_Size[inc-1]);
        }
#ifdef DISPLAY
        XResize(field);
#endif
        for (iter=0; iter<iteration; iter++)
        {
           double error = temperature_iterate(field);
           if (error<EPSILON)
           {
                   printf("Finished. iteration=%d, error=%lf\n", iter, error);

                break;
           }
           swapField = field;
           field = tempField;
           tempField = swapField;
#ifdef DISPLAY
           end_time
           if (time_elapsed_ns > FRAME_INTERVAL*1000000)
           {
                start_time;
                XRedraw(field);
           }
//                  puts("Field:");
//                  int i, j;
//                   for (i=0; i<field->x; ++i)
//                   {
//                           for (j=0; j<field->y; ++j)
//                               printf("%lf ", field->t[i][j]);
//                           puts("");
//                   }
#endif
        }
```

```
    }
    deleteField(field);
    deleteField(tempField);
    free(X_Size);
    free(Y_Size);
    printf("Finished in %d iterations.\n", iter_cnt
    end_time;
    printf("%lf\n", time_elapsed_s);
    return 0;
}
```

Listing 5: main_pthread_increment.h

```
#include "const.h"
#include "models.h"
#include "display.h"
#include <pthread.h>
#include <stdio.h>

#define legal(x, n) ( (x)>=0 && (x)<(n) )
#define start_time clock_gettime(CLOCK_MONOTONIC, &
#define end_time clock_gettime(CLOCK_MONOTONIC, &fi
#define time_elapsed_ns (long long)(finish.tv_sec-s
#define time_elapsed_s (double)(finish.tv_sec-start
#define NOT_FIRE_PLACE i

#define update(dx, dy) if ( legal(i+dx, field->x) &

int iteration, threads;
TemperatureField *field;
TemperatureField *tempField, *swapField;
pthread_t *threadPool;
pthread_mutex_t *subThreadWakeUp, *subThreadFinishe
int *threadID, terminate;
double *error;
int INCREMENT_TIME;
double INCREMENT, EPSILON;

int dx[4] = {0, -1, 0, 1};
int dy[4] = {1, 0, -1, 0};

int x, y, iter_cnt;

int min(int x, int y){ if (x<y) return x; return y

void* iterateLine(void* data)
{
    int threadID = *((int*)data);
    while (1)
    {
            pthread_mutex_lock(&subThreadWakeUp[thr
            if (terminate) break;
            int blockSize = field->x/threads + !!(
            int lineStart = blockSize*threadID;
            int lineEnd = min(blockSize*(threadID+
            error[threadID]=0;

            int i, j, d;
            for (i=lineStart; i<lineEnd; ++i)
                for (j=0; j<field->y; ++j)
                {
                        tempField->t[i][j] = 0;
                        for (d=0; d<4; ++d)
                        if ( legal(i+dx[d], field->x) && legal(j+dy[d], field->y))
                                tempField->t[i][j] += field->t[i+dx[d]][j+dy[d]];
                        else
                                tempField->t[i][j] += ROOM_TEMP;
                    tempField->t[i][j] /= 4;
                    if (NOT_FIRE_PLACE)
                        error[threadID] += fabs(tempField->t[i][j] - field->t[i][j]);
                }
            pthread_mutex_unlock(&subThreadFinished[threadID]);
    }
    pthread_exit(NULL);
}

double temperature_iterate()
{
        ++iter_cnt;
        refreshField(field, 0, 0, field->x, field->y, field->x, field->y);
        int i;

        for (i=0; i<threads; ++i)
                pthread_mutex_unlock(&subThreadWakeUp[i]);
        for (i=0; i<threads; ++i)
                pthread_mutex_lock(&subThreadFinished[i]);
        double sumError = 0;
        for (i=0; i<threads; ++i)
                sumError += error[i];

        return sumError;
}

int main(int argc, char **argv)
{
    struct timespec start, finish;
    start_time
    if (argc<8)
    {
            printf("Usage: %s x y iteration INCREMENT_TIME, INCREMENT threads EPSILON\n", argv[0]);
    }
    sscanf(argv[1], "%d", &x);
    sscanf(argv[2], "%d", &y);
    sscanf(argv[3], "%d", &iteration);
    sscanf(argv[4], "%d", &INCREMENT_TIME);
    sscanf(argv[5], "%lf", &INCREMENT);
    sscanf(argv[6], "%d", &threads);
    sscanf(argv[7], "%lf", &EPSILON);

    field = malloc(sizeof(TemperatureField));
    tempField = malloc(sizeof(TemperatureField));
    threadPool = malloc(sizeof(pthread_t)*threads);
    subThreadWakeUp = malloc(sizeof(pthread_mutex_t)*threads);
    subThreadFinished = malloc(sizeof(pthread_mutex_t)*threads);
    threadID = malloc(sizeof(int)*threads);
    error = malloc(sizeof(double)*threads);
    terminate = 0;
    field->x = y;
    field->y = x;
#ifdef DISPLAY
    XWindow_Init(field);
#endif

    int i;
    for (i=0; i<threads; ++i)
    {
```

```
        pthread_mutex_init(&subThreadWakeUp[i], NUL
        pthread_mutex_init(&subThreadFinished[i], N
        pthread_mutex_lock(&subThreadWakeUp[i]);
        pthread_mutex_lock(&subThreadFinished[i]);
        threadID[i] = i;
        pthread_create(&threadPool[i], NULL, iterat
    }

    int iter, inc;
    int *X_Size = malloc(sizeof(int)*INCREMENT_TIM
    int *Y_Size = malloc(sizeof(int)*INCREMENT_TIM
    X_Size[INCREMENT_TIME-1] = x;
    Y_Size[INCREMENT_TIME-1] = y;
    for (inc=INCREMENT_TIME-2; inc>=0; --inc)
    {
        X_Size[inc] = X_Size[inc+1] / INCREMENT;
        Y_Size[inc] = Y_Size[inc+1] / INCREMENT;
    }

    for (inc=0; inc<INCREMENT_TIME; ++inc)
    {
        if (!inc)
        {
            newField(field, X_Size[inc], Y_Size[inc
            newField(tempField, X_Size[inc], Y_Size
            initField(field);
        }
        else
        {
            newField(field, X_Size[inc], Y_Size[inc
            newField(tempField, X_Size[inc], Y_Size
        }
#ifdef DISPLAY
        XResize(field);
#endif
        for (iter=0; iter<iteration; iter++)
        {
           double error = temperature_iterate();
           if (error<EPSILON){
                   printf("Finished. iteration=%d,
                 break;
           }
           swapField = field;
           field = tempField;
           tempField = swapField;
#ifdef DISPLAY
           end_time
           if (time_elapsed_ns > FRAME_INTERVAL*10
           {
              start_time;
              XRedraw(field);
           }
#endif
        }
    }
#ifdef DISPLAY
    XRedraw(field);
    usleep(10000000);
#endif
    deleteField(field);
    deleteField(tempField);
    free(X_Size);
    free(Y_Size);
    free(threadPool);
    for (i=0; i<threads; ++i)
    {
            terminate = 1;
            pthread_mutex_unlock(&subThreadWakeUp[i]);
    }
    printf("Finished in %d iterations.\n", iter_cnt);
    end_time;
    printf("%lf\n", time_elapsed_s);
    pthread_exit(NULL);
    return 0;
}
```

Listing 6: main_mpi_increment.h

```
#include "const.h"
#include "models.h"
#include "display.h"
#include <mpi.h>
#include <math.h>
#include <assert.h>

#define start_time clock_gettime(CLOCK_MONOTONIC, &start);
#define end_time clock_gettime(CLOCK_MONOTONIC, &finish);
#define time_elapsed_ns (long long)(finish.tv_sec-start.tv_sec)*1000000000 + finish.tv_nsec - star
#define time_elapsed_s (double)(finish.tv_sec-start.tv_sec) + (double)(finish.tv_nsec - start.tv_n
#define fillReceiveBuffer for (i=0; i<line_buffer_size; ++i) recv_line_buffer[i]=ROOM_TEMP;

#define rank_x (world_rank/sq)
#define rank_y (world_rank%sq)
#define rank_id(x, y) ((x)*sq + (y))
#define sqr(x) ((x)*(x))
#define NOT_FIRE_PLACE (i||rank_x)

int iteration;
int INCREMENT_TIME;
double INCREMENT;
double EPSILON;
TemperatureField *field, *allField;
TemperatureField *tempField;
double *recv_line_buffer;
double *send_line_buffer1, *send_line_buffer2;
int line_buffer_size;

int dx[4] = {0, -1, 0, 1};
int dy[4] = {1, 0, -1, 0};

int x, y, iter_cnt;
int world_size, world_rank;
int sq;

int blockSizeX;
int blockSizeY;

int max(int a, int b){ return a>b ? a: b; }

double temperature_iterate(TemperatureField *field, int initX, int initY)
{
        int i, j, d;
        ++iter_cnt;
        refreshField(field, initX, initY, blockSizeX, blockSizeY, blockSizeX*sq, blockSizeY*sq);
        for (i=0; i<blockSizeX; ++i)
                for (j=0; j<blockSizeY; ++j)
                        tempField->t[i+1][j+1] = fi
        /* Start sending process... */
        //Up
        if (rank_x>0) MPI_Send(tempField->t[1]+1, b
        //Down
        if (rank_x<sq-1) MPI_Send(tempField->t[bloc
        //Left
        if (rank_y>0) {
                for (i=0; i<blockSizeX; ++i)
                        send_line_buffer1[i] = tem
                MPI_Send(send_line_buffer1, blockSi
        }
        //Right
        if (rank_y<sq-1) {
                for (i=0; i<blockSizeX; ++i)
                        send_line_buffer2[i] = tem
                MPI_Send(send_line_buffer2, blockSi
        }
        /* Start receiving process... */
        //Up
        if (rank_x<sq-1) MPI_Recv(recv_line_buffer
        else fillReceiveBuffer;
        for (i=0; i<blockSizeY; ++i) tempField->t[b
        //Down
        if (rank_x>0) MPI_Recv(recv_line_buffer, b
        else fillReceiveBuffer;
        for (i=0; i<blockSizeY; ++i) tempField->t[0
        //Left
        if (rank_y<sq-1) MPI_Recv(recv_line_buffer
        else fillReceiveBuffer;
        for (i=0; i<blockSizeX; ++i) tempField->t[i
        //Right
        if (rank_y>0) MPI_Recv(recv_line_buffer, bl
        else fillReceiveBuffer;
        for (i=0; i<blockSizeX; ++i) tempField->t[i

        /* Calculation */
        double ret = 0;
        for (i=0; i<blockSizeX; ++i){
                for (j=0; j<blockSizeY; ++j)
                {
                        field->t[i][j]=0;
                        for (d=0; d<4; ++d)
                            field->t[i][j] += temp
                        field->t[i][j] /= 4;
                        if (NOT_FIRE_PLACE)
                                ret += fabs(field->
                }
        }
        return ret;
}

///Dest must be full, i.e. X*Y
void scatter(TemperatureField *source, int X, int Y
{
    assert(dest->x==X && dest->y==Y);
    double *send_data;
    int i, j, k, cnt=0;
    if (world_rank==0) {
            send_data = malloc(sizeof(double)*X*Y*
            for (k=0; k<world_size; ++k)
                for (i=0; i<X; ++i)
                for (j=0; j<Y; ++j)
                    send_data[cnt++] = source->t[k/sq*blockSizeX+i][k%sq*blockSizeY+j];
    }
    MPI_Scatter(send_data, X*Y, MPI_DOUBLE, dest->storage, X*Y, MPI_DOUBLE, 0, MPI_COMM_WORLD);
    if (world_rank==0) free(send_data);
}

///Source must be full, i.e. X*Y
void gather(TemperatureField *dest, int X, int Y, TemperatureField *source)
{
    assert(source->x==X && source->y==Y);
    double *recv_data;
    int i, j, k, cnt=0;
    if (world_rank==0)
        recv_data = malloc(sizeof(double)*X*Y*world_size);
    MPI_Gather(source->storage, X*Y, MPI_DOUBLE, recv_data, X*Y, MPI_DOUBLE, 0, MPI_COMM_WORLD);
    if (world_rank==0)
    {
        for (k=0; k<world_size; ++k)
           for (i=0; i<X; ++i)
                for (j=0; j<Y; ++j)
                    dest->t[k/sq*blockSizeX+i][k%sq*blockSizeY+j] = recv_data[cnt++];
    }
    if (world_rank==0) free(recv_data);
}

int main(int argc, char **argv)
{
    struct timespec start, finish;
    if (world_rank==0) start_time
    if (argc<7)
    {
        printf("Usage: %s x y iteration INCREMENT_TIME, INCREMENT, EPSILON\n", argv[0]);
    }
    sscanf(argv[1], "%d", &x);
    sscanf(argv[2], "%d", &y);
    sscanf(argv[3], "%d", &iteration);
    sscanf(argv[4], "%d", &INCREMENT_TIME);
    sscanf(argv[5], "%lf", &INCREMENT);
    sscanf(argv[6], "%lf", &EPSILON);

    MPI_Init(NULL, NULL);
    MPI_Comm_size(MPI_COMM_WORLD, &world_size);
    MPI_Comm_rank(MPI_COMM_WORLD, &world_rank);
//    if (world_size<4)
//    {
//      puts("At least 4 processes.");
//      return 0;
//    }

    field = malloc(sizeof(TemperatureField));
    tempField = malloc(sizeof(TemperatureField));
    field->x = y;
    field->y = x;
#ifdef DISPLAY
    if (world_rank==0) XWindow_Init(field);
#endif

    int iter, inc, i, j;
    int *X_Size = malloc(sizeof(int)*INCREMENT_TIME);
    int *Y_Size = malloc(sizeof(int)*INCREMENT_TIME);
    X_Size[INCREMENT_TIME-1] = x;
    Y_Size[INCREMENT_TIME-1] = y;
```

```
sq = sqrt(world_size) + 0.001;
for (inc=INCREMENT_TIME-2; inc>=0; --inc)
{
    X_Size[inc] = X_Size[inc+1] / INCREMENT;
    Y_Size[inc] = Y_Size[inc+1] / INCREMENT;
    X_Size[inc] = ((X_Size[inc]/sq) + !!(X_Size
    Y_Size[inc] = ((Y_Size[inc]/sq) + !!(Y_Size
}
if (world_rank==0)
        allField = malloc(sizeof(TemperatureFi

for (inc=0; inc<INCREMENT_TIME; ++inc)
{
    MPI_Barrier(MPI_COMM_WORLD);
    blockSizeX = X_Size[inc]/sq;
    blockSizeY = Y_Size[inc]/sq;
    int lastX, lastY;
    if (!inc) lastX=0, lastY=0; else lastX = X
    line_buffer_size = max(blockSizeX, blockSiz
    recv_line_buffer = malloc(sizeof(double)*li
    send_line_buffer1 = malloc(sizeof(double)*
    send_line_buffer2 = malloc(sizeof(double)*

    newField(field, blockSizeX, blockSizeY, 0,
    newField(tempField, blockSizeX+2, blockSize
    if (world_rank==0)
    {
        newField(allField, X_Size[inc], Y_Size
        if (!inc) initField(allField);
    }
    scatter(allField, blockSizeX, blockSizeY,

    for (iter=0; iter<iteration; iter++)
    {
       double ret= temperature_iterate(field,
       double recvedRes = 0;
       MPI_Allreduce(&ret, &recvedRes, 1, MPI_D
       if (recvedRes<EPSILON)
       {
               if (world_rank==0) printf("Finis
                break;
       }
       MPI_Barrier(MPI_COMM_WORLD);
#ifdef DISPLAY
       if (iter%1==0)
       {
            gather(allField, blockSizeX, blockS
            if (world_rank==0){
                    XRedraw(allField);
//              printf("All field:\n", world_ra
//              for (i=0; i<allField->x; ++i)
//              {
//                      for (j=0; j<allField->y
//                          printf("%lf ", allF
//                      puts("");
//              }
            }
       }
#endif
    }
    free(recv_line_buffer);
    free(send_line_buffer1);
    free(send_line_buffer2);
```

```
        gather(allField, blockSizeX, blockSizeY, field);
//      puts("finish iteration");
//
//      if (world_rank==0)
//      {
//          printf("All field:\n", world_rank);
//          for (i=0; i<allField->x; ++i)
//          {
//                  for (j=0; j<allField->y; ++j)
//                      printf("%lf ", allField->t[i][j]);
//                  puts("");
//          }
//          puts("");
//      }
#ifdef DISPLAY
        if (world_rank==0) XRedraw(allField);
#endif
        deleteField(field);
        deleteField(tempField);
    }
    free(X_Size);
    free(Y_Size);
    if (world_rank==0)
    {
            printf("Finished in %d iterations.\n", iter_cnt);
            end_time;
            printf("%lf\n", time_elapsed_s);
    }
    MPI_Finalize();
#ifdef DISPLAY
        if (world_rank==0) usleep(100000000);
#endif
    return 0;
}
```